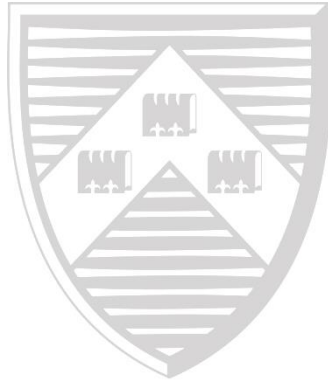


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**Estimating the Impact of House Prices on
Household Labour Supply in the UK**

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Estimating the Impact of House Prices on Household Labour Supply in the UK

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Abstract

This paper applies the British Household Panel Survey (BHPS) from 1997-2008 and study the impact of local authority district house prices on labour supply of couples via bivariate seemingly unrelated regression (SUR) probit and tobit models after imputing potential wages for both workers and non-workers using the Heckman selectivity approach. The use of local authority district house prices has the virtue of being disaggregate and exogenous to the individual. This avoids the potential simultaneity bias from using self-reported house prices (this and labour supply may both be affected by unobserved individual heterogeneity). We allow for the interdependent nature of the couple's labour supply decisions and enhance efficiency of estimation by exploiting the structure in the error terms of the two-equation system. We find heterogeneous responses of labour supply from different household types to house prices as well as the joint decision making of spouses/partners within a household on labour supply. Gender and age differences are present. Our results give interpretations on the role of the different channels through which this impact is driven, including wealth effects, borrowing constraints, precautionary savings, bequest motives and habit formation.

1 Introduction

Housing is both a consumption and an investment good, which constitutes a large proportion of wealth for most households in the UK. Figure 1 shows that in 2010-2012, net property wealth is the second largest proportion of aggregate total wealth, accounting for 37% of total wealth in Great Britain. There is an extensive literature on the impact of house prices on consumption based on both macro and micro data. A gap in the literature is the impact of housing wealth changes due to variations of house prices on household labour supply, which is of interest to both policy makers trying to affect the labour market and academics trying to understand household decision making.

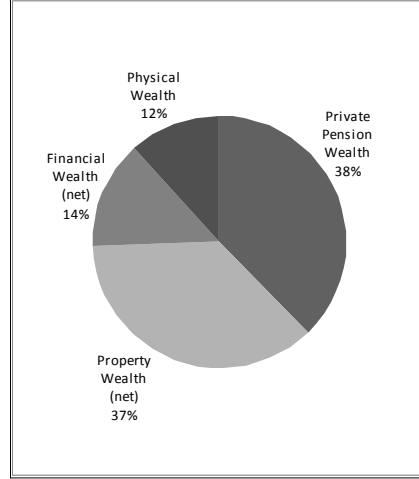


Figure 1 *Break down of aggregate total wealth in Great Britain 2010-2012*
Source: Wealth and Assets Survey - Office for National Statistics

As a major component of wealth for a typical household in the UK, housing wealth is an important source of wealth effects on household consumption of non-housing goods and leisure. In a life-cycle model, individuals will reallocate their resources over time once they get new information related to their life-time resource constraint such as a change in their housing wealth. Considering the dual nature of housing as both a component of financial asset and a consumption good, households who plan to purchase new houses or trade up their current houses can be thought of as "short" in housing, i.e., for them the fundamental value of house they own is smaller than the present discounted value of their planned future consumption of housing services (Buiter, 2008). On the other hand, households who plan to downsize their houses are "long" in housing. As Campbell and Cocco (2007) point out, in the absence of instruments that can insure these short and long positions, unexpected shocks to house prices has a redistributive wealth effect. To be specific, given the sequences of future income and interest rates, we should expect to see those "short" in housing cut their consumption or increase labour supply and those "long" in housing increase their consumption or decrease labour supply when house prices rise. These effects are expected to be significant due to the magnitude of the housing wealth as well as the volatility of house prices. Existing literature in support of this hypothesis includes Case et al. (2005), Campbell and Cocco (2007), Carroll et al. (2011) and Disney and Gathergood (2014). However, an increase of house prices does not necessarily mean an increase in homeowners' real wealth. Homeowners with long expected tenure of their houses are hedged against the fluctuations in house prices, i.e., increasing house prices compensate for the increase of the (implicit) price of their future housing needs. In this case, increasing house prices have no real wealth effect in household consumption (Sinai

and Souleles, 2005). This means we can't simply attribute the correlations between house prices and household non-housing consumption and labour supply to a pure housing wealth effect without further analysis (Campbell and Cocco, 2007; Browning et al., 2013). One alternative mechanism is the role of housing asset as collateral available to homeowners. An increase of house prices would improve the possibility for the borrowing constrained homeowners to borrow against the housing equity and allow them increase consumption and decrease labour supply. Ortalo-Magne and Rady (2006), Lustig and Van Nieuwerburgh (2006) and Campbell and Cocco (2007) find evidence for the collateral effect of housing.

Second, there could be common factors that drive house prices and consumption/ labour supply simultaneously. Unobserved macroeconomic factors such as income expectations may affect house prices and consumption simultaneously (King, 1990). Alternatively, financial liberalisations have general effects of both driving up house prices and stimulating consumption by relaxing borrowing constraints for everyone (Attanasio and Weber, 1994; Aron et al., 2007). If these common factors are indeed the main drivers of the simultaneous process of changes in house price and consumption/ labour supply, then there may be no causal relationship between house prices and consumption/ labour supply.

In our study, the effect of house prices on labour supply is estimated for three age groups, i.e., the young households (aged 18-39), middle aged households (aged 40-54) and old households (aged 55-75), separately. Such grouping is based on the distinct features of these three age groups that would probably be the main drivers of the correlation between house prices and labour supply. Young households are more likely to face borrowing constraints, be renters and plan to upsize their houses. For them an increase in house prices raises the need for income to finance a new/upgraded purchase so labour supply/participation should increase. Old households are likely to plan downsizing so they are better off from increased house prices. So just the wealth channel implies an increase in house prices sees young renting households increase their labour supply while old home owning households decrease labour supply or choose to retire, *cet par*. Meanwhile, we expect that some homeowners do not plan to move and house price increases are neutral (they are hedged) and have no real wealth effects. For all ages precautionary saving motives and bequest targets, can also affect labour supply adjustments in response to a house price change. Finally habit formation (preference for the current household labour supply pattern) can lead to inertia. Each channel may impact differentially on the male and female partners in the joint household labour supply determination.

We use the British Household Panel Survey (BHPS) from 1997-2008 and study the impact of local authority district house prices on labour supply of couples via bivariate seemingly unrelated regression (SUR) probit and tobit models after imputing potential wages for both workers and non-workers using the Heckman selectivity approach.

In contrast to a number of empirical studies that model the labour supply of married female treating the husband's labour supply as predetermined, we allow for the interdependent nature of the couple's labour supply decisions and

enhance efficiency of estimation by exploiting the structure in the error terms of the two-equation system. The rationale for this is that the shocks to labour supply of male and female in the same family should be correlated via some common unobserved factors¹. If decisions of partners are interrelated this is essential to gain unbiasedness and efficiency. In the process of estimation, we treat the panel data as cross sections of household-year observations, but allow for heteroskedasticity and general correlation over time for the same household in computing standard errors. Our model is static in the sense that we do not model the unobserved individual time-invariant heterogeneity in the error terms and do not take the selection of tenure choice into consideration. In other words, we just investigate the households' behaviours given their home ownership at a point in time².

The local authority district house prices cover 332 Districts and are assumed exogenous to the individual³. This avoids the potential simultaneity bias from using self-reported house prices (this and labour supply may both be affected by unobserved individual heterogeneity).

Another important advantage of our study is the use of individual self-reported financial expectation as an independent variable to control for income expectations as Disney and Gathergood (2014) do⁴, which avoids the reliance of a parametric specification of income process to control for income expectations in previous literature (Campbell and Cocco, 2007; Browning et al., 2013).

We first concentrate on finding the heterogeneous responses of labour supply from different household types to house prices as well as the joint decision making of spouses/partners within a household on labour supply. Our results give interpretations on the role of the different channels. Controlling for tenure, number of children, age, individual subjective financial expectations and other demographics, we find heterogeneous effects of house prices on labour supply at both extensive and intensive margins which differ by gender and age of partners. Overall, the labour supply of males is more responsive to house prices change than females.

The remainder of this paper is organised as follows: Section 2 describes the data we use. Section 3 discusses the econometric specification and estimation

¹Amemiya (1974) states that the multivariate regression system can be viewed as a reduced form of the simultaneous equation model. In the same paper, Amemiya derives the estimator for a simultaneous-equation tobit model and points out that this model can be applied in modelling the joint determination of work hours of husband and wife. However, the identification is subject to restrictions of parameters that are not necessarily satisfied in the context of household labour supply.

²The perfect intertemporal life cycle model considers labour supply, consumption, saving in each period as well as changes of house finance and tenure over time. However, given the fact that change of tenure is infrequent and our use of pooled data, we don't model the tenure choice with labour supply in this paper.

³In the whole sample, only 12% of households ever moved between local authority districts they live in. This means the majority of households stay in the same local authority districts through the period covered by our sample. And for this reason we can treat the localities they live as predetermined.

⁴As argued by Browning et al. (2013), a possible drawback of using the self-reported financial expectation is that it may include expectation of house prices.

strategy. Section 4 discusses the estimation results. Finally, Section 5 concludes.

2 The Data

We use micro data from the twelve waves in British Household Panel Survey (BHPS) from 1997-2008. This survey is based on a representative sample of more than 5000 households, where individuals aged above 16 years old are interviewed. All the individuals included in the survey are interviewed successively across years, and their new household members will also be included.

We restrict the sample to observations satisfying the following criteria:

- (1) Nobody is self-employed.
- (2) All the individuals are spouses/live-in partners who stayed together through the period observed.
- (3) All the couples include people with different genders⁵.
- (4) All the individuals are aged 18-75.

After such selection, we have a sample of 25294 household-year observations, with each observation containing the information of male and female partners and corresponding household characteristics.

In addition to the BHPS, we also include some macro variables from other sources:

- (1) Regional average earnings from a report by DCLG which was based on Annual Survey of Hours and Earnings (ASHE) data for average earnings.
- (2) Regional claimant count rates calculated using claimant count and claimant denominators from ONS.
- (3) Local authority district (LAD) house prices (mean) from Land Registry⁶.
- (4) Retail Prices Index from ONS.

Tables 1 and 2 show the summary statistics of the whole sample for individual level and household level variables, respectively. The age range of 18-75 means there are some retired couples in the sample who are benefit receivers and they may voluntarily choose not to work if they are satisfied with the benefit.

⁵The number of live-in partners with the same gender is very small compared to the sample size. The purpose of excluding them is to investigate the possible gender difference in family labour supply.

⁶This data is very disaggregate. In our sample there are 332 individual local authorities in England and Wales. The data excludes sales at less than market price (e.g. Right To Buy), sales below £1000 and sales above £20m.

Table 1 *Summary statistics of individual level variables*

individual level variables	male		female	
	mean	standard deviation	mean	standard deviation
employed	0.75	0.43	0.64	0.48
weekly hours of work	29.3	18.94	18.57	16.49
hourly gross pay (nominal)	13.38	15.97	9.62	12.28
age	47.37	14.53	45.2	14.24
negative financial expectation	0.11	0.31	0.1	0.3
positive financial expectation	0.29	0.45	0.25	0.43
married	0.82	0.38	0.82	0.38
divorced	0.04	0.19	0.04	0.19
never married	0.13	0.34	0.13	0.34
degree	0.2	0.4	0.17	0.38
hnd	0.2	0.4	0.15	0.36
a-level	0.22	0.41	0.3	0.46
gcse	0.3	0.46	0.33	0.47
excellent health status	0.25	0.43	0.22	0.41
good health status	0.47	0.5	0.48	0.5
poor health status	0.06	0.23	0.07	0.25
very poor health status	0.01	0.1	0.01	0.12

Notes: The question asked in the BHPS questionnaire about financial expectation is: "Looking ahead, how do you think you will be financially a year from now?" And interviewees can choose from the four answers :"(1) Better off; (2) Worse off than now; (3) About the same; (4) Don't know."

Table 2 *Summary statistics of household level variables*

household level variables	mean	standard deviation
owner	0.82	0.38
annual household non-labour income (nominal)	£6,865.45	8901.7
number of children	0.74	1.03

As we can see from Figure 2, there are discrepancies in terms of participation patterns among the three age groups. In particular, most of households aged under 54 have both partners working while most households aged above 54 have neither partner working. Among the three age groups, the middle aged couples (aged 40-54) have the highest percentage of both working and lowest percentage of neither working. For all age groups, it is very rare to find that only the female works in a household.

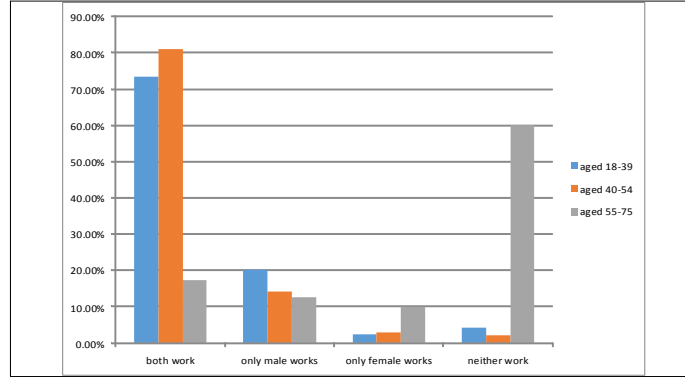


Figure 2 *Participation patterns in the sample*

Figure 3 shows the scatterplot of weekly working hours for male and female in the whole sample, the vertical axis shows hours of work for male, while the horizontal axis shows hours of work for female. The red line is the 45 degree reference line. As we can see from the scatterplot, the majority of the observations are to the right of the red line. This suggests that in most households, male partners have more hours of work than female. The most intensive area of observations (the darkest area) are the "band" with males working around 40 hours a week and females working less than 40 hours a week.

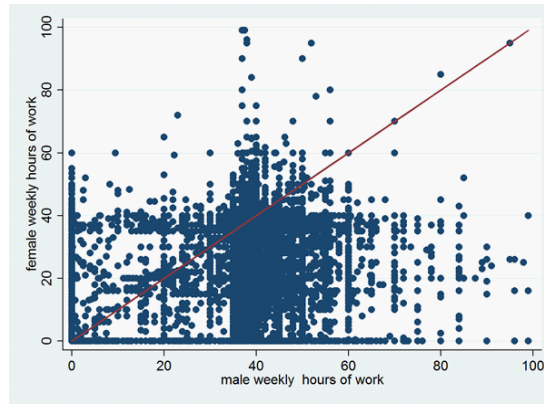


Figure 3 *Scatterplot of weekly working hours for male and female partners*
(horizontal axis-male weekly hours of work; vertical axis-female weekly hours of work)

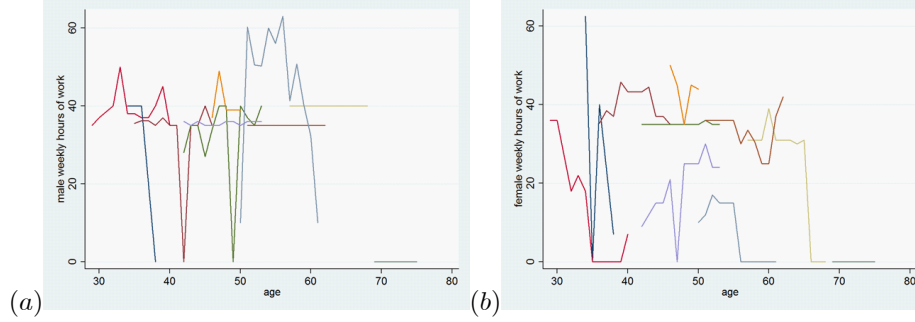


Figure 4 (a) *Evolutions of weekly hours of work for male partners*
(b) *Evolutions of weekly hours of work for female partners*
(horizontal axis-age; vertical axis-weekly hours of work)

As an example, Figure 4 plots the evolution of weekly hours of work for male and female partners from 10 randomly selected households in our sample. Each line corresponds to a household. Though these 10 households may not be representative for the population, we can observe some gender difference in hours of work: over the life cycle, males tend to choose to work full time (around 40 hours per week) or not to work at all, while females have more variations in their hours of work over their life. This may be due to the fertility and different commitments of family care. It is important to note that Figure 4 does not control for changes of characteristics such as number of children in the family, family non-labour income, etc..

We are also interested in the destinations when one exits the labour market in each age group, i.e., the job status of non-workers in each age group (Table 3). As we can see from Table 3, there are some gender differences in destinations in each age group. For those under 40 and aged 40-54, most male non-workers are unemployed while most female non-workers engage in family care. And for those aged above 54, most of the non-workers, either male or female, are retired. Therefore, in the older group, the decisions of not to work are more likely to be permanent than in other age groups, though we don't exclude the possibility that one will want to re-enter the labour market at some point after retirement.

Table 3 *Tabulation of job status of non-workers*

age	gender	unemployed	retired	family care	government training	other
aged 18-39	male	85.30%	0.60%	7.60%	1.10%	5.50%
	female	9.70%	0.10%	88.20%	0.10%	1.90%
aged 40-54	male	54.50%	29.50%	11.70%	0.40%	3.90%
	female	12.50%	5.10%	79.50%	0.40%	2.50%
aged 55-75	male	1.60%	97.50%	0.20%	0.02%	0.70%
	female	0.80%	84.10%	14.60%	0	0

As to house prices, here we pick up 4 cities and 4 rural areas from 332 local authority districts and show the real house prices overtime for each district. As is shown in Figure 5, the house prices in different localities all have a growing trend over time. It should be noticed that in London, it has higher mean.

3

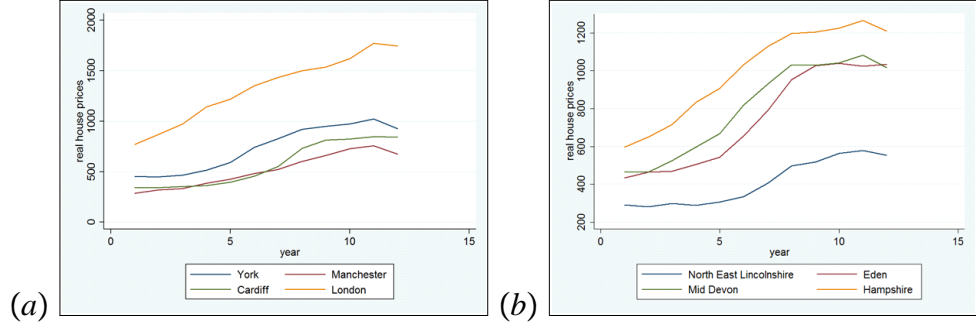


Figure 5 *Local Authority District real house prices across years*
(a) *Four cities: London, York, Cardiff and Manchester*
(b) *Four rural areas: Hampshire, Mid Devon, Eden and North East Lincolnshire*
(horizontal axis-year; vertial axis-real house prices)

3.1 The House Price Process

If we assume that households are forward-looking and have rational expectaions, then investigating the house price process is important in understanding how households predict their housing wealth in the future. We assume that the house prices follow a first order autoregressive (AR1) model with a deterministic part a , an individual effect e_l and an idiosyncratic error θ_{lt} :

$$\ln(P_{lt}) = a + \rho \ln(P_{l,t-1}) + e_l + \theta_{lt} \quad (1)$$

,where $\ln(P_{lt})$ is the log of real house price; the subscripts l denotes local authority districts and t denotes time.

The unit root hypothesis $\rho = 1$ in equation (1) is rejected using Levin-Lin-Chu test⁷. Therefore we conclude that real house prices are stationary and the shocks to house prices do not have a lasting effect. Households who have rational expectations of current house prices based on their observations of house prices in the past in the local authorities they live in should not be affected by variation

⁷ $l = 1, \dots, 332. t = 1, \dots, 12$. The period covered by our sample (12 years) may not be long enough to capture long term fluctuations of house prices which may be non-stationary.

of house prices. This finding will serve as evidence to explain the insignificant response of house prices in Section 4 where we present the estimation results.

4 Econometric Specification and Estimation Strategy

4.1 Econometric Specification

In a family with two workers or potential workers, the family utility maximisation problem ignoring intertemporal effects yields optimal hours of work of the two persons conditional on real market wages of the two persons and household non-labour income.

We assume a single mechanism to decide the behaviour of labour supply both on the extensive margin (participation) and intensive margin (hours of work), ignoring monetary or time fixed costs of work (Cogan,1981).

If both persons in the household work, then the interior solution of labour supply (constant time endowment minus leisure demand) of each person i can be written as a reduced form function of two real wages and household non-labour income. We specify the labour supply of person i as follows:

$$h_i^* = \alpha_i + \beta_{1i} \ln w_i + \beta_{2i} X_i + \beta_{3i} \ln w_j + \varepsilon_i, i, j = m, f$$

where h_i^* is the desired hours of work of person i , w_i and w_j are real wages of person i and his/her partner person j , respectively. X_i are a set of exogenous observable variables determining labour supply behaviours, ε_i is individual heterogeneity in tastes for work⁸, $\alpha_i, \beta_{1i}, \beta_{2i}$ and β_{3i} are individual preference parameters and the subscripts m and f denote male partner and female partner, respectively. This specification assumes linearity of hours of work in $\ln w_i$ and $\ln w_j$ and other exogenous regressors.

If we allow for the possibility that the family utility is higher with person i not working and consider the fact that negative hours of work is infeasible, then the optimal labour supply of person i becomes

$$h_i = \max\{h_i^*, 0\}, i = m, f$$

where h_i indicates observed hours of work of person i , h_i^* indicates desired hours of work of person i . $h_i = 0$ is the corner solution where desired hours of work are non positive and actual hours are zero. Basing the analysis on the subsample of workers will lead to sample selection bias with respect to the population distribution of desired hours of work given by the labour supply function. In particular, existing empirical work shows the elasticity of wage and income on hours of work would be misleading if we only consider the group of workers. Define the observable counterparts of the two latent variables h_m^* and

⁸We treat the data as pooled cross section data but correct for the autocorrelation of the same household over time and the heteroskedasticity across households in estimating the standard errors of coefficients.

h_f^* as h_m and h_f , respectively. Given the market wage w_i , the two-equation Seemingly Unrelated Regression (SUR) censored model for observed hours of work is:

$$\begin{aligned} h_{hm} &= \begin{cases} \alpha_m + \beta_{1m} \ln w_{hm} + \beta_{2m} X_{hm} + \beta_{3m} \ln w_{hf} + \varepsilon_{hm} & \text{if } RHS > 0 \\ 0 & \text{otherwise} \end{cases} \\ h_{hf} &= \begin{cases} \alpha_f + \beta_{1f} \ln w_{hf} + \beta_{2f} X_{hf} + \beta_{3f} \ln w_{hm} + \varepsilon_{hf} & \text{if } RHS > 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad (2)$$

where the subscript h indicates household, h_{hm} and h_{hf} are observed hours of work for male and female partners, respectively; X_{hi} ($i = m, f$) includes interaction of owner dummy and log of real house prices, renter dummy and log of real house prices, age, age squared, dummy of worse financial expectation, dummy of better financial expectation, dummies of marital status including being married, divorced, and never married, dummies of highest education qualifications including degree, hnd, alevel and gcse, number of children in the household, health status dummies including 4 categories, log of real annual household non-labour income.

Similarly, the two-equation Seemingly Unrelated Regression (SUR) censored model for participation is:

$$\begin{aligned} I_{hm} &= \begin{cases} 1 & \text{if } \alpha_m + \beta_{1m} \ln w_{hm} + \beta_{2m} X_{hm} + \beta_{3m} \ln w_{hf} + \varepsilon_{hm} > 0 \\ 0 & \text{otherwise} \end{cases} \\ I_{hf} &= \begin{cases} 1 & \text{if } \alpha_f + \beta_{1f} \ln w_{hf} + \beta_{2f} X_{hf} + \beta_{3f} \ln w_{hm} + \varepsilon_{hf} > 0 \\ 0 & \text{otherwise} \end{cases} \end{aligned} \quad (3)$$

The error terms ε_{hm} and ε_{fm} are assumed to be bivariate normally distributed with zero means and covariance matrix:

$$\Sigma = \begin{bmatrix} \sigma_{\varepsilon_{hm}}^2 & \sigma_{\varepsilon_{hm}\varepsilon_{hf}} \\ \sigma_{\varepsilon_{hm}\varepsilon_{hf}} & \sigma_{\varepsilon_{hf}}^2 \end{bmatrix}$$

At the same time, we also assume the wage equations for person i ($i=f,m$) in the household as follows:

$$\ln w_{hi} = \gamma_i Z_{hi} + \eta_{hi} \quad (4)$$

where Z_{hi} are a set of variables to determine real wages including dummies of highest education qualifications including degree, hnd, alevel and gcse, age, regional claimant count rate, log of real regional average earnings and year dummies; η_i is the error term which is normally distributed. Assume $E(\eta_{hi}) = 0$, the error terms in the wage equations for each partner are independent, i.e., $E(\eta_{hi}\eta_{hj}) = 0$, and allow for the correlation between ε_{hi} and η_{hi} . In other words, the covariance matrix of the error terms of equation systems (2) and (4) is:

$$\Omega = \begin{bmatrix} \sigma_{\varepsilon_{hm}}^2 & \sigma_{\varepsilon_{hm}\varepsilon_{hf}} & \sigma_{\varepsilon_{hm}\eta_{hf}} & 0 \\ \sigma_{\varepsilon_{hm}\varepsilon_{hf}} & \sigma_{\varepsilon_{hf}}^2 & 0 & \sigma_{\varepsilon_{hf}\eta_{hf}} \\ \sigma_{\varepsilon_{hm}\eta_{hm}} & 0 & \sigma_{\eta_{hm}}^2 & 0 \\ 0 & \sigma_{\varepsilon_{hf}\eta_{hf}} & 0 & \sigma_{\eta_{hf}}^2 \end{bmatrix}$$

4.2 Estimation Strategy

4.2.1 Imputation of wages

Before estimating the participation and hours of work decisions for households, an important issue to deal with is the missing wages for non-workers. The information on wage is required if we take the selection problem into account. However, there is still no consensus on particular solutions to wage-imputation problem (Heckman, 1993). Here we adopt the Heckman-style selectivity adjusted method. The idea underlying this method is the reservation wage condition, i.e., we assume that people decide not to work because the potential in-work wages are lower than for comparable workers. In other words, when people make the participation decisions, they simply consider the "average" wages of workers with the same observable characteristics as themselves. Compared to the entry wage measures, the Heckman selectivity approach captures the "long term" equilibrium wage for certain types of workers as long as they are long-lived enough to realise these "long run" wages (Myck and Reed, 2005).

In particular, we impute wages for everyone by the following procedures:

In the first step, we substitute the wage equations to the desired hours of work equation to have a reduced form for hours of work for each partner

$$h_{hi} = \begin{cases} \pi_i q_{hi} + v_{hi} & \text{if } RHS > 0 \\ 0 & \text{otherwise} \end{cases} \quad (5)$$

where $\pi_i = (\alpha, \beta_{1i}\gamma, \beta_{2i}, \beta_{3i}\gamma_j), q'_{hi} = (1, Z_{hi}, X_{hi}, Z_{hj}), v_{hi} = \varepsilon_{hi} + \beta_{1i}\eta_{hi} + \beta_{3i}\eta_{hj}$

Then we run a univariate probit model on equation (6) for each partner and find λ_{hi} to correct for the selection bias.

$$I_{hi} = \begin{cases} 1 & \text{if } \pi_i q_{hi} + v_{hi} > 0 \\ 0 & \text{otherwise} \end{cases} \quad (6)$$

In the second step, we use λ_{hi} as an extra regressor to predict $\ln(\text{wage})$ for everyone:

$$\ln w_i = \gamma_i Z_{hi} + \delta_i \lambda_{hi} + \eta_{hi}$$

where the extra term $\lambda_{hi} = \frac{\phi_{v_{hi}}(\widehat{\pi_i q_{hi}})}{\Phi_{v_{hi}}(\widehat{\pi_i q_{hi}})}$ is the predicted inverse Mills ratio from equation (6). As to the implementation of estimation, the idea is to assume η_{hi} and v_{hi} are bivariate normal, and it follows $E(\eta_{hi}|v_{hi}) = \delta_{hi}v_{hi}$, which allow the use of the individual inverse Mills ratio⁹.

⁹ A justification of using the individual inverse Mills ratio can be found in the Appendix.

4.2.2 Likelihood functions of the bivariate probit and tobit models

There are four possible participation regimes for a household: Both work (1,1), Only male works (1,0), Only female works (0,1), Neither work (0,0). That is, our data is: $[(0, 0), (h_{hm}, 0), (0, h_{hf}), (h_{hm}, h_{hf})]$.

We use maximum likelihood estimators in estimating household participation and hours of work.

The likelihood function of the bivariate tobit model (equation (7)) is a combination of the likelihood at the extensive and intensive margin. In other words, for each participation regime, the likelihood contribution can be decomposed into two parts: the probability of being in such regime and the probability of observing the particular hours conditional on being in this regime. Of course, when the individual doesn't work, his/her actual value of hours (which is zero) has no effect on the likelihood function.

$$\begin{aligned}
L_{bi-tobit} &= \prod_{1,1} \Pr(h_m, h_f | h_m > 0, h_f > 0) \Pr(h_m > 0, h_f > 0) \\
&\quad \prod_{1,0} \Pr(h_m | h_m > 0, h_f < 0) \Pr(h_m > 0, h_f < 0) \\
&\quad \prod_{0,1} \Pr(h_f | h_m < 0, h_f > 0) \Pr(h_m < 0, h_f > 0) \\
&\quad \prod_{0,0} \Pr(h_m < 0, h_f < 0)
\end{aligned} \tag{7}$$

Note that

$$\begin{aligned}
&\int_{h_m > 0, h_f > 0} \Pr(h_m, h_f | h_m > 0, h_f > 0) dh_m, dh_f \\
&= \int_{h_m > 0, h_f < 0} \Pr(h_m > 0, h_f < 0) dh_m, dh_f \\
&= \int_{h_m < 0, h_f > 0} \Pr(h_m < 0, h_f > 0) dh_m, dh_f \\
&= 1
\end{aligned} \tag{8}$$

Given (8), the bivariate probit likelihood function can be viewed as nested with the bivariate tobit likelihood function, in the sense that equation (9) can be thought of as the integration of equation (7) over hours of work in each participation regime:

$$\begin{aligned}
L_{bi-probit} &= \prod_{1,1} \left[\int_{h_m > 0, h_f > 0} \Pr(h_m, h_f | h_m > 0, h_f > 0) \Pr(h_m > 0, h_f > 0) dh_m, dh_f \right] \\
&\prod_{1,0} \left[\int_{h_m > 0, h_f < 0} \Pr(h_m | h_m > 0, h_f < 0) \Pr(h_m > 0, h_f < 0) dh_m, dh_f \right] \\
&\prod_{0,1} \left[\int_{h_m < 0, h_f > 0} \Pr(h_f | h_m < 0, h_f > 0) \Pr(h_m < 0, h_f > 0) dh_m, dh_f \right] \\
&\prod_{0,0} \left[\int_{h_m < 0, h_f < 0} \Pr(h_m < 0, h_f < 0) dh_m, dh_f \right] \\
&= \prod_{1,1} \Pr(h_m > 0, h_f > 0) \prod_{1,0} \Pr(h_m > 0, h_f < 0) \\
&\prod_{0,1} \Pr(h_m < 0, h_f > 0) \prod_{0,0} \Pr(h_m < 0, h_f < 0) \tag{9}
\end{aligned}$$

Equation (9) is the likelihood function of the bivariate probit.

We note that the likelihood function (7) corresponds to model (2), while the likelihood function (9) corresponds to model (3).

In principle, it gives us two ways of estimating the models. One is to estimate just the participation decisions of households, while the other is to estimate the household participation and the level of working hours jointly. Given the assumption of no involuntary unemployment, if both desired hours of work and selection mechanism are correctly specified, i.e., the models (2) and (3) are both correct, then estimating both models gives consistent results while the estimation result of bivariate tobit is more efficient than that of bivariate probit. However, if the mechanisms to decide the behaviour of labour supply on the extensive margin (participation) and intensive margin (hours of work) are different, then the specification of (2) is incorrect and the estimation of bivariate tobit would be inconsistent. On the other hand, so long as the participation model (3) is correctly specified, the estimation of the bivariate probit will be consistent¹⁰.

5 Estimation Results

In the estimations of both household participating and household hours of work, the three age groups are treated separately and the results are compared among them. In the estimations, we pooled the panel as if it is cross sectional, but allow

¹⁰It could well be the case that the desired hours of work is correctly specified while the selection mechanism is incorrectly specified, or that neither the desired hours of work nor the selection mechanism is correctly specified. In such cases, both models (2) and (3) are not reliable.

heteroskedasticity and general correlation over time for the same household¹¹, while independence over households is still assumed.

5.1 Estimation results for household participation

The Wald tests performed after estimations for the three age groups all reject the null hypotheses that $cov(\varepsilon_{hm}, \varepsilon_{hf}) = 0$, which shows evidence that cross equation correlations matter, i.e., the decisions of participation of both partners are interdependent via some unobserved household heterogeneity and/or the shocks to each partner's participation is likely to be mutually correlated. Table 4 shows the percentage correctly predicted after the bivariate probit estimations for household labour participation¹². We can see that the percentage correctly predicted for the subsamples in which both work are plausible for the younger and middle aged groups, while for the older group the participation of the subsamples in which neither work is well predicted.

Table 4 *Percentage correctly predicted*

age	overall	(1,1)	(1,0)	(0,1)	(0,0)
aged 18-39	57.40%	70%	24.20%	0	35.40%
aged 40-54	63.70%	77%	7.20%	1.20%	28.80%
aged 55-75	69.20%	47.90%	17.40%	5.50%	97%

Notes: (1,1),(1,0),(0,1) and (0,0) denote four participation patterns of both work, only male works, only female works and neither work, respectively.

As Table 5 shows, the predicted mean probabilities of the four participation patterns for each age group exhibit great differences among age groups. On average, the predicted probability of both working is high for younger and middle aged households but very low for the older group. On the other hand, the pattern of neither working dominates for the older households in general. The estimated histogram of participation patterns for older households are shown in Figure 5. These findings are consistent with the life cycle profile of the participation decision.

Table 5 *Predicted mean of probabilities of the four participation patterns*

age	Pr(0,0)	Pr(1,0)	Pr(0,1)	Pr(1,1)
aged 18-39	0.04	0.22	0.04	0.7
aged 40-54	0.02	0.15	0.03	0.8
aged 55-75	0.63	0.12	0.11	0.14

¹¹We use cluster-robust standard errors in the estimation.

¹²We assign the participation pattern with the highest predicted probability to each household and compare them with the actual participation patterns in the sample.

Notes: $\Pr(0,0)$, $\Pr(1,0)$, $\Pr(0,1)$ and $\Pr(1,1)$ denote the probabilities of neither work, only male works, only female works and both work, respectively.

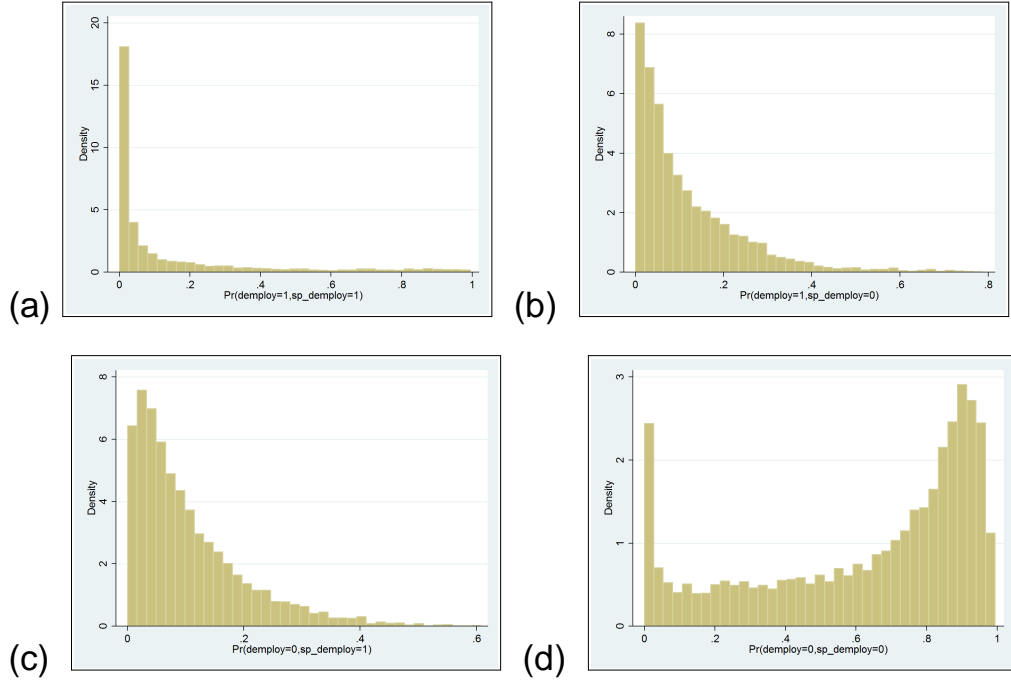


Figure 5 *Estimated histogram of participation patterns for households aged 55-75 (horizontal axis-probability; vertical axis-estimated density)*

- (a) *Histogram of estimated $\Pr(1,1)$*
- (b) *Histogram of estimated $\Pr(1,0)$*
- (c) *Histogram of estimated $\Pr(0,1)$*
- (d) *Histogram of estimated $\Pr(0,0)$*

Table 6 shows the marginal effects of house prices for an average renting/home owning household. In line with the existing empirical evidence, these marginal effects at the extensive margin of labour supply are biggest and most significant for those households at the margins of labour supply: the older households. For younger households, the significant negative signs of marginal effects on $\Pr(0,0)$ can be explained by the fact that most of them plan to purchase a new house or upgrade their houses thus are "short" in housing and for this reason would be worse off and decrease their consumption of leisure when house prices rise. And this wealth effect could be reinforced by the possibility that the younger households are borrowing constrained. According to Table 6, for

example, for an average young homeowners household¹³, a 1% increase in house prices is associated with a decrease of 0.002 in the probability of neither partner work. Similarly, for an average young renting household, a 1% increase in house prices is associated with a decrease of 0.01 in the probability of neither partner work. This greater magnitude of effect for renters could be due to the possibility that renters are more borrowing constrained and more "short" in housing than owners who want to upsize their houses¹⁴.

On the other hand, the marginal effects on $\Pr(0,0)$ for older home owning households are also negative. Since the older home owning households are more likely to downsize their houses ("long" housing), this negative marginal effect cannot be explained by a pure wealth effect. Moreover, an increase of house prices should make them better off and might allow some of them to achieve their bequest targets earlier and possibly bring forward their retirement timing. Thus this negative effect could only be attributed to alternative explanations such as common macro economic factors that drive house prices and labour market conditions simultaneously or a portfolio effect makes old people want to invest more in financial assets to balance the increased weight of housing in the portfolio.

For the middle aged households, the marginal effects are not significant except for that on $\Pr(0,1)$ for owners. This could be due to the fact that they are at prime working ages when their careers are relatively stable so that their participation decisions are irrelevant to house price changes. Alternatively, the insignificant response could be explained by our finding in Section 2.1 that house prices are stationary and the shocks of house prices would not have a lasting effect and for this reason those households with rational expectations that are not about to exit the housing market are not likely to respond to the temporary shocks of house prices (Browning et al., 2013).

Table 6 *Marginal effects of house prices (evaluated at means)*

age	$\Pr(0,0)$		$\Pr(1,0)$		$\Pr(0,1)$		$\Pr(1,1)$	
	owner	renter	owner	renter	owner	renter	owner	renter
aged 18-39	-0.002***	-0.01*	0.01	0.04	-0.009***	-0.03***	-0.0007	-0.005
aged 40-54	-0.002	0.006	-0.0006	0.03	-0.008*	-0.02	0.01	-0.002
aged 55-75	-0.16***	-0.14***	0.08***	0.06**	0.02	0.03	0.05***	0.04***

Notes: *, ** and *** indicate statistical significance at the 5%, 1% and 0.1% level, respectively.

It should also be noticed that the marginal effects differ when evaluated at different values of regressors. For example, although the marginal effects of

¹³By "an average young homeowners household", we refer to a homeowners household whose characteristics are the mean values of the young household sample, i.e., with the mean male/female age, mean number of children, mean wages, mean household non-labour income, etc.

¹⁴This is not necessarily true because we don't have information either on the renters' house purchase plan or on the owners' house upsizing plan.

house prices on $\Pr(0,0)$ for older households are negative, the absolute values begin to decline at the male age of 60 (Figure 5). The story is the same for this marginal effect evaluated at different values of log of real annual household non-labour incomes (Figure 6), i.e., as the household non-labour income increases, given an increase of house prices, the extent to which $\Pr(0,0)$ decreases becomes smaller. And these are true for both owners and renters.

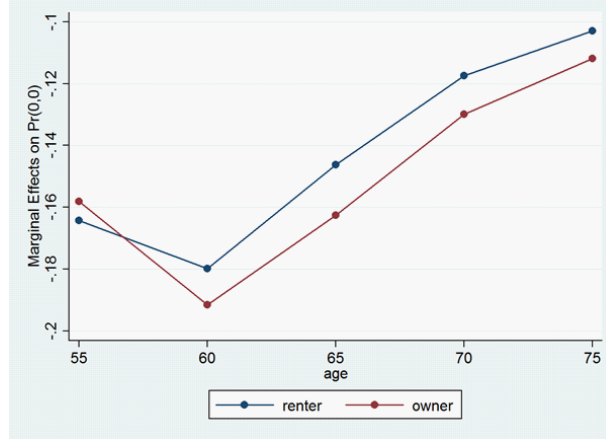


Figure 5 *Marginal effects on $\Pr(0,0)$ by male ages for households aged 55-75*

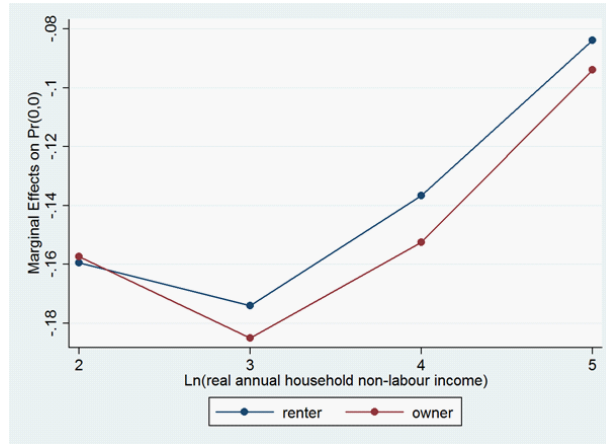


Figure 6 *Marginal effects on $\Pr(0,0)$ by household non-labour income for households aged 55-75*

5.2 Estimation results for household hours of work

For each age group, the likelihood-ratio test performed after the bivariate tobit estimation rejects the null hypothesis of zero correlation of cross-equation errors,

which supports the two-equation model as opposed to a single equation model. Like in the bivariate probit model above, this gives evidence that the hours of work decisions between partners are interrelated.

The bivariate tobit estimation reports the beta coefficients for the latent regression model which reflects the partial effects on the desired hours of work, because $E(h^*|x)$ is linear in x 's. However, we are often interested in how the reported estimation results can be translated into estimation of actually observed hours of work. To see how, we show the correct calculation for male partners as follows.

$$\begin{aligned}
E(h_m) &= E(h_m|h_f, (1, 1)) \Pr(1, 1) + E(h_m|(1, 0)) \Pr(1, 0) \\
&\quad + 0 * \Pr(0, 1) + 0 * \Pr(0, 0) \\
&= E(h_m|h_f, (1, 1)) \Pr(1, 1) + E(h_m|(1, 0)) \Pr(1, 0) \\
&= E(h_m|h_m > 0) \Pr(h_m > 0) \\
&= \Phi_1\left(\frac{\widehat{\beta}_m X_{mg}}{\widehat{\sigma}_1}\right) \widehat{\beta}_1 X_{mg} + \widehat{\sigma}_1 \phi_1\left(\frac{\widehat{\beta}_m X_{mg}}{\widehat{\sigma}_1}\right)
\end{aligned}$$

where $\widehat{\beta}_m = (\widehat{\alpha}_m, \widehat{\beta}_{1m}, \widehat{\beta}_{2m}, \widehat{\beta}_{3m})$, $X_{mg} = (\ln w_m, X_m, \ln w_f)$, Φ_1 and ϕ_1 are the marginal standard normal distribution function and density function of the error term ε_{hm} in male hours of work equation, respectively. $\widehat{\sigma}_1$ is the estimated standard deviation of ε_{hm} .

Goodness of fit measured by the correlation coefficient between fitted hours and actual hours are shown in Table 7.

Table 7 *Correlation coefficients between fitted hours and actual hours*

age	male	female
aged 18-39	0.32	0.59
aged 40-54	0.25	0.36
aged 55-75	0.68	0.58

Adjustment factor calculated as $n^{-1} \sum_{h=1}^n \Phi_i(\widehat{\beta}_i X_{igh}/\widehat{\sigma}_i)$ (where h denotes observation in each subsample) in each subsample are shown in Table 8.

Table 8 *Adjustment factors for each subsample*

	Aged 18-39		Aged 40-54		Aged 55-75	
	male	female	male	female	male	female
Adjustment factor	0.99	0.82	1	0.91	0.3	0.27

Table 9 shows the estimated coefficients in hours of work equations adjusted by adjustment factors, which suggests that the observed hours of work of

younger male and older male are most significantly responsive to house prices change. Regardless of how their female partners respond to house prices, they will increase their hours of work when house prices rise. Again, the effects of house prices are not statistically significant for the middle aged households. This is probably because of either most of them are already working full time¹⁵, therefore an increase of house prices cannot further increase their working hours considering the pure wealth effect. Alternatively, we can again attribute this insignificant response to their rational expectations on the stationary house prices and their plan of not exiting the housing market immediately as we discuss in Section 4.1.

On the other hand, we can see some gender differences in each age group: First, the working hours of female partners in the younger and middle aged households are not significantly responsive to house prices change. Second, the number of children in the household has a significant negative effect on female partners' hours of work in the younger and middle aged households, which reflects the possibility that more of the commitments of family care are undertaken by female partners.

¹⁵In our sample, both male and female partners in the middle aged households have the highest mean weekly hours of work (37.5 for male and 24.3 for female) and lowest variance of weekly hours of work (12.6 for male and 14.4 for female) compared to their younger and older counterparts.

Table 9 *Estimated coefficients in hours of work equations*
(adjusted by adjustment factors)

Independent variables	Aged 18-39 (n=6164)		Aged 40-54 (n=4457)		Aged 55-75 (n=5801)	
	Male	Female	Male	Female	Male	Female
Ln(wage)	3.24 (2.13)	-6.91 (4.67)	1.15 (2.87)	2 (5.34)	4.46 (8.55)	1.28 (10.18)
Partners' lnwage	-2.04 (1.49)	-0.94 (2.3)	-2.35 (1.43)	-2.46 (2.28)	-4.41*** (4.4)	0.76 (4.17)
Owner*ln(real house price)	2.45*** (0.57)	-0.68 (0.82)	0.34 (0.67)	-0.57 (1.03)	2.94*** (2.6)	1.28* (2.02)
Renter*ln(real house price)	1.43** (0.56)	-1.27 (0.82)	-0.49 (0.68)	-1.21 (1.07)	2.96*** (2.6)	1.36* (2.08)
Age	0.89 (0.62)	1.97*** (0.71)	0.16 (1.44)	3.14 (1.89)	-0.75 (3.6)	-2.4** (3.41)
Age^2	-0.01 (0.01)	-0.02* (0.01)	-0.004 (0.02)	-0.04* (0.02)	-0.001 (0.03)	0.01 (0.03)
Married	-1.76 (3)	-0.09 (2.91)	0.02 (3.56)	1.43 (6.14)	3.17 (13.04)	-0.41 (11.9)
Gcse	1.7 (1.49)	-7.04** (2.82)	-0.61 (2.59)	-7.84 (4.59)	-0.37 (8.01)	-1.66 (8.83)
Nkids	0.29 (0.33)	-5.89*** (0.47)	0.26 (0.4)	-4.01*** (0.58)	2.1 (4.12)	-1.81 (3.64)
Ln (real non-labour income)	-1.66*** (0.2)	-2.05*** (0.23)	-1.55*** (0.24)	-1.55*** (0.28)	-3*** (0.61)	-1.53*** (0.54)
rho12	0.07*** (0.02)		0.07** (0.03)		0.26*** (0.04)	

Notes: *, ** and *** indicate statistical significance at the 5%, 1% and 0.1% level, respectively.

Standard errors adjusted by clusters of households are in parentheses.

rho12 is the estimated correlation coefficient of errors in the two equations.

5.3 Caveats

Before closing, we point out some potential limitations in our analysis. First, since the bivariate probit and tobit models crucially rely on the assumption of bivariate normality of errors, a violation of this assumption would lead to difficulty in explaining the MLE results. Figure A1 in the Appendix plots the quantiles of truncated residuals of bivariate tobit¹⁶ against the quantiles of normal distribution for each gender in the three age groups, which shows a graphical

¹⁶The truncated residuals of bivariate tobit are calculated as actual working hours minus predicted desired working hours, standardised by the corresponding estimated standard deviation of error.

comparison between marginal distribution of truncated residuals and normal distribution. Except the young and middle aged male partners, the straight part of the plot shows evidence of truncated normality of residuals.

Second, by specifying the two-equation system for male and female partners' labour supply without cross-equation restriction, we allow for different parameters associated with the two genders. But to what extent these parameters differ between across equations for male and female needs to be further tested. Third, our modelling and estimation method are static in that we do not control for the individual time-invariant heterogeneity and the selection of housing tenure choice. On the other hand, while the house prices data on local authority district level exhibits rich variation across localities and over time, the period covered by our sample (12 years) may not be long enough to capture long term fluctuations of house prices which may be non-stationary and might affect household labour supply more significantly and differently in each age group. Finally, the implicit ad hoc assumption that every non-worker is voluntarily unemployed might be problematic if the labour market condition is not good enough to accommodate everyone who is willing to work. One solution is to model another hurdle (Blundell and Meghir, 1987).

6 Conclusion

This paper attempts to analyse the impact of house prices on couples' labour supply both at the extensive margin (participation) and intensive margin (hours of work). We impute wages for both workers and non-workers by adopting the Heckman selection model. With the information of predicted wages, we further estimate the participation equation and hours of work equation considering the interdependent nature of couple's labour supply with bivariate SUR probit and bivariate tobit model. Our setting of the two-equation system is an improvement on previous literature that apply a single-equation model to estimate individual labour supply, which may suffer from lack of behavioural appeal in the context of household decision making and loss of estimation efficiency.

The estimation results show heterogeneous effects of house prices on labour supply of different age groups. The sensitivity of labour supply to house prices depends on homeownership, age and gender. More precisely, at the extensive margin, all age groups show increased participation in response to an increase of house prices. This is consistent with an increase of house prices leading to negative wealth effect on consumption of leisure for those young household who are likely to be "short" in housing with borrowing constraints. On the other hand, the increased participation probabilities for old households who are likely to be "long" in housing imply the wealth effect is very small if present at all and probably offset by other effects such as a strong bequest motive. Alternatively, the house prices and individual labour supply could well be driven by a common macro factor: a boom of the economy could stimulate house prices to rise as well as create more working opportunities. Therefore, old people with a very strong preference for work may choose to work when they can find a job. In this

case, the correlation between house prices and labour supply does not reflect a causal relationship. For middle aged households, however, the effect of house prices on labour supply is not statistically significant. At the same time, we find that the house price process is stationary, which implies the shocks of house prices tend to fade away in the long run and for this reason households who have rational expectations on house prices and do not plan to exit the housing market immediately may not be affected by house prices variations. Gender difference is present. First, the working hours of female partners in the younger and middle aged households are not significantly responsive to house prices change. Second, the number of children in the household has a significant negative effect on female partners' hours of work in the younger and middle aged households, which reflects the possibility that more of the commitments of family care are undertaken by female partners.

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A Appendix

Hereafter we omit the subscript h which indicates household for notational convenience. If we ignore the possible endogeneity of wages and assume ε_i and η_i are independent of each other, then we can simply impute the wages for all non-workers using the fitted values of the wage equation as if wages are observed and correct for the standard errors. However, wages could be endogenous because of possible correlation between unobservables affecting tastes for work and unobservables affecting productivity hence wages (Blundell et al, 2007). The two step Heckman sample selection approach allows for the endogeneity of wages by considering the joint distribution for ε_i and η_i .

Substitute equation (4) to equation (2) to have a reduced form for desired hours of work for each person:

$$\begin{aligned} h_i^* &= \alpha_i + \beta_{1i}(\gamma_i Z_i + \eta_i) + \beta_{2i} X_i + \beta_{3i}(\gamma_j Z_j + \eta_j) + \varepsilon_i \\ &= \alpha_i + \beta_{1i} \gamma_i Z_i + \beta_{2i} X_i + \beta_{3i} \gamma_j Z_j + (\varepsilon_i + \beta_{1i} \eta_i + \beta_{3i} \eta_j) \\ &= \pi_i q_i + v_i \end{aligned} \tag{10}$$

where $\pi_i = (\alpha, \beta_{1i} \gamma, \beta_{2i}, \beta_{3i} \gamma_j)$, $q_i' = (1, Z_i, X_i, Z_j)$, $v_i = \varepsilon_i + \beta_{1i} \eta_i + \beta_{3i} \eta_j$. Equations (4) and (10) constitute a triangular system to completely describe labour supply and wages (Blundell et al., 2007).

Wages are only observed if person i participates, i.e. if desired hours of work for person i are positive.

The probability of person i participating conditional on his/her partner/spouse person j 's participation decision ($i, j=1, 2$), i.e., marginal probability of person i participating is:

$$\begin{aligned} \Pr(h_i^* > 0 | X_i, Z_i, h_j) &= \Pr(v_i > -\pi_i q_i, h_j > 0) + \Pr(v_i > -\pi_i q_i, h_j \leq 0) = \\ &= \Pr(v_i > -\pi_i q_i, v_j > -\pi_j q_j) + \Pr(v_i > -\pi_i q_i, v_j \leq -\pi_j q_j) \\ &= \Pr(v_i > -\pi_i q_i) = \Phi(\pi_i q_i) = \Phi(\alpha + \beta_{1i} \gamma_i Z_i + \beta_{2i} X_i + \beta_{3i} \gamma_j Z_j) \end{aligned}$$

where $\Phi(\cdot)$ is the marginal distribution of v_i .

The mean of log of wages for person i given he/she works is

$$\begin{aligned} &E(\ln w_i | h_1^* > 0, X_i, Z_i) \\ &= E(\gamma_i Z_i + \eta_i | v_i > -\pi_i q_i) \\ &= E(\gamma_i Z_i + \eta_i | \varepsilon_i + \beta_{1i} \eta_i + \beta_{3i} \eta_j > -\alpha - \beta_{1i} \gamma Z_i - \beta_{2i} X_i - \beta_{3i} \gamma_j Z_j) \\ &= E(\gamma_i Z | \varepsilon_i + \beta_{1i} \eta_i + \beta_{3i} \eta_j > -\alpha - \beta_{1i} \gamma Z_i - \beta_{2i} X_i - \beta_{3i} \gamma_j Z_j) \\ &\quad + E(\eta_i | \varepsilon_i + \beta_{1i} \eta_i + \beta_{3i} \eta_j > -\alpha - \beta_{1i} \gamma Z_i - \beta_{2i} X_i - \beta_{3i} \gamma_j Z_j) \end{aligned} \tag{11}$$

The correlation between the error term of reduced form participation equation and the error term of the wage equation for the same person is:

$$\begin{aligned}
cov(v_i, \eta_i) &= cov((\varepsilon_i + \beta_{1i}\eta_i + \beta_{3i}\eta_j), \eta_i) \\
&= E((\varepsilon_i + \beta_{1i}\eta_i + \beta_{3i}\eta_j)\eta_i) \\
&= E(\varepsilon_i\eta_i) + E(\beta_{1i}\eta_i^2) + E((\beta_{3i}\eta_j)\eta_i)
\end{aligned} \tag{12}$$

Assuming $E(\varepsilon_i) = 0$, $E(\eta_i) = 0$ and $E(\eta_i\eta_j) = 0$ ¹⁷, equation (12) becomes

$$cov(v_i, \eta_i) = cov(\varepsilon_i, \eta_i) + \beta_{1i}\sigma_{\eta_i}^2 \tag{13}$$

If either the covariance of ε_i and η_i is non-zero or variance of η_i is non-zero (which is for sure unless wages are non-random), the covariance of v_i and η_i is non-zero.

On the other hand, the mean of v_i is

$$E(v_i) = E(\varepsilon_i + \beta_{1i}\eta_i) = 0$$

Assume v_i and η_i are jointly normal distributed:

$$\begin{bmatrix} \eta_i \\ v_i \end{bmatrix} \sim N \left[\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_{\eta_i}^2 & \sigma_{\eta_i v_i} \\ \sigma_{\eta_i v_i} & \sigma_{v_i}^2 \end{bmatrix} \right]$$

where $\sigma_{\eta_i v_i} = \sigma_{\varepsilon_i \eta_i} + \beta_{1i}\sigma_{\eta_i}^2$ and

$$\sigma_{v_i}^2 = \sigma_{\varepsilon_i}^2 + \beta_{1i}^2\sigma_{\eta_i}^2 + \beta_{3i}^2\sigma_{\eta_j}^2 + 2\beta_{1i}\sigma_{\varepsilon_i \eta_i} + 2\beta_{1i}\beta_{3i}\sigma_{\eta_i \eta_j} + 2\beta_{3i}\sigma_{\varepsilon_i \eta_j} = \sigma_{\varepsilon_i}^2 + \beta_{1i}^2\sigma_{\eta_i}^2 + \beta_{3i}^2\sigma_{\eta_j}^2 + 2\beta_{1i}\sigma_{\varepsilon_i \eta_i} + 2\beta_{3i}\sigma_{\varepsilon_i \eta_j}$$

Since the conditional expectation of η_i given v_i is

$$E(\eta_i|v_i) = \frac{\sigma_{\eta_i v_i}}{\sigma_{v_i}^2} v_i$$

it follows that

$$\begin{aligned}
\eta_i &= \frac{\sigma_{\eta_i v_i}}{\sigma_{v_i}^2} v_i + \zeta \\
&= \frac{\sigma_{\varepsilon_i \eta_i} + \beta_{1i}\sigma_{\eta_i}^2}{\sigma_{\varepsilon_i}^2 + \beta_{1i}^2\sigma_{\eta_i}^2 + \beta_{3i}^2\sigma_{\eta_j}^2 + 2\beta_{1i}\sigma_{\varepsilon_i \eta_i} + 2\beta_{3i}\sigma_{\varepsilon_i \eta_j}} v_i + \zeta
\end{aligned} \tag{14}$$

Using equation (14) and assuming ζ and $v_i = \varepsilon_i + \beta_{1i}\eta_i + \beta_{3i}\eta_j$ are independent, equation (11) becomes

¹⁷ Assume the error terms in the wage equations for each partner are independent.

$$\begin{aligned}
& E(\ln w_i | h_i^* > 0, X_i, Z_i, Z_j) \\
&= E(\gamma_i Z_i | \varepsilon_i + \beta_{1i} \eta_i + \beta_{3i} \eta_j > -\alpha - \beta_{1i} \gamma Z_i - \beta_{2i} X_i - \beta_{3i} \gamma_j Z_j) + \\
& \quad E\left(\frac{\sigma_{\varepsilon_i \eta_i} + \beta_{1i} \sigma_{\eta_i}^2}{\sigma_{\varepsilon_i}^2 + \beta_{1i}^2 \sigma_{\eta_i}^2 + \beta_{3i}^2 \sigma_{\eta_j}^2 + 2\beta_{1i} \sigma_{\varepsilon_i \eta_i} + 2\beta_{3i} \sigma_{\varepsilon_i \eta_j}}\right) \\
& \quad (\varepsilon_i + \beta_{1i} \eta_i + \beta_{3i} \eta_j) + \zeta) \\
& \quad | \varepsilon_i + \beta_{1i} \eta_i + \beta_{3i} \eta_j > -\alpha - \beta_{1i} \gamma Z_i - \beta_{2i} X_i - \beta_{3i} \gamma_j Z_j) \\
&= \gamma_i Z_i + \left(\frac{\sigma_{\varepsilon_i \eta_i} + \beta_{1i} \sigma_{\eta_i}^2}{\sigma_{\varepsilon_i}^2 + \beta_{1i}^2 \sigma_{\eta_i}^2 + \beta_{3i}^2 \sigma_{\eta_j}^2 + 2\beta_{1i} \sigma_{\varepsilon_i \eta_i} + 2\beta_{3i} \sigma_{\varepsilon_i \eta_j}}\right) \\
& \quad E(\varepsilon_i + \beta_{1i} \eta_i + \beta_{3i} \eta_j | \varepsilon_i + \beta_{1i} \eta_i + \beta_{3i} \eta_j > -\alpha - \beta_{1i} \gamma Z_i - \beta_{2i} X_i - \beta_{3i} \gamma_j Z_j) \\
&= \gamma_i Z_i + \left(\frac{\sigma_{\varepsilon_i \eta_i} + \beta_{1i} \sigma_{\eta_i}^2}{\sigma_{\varepsilon_i}^2 + \beta_{1i}^2 \sigma_{\eta_i}^2 + \beta_{3i}^2 \sigma_{\eta_j}^2 + 2\beta_{1i} \sigma_{\varepsilon_i \eta_i} + 2\beta_{3i} \sigma_{\varepsilon_i \eta_j}}\right) \\
& \quad \frac{\varphi(-\alpha - \beta_{1i} \gamma Z_i - \beta_{2i} X_i - \beta_{3i} \gamma_j Z_j)}{1 - \Phi(-\alpha - \beta_{1i} \gamma Z_i - \beta_{2i} X_i - \beta_{3i} \gamma_j Z_j)} \quad (15)
\end{aligned}$$

where $\Phi(\cdot)$ and $\varphi(\cdot)$ are the marginal distribution and marginal density of v_i , respectively.

By symmetry of normal distribution,

$$\frac{\varphi(-\alpha - \beta_{1i} \gamma Z_i - \beta_{2i} X_i - \beta_{3i} \gamma_j Z_j)}{1 - \Phi(-\alpha - \beta_{1i} \gamma Z_i - \beta_{2i} X_i - \beta_{3i} \gamma_j Z_j)} = \frac{\varphi(\alpha + \beta_{1i} \gamma Z_i + \beta_{2i} X_i + \beta_{3i} \gamma_j Z_j)}{\Phi(\alpha + \beta_{1i} \gamma Z_i + \beta_{2i} X_i + \beta_{3i} \gamma_j Z_j)} = \lambda_i$$

Therefore equation (15) becomes:

$$\begin{aligned}
& E(\ln w_i | h_i^* > 0, X_i, Z_i, Z_j) = \\
& \gamma_i Z_i + \left(\frac{\sigma_{\varepsilon_i \eta_i} + \beta_{1i} \sigma_{\eta_i}^2}{\sigma_{\varepsilon_i}^2 + \beta_{1i}^2 \sigma_{\eta_i}^2 + \beta_{3i}^2 \sigma_{\eta_j}^2 + 2\beta_{1i} \sigma_{\varepsilon_i \eta_i} + 2\beta_{3i} \sigma_{\varepsilon_i \eta_j}}\right) \lambda_i
\end{aligned}$$

To be specific, the Heckman model corrects for the selection bias by adding an extra term to the wage model:

$$\ln w_i = \gamma_i Z_i + \delta_i \lambda_i + \eta_i \quad (16)$$

where the extra term $\lambda_i = \frac{\phi(\widehat{\pi_i q_i})}{\Phi(\widehat{\pi_i q_i})}$ is the predicted inverse Mills ratio from the reduced form participation/selection equation as derived above.

B Appendix

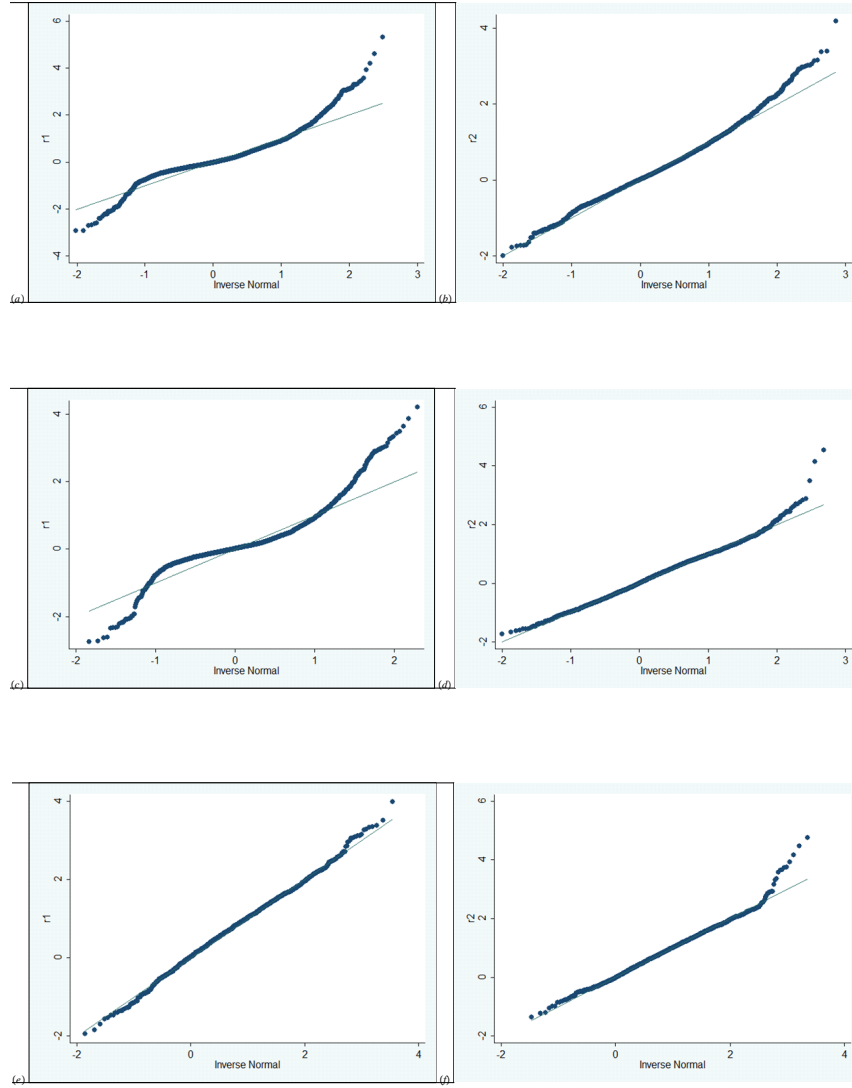


Figure A1 Plot of quantiles of truncated residuals against quantiles of normal distribution

(horizontal axis-quantiles of normal distribution , vertical axis-quantiles of truncated residuals of the bivariate tobit)

- (a) young male; (b) young female;
(c) middle aged male; (d) middle aged female;
(e) old male; (f) old female.