Aula 12

2024-12-05

Integral definida, antiderivada ou primitiva

$$F'(x) = f(x)$$

$$F(x) = x^2 + C; C \in \mathbb{R}$$

$$F(x) = G(x) + C; C \in \mathbb{R}$$

Se $f(x) = \cos x$, então

$$F \underbrace{(x)}_{\text{primitiva}} = \sin \underbrace{x}_{\text{primtiva}} + C; C \in \mathbb{R}$$

$$\int f(x) \underbrace{d}_{\substack{\text{diferencial} \\ \text{em x}}} x = F(x) + \underbrace{C}_{\substack{\text{constante} \\ \text{de integração}}} ; C \in \mathbb{R}$$

$$(F(x) + C)' = f(x)$$

$$y = (\sin x)^2$$

$$u = \sin x \Rightarrow \frac{du}{dx}$$

Derivada e integral são processos inversos. Assim,

$$\frac{d}{dx} \int f(x) dx = f(x)$$

$$\int f'(x) dx = f(x)$$

$$\int 3x^2 dx = \frac{(3x)^{2+1}}{2+1} + C = \frac{\cancel{3}x^3}{\cancel{3}} + C = x^3 + C$$

$$\int x^n dx = \frac{x^{2+1}}{n+1} + n \neq -1$$

Exemplo

3

$$\text{integral } (3\pi)^t dt =$$

Exemplos

1) $\int x^d t$

$$3)$$

interal (3 pi)^t d t = (3 pi)^t/n t-

3) $\int (3\pi)^t dt = \frac{(3m)^3}{\ln}(3p + C)$

Propriedades da Integral

Sejam $f(x)$ e $g(x)$ duas funções e seja K uma constante real, enão.

1) $\int K(fx) dx = K \int (fx) dx = K.F(x) + C$

2) $\int[f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)ox = F(x) \pm GW$

Exemplos

1)

$$\int 5(x^3 + \cos(x))dx = \int 5x^3dx = \int \cos x dx =$$
$$= 5 \int x^3dx + \int \cos(x)dx = 5 \cdot \frac{x^{3+1}}{3+1}) + C_1 + \sin x + C_2$$
$$= \frac{5x^4}{4} + \sin x + C; C = C_1 + C_2; C \in \mathbb{R}$$

3)

$$\int \frac{\left(u^2 - 1\right)^2}{u^2} \cdot du = \int \frac{\left(u^4 - 2u^2 + 1\right)}{u^2}du =$$
$$\int \frac{u^4}{u^2}du - \int \frac{2u^2}{u^2}du + \int \frac{1}{u^2}du =$$
$$\int u^2du - 2\int du + \int u^{-2}du = \frac{u^{2+1}}{2+1} + C_1 - 2u + Ca +$$
$$\frac{u^{-2+1}}{-2+1} + C_3 = \frac{u^3}{3} - 24 + \frac{u}{-1} + C, C = C_1 + C_2 + C_3$$

Funções compostas

Exemplo 1

$$\int \sin(2t)dt$$
$$u = 2t$$
$$\frac{du}{dt} = 2 \Rightarrow dt = \frac{du}{2}$$
$$\int \sin(2t)dt = \int \sin u \frac{du}{2} = \frac{1}{2}\int \sin u du$$
$$= \frac{1}{2} \cdot (-\cos u) + C$$
$$= -\frac{1}{2}\cos(2t) + C \ ; \ C \in \mathbb{R}$$

Exemplo 2

$$\int \pi^{4w} dw$$

$$u = 4w$$

$$\frac{du}{dw} = 4 \Rightarrow dw = \frac{du}{4}$$

$$\int \pi^{4w} dw = \int \pi^4 \frac{du}{4} = \frac{1}{4} \cdot \int \pi^u du$$

$$= \frac{1}{4} \frac{\pi^u}{\ln \pi}$$

$$= \frac{1}{4} \frac{\pi^{4w}}{\ln \pi} + C$$

Exemplo 3

$$\int \sqrt{3x+4} dx$$

$$u = 3x + 4$$

$$\frac{du}{dx} = 3 \Rightarrow dx = \frac{du}{3}$$

$$\int \sqrt{3x+4} dx = \int u^{\frac{1}{2}} \frac{du}{3} = \frac{1}{3} \int u^{\frac{1}{2}} du$$

$$= \frac{1}{3} \frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= \frac{1}{3} \cdot \frac{u^{\frac{3}{2}}}{\frac{3}{2}} + C$$

$$= \frac{1}{3} \cdot \frac{2}{3} u^{\frac{3}{2}} + C$$

$$= \frac{2}{9} (3x+4)^{\frac{3}{2}} + C$$

$$= \left(\frac{2}{9}\right) \sqrt{(3x+4)^3} + C$$

Exemplo 4

$$\int x\cos(x^2)dx$$

$$u = x^2$$

$$\frac{du}{dx} = 2x$$

$$\frac{du}{2} = xdx$$

$$\int x\cos(x^2)dx = \int \cos u \frac{du}{2} = \frac{1}{2}\int \cos u du$$

$$= \frac{1}{2}\sin u + C$$

$$= \frac{1}{2}\sin(x^2) + C$$

Exemplo 5

$$\int x^2 \cdot \sin\left(\frac{x^3}{\pi}\right)dx$$

$$u = \frac{x^3}{\pi} \Rightarrow \frac{du}{dx} = \frac{1}{\pi}3x^2 = \frac{3}{\pi}x^2$$

$$\frac{du}{dx} = \frac{3}{\pi}x^2$$

$$\frac{\pi}{3}du = x^2dx$$

$$\int x^2 \cdot \sin\left(\frac{x^3}{\pi}\right)dx = \int \sin u \frac{\pi}{3}du = \frac{\pi}{3}\int \sin u du$$

$$= \frac{\pi}{3}(-\cos u) + C$$

$$= \frac{\pi}{3}\cos\left(\frac{x^{\frac{2}{3}}}{\pi}\right) + C$$

Exemplo 6

$$\int \frac{\left(\sqrt{7}+\frac{1}{y}\right)^{10}}{y^2}dy = \int \left(\sqrt{7}+y^{-1}\right)^{10} \cdot y^{-2} \cdot dy$$

$$u = \sqrt{7}+y^{-1}$$

$$\frac{du}{dy} = -1y^{-2} \Rightarrow \frac{du}{-1} = y^{-2}dy$$

$$= \int u^{10} \cdot \frac{du}{-1} = -\int u^{10}du$$

$$= -\frac{u^{10+1}}{10+1} + C$$

$$= -\frac{u^{11}}{11} + C$$

$$= -\frac{\left(\sqrt{7}+y^{-1}\right)^{11}}{11} + C$$

Integração por partes

$$[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

$$(u \cdot v)' = u'v + u \cdot v'$$

$$\int [f(x) \cdot g(x)]\cancel{'}\,\cancel{dx} = \int [f'(x) \cdot g(x) + f(x) \cdot g'(x)]dx$$

$$f(x) \cdot g(x) = \int f'(x)g(x)dx + \int f(x) \cdot g'(x)dx$$

$$u \cdot v = \int u'vdu + \int uv'dv$$

$$u = f(x)$$

$$\frac{du}{dx} = f'(x) \Rightarrow f'(x)dx \cdot duv = g(x)$$

$$\frac{dv}{dx} = g'(x)$$

$$dv = g'(x)dx$$

$$\int v \cdot du + \int u \cdot dv \Rightarrow \int udv = uv - \int vdu$$

Exemplo

$$\int x \cdot \sin x dx = \int u \cdot \mathtt{op}\ dv = u \cdot v - \int v \cdot du$$

$$u = x \Rightarrow \frac{du}{dx} = 1 \Rightarrow du = dx$$

$$\int dv = \int \sin x dx \to v = -\cos x$$

$$= x \cdot (-\cos x) - \int -\cos x \cdot dx$$

$$= -x \cdot \cos x + \int \cos x dx$$

$$= -x \cdot \cos x + \sin x + C$$

$$(-x \cos x + \sin x)' = -\cos x + (-x) \cdot (-\sin x)$$