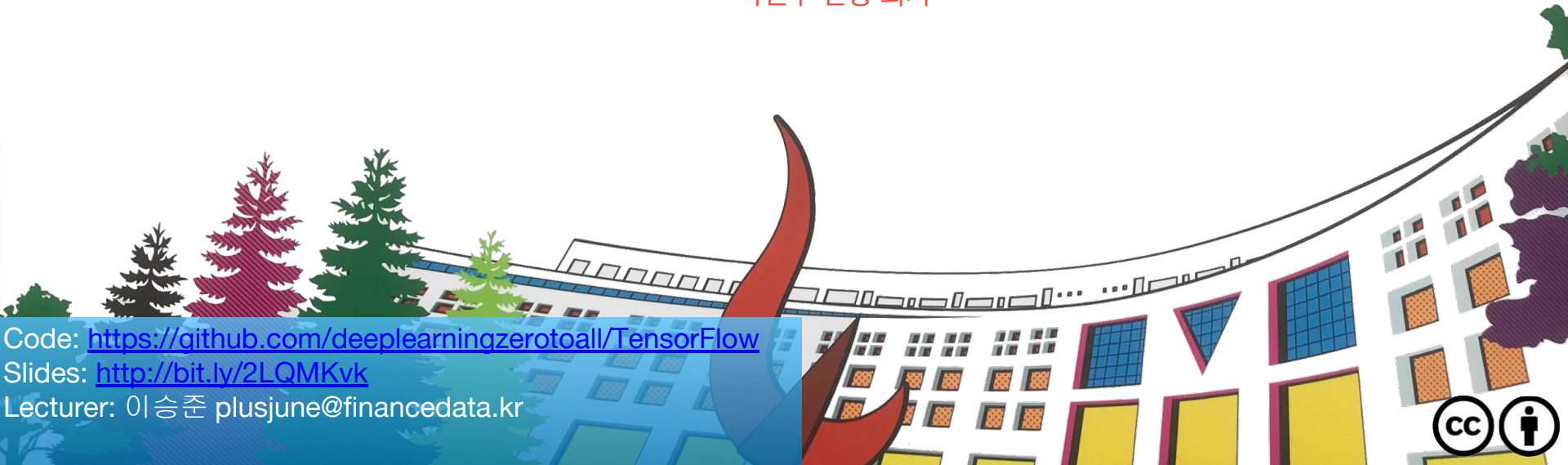
ML/DL for Everyone Season2

with TensorFlow

04 - Multi-variable linear regression

다변수 선형 회기



Code: https://github.com/deeplearningzerotoall/TensorFlow
Slides: http://bit.ly/2LQMKvk
Lecturer: 이승준 plusjune@financedata.kr

Recap

- Hypothesis
- Cost function
- Gradient descent

Recap

- Hypothesis
  예측

$$H(x) = Wx + b$$

- Cost function
  예측과 실제의 차이

cost가 되는 W를 찾아가는 과정

$$cost(W) = \frac{1}{m}\sum_{i=1}^{m}(Wx_i - y_i)^2$$

-값이 나오지 않도록 제곱

- Gradient descent

$$W := W - \alpha\frac{1}{m}\sum_{i=1}^{m}(W(x_i) - y_i)x_i$$

Predicting exam score:
regression using one input (x)

One-variable
(One-feature)

x (hours)	y (score)
10	90
9	80
3	50
2	60
11	40

Predicting exam score:
regression using three inputs (x1, x2, x3)

Multi-variable (Multi-feature)

x_1 (quiz 1)	x_2 (quiz 2)	x_3 (midterm 1)	y (final)
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

Hypothesis

$$H(x) = Wx + b$$

Hypothesis

$$H(x) = Wx + b$$

$$H(x_1, x_2, x_3) = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

변수가 늘어난 개수만큼 가중치(x)의 개수도 늘어나

Cost function

$$H(x_1, x_2, x_3) = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

$$cost(W, b) = \frac{1}{m} \sum_{i=1}^{m} \left(H(x_1, x_2, x_3) - y_i\right)^2$$

Multi-variable

$$H(x_1, x_2, x_3) = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

$$H(x_1, x_2, x_3, \ldots, x_n) = w_1 x_1 + w_2 x_2 + w_3 x_3 + \ldots + w_n x_n + b$$

Matrix

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + \ldots + w_n x_n$$

Matrix multiplication

https://www.mathsisfun.com/algebra/matrix-multiplying.html

Hypothesis using matrix

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + \ldots + w_n x_n$$

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_1 w_1 + x_2 w_2 + x_3 w_3 \end{pmatrix}$$

$$H(X) = XW$$

일반적으로 메트릭스는 대문자

행과 열을 연산하기 때문에 X가 앞에 온다

Hypothesis using matrix

x_1	x_2	x_3	y
73	80	75	152
93	88	93	185
89	91	90	180
96	98	100	196
73	66	70	142

Test Scores for General Psychology

$$H(x_1, x_2, x_3) = w_1 x_1 + w_2 x_2 + w_3 x_3$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix

x_1	x_2	x_3	y
73	80	75	152
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$$H(X) = XW$$

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Many x instances

x_1	x_2	x_3	y
73	80	75	152
93	88	93	185
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Test Scores for General Psychology

$$\begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_1 w_1 + x_2 w_2 + x_3 w_3 \end{pmatrix}$$

http://college.cengage.com/mathematics/brase/understandable_statistics/7e/students/datasets/mlr/frames/frame.html

x_1	x_2	x_3	y
73	80	75	152
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Hypothesis using matrix

매트릭스를 사용하면 데이터의 개수가 많든 적든 상관 없다

$$w_1 x_1 + w_2 x_2 + w_3 x_3 + \ldots + w_n x_n$$

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

[5, 3] [3, 1] [5, 1]

동일해야 함

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix

$$\begin{pmatrix} \mathbf{X} \end{pmatrix} \times \begin{pmatrix} \mathbf{W} \end{pmatrix} = \begin{pmatrix} \mathbf{H(X)} \end{pmatrix}$$

[5, 3] [?, ?] [5, 1]

$$H(X) = XW$$

Hypothesis using matrix

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ w_3 \end{pmatrix} = \begin{pmatrix} x_{11}w_1 + x_{12}w_2 + x_{13}w_3 \\ x_{21}w_1 + x_{22}w_2 + x_{23}w_3 \\ x_{31}w_1 + x_{32}w_2 + x_{33}w_3 \\ x_{41}w_1 + x_{42}w_2 + x_{43}w_3 \\ x_{51}w_1 + x_{52}w_2 + x_{53}w_3 \end{pmatrix}$$

[n, 3] [3, 1] [n, 1]

동일

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix (n output)

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot ? = \begin{pmatrix} x_{11}w_{11} + x_{12}w_{21} + x_{13}w_{31} & x_{11}w_{12} + x_{12}w_{22} + x_{13}w_{32} \\ x_{21}w_{11} + x_{22}w_{21} + x_{23}w_{31} & x_{21}w_{12} + x_{22}w_{22} + x_{23}w_{32} \\ x_{31}w_{11} + x_{32}w_{21} + x_{33}w_{31} & x_{31}w_{12} + x_{32}w_{22} + x_{33}w_{32} \\ x_{41}w_{11} + x_{42}w_{21} + x_{43}w_{31} & x_{41}w_{12} + x_{42}w_{22} + x_{43}w_{32} \\ x_{51}w_{11} + x_{52}w_{21} + x_{53}w_{31} & x_{51}w_{12} + x_{52}w_{22} + x_{53}w_{32} \end{pmatrix}$$

Weight의 크기는 입력 데이터의 column 개수와 출력 데이터와 column 개수에 따라 결정

[n, 3] [?, ?] [n, 2]

3, 2
변수(feature)의 개수, 출력의 개수

$$H(X) = XW$$

https://www.symbolab.com/solver/matrix-calculator

Hypothesis using matrix (n output)

$$\begin{pmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \\ x_{41} & x_{42} & x_{43} \\ x_{51} & x_{52} & x_{53} \end{pmatrix} \cdot \begin{pmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \\ w_{31} & w_{32} \end{pmatrix} = \begin{pmatrix} x_{11}w_{11} + x_{12}w_{21} + x_{13}w_{31} & x_{11}w_{12} + x_{12}w_{22} + x_{13}w_{32} \\ x_{21}w_{11} + x_{22}w_{21} + x_{23}w_{31} & x_{21}w_{12} + x_{22}w_{22} + x_{23}w_{32} \\ x_{31}w_{11} + x_{32}w_{21} + x_{33}w_{31} & x_{31}w_{12} + x_{32}w_{22} + x_{33}w_{32} \\ x_{41}w_{11} + x_{42}w_{21} + x_{43}w_{31} & x_{41}w_{12} + x_{42}w_{22} + x_{43}w_{32} \\ x_{51}w_{11} + x_{52}w_{21} + x_{53}w_{31} & x_{51}w_{12} + x_{52}w_{22} + x_{53}w_{32} \end{pmatrix}$$

[n, 3] [3, 2] [n, 2]

$$H(X) = XW$$

입력 개수와 출력 개수에 상관없이 우리의 가설(H(x))은 항상 XW로 변함이 없다

https://www.symbolab.com/solver/matrix-calculator

WX vs **XW**

- Lecture (theory)

$$H(x) = Wx + b$$

왼쪽의 Wx + b와 동일한 의미

$$h_\theta(x) = \theta_1 x + \theta_0$$

$$f(x) = ax + b$$

- Implementation (TensorFlow)

$$H(X) = XW$$

텐서플로우 코드에서는 matrix의 곱으로 XW로 씀

What’s Next?

- Logistic (Regression) Classification