TSDT14 Signal Theory

Lecture 10

Reconstruction in CD Players and Exam Problems

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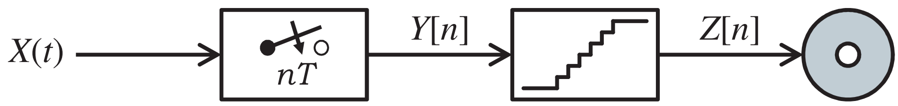
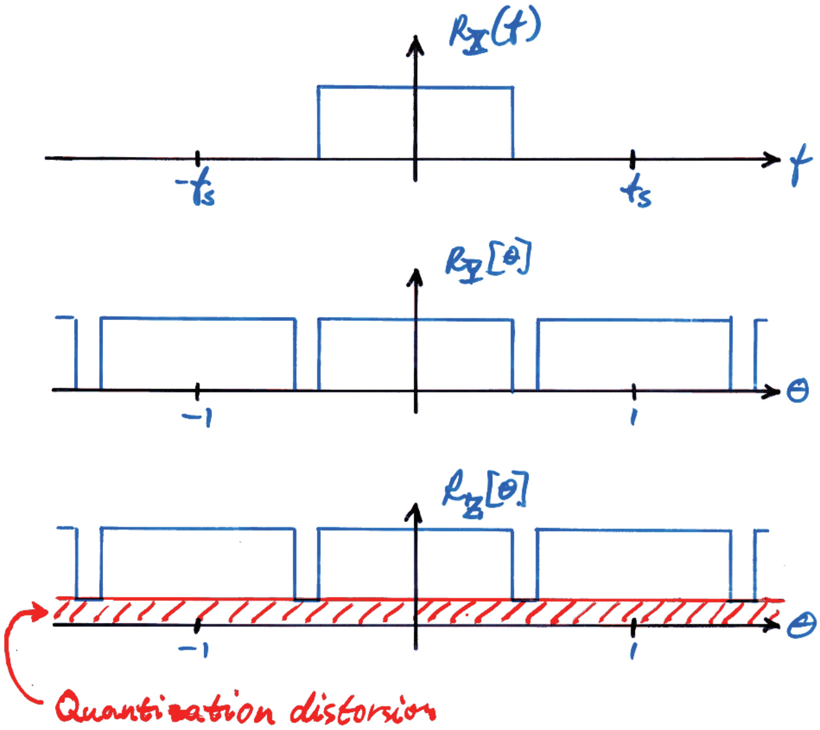
CDs at Recording 1(2)



Band-limited input:	$R_X(f) \approx 0,$	$\lvert f \rvert \geq B = 20\ \mathrm{kHz}$	
Sampling frequency:	$f_s = 44.1\ \mathrm{kHz}$		
Uniform quantization:	16 bits	$\Rightarrow$	$N = 2^{16}$ steps
Saturation level:	A		$\frac{\Delta^2}{12} = \frac{A^2}{2^{32} \cdot 3}$
Quantization step height:	Δ		

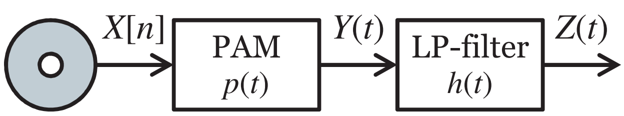
Signal-to-Distorsion Ratio: $\mathrm{SDR}_{\max} = 10 \log_{10}(2^{32}) \approx 96\ \mathrm{dB}$

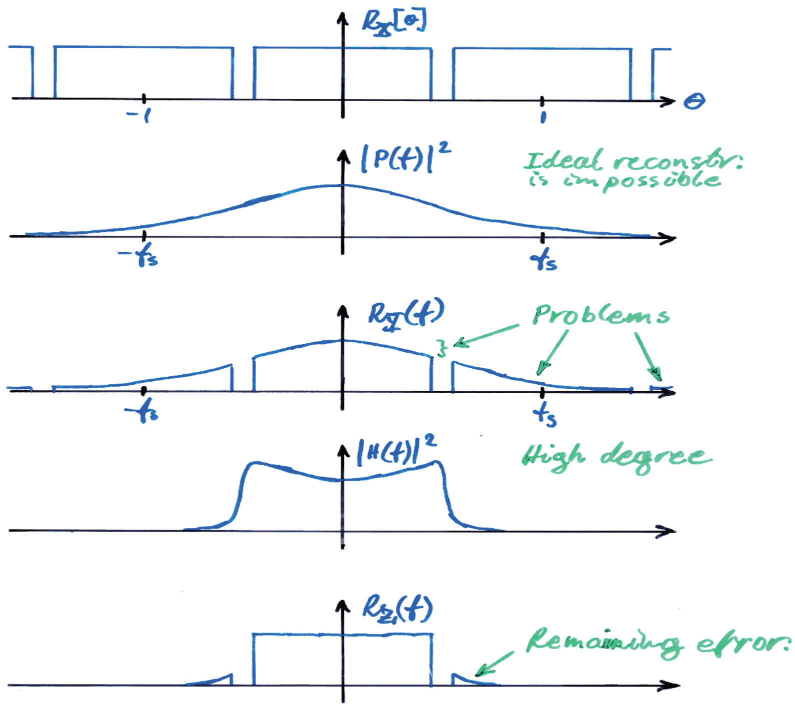
CDs at Recording 2(2)



Listening to CDs – 1

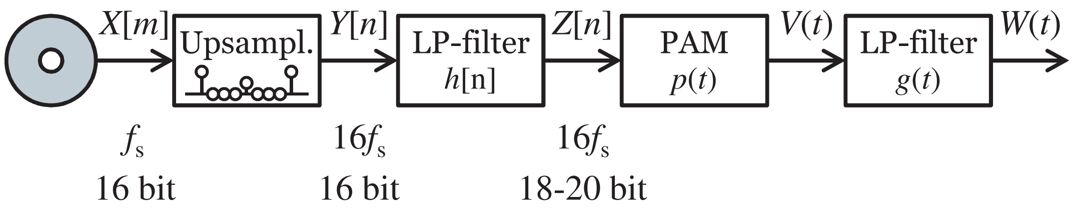
Direct PAM





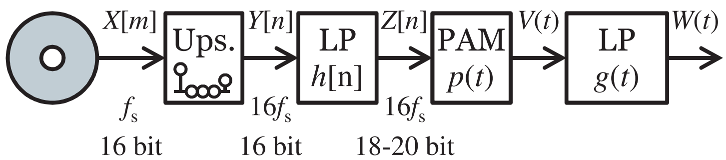
Listening to CDs – 2

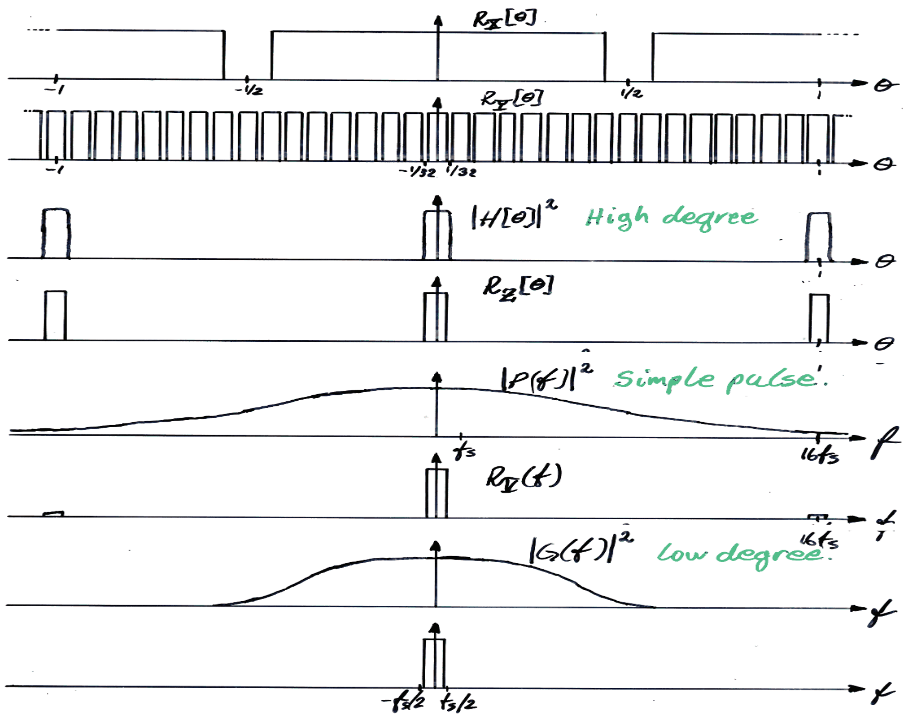
Oversampling 1(2)



Listening to CDs – 2

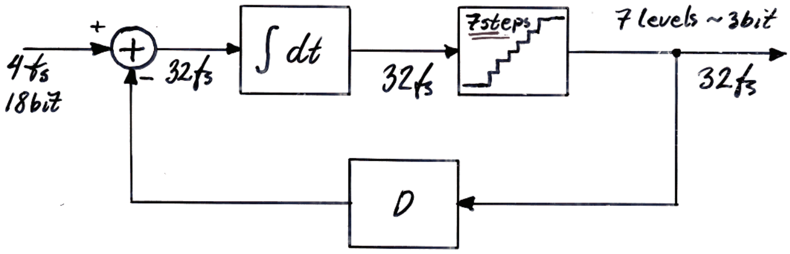
Oversampling 2(2)

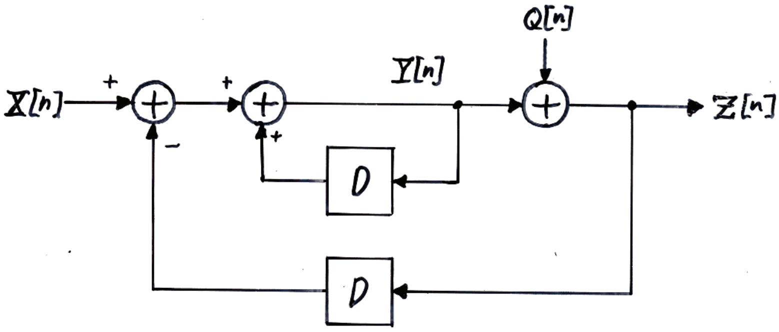




Listening to CDs – 3

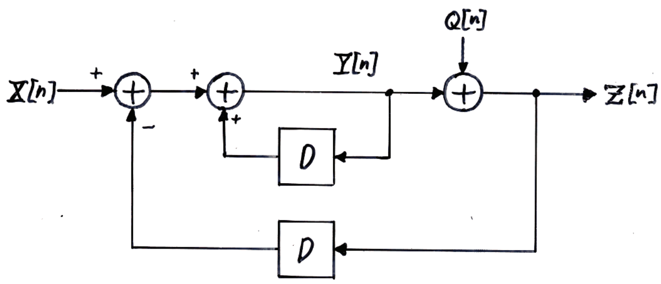
Noise Shaping 1(4) – Principle of First-Order Noise Shaper





Listening to CDs – 3

Noise Shaping 2(4) – Analysis 1



If no $Q[n]$:

$$Z[n] = Y[n] = X[n] + Y[n-1] - Z[n-1] = X[n]$$

Error: $S[n] = Z[n] - X[n]$

Fig $\Rightarrow$ $Y[n] = X[n] + Y[n-1] - Z[n-1]$

$Z[n] = Y[n] + Q[n]$ $\Big\} \Rightarrow$

$$\Rightarrow \quad S[n] = \big(X[n] + \underbrace{Y[n-1] - Z[n-1]}_{-Q[n-1]}\big) + Q[n] - X[n]$$

$$= Q[n] - Q[n-1]$$

Interpretation:
The quantization noise is filtered.

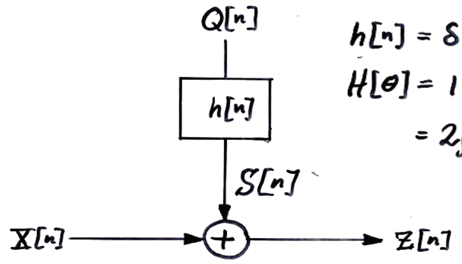
$$h[n] = \delta[n] - \delta[n-1]$$

$$H[\theta] = 1 - e^{-j2\pi\theta}$$

$$= 2j e^{-j\pi\theta} \sin(\pi\theta)$$

Listening to CDs – 3

Noise Shaping 3(4) – Analysis 2



$$h[n] = \delta[n] - \delta[n-1]$$

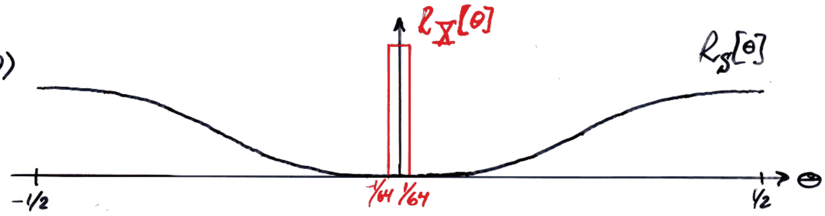
$$H[\theta] = 1 - e^{-j2\pi\theta} = 2j e^{-j\pi\theta} \sin(\pi\theta)$$

Quantization error: $Q[n]$:

$$\frac{\Delta^2}{12} = \frac{A^2}{7^2}\cdot\frac{1}{3} = \frac{A^2}{147} \Rightarrow r_Q[k] = \frac{A^2}{147}\delta[k], \quad R_Q[\theta] = \frac{A^2}{147}$$

Error: $S[n]$:

$$R_S[\theta] = |H[\theta]|^2 R_Q[\theta] = \frac{4A^2}{147}\sin^2(\pi\theta)$$



Error power in audio range ($\sin x \approx x$ for small x)

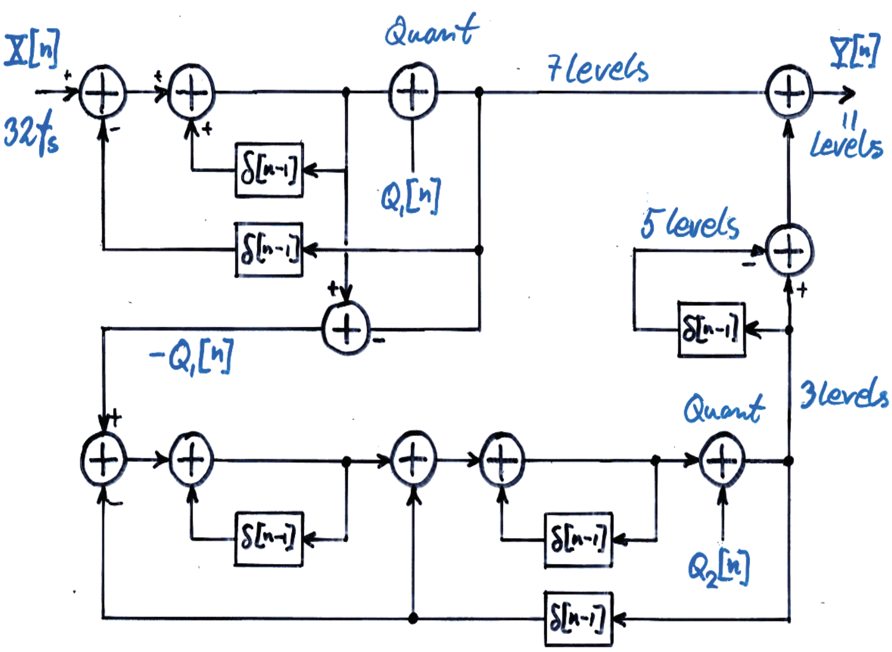
$$\int_{-1/64}^{1/64} R_S[\theta]\,d\theta = 2\int_0^{1/64} \frac{4A^2}{147}\sin^2(\pi\theta)\,d\theta \approx \frac{A^2}{20}\int_0^{1/64}(\pi\theta)^2 d\theta$$

$$\approx \frac{A^2}{2}\int_0^{1/64}\theta^2 d\theta = \frac{A^2}{2}\left[\frac{\theta^3}{3}\right]_0^{1/64} = \frac{A^2}{2^{19}\cdot 3} \approx 57dB$$

Approx. 9.5 bits

Listening to CDs – 3

Noise Shaping 4(4) – Implementation of Third-Order Noise Shaper



Error PSD:

$$R_S[\theta] = \frac{64A^2}{147}\cdot\sin^6(\pi\theta)$$

Error Power in Audio Range:

$$2\int_0^{1/64}\frac{64A^2}{147}(\pi\theta)^6 d\theta = \dots \approx$$

$$\approx \frac{A^2}{2^{33}\cdot 3} \approx 100dB$$

Corresponds to:

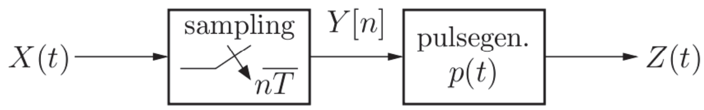
Approx. 16.5 bits

2010-08-28 – Problem 4

The time-continuous process $X(t)$ is bandlimited white noise with mean $m_X = 0$ and PSD

$$R_X(f) = \begin{cases} 1, & |f| \leq W, \\ 0, & \text{elsewhere.} \end{cases}$$

The signal $X(t)$ is sampled and pulse-amplitude-modulated according to the figure below.

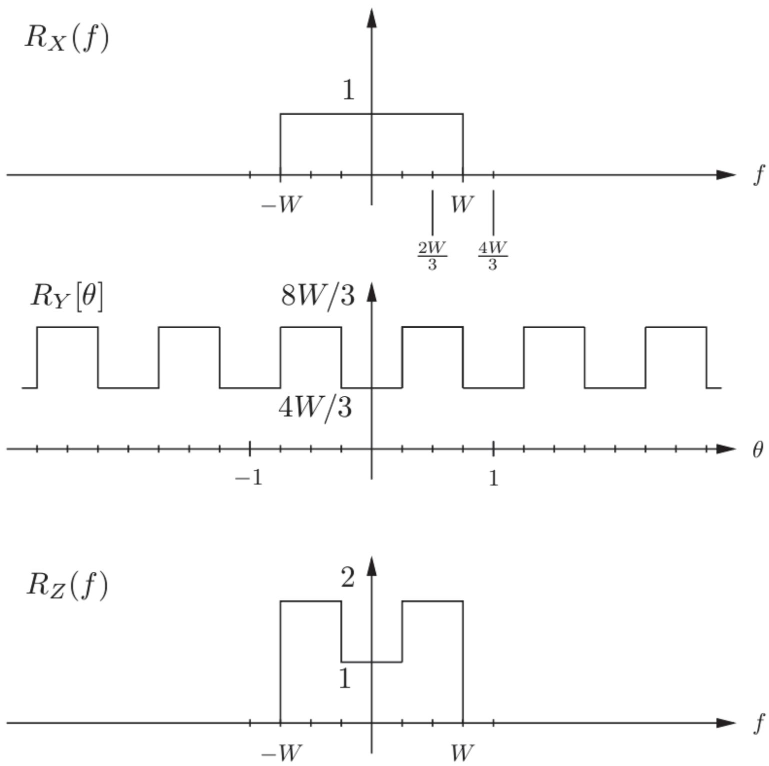


The sampling frequency is $f_s = \dfrac{1}{T} = \dfrac{4W}{3}$ and the pulse shape of the PAM is $p(t) = \dfrac{3\sin(2\pi W t)}{4\pi W t}$.

a) Determine and draw the PSDs $R_Y[\theta]$ and $R_Z(f)$. (3 p)

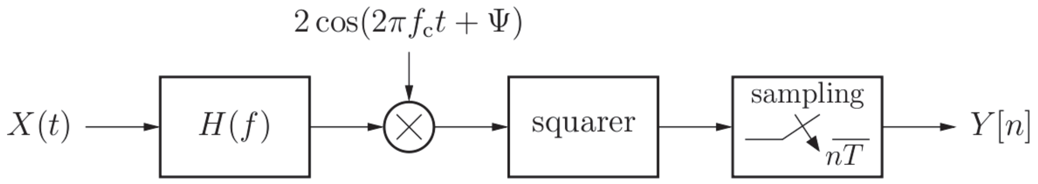
b) Determine the reconstruction error $\varepsilon^2 = E\{(Z(t) - X(t))^2\}$. (2 p)

2010-08-28 – Problem 4 – PSDs



2009-10-23 – Problem 3

The input $X(t)$ to the system below is a strictly stationary white process with PSD R_0, and the stochastic variable Ψ is as usual uniformly distributed on $[0, 2\pi)$ and independent of $X(t)$. The input is Gaussian with mean 0.



The initial filter has frequency response

$$H(f) = \begin{cases} 2, & |f| < f_0, \\ 0, & \text{elsewhere.} \end{cases}$$

The carrier frequency is $f_c = 2f_0$, while the sampling frequency is $f_s = 3f_0$.

Calculate the power P_Y of the output $Y[n]$. (5 p)

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