

Exercise 1

A small 25 W light bulb has an efficiency of 20%. How many photons are approximately emitted per second?

Assume in the calculations that we only use average photons of wavelength 500 nm.

The sum of the energies of the object's emitted photons (per second) is equal to the object's electrical power. The lightbulb has a power P of 25W (taking into account efficiency; 5W).

The energy of a photon is

$$E = hf = \frac{hc}{\lambda} [\text{J}]$$

We start with the equation for radiant flux $\phi = \frac{dQ}{dt}$. From this we see that to achieve radiant flux ϕ , there has to be n photons producing equal energy E within one second.

The number of photons therefore approximately is

$$n_{\text{photons}} = \frac{P}{\frac{hc}{\lambda}} [\text{s}^{-1}] = \frac{5}{\frac{6.626 \cdot 10^{-34} \cdot 3 \cdot 10^8}{500 \cdot 10^{-9}}} = 1.26 \cdot 10^{19}$$

Exercise 2

A light bulb (2.4 V and 0.7 A), which is approximately sphere-shaped with a diameter of 1 cm, emits light equally in all directions. Find the following entities (ideal conditions assumed)

- Radiant flux
- Radiant intensity

- Radiant exitance
- Emitted energy in 5 minutes

Use W for Watt, J for Joule, m for meter, s for second and sr for steradian.

Radiometry is used for measuring the properties of the emissive object.

The light bulb is assumed to be 100% efficient.

- The radiant flux of the object is $\phi = P = V \cdot C = 1.68 \text{ [W]}$.
- The radiant intensity of the object is $I = \frac{d\phi}{d\omega}$. A sphere has 4π steradians. The intensity is then $I = 1.68/4\pi = 0.1337 \text{ [W/sr]}$.
- The exitance is $M = \frac{d\phi}{dA}$. The area of the sphere is $4\pi \cdot 0.005^2 \text{ [m}^2\text{]}$. The exitance is therefore $5347.61 \text{ [} \frac{\text{W}}{\text{m}^2} \text{]}$.
- The emitted energy over the course of five minutes is equal to the power of the bulb multiplied by 300 seconds,

$$E_{\text{emitted}} = P \cdot 300 = \phi \cdot 300 = 504 \text{ [J]}.$$

Exercise 3

The light bulb from above is observed by an eye, which has an opening of the pupil of 6 mm and a distance of 1 m from the light bulb. Find the irradiance received in the eye.

Irradiance is given by $E = \frac{d\phi}{dA}$. The equation has to be in density form as to "collect" the packets of light from the emissive object.

We assume the observer (eye) is looking directly at the bulb (do not replicate in real life), so that the angle between the central ray and the surface is zero ($\cos \theta = 1$).

We calculate the irradiance by propagating the bulb's flux to a sphere of 1m. Irradiance can also be represented as $E = I \frac{\cos \theta}{d^2}$, which given the light bulb is an isotropic source of light is equal to $\frac{\phi}{4\pi 1^2} = 0.1337 \frac{\text{W}}{\text{m}^2}$.

Exercise 4

A 200 W spherically shaped light bulb (20% efficiency) emits red light of wavelength 650 nm equally in all directions. The light bulb is placed 2 m above a table. Calculate the irradiance at the table.

Photometric quantities can be calculated from radiometric ones based on the equation

Photometric = Radiometric $\cdot 685 \cdot V(\lambda)$ in which $V(\lambda)$ is the luminous efficiency curve.

At 650 nm, the luminous efficiency curve has a value of 0.1. Calculate the illuminance.

Illuminance, the photometric equivalent of the radiometric irradiance, is measured by $E = \frac{d\phi}{dA}$ in $\frac{\text{lm}}{\text{m}^2}$.

Initially we calculate the radiometric quantities (as the input measurements are provided in radiometric-compatible units). The bulb's output power is 40W given its efficiency at 20%.

The bulb's (spherical) irradiance on the table is $\frac{\phi_{\text{bulb}}}{4\pi d^2} = \frac{40}{4\pi 2^2} \frac{\text{W}}{\text{m}^2}$.

Converting this into its photometric equivalent we find

$$\frac{40}{4\pi 2^2} \cdot 685 \cdot V(650) = 54.5106 \frac{\text{lm}}{\text{m}^2}.$$

Exercise 5

In a simple arrangement the luminous intensity of an unknown light source is determined from a known light source. The light sources are placed 1 m from each other and illuminate a double sided screen placed between the light sources. The screen is moved until both sides are equally illuminated as observed by a photometer. At the position of match, the screen is 35 cm from the known source with luminous

intensity $I_s = 40 \text{ lm/sr} = 40 \text{ cd}$ and 65 cm from the unknown light source. What is the luminous intensity I_x of the unknown source?

Luminous intensity is measured by $I = \frac{d\phi}{d\omega}$.

The illuminance on both sides of the screen are equal from both sources giving the relation $E_k = E_u \iff \frac{I_k}{d_k^2} = \frac{I_u}{d_u^2}$. We can assume $\cos \theta = 1$ as the sources are directly illuminating the screen.

Solving for I_u :

$$I_u = I_k \frac{d_u^2}{d_k^2} = 40 \frac{0.65^2}{0.35^2} = 137.96 \text{ cd}$$

Exercise 6

The radiance L from a diffuse light source (emitter) of $10 \times 10 \text{ cm}$ is $5000 \text{ W/(sr m}^2\text{)}$. Calculate the radiosity (radiant exitance). How much energy is emitted from the light source?

Radiance is measured by $L = \frac{d\phi}{dA_{\text{perp}}d\omega}$. Radiosity is given by $B = \frac{d\phi}{dA}$. The area of the emissive object is 100 square centimetres.

The perpendicular area can be assumed to be facing the light source ($\cos \theta = 1$).

For a diffuse emitter, the integral form of total emitted radiant flux and hence radiosity are

$$\phi = \int_A \int_{2\pi} L \cos \theta d\omega dA = LA\pi, \quad B = \int_{2\pi} L \cos \theta d\omega = L\pi$$

Therefore the radiosity is $B = 5000 \cdot \pi \frac{\text{W}}{\text{m}^2}$ and the flux is

$$\phi = B \cdot A = 50 \cdot \pi \frac{\text{J}}{\text{s}}.$$

Exercise 7

The radiance $L = 6000 \cos \theta \text{ W/(m}^2 \text{ sr)}$ for a non-diffuse emitter of area 10 by 10 cm. Find the radiant exitance. Also, find the power of the entire light source.

Radiance is measured by $L = \frac{d\phi}{dA_{\text{perp}} d\omega}$. Radiant exitance is given by $M = \frac{d\phi}{dA}$. The power of the source is given by $\phi = \frac{dQ}{dt}$. The area of the emissive object is 100 square centimetres.

The radiance may be re-written to $L = \frac{d\phi}{\cos \theta dA d\omega}$. Substituting for L : $6000 \cdot \cos \theta = \frac{d\phi}{\cos \theta dA d\omega}$. With further transformations $M = 6000 \cdot \cos^2 \theta d\omega$ and integration

$$M = \int_0^{\frac{\pi}{2}} 6000 \cos^2 \theta d\omega = 6000 \int_{2\pi} \int_0^{\frac{\pi}{2}} \cos^2 \theta \sin \theta d\theta d\phi = 6000 \cdot 2\pi \cdot \frac{1}{3} = 4000 \cdot \pi$$

The power is then $\phi = \int_A M dA = M \cdot 0.01 = 40\pi$.