CSCD 327: Relational Database Systems

Relational Algebra

Instructor: Dr. Dan Li

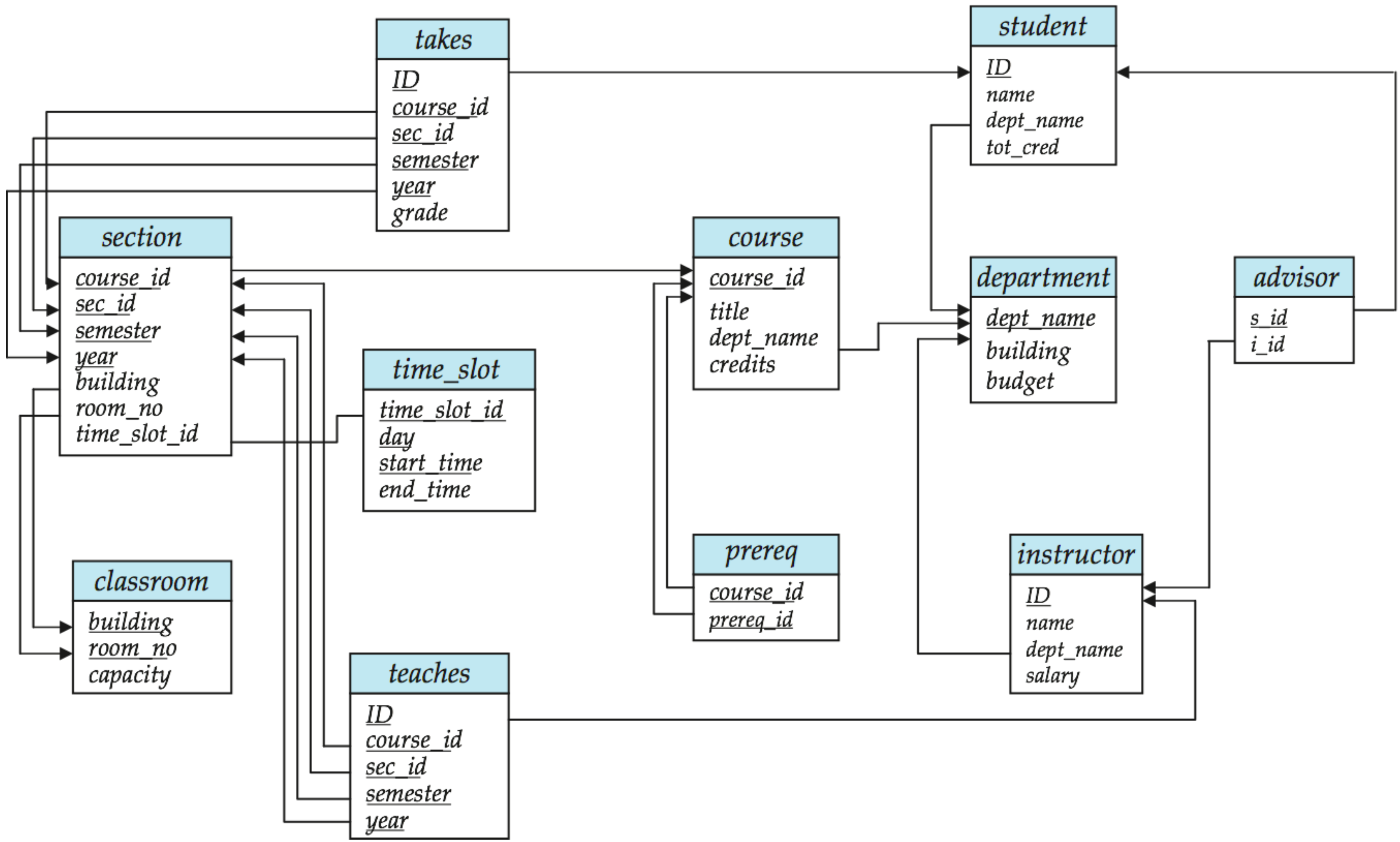
Formal Relational Query Languages

- Two mathematical Query Languages form the basis for "real" languages (e.g. SQL), and for implementation:
  - *Relational Algebra*: procedural, tells you how to process a query, very useful for representing execution plans.
  - *Relational Calculus*: Lets users describe what they want, rather than how to compute it. (Non-procedural, *declarative*.)

Relational Algebra

- Six basic operators
  - select: σ
  - project: $\prod$
  - union: $\cup$
  - set difference: –
  - Cartesian product: x
  - rename: ρ
- The operators take one or two relations as inputs and produce a new relation as a result.

Running Example: Schema Diagram for University Database



Select Operation

- Notation: $\sigma_p(r)$
- p is called the **selection predicate**
- Defined as:

$$\sigma_p(\mathbf{r}) = \{t \mid t \in r \textbf{ and } p(t)\}$$

Where p is a formula in propositional calculus consisting of **terms** connected by : $\wedge$ (**and**), $\vee$ (**or**), $\neg$ (**not**)
Each **term** is one of:

<attribute> *op* <attribute> or <constant>

where *op* is one of: $=, \neq, >, \geq. <. \leq$

- Example of selection:

$$\sigma_{dept\_name=\text{“}Physics\text{”}}(instructor)$$

Select Operation – Example

- Relation r

A	B	C	D
α	α	1	7
α	β	5	7
β	β	12	3
β	β	23	10

- $\sigma_{A=B \wedge D > 5}(r)$

A	B	C	D
α	α	1	7
β	β	23	10

Project Operation

- Notation: $$\prod_{A_1, A_2, \ldots, A_k} (r)$$

  where A_1, A_2 are attribute names and r is a relation name.
- The result is defined as the relation of k columns obtained by erasing the columns that are not listed
- Duplicate rows removed from result, since relations are sets
- Example: To eliminate the *dept_name* attribute of *instructor*

$$\prod_{ID,\ name,\ salary} (instructor)$$

Project Operation – Example

- Relation *r*:

A	B	C
α	10	1
α	20	1
β	30	1
β	40	2

- $\Pi_{A,C}(r)$

A	C
α	1
α	1
β	1
β	2

=

A	C
α	1
β	1
β	2

Union Operation

- Notation: $r \cup s$
- Defined as:

$$r \cup s = \{t \mid t \in r \text{ or } t \in s\}$$

- For $r \cup s$ to be valid.

  1. r, s must have the *same* **arity/degree** (same number of attributes)

  2. The attribute domains must be **compatible** (example: 2nd column
     of r deals with the same type of values as does the 2nd column of s)

- Example: to find all courses taught in the Fall 2009 semester, or in the Spring 2010 semester, or in both

$$\Pi_{course\_id} (\sigma_{semester=\text{"Fall"} \land year=2009} (section)) \cup$$

$$\Pi_{course\_id} (\sigma_{semester=\text{"Spring"} \land year=2010} (section))$$

Union Operation – Example

- Relations *r, s:*

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- r ∪ s:

A	B
α	1
α	2
β	1
β	3

Set Difference Operation

- Notation $r - s$
- Defined as:

$$r - s = \{t \mid t \in r \textbf{ and } t \notin s\}$$

- Set differences must be taken between **compatible** relations.
  - r and s must have the same arity
  - attribute domains of r and s must be compatible
- Example: to find all courses taught in the Fall 2009 semester, but not in the Spring 2010 semester

$$\Pi_{course\_id}(\sigma_{semester=\text{“Fall”} \wedge year=2009}(section)) -$$

$$\Pi_{course\_id}(\sigma_{semester=\text{“Spring”} \wedge year=2010}(section))$$

Set difference of two relations

- Relations *r*, *s*:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- *r* – *s:*

A	B
α	1
β	1

Cartesian-Product Operation

- Notation r x s
- Defined as:

$$r \times s = \{t\, q \mid t \in r \textbf{ and } q \in s\}$$

- Assume that attributes of r(R) and s(S) are disjoint. (That is, $R \cap S = \emptyset$).
- If attributes of *r(R)* and *s(S)* are not disjoint, then renaming must be used.

Cartesian-Product Operation – Example

- Relations *r, s*:

A	B
α	1
β	2

r

C	D	E
α	10	a
β	10	a
β	20	b
γ	10	b

s

- *r* x *s*:

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

Composition of Operations

- Can build expressions using multiple operations
- Example: $\sigma_{A=C}(r \times s)$
- *r x s*

A	B	C	D	E
α	1	α	10	a
α	1	β	10	a
α	1	β	20	b
α	1	γ	10	b
β	2	α	10	a
β	2	β	10	a
β	2	β	20	b
β	2	γ	10	b

- $\sigma_{A=C}(r \times s)$

A	B	C	D	E
α	1	α	10	a
β	2	β	10	a
β	2	β	20	b

Rename Operation

- Allows us to refer to a relation by more than one name.
- Example:

$$\rho_x (E)$$

returns the expression *E* under the name *X*

- If a relational-algebra expression *E* has degree *n*, then

$$\rho_{x(A_1, A_2, \ldots, A_n)}(E)$$

returns the result of expression *E* under the name *X*, and with the attributes renamed to A_1, A_2, …., A_n .

Example Query

- Find the largest salary in the university
  - Step 1: find instructor salaries that are less than some other instructor salary (i.e. not maximum)
    - using a copy of *instructor* under a new name *d*
    - $\prod_{instructor.salary} (\sigma_{instructor.salary < d,salary} (instructor \times \rho_d (instructor)))$
  - Step 2: Find the largest salary
    - $\prod_{salary} (instructor) - \prod_{instructor.salary} (\sigma_{instructor.salary < d,salary} (instructor \times \rho_d (instructor)))$

Example Queries

- Find the names of all instructors in the Physics department, along with the *course_id* of all courses they have taught

    - Query 1

$$\Pi_{instructor.ID,course\_id}\,(\sigma_{dept\_name=\text{“Physics”}}\,(\sigma_{instructor.ID=teaches.ID}\,(instructor \times teaches)))$$

    - Query 2

$$\Pi_{instructor.ID,course\_id}\,(\sigma_{instructor.ID=teaches.ID}\,(\sigma_{dept\_name=\text{“Physics”}}\,(instructor) \times teaches))$$

Additional Operations

We define additional operations that do not add any power to the relational algebra, but that simplify common queries.

- Set intersection
- Natural join
- Division
- Assignment

Set-Intersection Operation

- Notation: $r \cap s$
- Defined as:
- $r \cap s = \{ t \mid t \in r \textbf{ and } t \in s \}$
- Assume:
  - r, s have the *same arity*
  - attributes of r and s are compatible
- Note: $r \cap s = r - (r - s)$

Set-Intersection Operation – Example

- Relation *r, s*:

A	B
α	1
α	2
β	1

r

A	B
α	2
β	3

s

- $r \cap s$

A	B
α	2

Natural-Join Operation

- Notation: $r \bowtie s$

- Let *r* and *s* be relations on schemas *R* and *S* respectively. Then, $r \bowtie s$ is a relation on schema $R \cup S$ obtained as follows:
  - Consider each pair of tuples t_r from *r* and t_s from *s*.
  - If t_r and t_s have the same value on each of the attributes in $R \cap S$, add a tuple *t* to the result, where
    - *t* has the same value as t_r on *r*
    - *t* has the same value as t_s on *s*
- Example:

  $R = (A, B, C, D)$

  $S = (E, B, D)$

  - Result schema = (A, B, C, D, E)
  - $r \bowtie s$ is defined as:

  $\Pi_{r.A,\ r.B,\ r.C,\ r.D,\ s.E}\ (\sigma_{r.B\ =\ s.B\ \wedge\ r.D\ =\ s.D}\ (r \times s))$

Natural Join Example

- Relations r, s:

A	B	C	D
α	1	α	a
β	2	γ	a
γ	4	β	b
α	1	γ	a
δ	2	β	b

r

B	D	E
1	a	α
3	a	β
1	a	γ
2	b	δ
3	b	ε

s

- $r \bowtie s$

A	B	C	D	E
α	1	α	a	α
α	1	α	a	γ
α	1	γ	a	α
α	1	γ	a	γ
δ	2	β	b	δ

Another Natural Join Example

- Find the names of all instructors in the Comp. Sci. department together with the course titles of all the courses that the instructors teach
  - $\Pi_{name,\ title}$ ($\sigma_{dept\_name=\text{"Comp. Sci."}}$ (*instructor* $\bowtie$ *teaches* $\bowtie$ *course*))
- Natural join is associative
  - (*instructor* $\bowtie$ *teaches*) $\bowtie$ *course* is equivalent to *instructor* $\bowtie$ (*teaches* $\bowtie$ *course*)
- Natural join is commutative
  - *instruct* $\bowtie$ *teaches* is equivalent to *teaches* $\bowtie$ *instructor*

Division Operation

- Notation: $r \div s$
- Suited to queries that include the phrase "for all".
- Let *r* and *s* be relations on schemas *R* and *S* respectively where
  - $R = (A_1, \ldots, A_m, B_1, \ldots, B_n)$
  - $S = (B_1, \ldots, B_n)$

  The result of $r \div s$ is a relation on schema

  $R - S = (A_1, \ldots, A_m)$

  A tuple *t* is in $r \div s$ if and only if both of two conditions hold:

  1. *t* is in $\prod_{R\text{-}S}(r)$
  2. For every tuple t_s in *S*, there is a tuple t_r in *R* satisfying both of the following:
     1. $t_r[S] = t_s[S]$
     2. $t_r[R\text{-}S] = t$

Division Operation – Example

- Relations *r*, *s*:

A	B
α	1
α	2
α	3
β	1
γ	1
δ	1
δ	3
δ	4
$\in$	6
$\in$	1
β	2

r

B
1
2

s

- $r \div s$:

A
α
β

Another Division Example

- Relations *r*, *s*:

A	*B*	*C*	*D*	*E*
α	a	α	a	*1*
α	a	γ	a	*1*
α	a	γ	b	*1*
β	a	γ	a	*1*
β	a	γ	b	*3*
γ	a	γ	a	*1*
γ	a	γ	b	*1*
γ	a	β	b	*1*

r

D	*E*
a	*1*
b	*1*

s

- $r \div s$:

A	*B*	*C*
α	a	γ
γ	a	γ

Assignment Operation

- The assignment operation ($\leftarrow$) provides a convenient way to express complex queries.
  - Write query as a sequential program consisting of
    - a series of assignments
    - followed by an expression whose value is displayed as a result of the query.
  - Assignment must always be made to a temporary relation variable.
- Example: Write $r \cap s$ as

$$temp \leftarrow r - s$$

$$result \leftarrow r - temp$$

  - The result to the right of the $\leftarrow$ is assigned to the relation variable on the left of the $\leftarrow$.