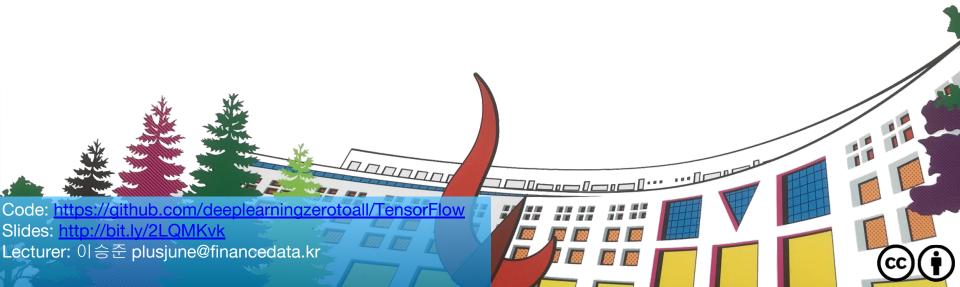
# ML/DL for Everyone Season2



03 - How to minimize cost



### **Hypothesis and Cost**

Hypothesis H(x) = Wx + b

Cost  $cost(W,b) = rac{1}{m} \sum_{i=1}^m \left(H(x_i) - y_i
ight)^2$ 

### Simplified hypothesis

Hypothesis H(x)=Wx

Cost  $cost(W) = rac{1}{m} \sum_{i=1}^m (Wx_i - y_i)^2$ 

$$cost(W) = rac{1}{m} \sum_{i=1}^m {(Wx_i - y_i)^2}$$

• W = 0, cost(W) = ?

Х	У
1	1
2	2
3	3

$$cost(W) = rac{1}{m} \sum_{i=1}^m \left(Wx_i - y_i
ight)^2$$

X	У
1	1
2	2
3	3

• W = 0, cost(W) = 4.67  

$$\frac{1}{3}((0*1-1)^2 + (0*2-2)^2 + (0*3-3)^2))$$

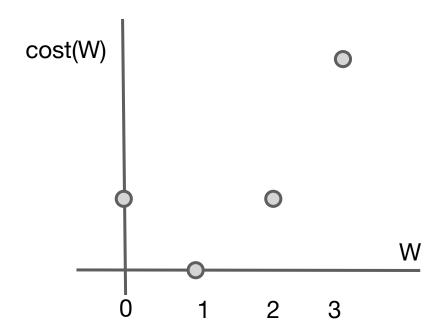
$$cost(W) = rac{1}{m} \sum_{i=1}^m (Wx_i - y_i)^2$$

Х	у
1	1
2	2
3	3

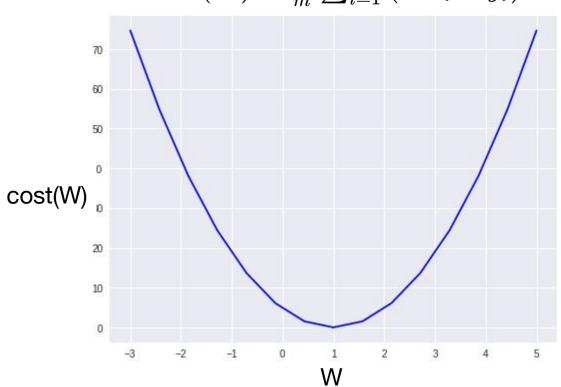
- W = 0, cost(W) = 4.67  $\frac{1}{3}((0*1-1)^2 + (0*2-2)^2 + (0*3-3)^2))$
- W = 1, cost(W) = 0  $\frac{1}{3}((1*1-1)^2 + (1*2-2)^2 + (1*3-3)^2))$
- W = 2, cost(W) = 4.67  $\frac{1}{3}((2*1-1)^2 + (2*2-2)^2 + (2*3-3)^2))$
- W = 3, cost(W) = 18.67  $\frac{1}{3}((3*1-1)^2 + (3*2-2)^2 + (3*3-3)^2))$

- W = 0, cost(W) = 4.67
- W = 1, cost(W) = 0
- W = 2, cost(W) = 4.67
- W = 3, cost(W) = 18.67

- W = 0, cost(W) = 4.67
- W = 1, cost(W) = 0
- W = 2, cost(W) = 4.67
- W = 3, cost(W) = 18.67

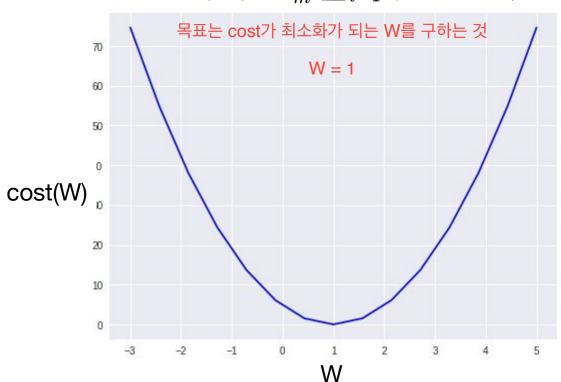


$$cost(W) = rac{1}{m} \sum_{i=1}^m {(Wx_i - y_i)^2}$$



### How to minimize cost?

$$cost(W) = rac{1}{m} \sum_{i=1}^m {(Wx_i - y_i)^2}$$



경사를 따라 내려가면서 최저점을 찾도록 설계된 알고리즘

## Gradient descent algorithm

경사 하강법 / 경사 하강 알고리즘

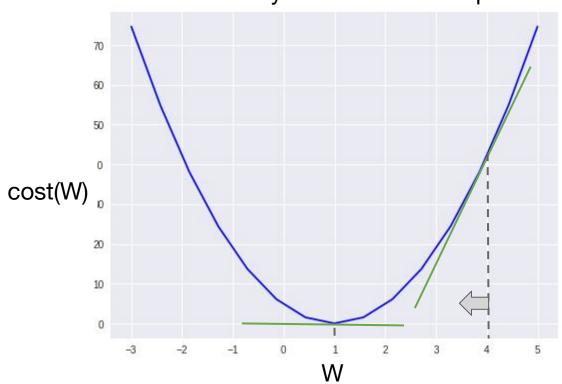
- Minimize cost function
- Gradient descent is used many minimization problems
- For a given cost function, cost (W, b), it will find W, b to minimize cost
- It can be applied to more general function: cost (w1, w2, ...)

변수가 여러 개 일 때도 사용 가능

엔지니어링 문제는 최적화 문제이고, 손실을 최소화하거나 이득을 최대화하는 것

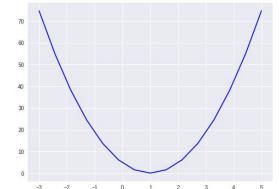
#### How it works?

How would you find the lowest point?

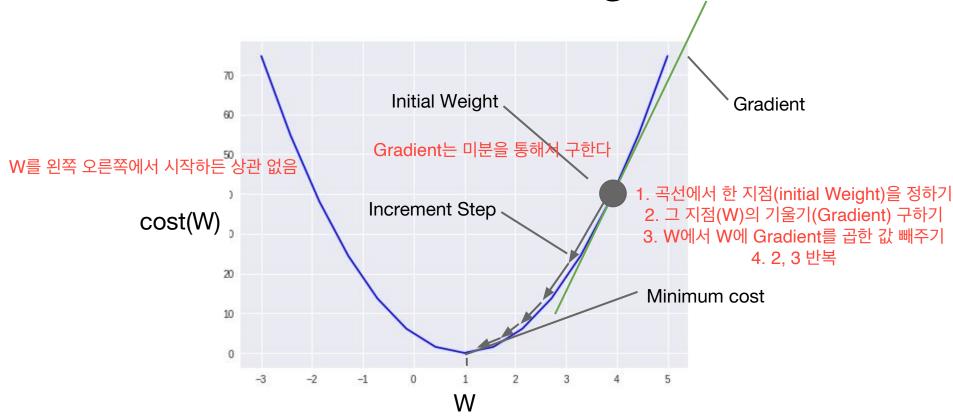


#### How it works?

- Start with initial guesses
  - Start at 0,0 (or any other value)
  - Keeping changing W and b a little bit to try and reduce cost(W, b)
- Each time you change the parameters, you select the gradient which reduces cost(W, b) the most possible
- Repeat
- Has an interesting property
  - Where you start can determine which minimum you end up



**Gradient descent algorithm** 

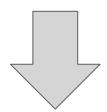


최저점에 다가갈 수록 기울기 작아진다

#### **Formal definition**

전체 개수 m으로 나눴는데, 이게 2m, 4m이든 cost의 특성에는 영향을 주지 않는다

$$cost(W,b) = rac{1}{m} \sum_{i=1}^m \left(H(x_i) - y_i
ight)^2$$



$$cost(W,b) = rac{1}{2m} \sum_{i=1}^m \left(H(x_i) - y_i
ight)^2$$

#### **Formal definition**

알파값에 따라 W값이 얼마나 빠르게 변할지 결정된다.

알파값은 작은 상수로 learning rate - 우리가 구한 값을 얼마나 반영해서 W에서 뺄지 결정하는 배수같은 것

$$W := W - lpha rac{\partial}{\partial W} rac{1}{2m} \sum_{i=1}^m \left(W(x_i) - y_i
ight)^2$$

W에 대해서만 미분하겠다는 편미분

$$W := W - lpha rac{1}{2m} \sum_{i=1}^m 2(W(x_i) - y_i) x_i$$

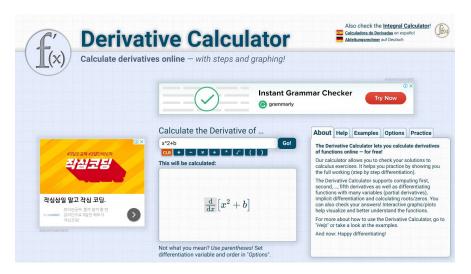
$$W := W - lpha rac{1}{m} \sum_{i=1}^m (W(x_i) - y_i) x_i$$

#### **Formal definition**

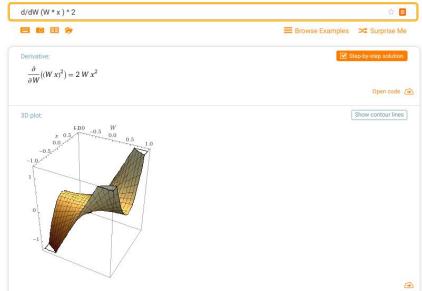
$$cost(W,b) = rac{1}{2m} \sum_{i=1}^m \left(H(x_i) - y_i
ight)^2$$

$$W := W - lpha rac{\partial}{\partial W} cost(W)$$

:= 는 '정의된다'는 의미



#### **Wolfram Alpha** computational intelligence.

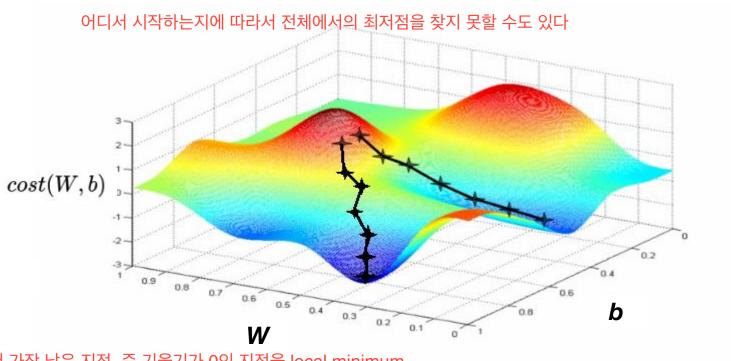


### **Gradient descent algorithm**

$$W := W - lpha rac{1}{m} \sum_{i=1}^m \left(W(x_i) - y_i
ight) x_i$$

적절한 알파값을 지정하는 것도 과제

#### **Convex function**



주변에서 가장 낮은 지점, 즉 기울기가 0인 지점을 local minimum 여기서는 지금 여러 개 있다 이런 상황에서는 gradient descent는 이런 상황에서 쓸 수 없다.

http://www.holehouse.org/mlclass/

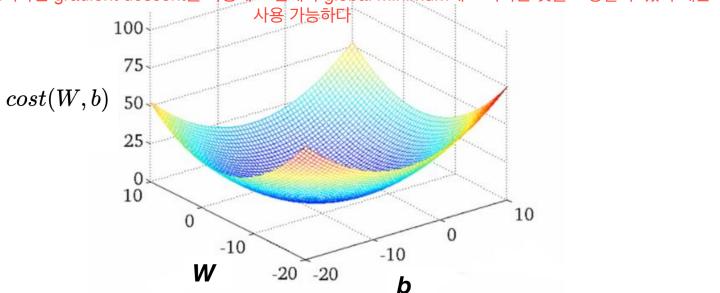
#### **Convex function**

global minimum와 local minimum이 일치한는 함수

$$cost(W,b) = rac{1}{m} \sum_{i=1}^m \left( H(x_i) - y_i 
ight)^2$$

cost function이 항상 convex function인 것은 아니지만,

convex function이라면 gradient descent를 사용해도 언제나 global minimum에 도착하는 것을 보장할 수 있기 때문에



#### What's Next?

• Multi-Variable Linear regression