Fourier analysis

Requirements

1. The report should be written in English.
2. Include your student number in each figure title as 'No. XXXXXXX'. And include your codes in the appendix with the question numbers.
3. Please submit your report in PDF format.

1. Fourier transform properties

Create a gate function $g(t)$, where D = 8, H = 5 over t = [-D, D] with a time interval 0.001.

$$g(t) = \begin{cases} H & , -D/2 \le t \le D/2 \\ 0 & , t < -D/2,\ t > D/2 \end{cases} \tag{1}$$

a) Create a Continuous-time Fourier transform (CTFT) function ***Xw* = CTFT (*x*, *t*, *w*)**, where input the time-domain signal x and t, and the frequency vector in radians, and output the CTFT coefficients. (Note that here w is to represent ω in programming.)

b) $g_2(t)$ is defined as a time shift of $g(t)$: $g_2(t) = y(t - D/2)$. Compare $g(t)$ and $g_2(t)$ in one plot.

c) Calculate CTFT of $g(t)$ and $g_2(t)$, denoted as Gw and $Gw2$, using the CTFT function. Compare the module and phase plots, and the real and imaginary plots of Gw and $Gw2$ in w = -10π~10π, respectively. Verify the time-shift properties of Fourier transform.

d) $y(t)$ is defined as $y(t) = g(t) \times \cos(4\pi t)$. Compare $g(t)$ and $y(t)$ over t = [-15, 15].

e) Compare the modulus and phase of CTFT of $y(t)$ and $g(t)$ in w= -10π~10π. Verify the modulation properties of Fourier transform.

f) According to the Parseval's formula, calculate and compare the energy of $y(t)$ in both the time and frequency domains. Are they the same? Why?

2. Fourier transform properties

A gate function $g(t)$, where $D = 8$, $H = 2$ over $t = [-D, D]$.

$$g(t) = \begin{cases} H & , \ -D/2 \le t \le D/2 \\ 0 & , \ t < -D/2,\ t > D/2 \end{cases} \tag{2}$$

a) Create a Discrete-time Fourier transform (DTFT) function ***Xw* = DTFT (*nT*, x*n*, *ω*)**, where input the time-domain signal nT and xn and the frequency vector in radians ω, and output the DTFT coefficients.

b) Set the sampling interval T as $D/80$ and $D/40$, respectively, and sample $g(t)$ as $g_1[n]$ and $g_2[n]$. Calculate DTFT of $g_1[n]$ and $g_2[n]$, denoted as Gw_1 and Gw_2, using the DTFT function. Compare the module and phase of Gw_1 and Gw_2 using different digital frequencies, i.e., f, f/f_s, ω/ω_s, in a Nyquist interval.

c) Deduce the theoretical CTFT function of g(t) and compare it with the Gw_1 and Gw_2 in $f = -3f_s \sim 3f_s$, analyzing the result.

d) Inverse the DTFT on Gw_1 and Gw_2 and compare the result with $g_1[n]$ and $g_2[n]$.

e) Can the Parseval's formula still be validated, why?