Aula 7

2024-10-17

Propriedades do Limite

1)

$$\lim[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$$

Exemplo:

$$\lim_{x \to 2} 3x + 5 = \lim_{x \to 2} 3x + \lim_{x \to 2} 5 = 3 \cdot 2 + 5 = 11$$

2)

$$\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

Exemplo:

$$\begin{aligned} \lim_{x \to -3} (x+4) \cdot (3x-1) &= \lim_{x \to -3} (x+4) \cdot \lim_{x \to -3} (3x-1) \\ &= (-3+4) \cdot (3 \cdot (-3) - 1) \\ &= 1 \cdot (-9-1) = -10 \end{aligned}$$

3)

$$\lim \left[\frac{f(x)}{g(x)}\right]; g(x) \neq 0$$

Exemplo:

$$\lim_{x \to 7} \frac{3x+2}{x+8}; x \neq -8$$

$$\lim_{x \to 7} \frac{3x+2}{x+8} = \frac{\lim_{x \to 7} 3x+2}{\lim_{x \to 7} x+8} = \frac{3 \cdot 7 + 2}{7+8} = \frac{23}{15}$$

4)

$$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n; n \in \mathbb{Q}$$

Exemplo:

$$\lim_{x \to \frac{\pi}{4}} (\sin x)^2 = \left[\lim_{x \to \frac{\pi}{4}} \sin x\right]^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{\left(\cancel{\sqrt{2}}\right)^{\cancel{2}}}{2^2} = \frac{2}{4} = \frac{1}{2}$$

Formas Indeterminadas

Temos sete formas clássicas de indeterminação, a saber:

$$\frac{0}{0} \ ; \frac{\infty}{\infty} \ ; 0^0 \ ; 0 \cdot \infty \ ; \infty^0, \infty - \infty \ ; 1^\infty$$

Exemplo:

$$\lim_{x\to 7} \frac{x-7}{x^2-49} = \frac{7-7}{7^2-49} = \cancel{\frac{0}{0}}$$

$$(a+b)\cdot(a-b) = a^2 - b^2$$

$$\lim_{x\to 7} \frac{1}{x+7} = \frac{1}{14}$$

Resoluções da Lista

11)

$$\lim_{x\to -7}(2x+5) = 2\cdot(-7)+5 = -14+5 = -9$$

22)

$$\lim_{h\to 0} \frac{\left(\sqrt{5h+4}-2\right)}{h} \cdot \frac{\left(\sqrt{5h+4}+2\right)}{\left(\sqrt{5h+4}+2\right)}$$

$$= \lim_{h\to 0} \frac{\left(\sqrt{5h+4}\right)^2 - 2^2}{h\cdot\left(\sqrt{5h+4}+2\right)}$$

$$= \lim_{h\to 0} \frac{5h+\cancel{4}-\cancel{4}}{h\cdot\left(\sqrt{5h+4}+2\right)}$$

$$= \lim_{h\to 0} \frac{5h}{h\cdot\left(\sqrt{5h+4}+2\right)} = \lim_{h\to 0} \frac{5}{\sqrt{5h+4}^{0}+2} = \frac{5}{\sqrt{4}+2} = \frac{5}{4}$$

24)

$$\lim_{x\to -3} \frac{x+3}{x^2+4x+3}$$

$$p(x) = a\cdot(x-x_1)\cdot(x-x_2)$$

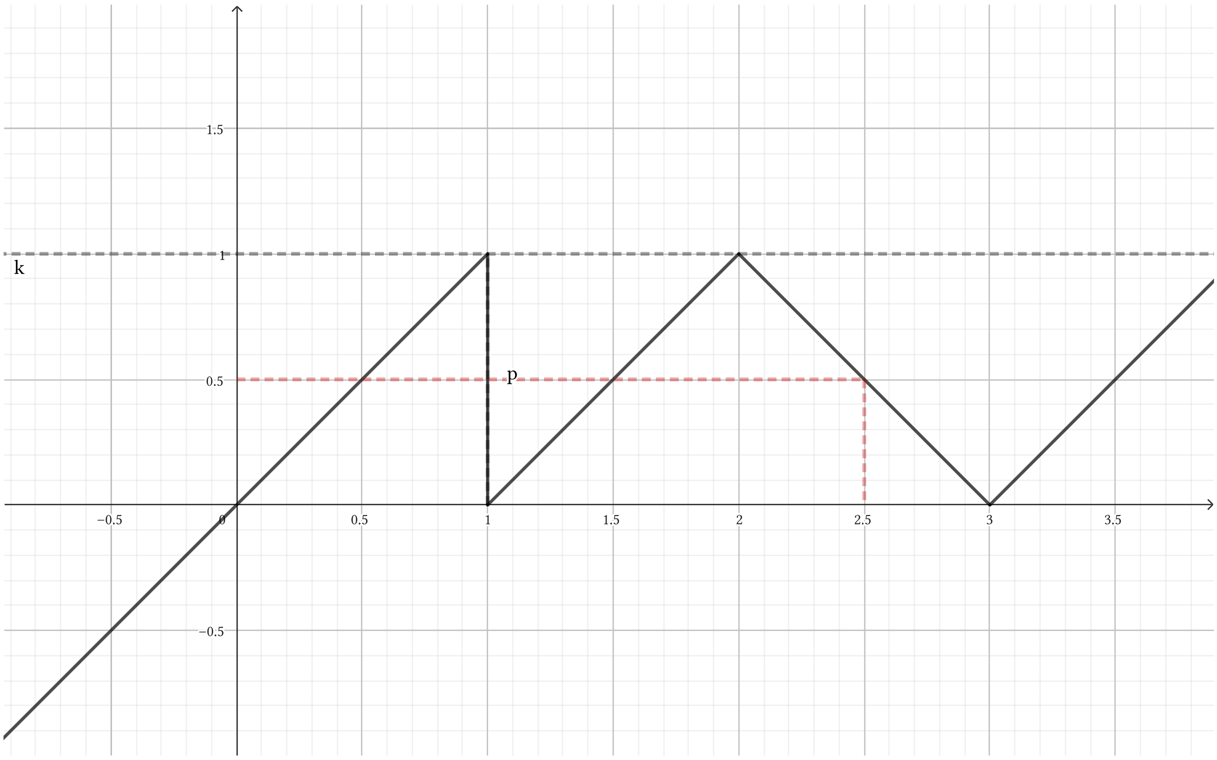
$$x^2+4x+3=0$$

$$\begin{cases} x_1+x_2=-4 \\ x_1\cdot x_2 = 3 \end{cases} \Rightarrow \begin{cases} x_1=-1 \\ x_2=-3 \end{cases}$$

$$= \lim(x\to -3)\frac{x+3}{(x+1)\cdot(x+3)}$$

$$= \lim(x\to -3)\frac{1}{x+1} = \frac{1}{-3+1} = \frac{1}{2}$$

1)



a)

$$\nexists \lim_{x\to 1} g(x)$$

$$\lim_{x\to 1^-} g(x) = 1$$

$$\lim_{x\to 1^+} g(x) = 0$$

d)

$$\lim_{x\to 2,5} g(x) = 0,5$$

5)

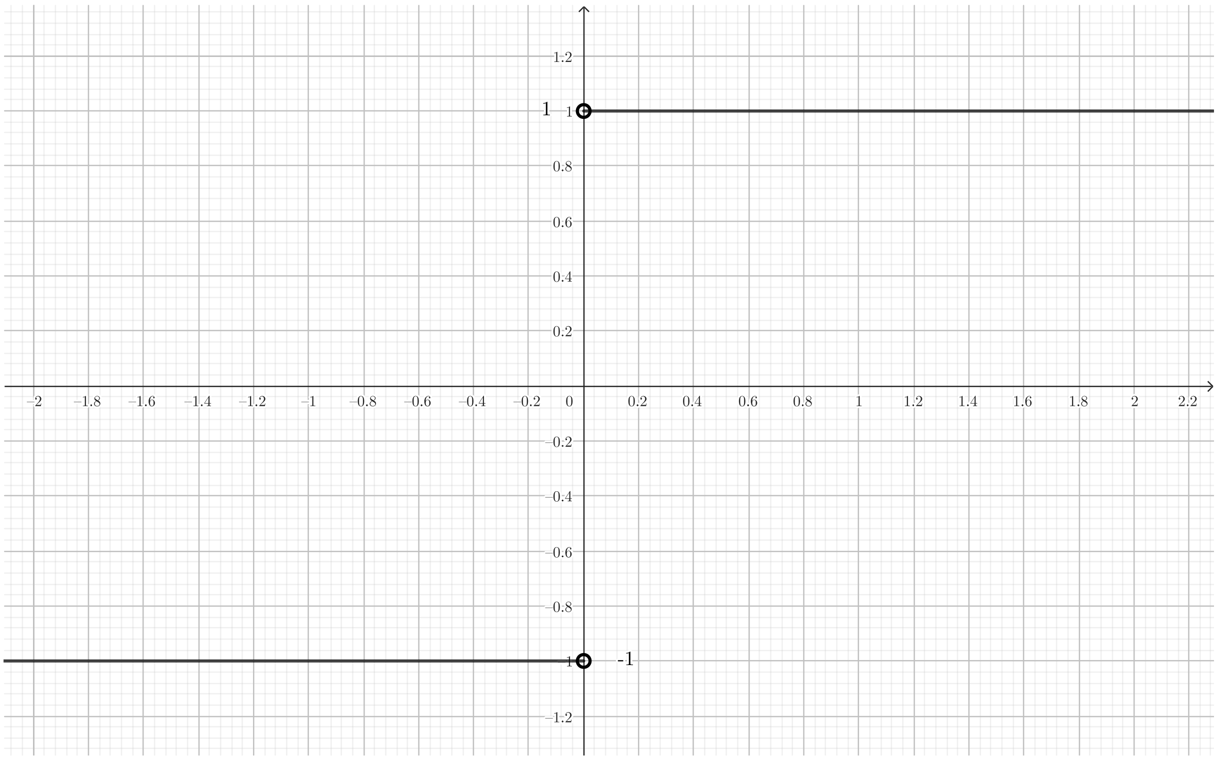
$$\lim_{x \to 0} \frac{x}{|x|}$$

$$|x| = \begin{cases} x, \text{ se } x_{=}0 \\ -x, \text{ se } x < 0 \end{cases}$$

$$|5| = 5$$

$$|-2| = 2$$

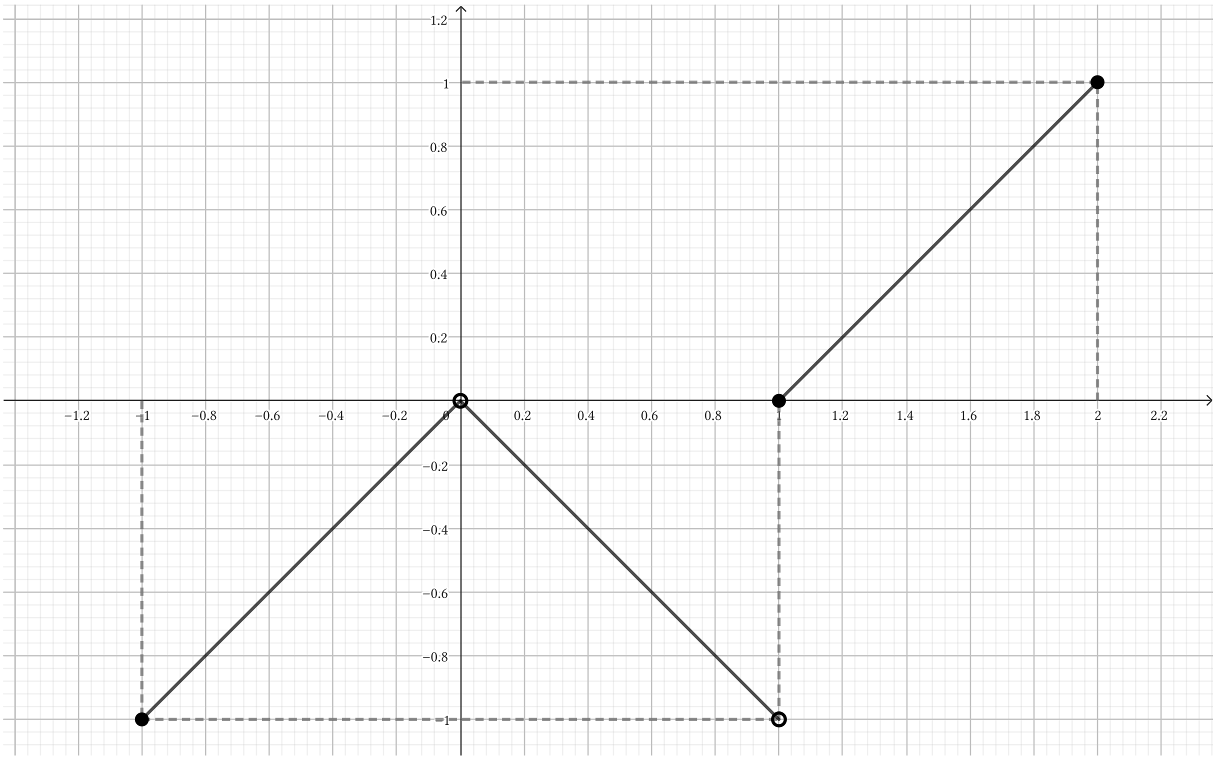
$$\frac{x}{|x|} = \begin{cases} \frac{x}{x} = 1; x > 0 \\ \frac{x}{-x} = -1; x < 0 \end{cases}$$



$$\lim_{x \to 0^-} \frac{x}{-|x|} = -1$$

$$\lim_{x \to 0^+} \frac{x}{|x|} = +1$$

3)



a) $\lim_{x \to 0} f(x)$ existe (V)

b) $\lim_{x \to 0} f(x) = 0$ (V)

c) $\lim_{x \to 0} f(x) = 1$ (F)

d) $\lim_{x \to 1} f(x) = 1$ (F)

e) $\lim_{x \to 1} f(x) = 0$ (F)

f) $\lim_{x \to 0} f(x)$ existe para todo $x \in]-1, 1[$