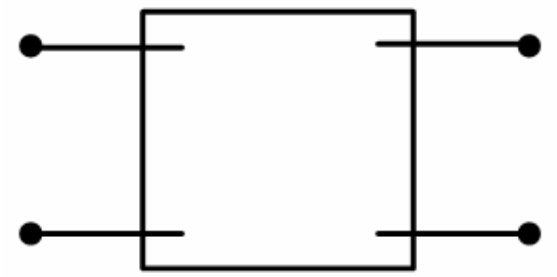
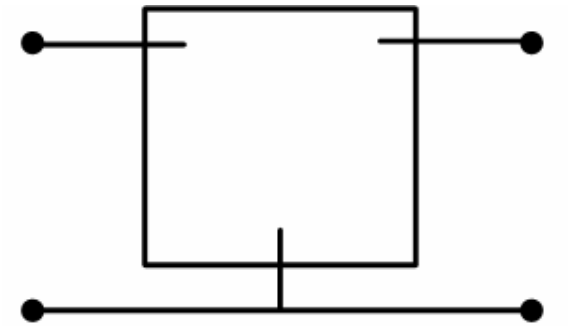
BAB XIII
KUTUB EMPAT

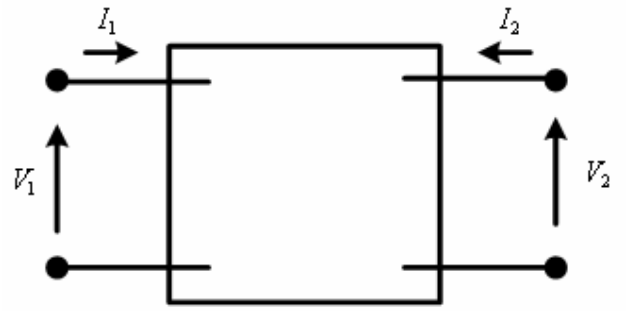
Bentuk umum :
Jaringan 2 port dengan 4 terminal



Jaringan 2 port dengan 3 terminal



Parameter Z



Misalkan :
I_1 dan I_2 adalah input
V_1 dan V_2 adalah output
Maka :

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Jika port 2 open circuit ($I_2 = 0$), sehingga :

$$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0}$$

$$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0}$$

Jika port 1 open circuit ($I_1 = 0$), sehingga :

$$Z_{21} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

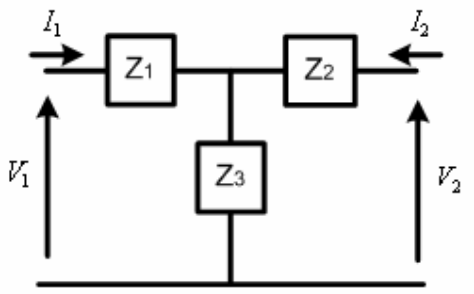
$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

Impedansi yang dihasilkan sebagai impedansi *open circuit* atau parameter *open circuit* atau parameter Z.

Z_{11} = impedansi port primer ketika port sekunder *open circuit*
Z_{22} = impedansi port sekunder ketika port primer *open circuit*
$Z_{12} = Z_{21}$ = impedansi transfer dimana perbandingan tegangan disatu port dibandingkan arus di port lainnya.

Contoh latihan :

1. Tentukan parameter Z !



Jawaban :

Ketika port 2 OC ($I_2 = 0$), maka :

$$\frac{V_1}{I_1} = Z_1 + Z_3$$

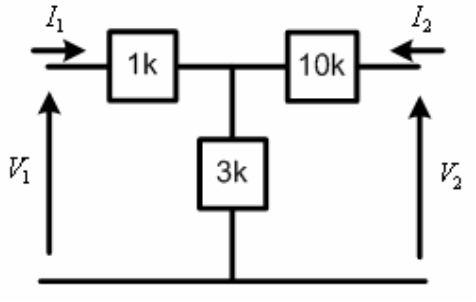
$$\frac{V_2}{I_1} = Z_3$$

Ketika port 1 OC ($I_1 = 0$), maka :

$$\frac{V_2}{I_2} = Z_2 + Z_3$$

$$\frac{V_1}{I_2} = Z_3$$

2. Tentukan parameter Z !



Jawaban :

$V_1 = (1k + 3k)I_1 + 3kI_2 = 4kI_1 + 3kI_2$

$V_2 = (10k + 3k)I_2 + 3kI_1 = 3kI_1 + 13kI_2$

maka :

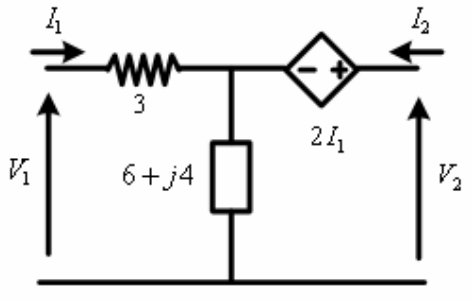
$Z_{11} = 4k$

$Z_{12} = 3k$

$Z_{21} = 3k$

$Z_{22} = 13k$

3. Tentukan parameter Z !



Jawaban :

$V_1 = (3 + 6 + j4)I_1 + (6 + j4)I_2$

$V_1 = (9 + j4)I_1 + (6 + j4)I_2$

$V_2 = 2I_1 + (6 + j4)I_2 + (6 + j4)I_1$

$V_2 = (8 + j4)I_1 + (6 + j4)I_2$

maka :

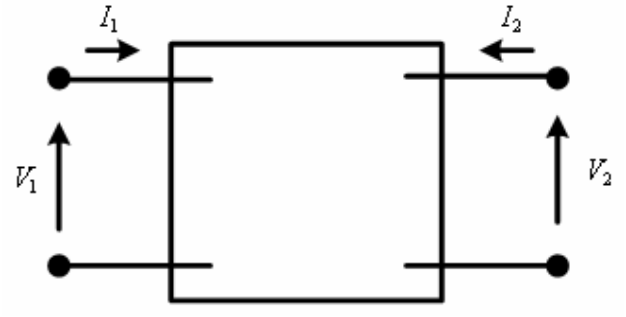
$Z_{11} = 9 + j4$

$Z_{12} = 6 + j4$

$Z_{21} = 8 + j4$

$Z_{22} = 6 + j4$

Parameter Y



Misalkan :
V_1 dan V_2 adalah input
I_1 dan I_2 adalah output
Maka :

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Jika port 2 short circuit ($V_2 = 0$), sehingga :

$$Y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0}$$

$$Y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Jika port 1 short circuit ($V_1 = 0$), sehingga :

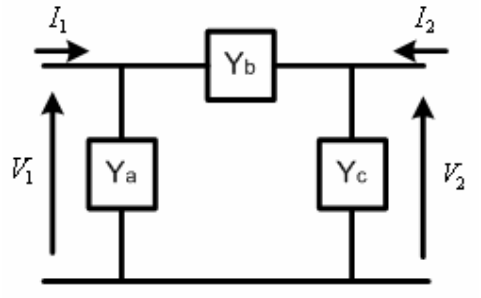
$$Y_{21} = \left.\frac{I_1}{V_2}\right|_{V_1=0}$$

$$Y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=0}$$

Admitansi yang dihasilkan sebagai admitansi *short circuit* atau parameter *short circuit* atau parameter Y.

Contoh latihan :

1. Tentukan parameter Y !



Jawaban :

Ketika port 2 SC ($V_2 = 0$), maka :

$$\frac{I_1}{V_1} = Y_a + Y_b$$

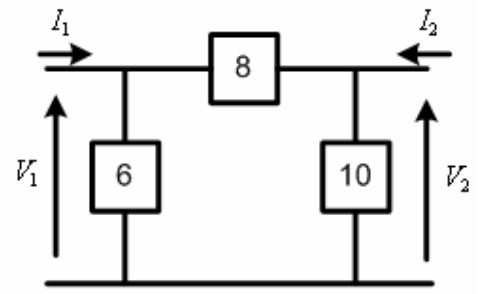
$$\frac{I_2}{V_1} = -Y_b$$

Ketika port 1 SC ($V_1 = 0$), maka :

$$\frac{I_1}{V_2} = -Y_b$$

$$\frac{I_2}{V_2} = Y_b + Y_c$$

2. Tentukan parameter Y !



Jawaban :

$I_1 = 14V_1 - 8V_2$

$I_2 = -8V_1 + 18V_2$

maka :

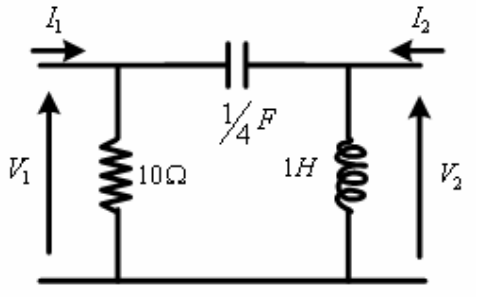
$Y_{11} = 14$

$Y_{12} = -8$

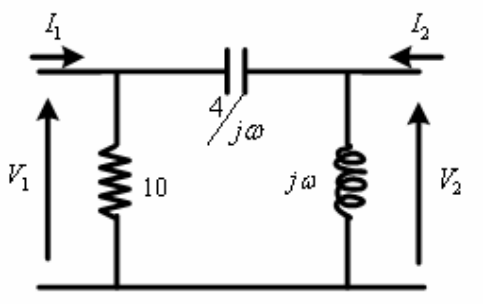
$Y_{21} = -8$

$Y_{22} = 18$

3. Tentukan parameter Y dalam domain $j\omega$!



Jawaban :



$$Y_{11} = \frac{I_1}{V_1} = \frac{1}{10} + \frac{1}{\frac{4}{j\omega}} = \frac{1}{10} + \frac{j\omega}{4}$$

$$Y_{12} = \frac{I_2}{V_1} = -\frac{j\omega}{4}$$

$$Y_{21} = \frac{I_1}{V_2} = -\frac{j\omega}{4}$$

$$Y_{22} = \frac{I_2}{V_2} = \frac{1}{j\omega} + \frac{1}{\frac{4}{j\omega}} = \frac{1}{j\omega} + \frac{j\omega}{4}$$

Parameter Hybrid (h) / Gabungan Parameter Z dan Y

$V_1 = h_{11} I_1 + h_{12} V_2$

$I_2 = h_{21} I_1 + h_{22} V_2$

dan

$I_1 = g_{11} V_1 + g_{12} I_2$

$V_2 = g_{21} V_1 + g_{22} I_2$

dimana :

$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0}$$

$$h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0}$$

$$h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0}$$

$$h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

dan

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I_2=0}$$

$$g_{12} = \left.\frac{I_1}{I_2}\right|_{V_1=0}$$

$$g_{21} = \left.\frac{V_2}{V_1}\right|_{I_2=0}$$

$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V_1=0}$$

Parameter Transmisi (Parameter ABCD)

$$V_1 = AV_2 - BI_2$$

$$I_1 = AV_2 - BI_2$$

parameter ini penting untuk *engineering* transmisi sebab disisi primer (pengirim) terdiri dari variable V_1 dan I_1, sedangkan disisi sekunder (penerima) terdiri dari variabel V_2 dan I_2 (negatif I_2 karena arus masuk ke beban penerima).

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0}$$

$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=0}$$

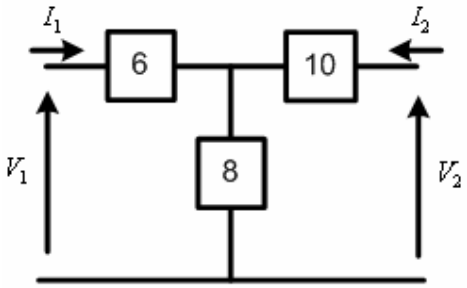
$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0}$$

$$D = \left.\frac{I_1}{-V_2}\right|_{V_2=0}$$

A = perbandingan tegangan ketika sekunder *open circuit*
B = transfer impedansi ketika sekunder *short circuit*
C = transfer admitansi ketika sekunder *open circuit*
D = perbandingan arus ketika sekunder *short circuit*

Contoh latihan :

Tentukan parameter transmisi !



Jawaban :

Parameter transmisi :

$V_1 = AV_2 - BI_2$

$I_1 = CV_2 - DI_2$

Pada saat V_2 *open circuit* ($I_2 = 0$) :

$$V_1 = AV_2 \rightarrow A = \frac{V_1}{V_2}$$

dim*ana* :

$$V_2 = \frac{Z_2}{Z_1 + Z_2} V_1$$

$$A = \frac{V_1}{V_2} = \frac{Z_1 + Z_2}{Z_2} = \frac{6+8}{8} = \frac{14}{8}$$

$$I_1 = CV_2 \rightarrow C = \frac{I_1}{V_2}$$

dim*ana* :

$$V_2 = Z_2 I_1$$

$$C = \frac{I_1}{V_2} = \frac{1}{Z_2} = \frac{1}{8}$$

Pada saat V_2 *short circuit* ($V_2 = 0$) :

$$V_1 = -BI_2 \rightarrow B = -\frac{V_1}{I_2}$$

dim *ana* :

$$V_{Z_{23}} = \frac{\frac{Z_2.Z_3}{Z_2+Z_3}}{Z_1+\frac{Z_2.Z_3}{Z_2+Z_3}} V_1$$

$$V_{Z_{23}} = -Z_3 I_2$$

$$\frac{\frac{Z_2.Z_3}{Z_2+Z_3}}{Z_1+\frac{Z_2.Z_3}{Z_2+Z_3}} V_1 = -Z_3 I_2$$

$$B = -\frac{V_1}{I_2} = \frac{Z_1(Z_2+Z_3)+Z_2Z_3}{Z_2} = \frac{188}{8}$$

$$I_1 = -DI_2 \rightarrow D = -\frac{I_1}{I_2}$$

dim *ana* :

$$I_2 = -\frac{Z_2}{Z_2+Z_3} I_1$$

$$D = -\frac{I_1}{I_2} = \frac{Z_2+Z_3}{Z_2} = \frac{18}{8}$$

sehingga :

$$A = \frac{14}{8}$$

$$B = \frac{188}{8}$$

$$C = \frac{1}{8}$$

$$D = \frac{18}{8}$$

Konversi Parameter Y ke Parameter Z

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

$$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$V_1 = \frac{\begin{vmatrix} I_1 & Y_{12} \\ I_2 & Y_{22} \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{Y_{22}I_1 - Y_{12}I_2}{\Delta Y} = \frac{Y_{22}}{\Delta Y}I_1 - \frac{Y_{12}}{\Delta Y}I_2$$

$$V_2 = \frac{\begin{vmatrix} Y_{11} & I_1 \\ Y_{21} & I_2 \end{vmatrix}}{\begin{vmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{vmatrix}} = \frac{-Y_{21}I_1 + Y_{11}I_2}{\Delta Y} = -\frac{Y_{21}}{\Delta Y}I_1 + \frac{Y_{11}}{\Delta Y}I_2$$

sehingga :

$$Z_{11} = \frac{Y_{22}}{\Delta Y}$$

$$Z_{12} = -\frac{Y_{12}}{\Delta Y}$$

$$Z_{21} = -\frac{Y_{21}}{\Delta Y}$$

$$Z_{22} = \frac{Y_{11}}{\Delta Y}$$

Konversi Parameter Z ke Parameter Y

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

$$\begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

$$I_1 = \frac{\begin{vmatrix} V_1 & Z_{12} \\ V_2 & Z_{22} \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{Z_{22}V_1 - Z_{12}V_2}{\Delta Z} = \frac{Z_{22}}{\Delta Z}V_1 - \frac{Z_{12}}{\Delta Z}V_2$$

$$V_2 = \frac{\begin{vmatrix} Z_{11} & V_1 \\ Z_{21} & V_2 \end{vmatrix}}{\begin{vmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{vmatrix}} = \frac{-Z_{21}V_1 + Z_{11}V_2}{\Delta Z} = -\frac{Z_{21}}{\Delta Z}V_1 + \frac{Z_{11}}{\Delta Z}V_2$$

sehingga :

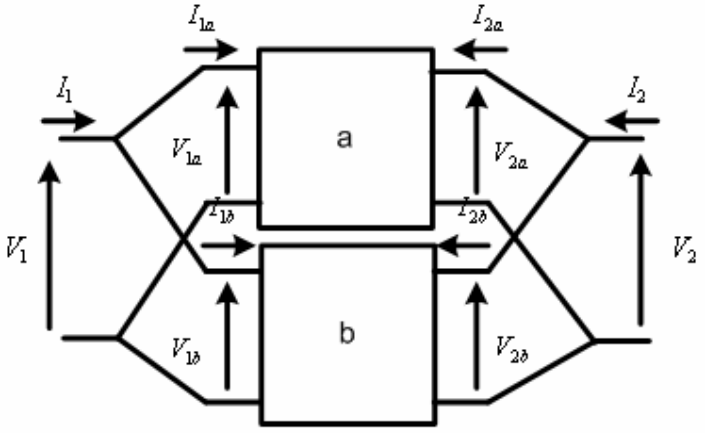
$$Y_{11} = \frac{Z_{22}}{\Delta Z}$$
$$Y_{12} = -\frac{Z_{12}}{\Delta Z}$$
$$Y_{21} = -\frac{Z_{21}}{\Delta Z}$$
$$Y_{22} = \frac{Z_{11}}{\Delta Z}$$

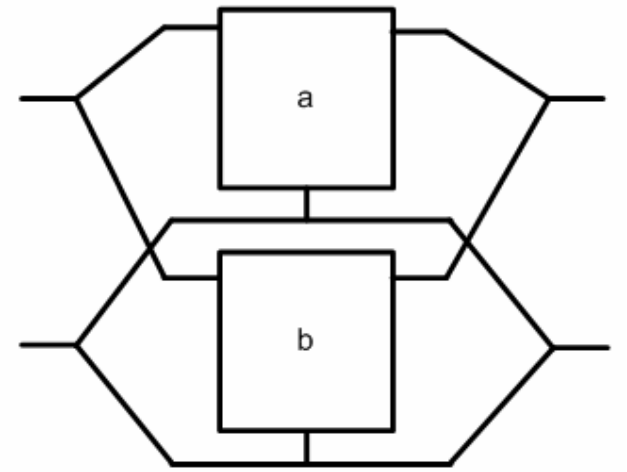
Tabel Konversi

$\begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix}$	$\begin{pmatrix} \frac{Y_{22}}{\Delta Y} & -\frac{Y_{12}}{\Delta Y} \\ -\frac{Y_{21}}{\Delta Y} & \frac{Y_{11}}{\Delta Y} \end{pmatrix}$	$\begin{pmatrix} \frac{A}{C} & \frac{\Delta T}{C} \\ \frac{1}{C} & \frac{D}{C} \end{pmatrix}$	$\begin{pmatrix} \frac{\Delta h}{h_{22}} & \frac{h_{12}}{h_{22}} \\ -\frac{h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{pmatrix}$
$\begin{pmatrix} \frac{Z_{22}}{\Delta Z} & -\frac{Z_{12}}{\Delta Z} \\ -\frac{Z_{21}}{\Delta Z} & \frac{Z_{11}}{\Delta Z} \end{pmatrix}$	$\begin{pmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{pmatrix}$	$\begin{pmatrix} \frac{D}{B} & -\frac{\Delta T}{B} \\ -\frac{1}{B} & \frac{A}{B} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{h_{11}} & -\frac{h_{12}}{h_{11}} \\ \frac{h_{21}}{h_{11}} & \frac{\Delta h}{h_{11}} \end{pmatrix}$
$\begin{pmatrix} \frac{Z_{11}}{Z_{21}} & \frac{\Delta Z}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{pmatrix}$	$\begin{pmatrix} -\frac{Y_{22}}{Y_{21}} & -\frac{1}{Y_{21}} \\ -\frac{\Delta Y}{Y_{21}} & -\frac{Y_{11}}{Y_{21}} \end{pmatrix}$	$\begin{pmatrix} A & B \\ C & D \end{pmatrix}$	$\begin{pmatrix} -\frac{\Delta h}{h_{21}} & -\frac{h_{11}}{h_{21}} \\ -\frac{h_{22}}{h_{21}} & -\frac{1}{h_{21}} \end{pmatrix}$
$\begin{pmatrix} \frac{\Delta Z}{Z_{22}} & \frac{Z_{12}}{Z_{22}} \\ -\frac{Z_{21}}{Z_{22}} & \frac{1}{Z_{22}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{Y_{11}} & -\frac{Y_{12}}{Y_{11}} \\ \frac{Y_{21}}{Y_{11}} & \frac{\Delta Y}{Y_{11}} \end{pmatrix}$	$\begin{pmatrix} \frac{B}{D} & \frac{\Delta T}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{pmatrix}$	$\begin{pmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{pmatrix}$

Interkoneksi Kutub Empat

1. Koneksi paralel





$$I_{1a} = Y_{11a}V_{1a} + Y_{12a}V_{2a}$$
$$I_{2a} = Y_{21a}V_{1a} + Y_{22a}V_{2a}$$
$$I_{1b} = Y_{11b}V_{1b} + Y_{12b}V_{2b}$$
$$I_{2b} = Y_{21b}V_{1b} + Y_{22b}V_{2b}$$

dimana :

$$V_1 = V_{1a} = V_{1b}$$
$$V_2 = V_{2a} = V_{2b}$$
$$I_1 = I_{1a} + I_{1b}$$
$$I_2 = I_{2a} + I_{2b}$$

maka :

$$I_1 = I_{1a} + I_{1b} = Y_{11a}V_{1a} + Y_{12a}V_{2a} + Y_{11b}V_{1b} + Y_{12b}V_{2b} = Y_{11a}V_{1a} + Y_{11b}V_{1b} + Y_{12a}V_{2a} + Y_{12b}V_{2b}$$
$$I_1 = (Y_{11a} + Y_{11b})V_1 + (Y_{12a} + Y_{12b})V_2$$
$$I_2 = I_{2a} + I_{2b} = Y_{21a}V_{1a} + Y_{22a}V_{2a} + Y_{21b}V_{1b} + Y_{22b}V_{2b} = Y_{21a}V_{1a} + Y_{21b}V_{1b} + Y_{22a}V_{2a} + Y_{22b}V_{2b}$$
$$I_2 = (Y_{21a} + Y_{21b})V_1 + (Y_{22a} + Y_{22b})V_2$$

dengan demikian :

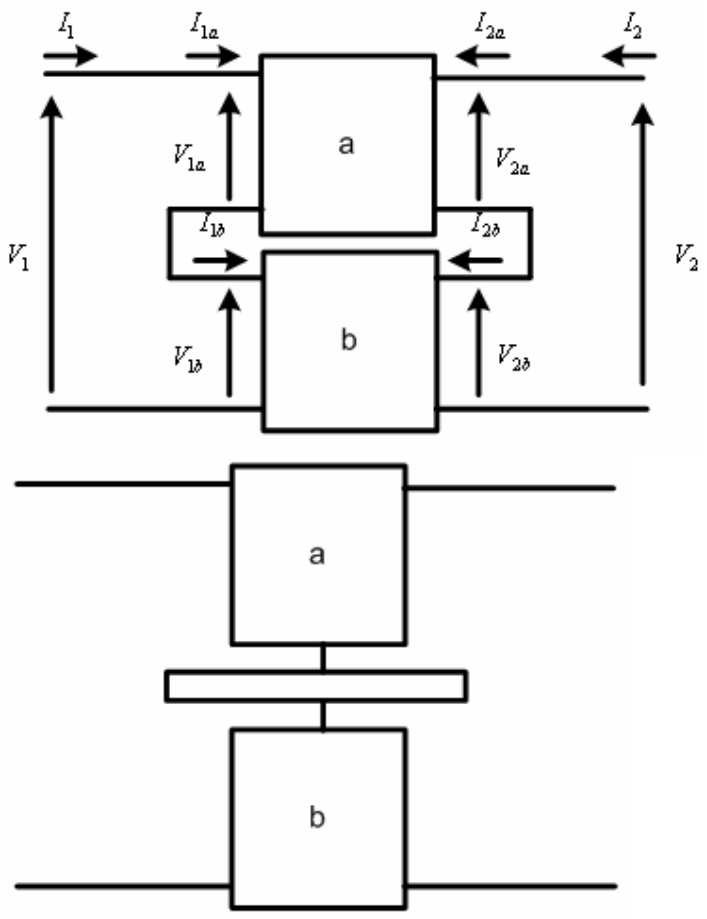
$Y_{11} = Y_{11a} + Y_{11b}$

$Y_{12} = Y_{12a} + Y_{12b}$

$Y_{21} = Y_{21a} + Y_{21b}$

$Y_{22} = Y_{22a} + Y_{22b}$

2. Koneksi seri



$V_{1a} = Z_{11a} I_{1a} + Z_{12a} I_{2a}$

$V_{2a} = Z_{21a} I_{1a} + Z_{22a} I_{2a}$

$V_{1b} = Z_{11b} I_{1b} + Z_{12b} I_{2b}$

$V_{2b} = Z_{21b} I_{1b} + Z_{22b} I_{2b}$

dimana :

$I_1 = I_{1a} = I_{1b}$

$I_2 = I_{2a} = I_{2b}$

maka :

$$V_1 = V_{1a} + V_{1b} = Z_{11a}I_{1a} + Z_{12a}I_{2a} + Z_{11b}I_{1b} + Z_{12b}I_{2b} = Z_{11a}I_{1a} + Z_{11b}I_{1b} + Z_{12a}I_{2a} + Z_{12b}I_{2b}$$

$$V_1 = (Z_{11a} + Z_{11b})I_1 + (Z_{12a} + Z_{12b})I_2$$

$$V_2 = V_{2a} + V_{2b} = Z_{21a}I_{1a} + Z_{22a}I_{2a} + Z_{21b}I_{1b} + Z_{22b}I_{2b} = Z_{21a}I_{1a} + Z_{21b}I_{1b} + Z_{22a}I_{2a} + Z_{22b}I_{2b}$$

$$V_2 = (Z_{21a} + Z_{21b})I_1 + (Z_{22a} + Z_{22b})I_2$$

dengan demikian :

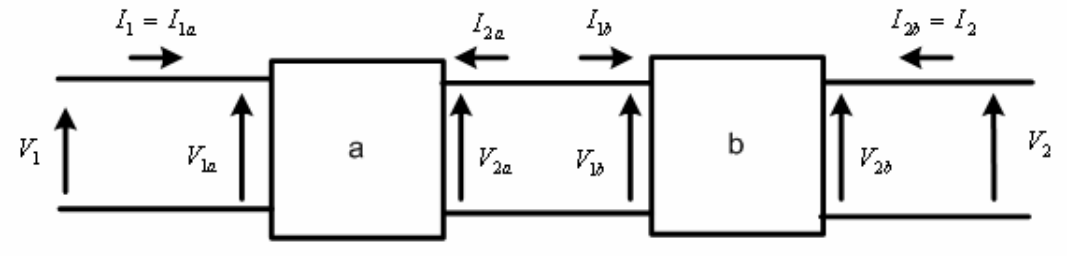
$$Z_{11} = Z_{11a} + Z_{11b}$$

$$Z_{12} = Z_{12a} + Z_{12b}$$

$$Z_{21} = Z_{21a} + Z_{21b}$$

$$Z_{22} = Z_{22a} + Z_{22b}$$

3. Koneksi Kaskade



$$V_1 = V_{1a} = A_aV_{2a} - B_aI_{2a} = A_aV_{1b} + B_aI_{1b} = A_a(A_bV_{2b} - B_bI_{2b}) + B_a(C_bV_{2b} - D_bI_{2b})$$

$$V_1 = (A_aA_b + B_aC_b)V_{2b} - (A_aB_b + B_aD_b)I_{2b}$$

$$I_1 = (C_aA_b + D_aC_b)V_2 - (C_aB_b + D_aD_b)I_2$$

dimana :

$$A = A_aA_b + B_aC_b$$

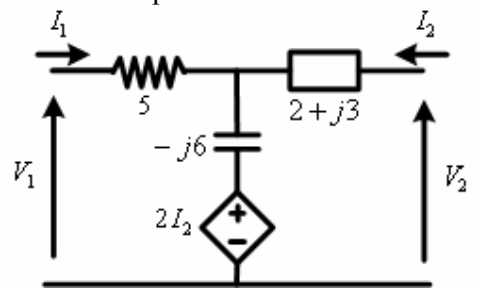
$$B = A_aB_b + B_aD_b$$

$$C = C_aA_b + D_aC_b$$

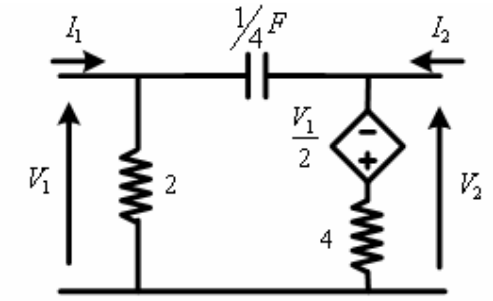
$$D = C_aB_b + D_aD_b$$

Soal – soal :

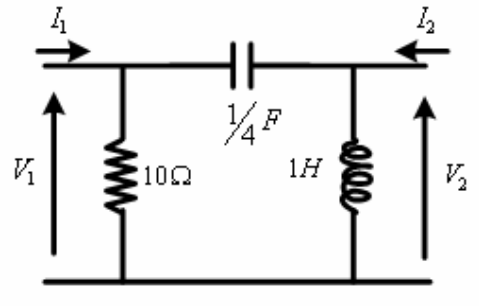
1. Tentukan parameter Z !



2. Tentukan parameter Y dalam $j\omega$!



3. Tentukan parameter Z dalam $j\omega$!

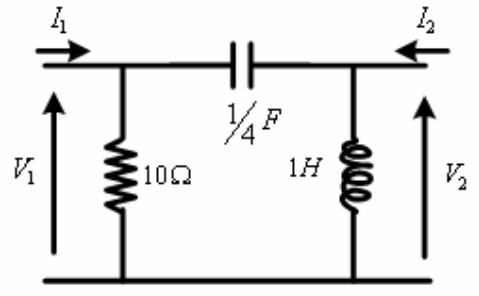


4. Jika parameter g dituliskan sebagai berikut :

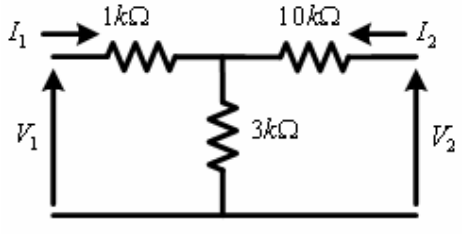
$\mathbf{I_1 = g_{11}V_1 + g_{12}I_2}$

$\mathbf{V_2 = g_{21}V_1 + g_{22}I_2}$

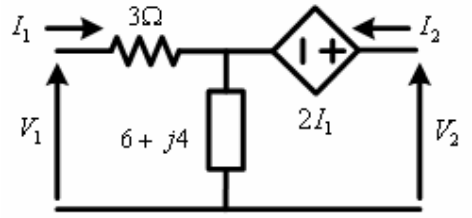
Tentukan g_{11}, g_{12}, g_{21}, dan g_{22} dari rangkaian disamping dalam domain $j\omega$!



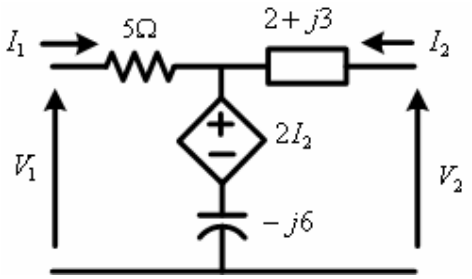
5. Tentukan paraameter Z rangkain berikut :



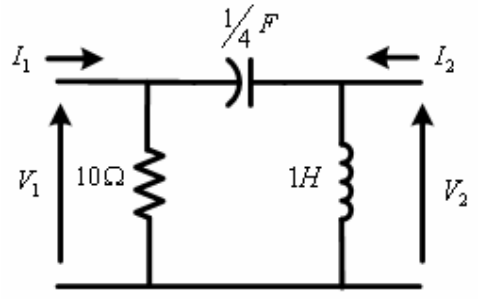
6. Tentukan parameter Z rangkaian berikut :



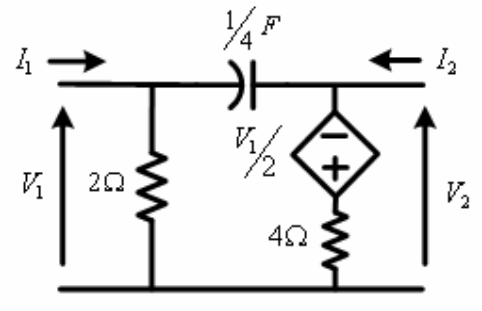
7. Tentukan parameter Z rangkain berikut :



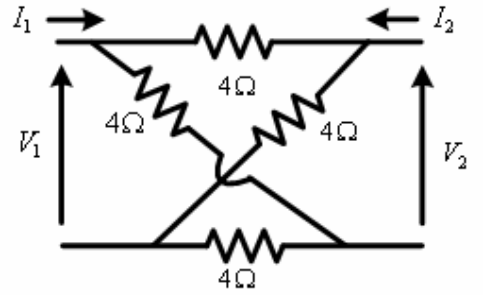
8. Tentukan parameter Y berikut dalam domain s :



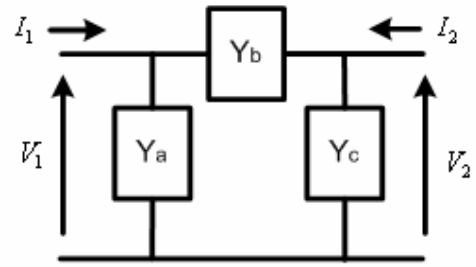
9. Tentukan parameter Y dalam domain s :



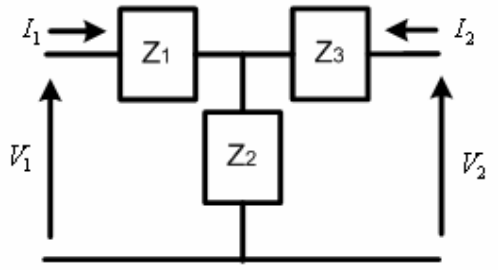
10. Tentukan parameter Z pada rangkaian berikut :



11. Tentukan parameter hibrid pada rangkaian berikut :



12. Tentukan parameter transmisi (ABCD) pada rangkaian berikut :



13. Jika parameter g dituliskan sebagai berikut :

$i_1 = g_{11}V_1 + g_{12}i_2$

$V_2 = g_{21}V_1 + g_{22}i_2$

Tentukan masing-masing parameter g pada gambar rangkain berikut dalam domain s