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Report
- Investigation on Deviations in Calibration Data Sets -

Introduction

- 4 data sets of the calibration of a laser-based stereoendoscopic system show several deviations in paramteres of the laser model that was fitted to observations.

Data set	KK17_alt_Jonas	KK17_mit_Rotation	KK10	KK2_mit_Rotation
Identifier	**1**	**2**	**3**	**4**
Issue	translation shows deviation	direction vectors show deviation	direction vectors show deviation	translation shows deviation

- 20 investigations were carried out that can be divided into 4 groups:
  - Data Consistency
  - Laser Model
  - Computation of Initial Values
  - Optimization Task

Introduction

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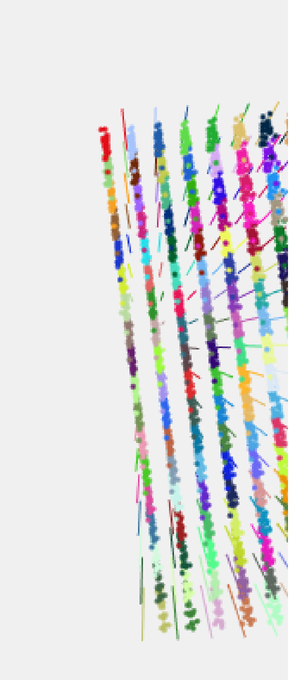
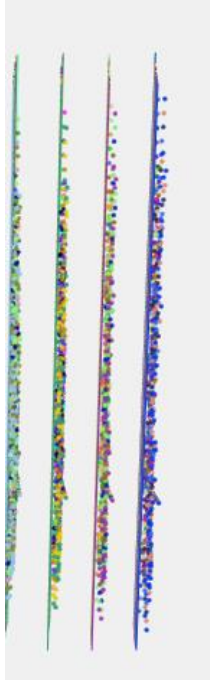
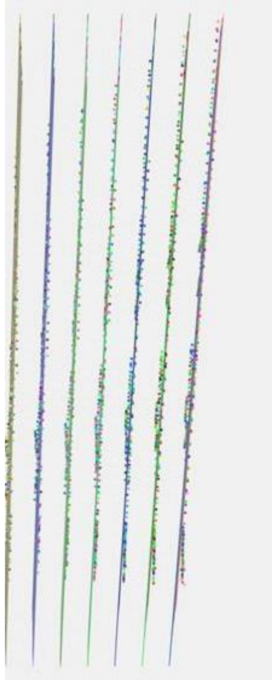
- Data set **1**
- Data set **2**
- Data set **3**
- Data set **4**







Data Consistency

1. Check Identity – Data Set 1 & 2

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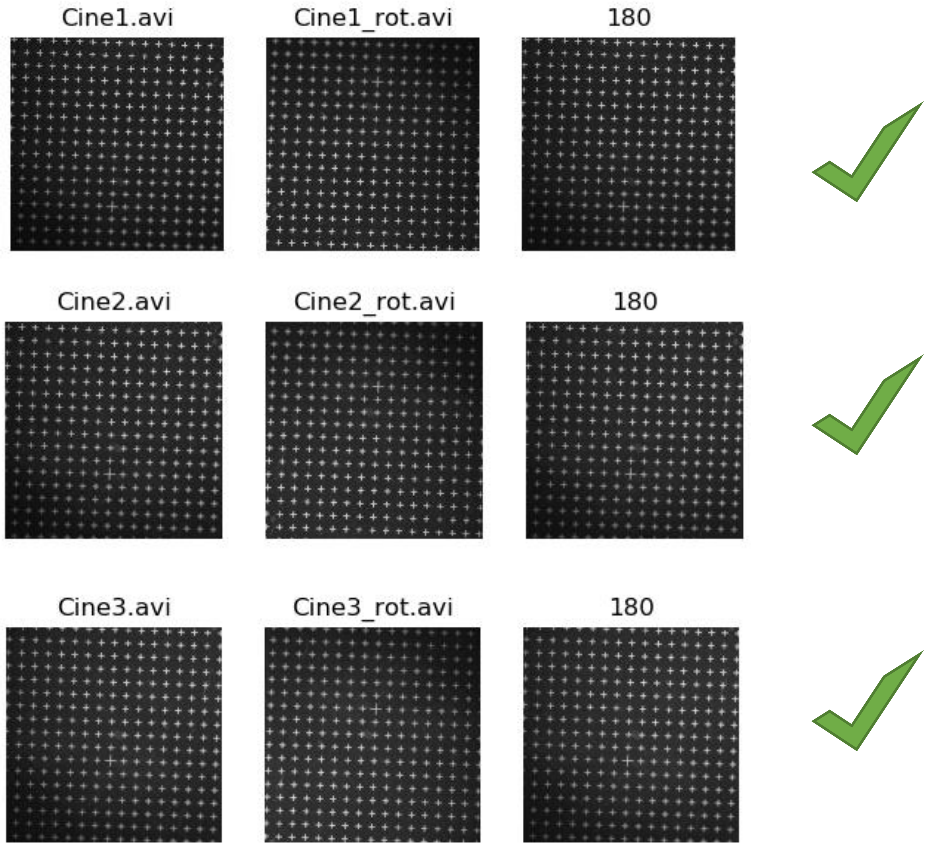
- Camera Calibration: KK18_cal_lsr_Cam_16904_Cine1.avi
- Laser Calibration: KK17_cal_cam_Cam_16904_Cine1.avi

?

Solved issue from data set: 1 – 2 – 3 – 4

2. Check Rotation Camera – Data Set 1 & 2

- File: KK17_cal_cam_Cam_16904_
- The first column shows images of data set 1, the second column shows images of data set 2 and the third row shows the second column rotated by 180°.
- Rotation of camera calibration images was done in a feasible way.

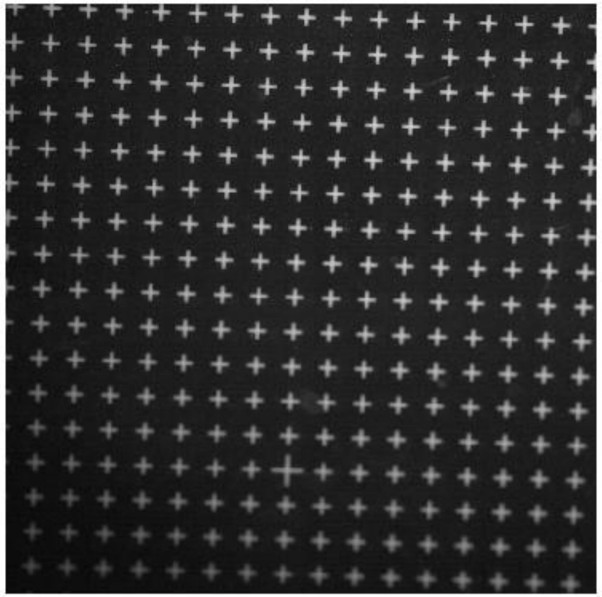


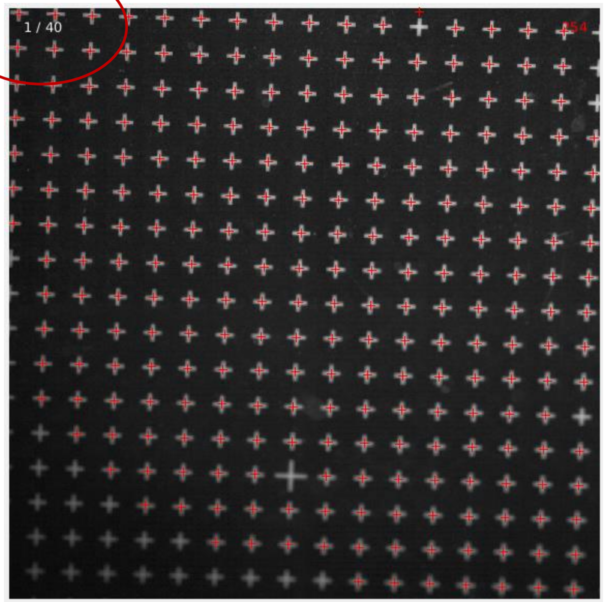
Solved issue from data set: 1 – 2 – 3 – 4

3. Check Images Camera – Data Set 1

- Same images are present but indices are not neccesarily matching (e.g. 2 -> 10).

Cine1.avi

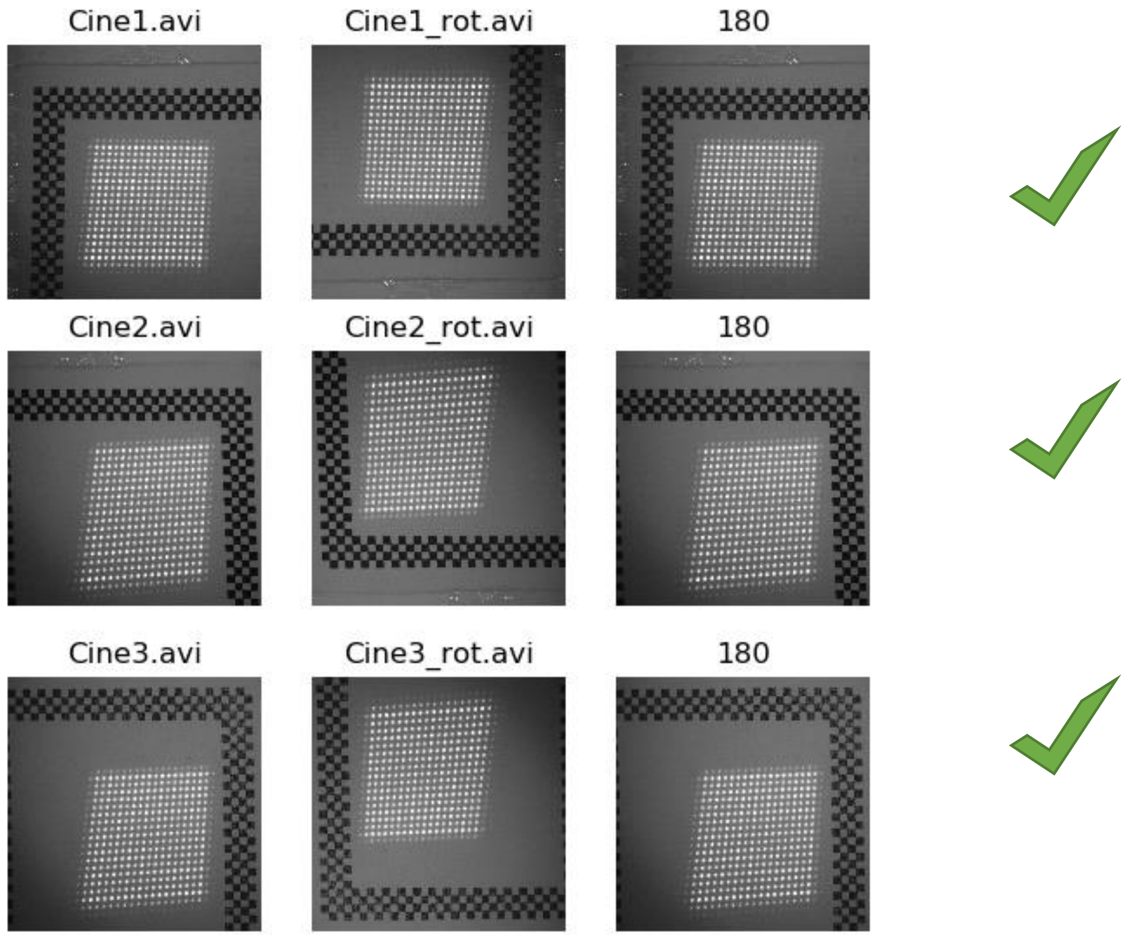




Solved issue from data set: 1 – 2 – 3 – 4

4. Check Rotation Laser – Data Set 1 & 2

- File: KK18_cal_lsr_Cam_16904_
- The first column shows images of data set 1, the second column shows images of data set 2 and the third row shows the second column rotated by 180°.
- Rotation of laser calibration images was done in a feasible way.



Solved issue from data set: 1 – 2 – 3 – 4

5. Check Images Laser – Data Set 1

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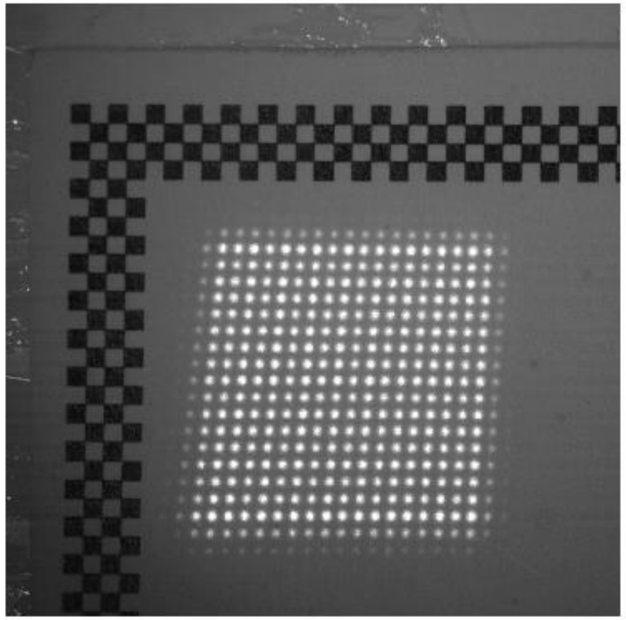
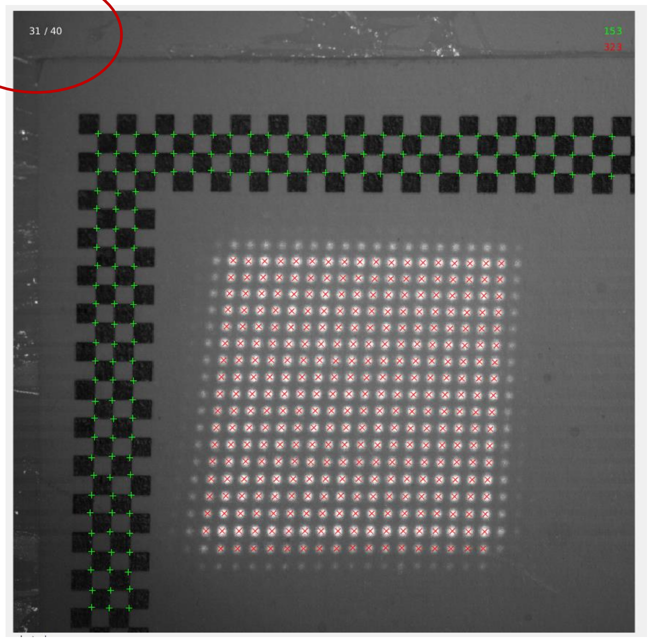
- Same images are present but indices are not neccesarily matching (e.g. 2 -> 10).

Cine1.avi





Solved issue from data set: 1 – 2 – 3 – 4

6. Check Matching Laser – Data Set 1

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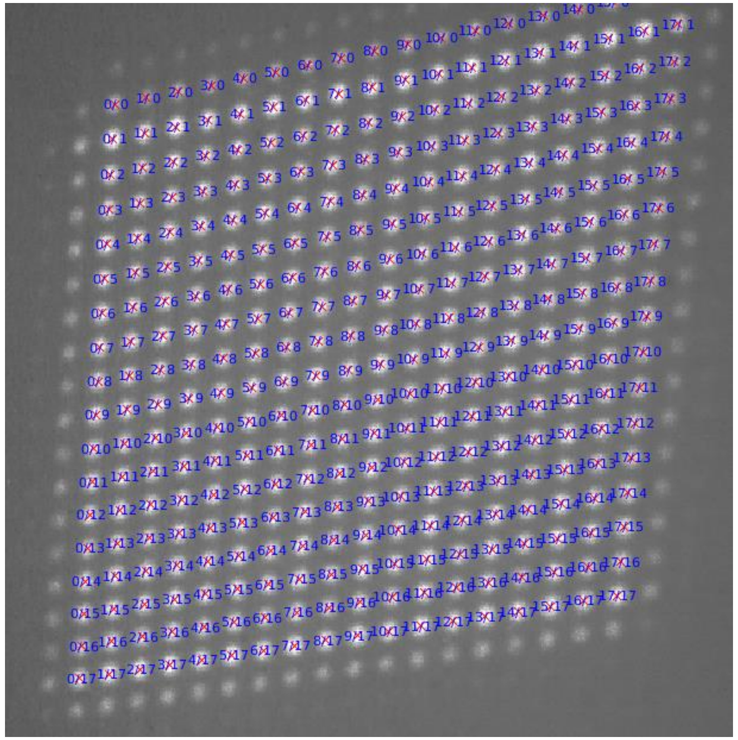
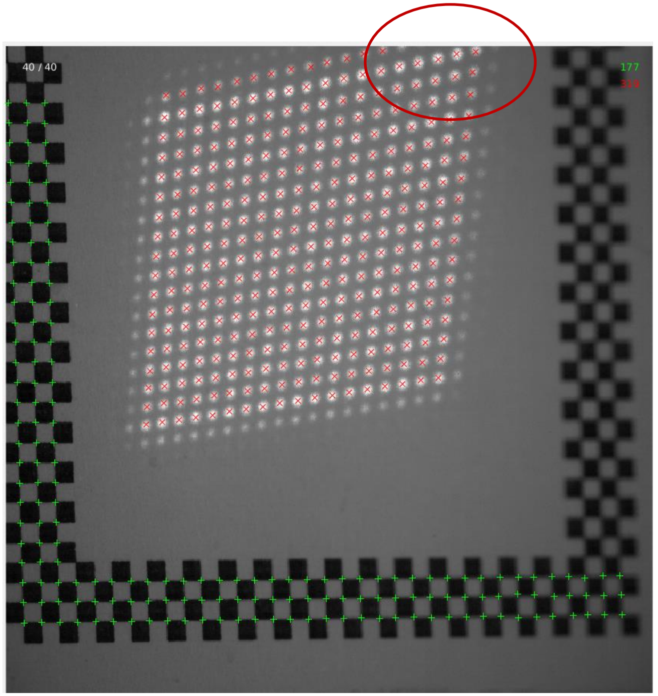
- All frames were checked, matching seems reasonable.



Solved issue from data set: 1 – 2 – 3 – 4

7. Check Incomplete Data – Data Set 2

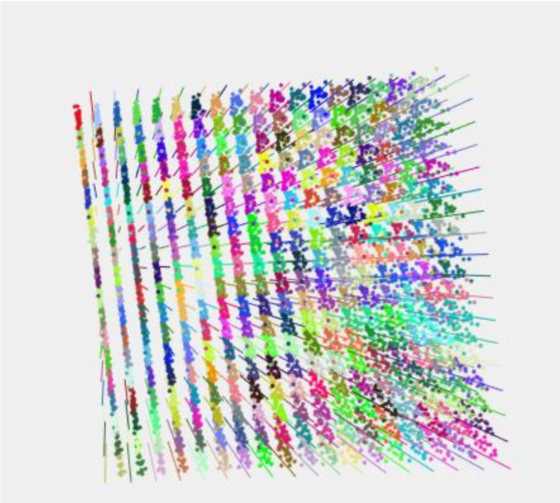
- Frames were removed from calibration where detection was not complete.
- No major influence on the parameter fitting could be found.



Solved issue from data set: 1 – 2 – 3 – 4

8. Compact Data Set – Data Set 2

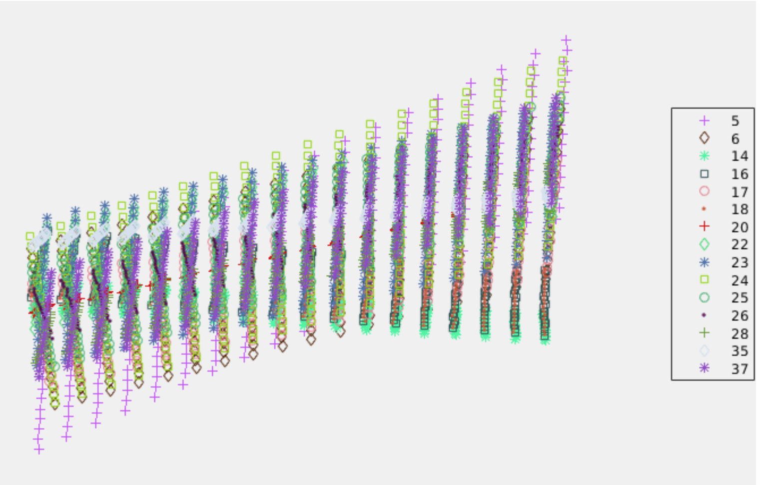
- A compact data set with only 15 images was created.
- No major influence on the parameter fitting could be found.

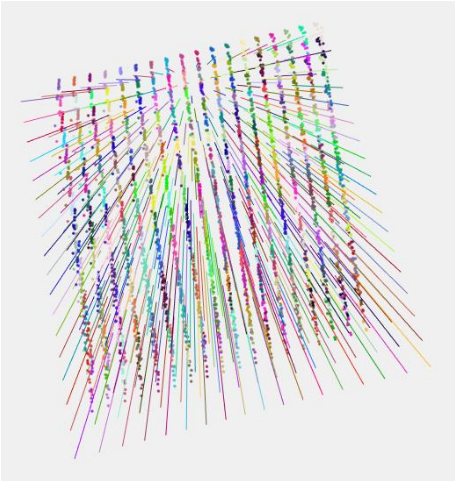


Solved issue from data set: 1 – 2 – 3 – 4

9. Small Variance Data Set – Data Set 2

- A data set was created where a smaller variance is present.
- No major influence on the parameter fitting could be found.

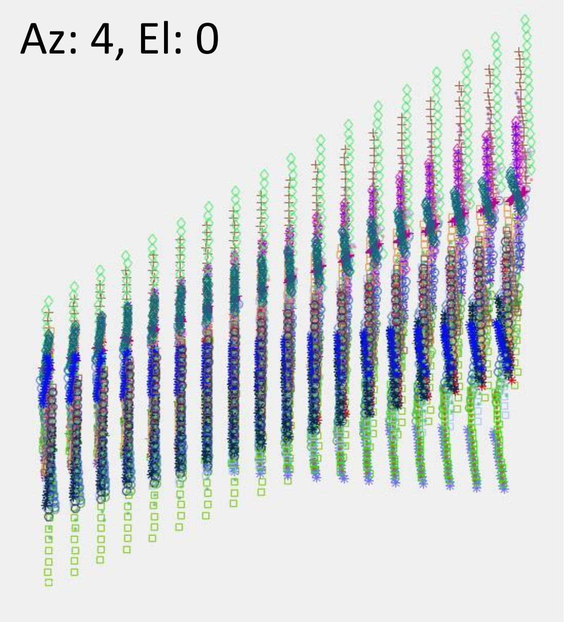


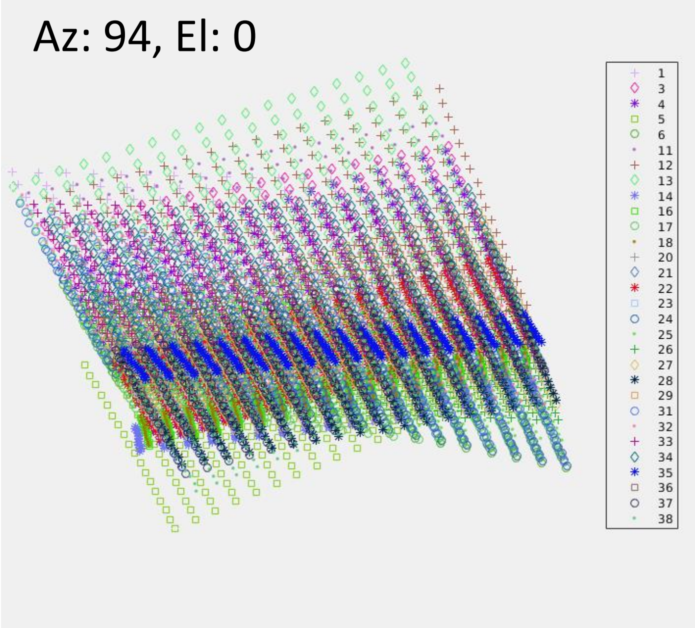


Solved issue from data set: 1 – 2 – 3 – 4

10. Removed Outliers – Data Set 2

- Why does data under 90° show more outliers and higher variance. **?**
- A data set with less outliers should be created. **!**





Solved issue from data set: 1 – 2 – 3 – 4

11. Intrinsic Parameters – Data Set 1 & 2

- The computation of the intrinsic parameters seems feasible.

Without rotation:

```
Focal Length:          fc = [ 16227.67875   16227.71764 ] � [ 0.00000   0.00000 ]
Principal point:       cc = [ 360.05020   371.22787 ] � [ 0.00000   0.00000 ]
Skew:             alpha_c = [ 0.00000 ] � [ 0.00000  ]   => angle of pixel axes = 90.00000 � 0.00000
degrees
Distortion:            kc = [ 4.09138   -1680.78743   0.00385   -0.00437  0.00000 ] � [ 0.00000
0.00000   0.00000   0.00000  0.00000 ]
Note: The numerical errors are approximately three times the standard deviations (for reference).
```

With rotation:

```
Focal Length:          fc = [ 16196.30671   16196.40637 ] � [ 0.00000   0.00000 ]
Principal point:       cc = [ 405.98353   397.01651 ] � [ 0.00000   0.00000 ]
Skew:             alpha_c = [ 0.00000 ] � [ 0.00000  ]   => angle of pixel axes = 90.00000 � 0.00000
degrees
Distortion:            kc = [ 4.09712   -1686.72189   -0.00400   0.00384  0.00000 ] � [ 0.00000
0.00000   0.00000   0.00000  0.00000 ]
Note: The numerical errors are approximately three times the standard deviations (for reference).
```

Solved issue from data set: 1 – 2 – 3 – 4

12. Detection Errors – Data Set 2

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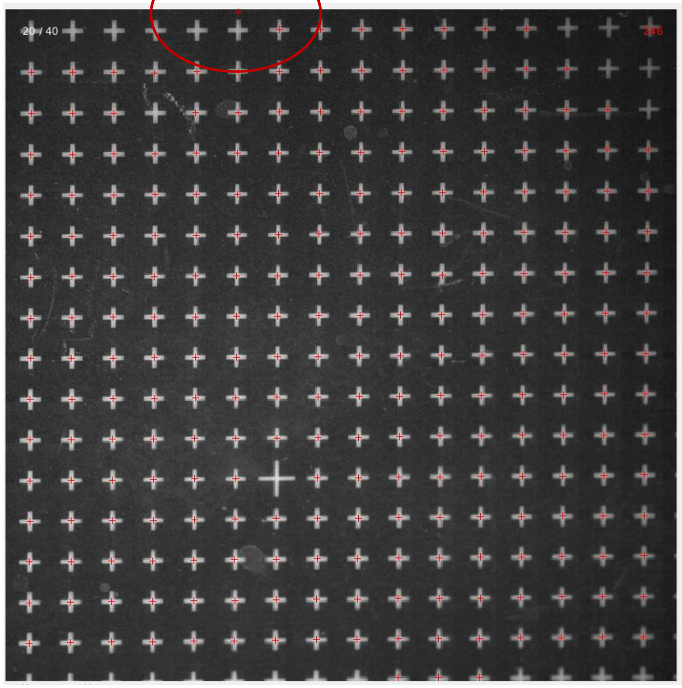
- For the camera calibration always all images are considered.
- An experiment could be carried out, where images with detection errors are removed from the set. **!**

```
Initialization of the intrinsic parameters - Number of images: 40


Calibration parameters after initialization:

Focal Length:          fc = [ 16012.50464   16012.50464 ]
Principal point:       cc = [ 383.50000   383.50000 ]
Skew:             alpha_c = [ 0.00000 ]   => angle of pixel = 90.00000 degrees
Distortion:            kc = [ 0.00000   0.00000   0.00000   0.00000   0.00000 ]

Main calibration optimization procedure - Number of images: 40
Gradient descent iterations:
1...2...3...4...5...6...7...8...9...10...11...12...13...14...15...16...17...18...19...20...21...22...2
done
```



Solved issue from data set: 1 – 2 – 3 – 4

Laser Model

13. Model

- The extrinsic parameters (the laser ray model) considers the rotation only around **one** axis. It could be further investigated if a second rotation has to be considered, i.e. a rotation around the imaging axis of the laser. !
- Could the computation of the distortion have an influence? That should be checked as well. !

Solved issue from data set: 1 – 2 – 3 – 4

Computation of Initial Values

14. System of Linear Equations

- A system of linear equations is solved prior to running an optimization for parameter fitting. The retrieved parameters are used as initial values for the optimization.
- The following methods could lead to errors: **!**
  - Orthogonal regression
  - Centroid of each ray
  - Centroid of all rays

- It is especially interesting how robust those methods are against outliers.

Solved issue from data set: 1 – 2 – 3 – 4

Optimization Task

15. Optimization Model

- A rough overview of the optimization task is given as:

$\min F(\mathbf{f}(\mathbf{x}))$

minimize the functional F with

$\mathbf{x} = [r_1, r_2, r_3, t_1, t_2, t_3]^T$

or

$\mathbf{x} = [r_1, r_2, r_3, t_1, t_2, t_3, \alpha, x_{11}, x_{21}, ..., x_{1p}, x_{2p}]^T$

where

$\mathbf{r} = [r_1, r_2, r_3]^T$ rotation vector

Euler-Rodriguez-Vector

$\mathbf{t} = [t_1, t_2, t_3]^T$ translation vector

$\alpha =$ divergence angle

$\mathbf{x} = [x_{11}, x_{21}, ..., x_{1p}, x_{2p}]^T$ ray offset

$\mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{x}, \mathbf{x}^c, \mathbf{c}^c, \mathbf{u}^c) \in \mathbb{R}^n, n = n_r n_i n_{sd}, n_{sd} = 3$

error from each observation in each direction where

$\mathbf{x}^c = [\mathbf{x}_1^{c(1)}, ..., \mathbf{x}_r^{c(i)}]^T$ coordinates of each observation in image i and ray r

$\mathbf{c}^c = [\mathbf{c}_1^{c(1)}, ..., \mathbf{c}_r^{c(i)}]^T$ centroid of each observation in image i and ray r

$\mathbf{u}^c = [\mathbf{u}_1^{c(1)}, ..., \mathbf{u}_r^{c(i)}]^T$ unit vector of each observation in image i and ray r

✓

Solved issue from data set: 1 – 2 – 3 – 4

16. Optimization Variables – Data Set 2

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- When carrying out the optimization with **precise = false**, fewer variables are present in the optimization model.
- The optimization does not stop in the local extremum from **experiment 7** (precise = true).
- A new local extremum is found.

Experiment 7

Iteration	Func	f(x)	step	optimality
0	656	8323.35		5.1e+06
1	1312	8323.35	10	5.1e+06
2	1968	3015.72	2.5	1.25e+06
3	2624	3015.72	5	1.25e+06
4	3280	2928.59	1.25	5.22e+05
5	3936	2928.59	1.25	5.22e+05
6	4592	2863.33	0.3125	1.53e+05

Experiment 11

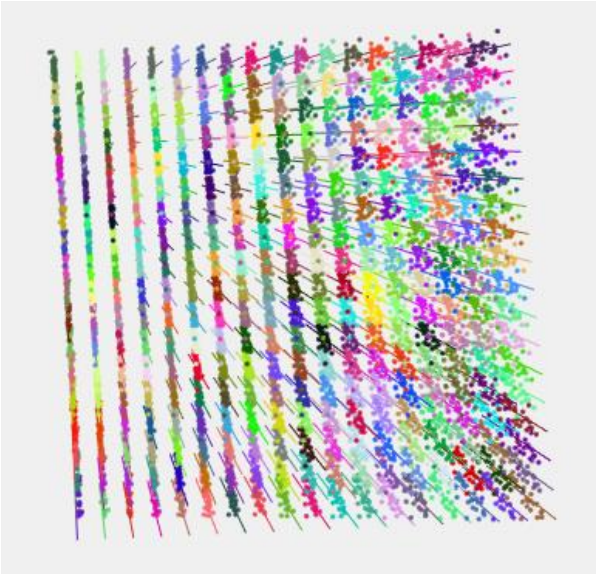
Iteration	Func	f(x)	step	optimality
0	8	8128.36		5.1e+06
1	16	2143.16	10	1.21e+06
2	24	2143.16	18.0532	1.21e+06
3	32	1578.08	4.51331	1.41e+05
4	40	1215.9	9.02662	1.03e+05
5	48	1103.04	7.31152	2.03e+05
6	56	1083.56	0.603121	267

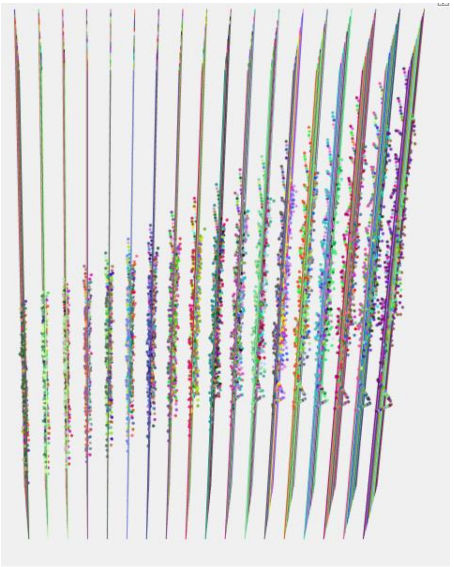
✓

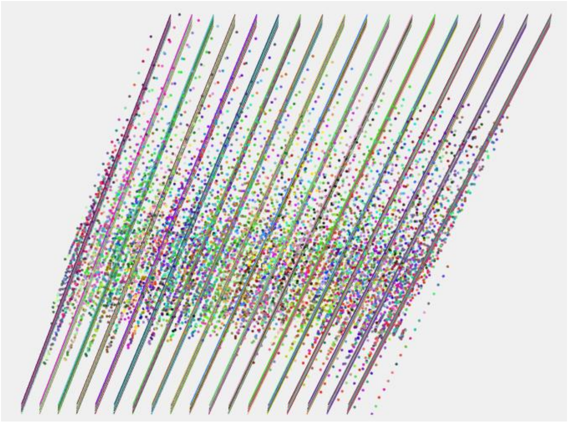
Solved issue from data set: 1 – 2 – 3 – 4

16. Optimization Variables

- Good qualitative are found.
- The issue from data set 2 can be solved.







Solved issue from data set: 1 – 2 – 3 – 4

17. Optimizer

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- A nonlinear data fitter from [1] is used.
- The following options are relevant:
  - TolFun is a lower bound on the change in the value of the objective function during a step. If |f(xi) – f(xi+1)| < TolFun, the iterations end. Solvers generally use TolFun as a relative bound, meaning iterations end when |f(xi) – f(xi+1)| < TolFun(1 + |f(xi)|), or a similar relative measure.
  - MaxIter is a bound on the number of solver iterations. MaxFunEvals is a bound on the number of function evaluations.

Solved issue from data set: 1 – 2 – 3 – 4

17. Optimizer – Data Set 2

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- The two differnet optimization algorithms **trust-region-reflective** (default) and **levenberg-marquardt** were tested.
- Levenberg-marquardt shows faster and more robust convergence.

trust-region-reflective

Iteration	Func	f(x)	step	optimality
0	8	8128.36		5.1e+06
1	16	2143.16	10	1.21e+06
2	24	2143.16	18.0532	1.21e+06
3	32	1578.08	4.51331	1.41e+05
4	40	1215.9	9.02662	1.03e+05
5	48	1103.04	7.31152	2.03e+05
6	56	1083.56	0.603121	267

levenberg-marquardt

Iteration	Func	Residual	optimality	Lambda	step
0	8	8128.36	5.1e+06	0.01	
1	16	4706.63	2.96e+06	0.001	17.3176
2	24	1107.06	2.76e+05	0.0001	12.5721
3	32	1083.56	4.99e+03	1e-05	1.54558

Solved issue from data set: 1 – 2 – 3 – 4

18. Initial Values – Data Set 2

- Two subsequent optimizations were carried out where the first optimization the ray offset were fixed variables (precise = false) and in the second optimization the ray offset was variable (precise = true). The resulting variables from the first optimization were used as initial values for the second optimization.
- In the extremum similar values for the objective are reached.

precise = false

Iteration	Func	f(x)	step	optimality
0	8	8128.36		5.1e+06
1	16	2143.16	10	1.21e+06
2	24	2143.16	18.0532	1.21e+06
3	32	1578.08	4.51331	1.41e+05
4	40	1215.9	9.02662	1.03e+05
5	48	1103.04	7.31152	2.03e+05
6	56	1083.56	0.603121	267

precise = true

1e-3 initial ray offset

Iteration	Func	f(x)	step	optimality
0	656	1394.36		1.6e+05
1	1312	1394.36	10	1.6e+05
2	1968	1394.36	2.5	1.6e+05
3	2624	1083.59	0.625	5.18e+04
4	3280	1083.59	1.25	5.18e+04
5	3936	1083.51	0.3125	2.68e+04
6	4592	1083.51	0.3125	2.68e+04
7	5248	1083.43	0.078125	1.88e+03

Solved issue from data set: 1 – 2 – 3 – 4

18. Initial Values – Data Set 2

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- The initial ray offset was lowered.
- This proofs that a local minimum is found. Qualitative evaluation seems reasonable. Large step size in the first iterations shows that it is unlikely that another local minimum is close.

1e-3 initial ray offset

Iteration	Func	f(x)	step	optimality
0	656	1394.36		1.6e+05
1	1312	1394.36	10	1.6e+05
2	1968	1394.36	2.5	1.6e+05
3	2624	1083.59	0.625	5.18e+04
4	3280	1083.59	1.25	5.18e+04
5	3936	1083.51	0.3125	2.68e+04
6	4592	1083.51	0.3125	2.68e+04
7	5248	1083.43	0.078125	1.88e+03

Iteration	Func	f(x)	step	optimality
0	656	1083.56		578
1	1312	1083.56	10	578
2	1968	1083.56	2.5	578
3	2624	1083.56	0.625	578
4	3280	1081.9	0.15625	1.24e+05

Solved issue from data set: 1 – 2 – 3 – 4

19. Reduced Tolerance – Data Set 2

- A reduced tolerance for the convergence criteria further proves that a local extremum is found.

Tolerance = 0.1

Iteration	Func	f(x)	step	optimality
0	8	8128.36		5.1e+06
1	16	2143.16	10	1.21e+06
2	24	2143.16	18.0532	1.21e+06
3	32	1578.08	4.51331	1.41e+05
4	40	1215.9	9.02662	1.03e+05
5	48	1103.04	7.31152	2.03e+05
6	56	1083.56	0.603121	267

Tolerance = 0.01

Iteration	Func	f(x)	step	optimality
0	8	8128.36		5.1e+06
1	16	2143.16	10	1.21e+06
2	24	2143.16	18.0532	1.21e+06
3	32	1578.08	4.51331	1.41e+05
4	40	1215.9	9.02662	1.03e+05
5	48	1103.04	7.31152	2.03e+05
6	56	1083.56	0.603121	267
7	64	1083.56	0.0165264	0.725

Solved issue from data set: 1 – 2 – 3 – 4

19. Reduced Tolerance – Data Set 3

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- A tolerance of 1.0 does not deliver feasible results.
- The deviations of data set 3 can therefore be explained and lowered with a smaller tolerance.

Tolerance = 1.0

Iteration	Func	f(x)	step	optimality
0	656	25720.1		1.97e+06
1	1312	10833.2	10	2.35e+06
2	1968	2425.37	20	2.65e+06
3	2624	1286.74	19.6511	2.21e+06

Tolerance = 0.1

Iteration	Func	f(x)	step	optimality
0	656	25720.1		1.97e+06
1	1312	10833.2	10	2.35e+06
2	1968	2425.37	20	2.65e+06
3	2624	1286.74	19.6511	2.21e+06
4	3280	1003.97	2.74823	1.37e+03
5	3936	1003.97	0.0640879	20.3

Solved issue from data set: 1 – 2 – 3 – 4

20. Outliers

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- Outliers could be the reason for the deviation of the laser origin, which is visible in 1,2 and 4. Possible solutions could be:

! – Remove outliers -> So far no outliers could be visually identified

! – Use other objective -> Would need a new implementation in MATLAB

! – Use different fitting -> RANSAC

Solved issue from data set: **1 – 2 – 3 – 4**

21. RANSAC

The *random sample consensus* is a paradigm for model fitting, which takes the presence of outliers in the underlying data set into account. It was first proposed by [6] and has since then widely been used [2nd source]. Outliers, which are points in the data set, are present if either

- their location deviates significantly from the ground truth, or
- their correspondence to a subset of the data set is wrong.

Why does this apply to our case -> explanation
Why can we not simply remove outliers -> explanation

Solved issue from data set: 1 – 2 – 3 – 4

21. RANSAC

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The RANSAC can be described as an algorithm that follows the steps:

1. Draw samples uniformly and at random from the input data set.
2. Cardinality of sample set is the smallest size sufficient to estimate the model parameters.
3. Compute the model parameters for each element of the sample data.
4. Evaluate the quality of the hypothetical models on the full data set.
   - Cost function for the evaluation of the quality of the model
   - Inliers: data points which agree with the model within an error tolerance
5. The hypothesis which gets the most support from the data set is taken as the best estimate.

- precise = false -> 2 points of 2 rays (4 points)
- precise = true -> 2 points of 2 rays (4 points) + 1 point of each left over ray

Previously: Approximation
Now: Interpolation with support

number of samples

probability outlier free desired

sample size

$$n = \frac{\log(1-p)}{\log(1-(1-\epsilon)^s)}$$

probability outlier present

Solved issue from data set: 1 – 2 – 3 – 4

Conclusion

- Several issues could be solved and a deeper understanding of the parameter fitting could be gained. This was marked with ✓.
- The deviation in all **direction vectors** in data set 2 and 3 could be solved
- The issue of a deviation of the **translation vector** in data set 1, 2 and 4 still remains!
- All propsals marked with **!** should further be considered.
- All questions marked with **?** should be answered.

Solved issue from data set: 1 – 2 – 3 – 4

Reference

- [1] https://de.mathworks.com/help/optim/ug/lsqnonlin.html