Name: _____ Date: _____

Practice Questions Set 1

1. Suppose one thousand tickets are sold at \$10 each to win a used car valued at \$5000. What is the expected value of the gain if a person purchases one ticket? [1]

2. Harry spins a roulette which consists of integers from 1 through 14 (not necessarily in order). What is the variance of the outcome he gets? [2]
(Note: the areas that corresponds to each outcome are equal)

3. The discrete random variable Xhas the probability density function

$$P(X = x) = \begin{cases} kx & x = 2, 4, 6 \\ k(x-2) & x = 8 \\ 0 & \text{otherwise} \end{cases}$$

(a) Find the value of k. [1]
(b) Find the exact value of $E(X)$. [2]
(c) Calculate $Var(3 - 4X)$. [2]

4. [2017 SRJC MYE P2 Q10]
(a) It is known that X is discrete random variable such that

$$4E(X^2) = Var(3 + 2X) + 1.$$

Find $E(X)$. [2]

(b) To promote the sales of a snack for children from March to August, one game card is inserted into each box of the snack. For each month, the game cards feature a different series of comic book characters and there are twelve distinct game cards in each series. Andy buys four boxes of the snack in March.
The random variable X denotes the number of distinct game cards that Andy will get from his purchases.

(i) Show that $P(X = 3) = \frac{55}{144}$ and $P(X = 2) = \frac{77}{1728}$. Hence, tabulate the probability distribution of X. [4]

(ii) Find $E(X)$ and $Var(X)$. [2]

The game cards can be used to redeem points in a game run by the distributor of the snack. For each series, one point can be redeemed for every three different game cards submitted to the distributor. Game cards from different series cannot be combined in any redemption. Andy buys four boxes each month during the promotion period from March to August.

(iii) Find the probability that Andy will claim three points in total during the promotion period. [3]

5. [2017 CJC MYE Q10]
In a game show, a contestant spins a game wheel and receives a score, X, for each spin based on where the wheel stops. The contestant is given 2 spins and the final score is the average score of the 2 spins. The outcomes of the a spin of the game are illustrated in the table below and the outcomes of the spins are independent of each other.

Score (x)	1	2	3	4	5
$P(X = x)$	$\frac{1}{3}$	$\frac{4}{15}$	$\frac{1}{5}$	$\frac{2}{15}$	$\frac{1}{15}$

The discrete random variable Y denotes the final score of the contestant.

(i) Evaluate $E(X)$ and $Var(X)$. [2]
(ii) Hence, by writing Y in terms of X, state the values of $E(Y)$ and $Var(Y)$. [3]

The contestant wins the grand prize if the final score is greater than 2. If the final score is less than or equal to 2, the contestant is given an additional spin and the contestant wins a consolation prize if the score of the additional spin is greater than 1.

(iii) Find the probability that the contestant wins the grand prize. [3]
(iv) Find the probability that the contestant wins a prize. [3]
(v) Given that the contestant won a prize, find the probability that it is the consolation prize. [3]

6. In a game, the probability of a player scoring with a shot is $\frac{1}{4}$. Let X be the number of shots the player takes to score, including the winning shot. (You can assume each shot is independent of the others).
(a) Find $P(X = 3)$. [2]
(b) Find the probability that the player will have at least three misses before scoring **twice**. [4]
(c) Prove that the expected value of X is 4. [5]
(Hint: You can use a Maclaurin's expansion to evaluate your expression)

7. A certain traffic light remains red for 40 seconds at a time. You arrive (at random) at the light and find it red. What is the probability that you will have to wait at least 15 seconds for the light to turn green? [6]

Answers

1. -\$5 2. $\frac{65}{4}$ 3. (a) $k = \frac{1}{18}$ (b) $\frac{52}{9}$ (c) $\frac{5120}{81}$
4. (a) $\pm\frac{1}{2}$ (b)(ii) $E(X) = \frac{6095}{1728}$, $Var(X) = 0.342$ (ii)0.00160
5. (i) $E(X) = \frac{7}{3}, Var(X) = \frac{14}{9}$ (ii)$E(Y) = \frac{7}{3}, Var(Y) = \frac{7}{9}$ (iii) $\frac{38}{75}$
(iv) 0.836 (v) 0.394 6. (a) $\frac{9}{64}$ (b) $\frac{189}{256}$ 7. $\frac{5}{8}$