Laboratory Report of Digital Signal Processing

Name: Junjie Fang

Student ID: 521260910018

Date: 2024/3/4

Score:

Source Code: https://github.com/julyfun/dsp-lab2

Contents

1. Signal operations

Given the parameters $A = 3, B = 4, D = 8$, the three gate functions are defined by:

$$g_0(t) := \begin{cases} 4 \text{ if } 0 \leq t \leq 3 \\ 0 \text{ otherwise} \end{cases}$$

$$g_1(t) := \begin{cases} 4 \text{ if} - \frac{3}{8} \leq t \leq -\frac{3}{5} \\ 0 \text{ otherwise} \end{cases}$$

$$g_2(t) := \begin{cases} 8 \text{ if } 8 \leq t \leq 11 \\ 0 \text{ otherwise} \end{cases}$$

And we know:

$$x(t) := \sum_{i=0}^{2} g_i(t)$$

We plot the x function in the figure below:

It can be seen that the images of the three gate functions do not overlap.

In practice, we use python's `matplotlib` to draw function images. For scalability, we use the `gate_func()`, `func_transform()` and `add_func()` to generate, transform and add functions.

2. Aliasing phenomenon in sampling process

Let the frequencies corresponding to the two peaks in the image be $f_{a1} = 14, f_{a2} = 3$ and the function values be $X_1 = 2, X_2 = 1$. The sampling frequency is $f_s = 100\text{Hz}$. According to the sampling theorem, we have:

$$f_{a1} = \pm f_1 - k_1 f_s$$
$$f_{a2} = \pm f_2 - k_2 f_s$$

where:

$$k_1, k_2 \neq 0$$
$$800\text{Hz} \leq f_1, f_2 \leq 850\text{Hz}$$

Plug the data $f_{a1} = 14, f_{a2} = 3$ into the equation and we can determine that the only solution is:

$$k_1 = 8, f_1 = 814\text{Hz}$$
$$k_2 = 8, f_2 = 803\text{Hz}$$

Next, we can determine the amplitudes A_1 and A_2 by reviewing some of the properties of *Contiunous-Time Fourier Transform (CTFT)*. The Fourier Transform used in this question is in the form:

$$X(jf) = \int_{-\infty}^{+\infty} x(t) e^{-j2\pi f t} \,\mathrm{d}t$$

$$x(t) = \int_{-\infty}^{+\infty} X(jf) e^{j2\pi f t} \,\mathrm{d}f$$

To sample the function from $0s$ to $5s$ is equivalent to

In this form, the sine wave with amplitude 1 and the following sum of two impulse function form a Fourier Transform pair, and we take modulo in this formula:

$$\|\mathcal{F}[\sin(2\pi f_0 t)]\| = \frac{1}{2}(\delta(f - f_0) + \delta(f + f_0))$$

Due to linearity of Fourier Transform, we have:

$$\begin{aligned} \|\mathcal{F}[x(t)]\| &= \|\mathcal{F}[A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t)]\| \\ &= \frac{A_1}{2}(\delta(f - f_1) + \delta(f + f_1)) + \frac{A_2}{2}(\delta(f - f_2) + \delta(f + f_2)) \end{aligned}$$

Sampling the function from $0s$ to $5s$ is equivalent to multiplying a gate function with height 1 and width 5, that is, the Fourier tranform of the two functions is convolved in the frequency domain. Suppose this gate function is g, and we have:

$$\|\mathcal{F}[g(t)]\| = |5 \operatorname{sinc}(5f)|$$

Where 5 is the width of the gate function. From the above conclusion, we can infer that the image of *CTFT* is the convolution of the two:

$$\begin{aligned} \|\mathcal{F}[x(t)]\| &= \|\mathcal{F}[g(t)] * \mathcal{F}[A_1 \sin(2\pi f_1 t) + A_2 \sin(2\pi f_2 t)]\| \\ &= \left\| 5 \operatorname{sinc}(5f) * \left(\frac{A_1}{2}(\delta(f - f_1) + \delta(f + f_1)) + \frac{A_2}{2}(\delta(f - f_2) + \delta(f + f_2)) \right) \right\| \\ &= \left\| \frac{5}{2}(A_1 \operatorname{sinc}(5(f - f_1)) + A_1 \operatorname{sinc}(5(f + f_1)) + A_2 \operatorname{sinc}(5(f - f_2)) + A_2 \operatorname{sinc}(5(f + f_2))) \right\| \end{aligned}$$

Therefore, the peak values X_1, X_2 is $\frac{5}{2}$ times the amplitudes A_1 and A_2:

$$A_1 = \frac{2}{5} X_1 = \frac{4}{5}$$

$$A_2 = \frac{2}{5} X_2 = \frac{2}{5}$$