# Numerical methods I (2019)

## PROJECT II

J.M.McNamee– *Numerical methods for roots of polynomials, part I*, BG PW, No. C.133451.

**You have to plot the function $y = f(x)$ between specified limits!**

- The Newton method for solving the nonlinear equation $f(x) = 0$.
  Construct a sequence $\{x_k\}$ such that $x_k \approx \alpha$, where $f(\alpha) = 0$:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}.$$

- The secant method for solving the nonlinear equation $f(x) = 0$.

$$x_{k+1} = x_k - \frac{f(x_k)(x_k - x_{k-1})}{f(x_k) - f(x_{k-1})}.$$

- The Steffensen method for solving the nonlinear equation $f(x) = 0$.
  Construct a sequence $\{x_k\}$ such that $x_k \approx \alpha$, where $f(\alpha) = 0$:

$$\begin{aligned} y_k &= x_k + f(x_k), \\ x_{k+1} &= x_k - \frac{(f(x_k))^2}{f(y_k) - f(x_k)}. \end{aligned}$$

- The Halley method for solving the nonlinear equation $f(x) = 0$.
  Construct a sequence $\{x_k\}$ such that $x_k \approx \alpha$, where $f(\alpha) = 0$:

$$x_{k+1} = x_k - \frac{2f(x_k)f'(x_k)}{2(f'(x_k))^2 - f(x_k)f''(x_k)}.$$

- The Chebyshev method for solving the nonlinear equation $f(x) = 0$.

  Construct a sequence $\{x_k\}$ such that $x_k \approx \alpha$, where $f(\alpha) = 0$:

$$y_k = x_k - \frac{f(x_k)}{f'(x_k)},$$
$$x_{k+1} = y_k - \frac{f''(x_k)(y_k - x_k)^2}{2f'(x_k)}.$$

- The Homeier method for solving the nonlinear equation $f(x) = 0$.

  Construct a sequence $\{x_k\}$ such that $x_k \approx \alpha$, where $f(\alpha) = 0$:

$$y_k = x_k - \frac{f(x_k)}{f'(x_k)},$$

$$x_{k+1} = x_k - \frac{1}{2}\, f(x_k) \left( \frac{1}{f'(x_k)} + \frac{1}{f'(y_k)} \right).$$

- The King method for solving the nonlinear equation $f(x) = 0$.

  Construct a sequence $\{x_k\}$ such that $x_k \approx \alpha$, where $f(\alpha) = 0$:

$$y_k = x_k - \frac{f(x_k)}{f'(x_k)},$$
$$x_{k+1} = y_k - \frac{f(y_k)}{f'(y_k)} - \frac{f(x_k)}{f'(x_k)} \left( \frac{f(y_k)}{f(x_k)} \right)^3.$$

- The Jarratt method for solving the nonlinear equation $f(x) = 0$.

  Construct a sequence $\{x_k\}$ such that $x_k \approx \alpha$, where $f(\alpha) = 0$:

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k - \frac{1}{2}\, \frac{f(x_k)}{f'(x_k)})}.$$

- The corrected Newton method for finding a root $\alpha$ of the function $y = f(x)$.

  Let $m$ be the multipicity of $\alpha$. Construct a sequence $\{x_k\}$ such that $x_k \approx \alpha$, where $f(\alpha) = 0$:

$$x_{k+1} = x_k - m\, \frac{f(x_k)}{f'(x_k)}.$$

- The Weierstrass method for finding all roots of the polynomial

$$w(x) = x^n + p_2 x^{n-1} + \ldots + p_{n+1}.$$

We assume that the roots $\alpha_1, \alpha_2, \ldots, \alpha_n$ of $w(x)$ are simple.

For $k = 0, 1, \ldots$ we compute $x_i^{(k)} \approx \alpha_i, \quad i = 1, 2, \ldots, n.$

for $k = 0, 1, \ldots$
  for $i = 1, 2, \ldots, n$

$$x_i^{(k+1)} = x_i^{(k)} - \frac{w(x_i^{(k)})}{\prod_{j=1, j\neq i}^{n} (x_i^{(k)} - x_j^{(k)})}.$$

- The simultaneous Ehrlich-Aberth method for finding all roots of the polynomial

$$w(x) = x^n + p_2 x^{n-1} + \ldots + p_{n+1}.$$

We assume that the roots $\alpha_1, \alpha_2, \ldots, \alpha_n$ of $w(x)$ are simple.

For $k = 0, 1, \ldots$ we compute $x_i^{(k)} \approx \alpha_i, \quad i = 1, 2, \ldots, n.$

for $k = 0, 1, \ldots$
  for $i = 1, 2, \ldots, n$

$$x_i^{(k+1)} = x_i^{(k)} - \frac{1}{\frac{w'(x_i^{(k)})}{w(x_i^{(k)})} - \sum_{j=1, j\neq i}^{n} \frac{1}{(x_i^{(k)} - x_j^{(k)})}}.$$

**Apply the recurrence relations for the orthogonal polynomials. Don't transform these polynomials to the basis $1, x, x^2, \ldots, x^n$!**

- The Chebyshev polynomials of the first order satisfy the recurrence relations:

$$T_0(x) = 1,\ T_1(x) = x, \tag{1}$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x),\ \ n = 2, 3, \ldots. \tag{2}$$

- The Chebyshev polynomials of the second order satisfy the recurrence relations:

$$U_0(x) = 1,\ U_1(x) = 2x, \tag{3}$$

$$U_n(x) = 2xU_{n-1}(x) - U_{n-2}(x),\ \ n = 2, 3, \ldots. \tag{4}$$

- The Legendre polynomials satisfy the recurrence relations:

$$P_0(x) = 1,\ P_1(x) = x, \tag{5}$$

$$(n+1)P_{n+1}(x) = (2n+1)xP_n(x) - nP_{n-1}(x),\ \ n = 1, 2, 3, \ldots. \tag{6}$$

- The Hermite polynomials satisfy the recurrence relations:

$$H_0(x) = 1,\ H_1(x) = 2x, \tag{7}$$

$$H_{n+1}(x) = 2xH_n(x) - 2n\,H_{n-1}(x),\ \ n = 1, 2, 3, \ldots. \tag{8}$$