BAB X
RESPON FREKUENSI DAN RESONANSI

Respon frekuensi merupakan hubungan atau relasi frekuensi tak bebas pada kedua besaran magnitude dan phasa diantara input sinusoidal *steady state* dan output sinusoidal *steady state*.
Direpresentasikan sebagai perbandingan output respon $Y(j\omega)$ terhadap input sinusoidal $X(j\omega)$ atau yang lebih dikenal dengan fungsi transfer dalam domain $j\omega$:

$$H(j\omega) = \frac{Y(j\omega)}{X(j\omega)}$$

dimana :

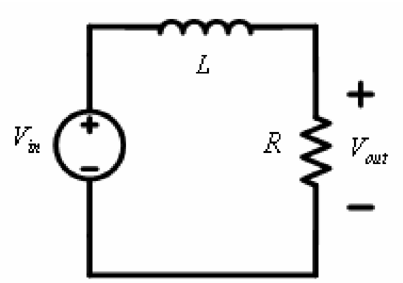
$$|H(j\omega)| = \frac{|Y(j\omega)|}{|X(j\omega)|}$$

$$\angle H(j\omega) = \frac{\angle Y(j\omega)}{\angle X(j\omega)} = \angle Y(j\omega) - \angle X(j\omega)$$

Misalkan :
Input $v_{in}(t) = A\cos(\omega_0 t + \theta)$ maka output $v_{out}(t) = A|H(j\omega)|\cos(\omega_0 t + \theta + \angle H(j\omega))$

Rangkaian RL

Jika komponen R sebagai output tegangan :



Fungsi transfer dalam domain s :

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + sL} = \frac{1}{1 + sL/R}$$

Jika $s = j\omega$, maka fungsi transfernya menjadi :

$$H(j\omega) = \frac{1}{1 + j\omega L/R}$$

sehingga respon frekuensi :

$$|H(j\omega)| = \frac{1}{\sqrt{1 + (\omega L/R)^2}}$$

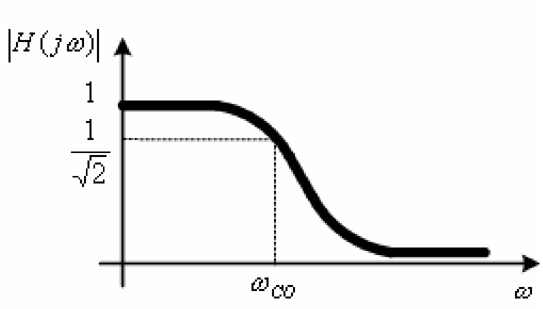
$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega L}{R}\right)$$

Gambar respon frekuensi magnitude :
saat :

$$\omega = 0 \Rightarrow |H(j\omega)| = 1$$

$$\omega = \infty \Rightarrow |H(j\omega)| = 0$$

$$\omega = \frac{R}{L} \Rightarrow |H(j\omega)| = \frac{1}{\sqrt{2}} \Rightarrow \textit{frekuensi..cut..off}$$

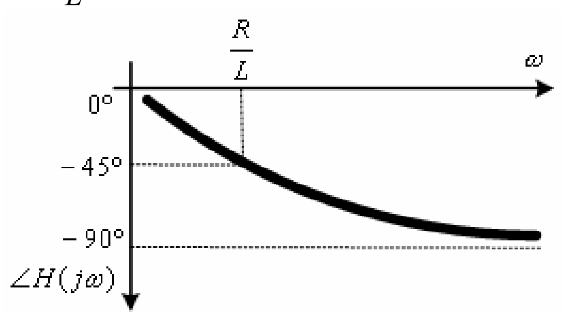


Gambar respon frekuensi phasa :
saat :

$$\omega = 0 \Rightarrow \angle H(j\omega) = 0°$$

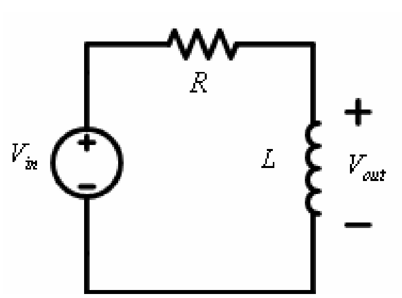
$$\omega = \infty \Rightarrow \angle H(j\omega) = -90°$$

$$\omega = \frac{R}{L} \Rightarrow \angle H(j\omega) = -45° \Rightarrow \textit{frekuensi..cut..off}$$



Rangkaian RL diatas sebagai *Low Pass Filter (LPF)*.

Jika komponen L sebagai output :



Fungsi transfer dalam domain s :

$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{sL}{sL+R}=\frac{1}{1+R/sL}$$

Jika $s=j\omega$, maka fungsi transfernya menjadi :

$$H(j\omega)=\frac{1}{1+R/j\omega L}=\frac{1}{1-jR/\omega L}$$

sehingga respon frekuensi :

$$|H(j\omega)|=\frac{1}{\sqrt{1+\left(R/\omega L\right)^2}}$$

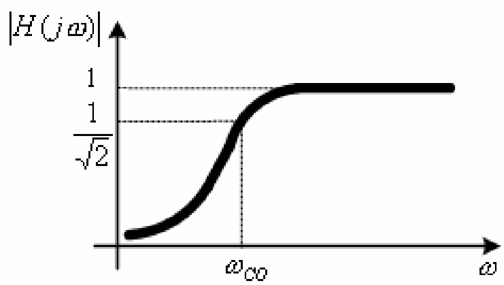
$$\angle H(j\omega)=-\tan^{-1}\left(-\frac{R}{\omega L}\right)$$

Gambar respon frekuensi magnitude :
saat :

$$\omega=0\Rightarrow\left|H\left(j\omega\right)\right|=0$$

$$\omega=\infty\Rightarrow\left|H\left(j\omega\right)\right|=1$$

$$\omega=\frac{R}{L}\Rightarrow\left|H\left(j\omega\right)\right|=\frac{1}{\sqrt{2}}\Rightarrow \mathit{frekuensi..cut..off}$$

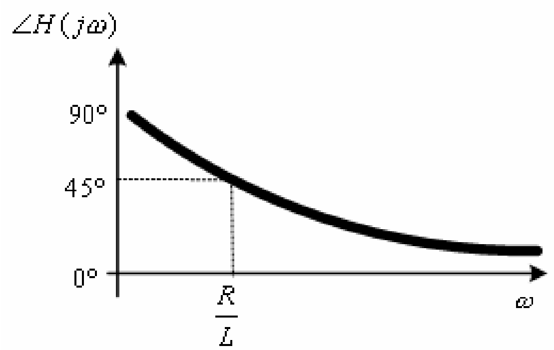


Gambar respon frekuensi phasa :
saat :

$$\omega=0\Rightarrow\angle H\left(j\omega\right)=90^\circ$$

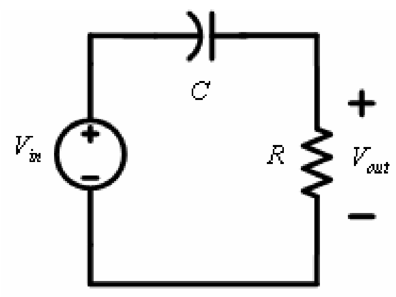
$$\omega=\infty\Rightarrow\angle H\left(j\omega\right)=0^\circ$$

$$\omega=\frac{R}{L}\Rightarrow\angle H\left(j\omega\right)=45^\circ\Rightarrow \mathit{frekuensi..cut..off}$$



Rangkaian RL diatas sebagai *High Pass Filter (HPF)*.

<u>Rangkaian RC</u>
Jika komponen R sebagai output tegangan :



Fungsi transfer dalam domain s :

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + \frac{1}{sC}} = \frac{1}{1 + \frac{1}{sCR}}$$

Jika $s = j\omega$, maka fungsi transfernya menjadi :

$$H(j\omega) = \frac{1}{1 + \frac{1}{j\omega CR}} = \frac{1}{1 - \frac{j}{\omega CR}}$$

sehingga respon frekuensi :

$$\left|H(j\omega)\right| = \frac{1}{\sqrt{1 + \left(\frac{1}{\omega CR}\right)^2}}$$

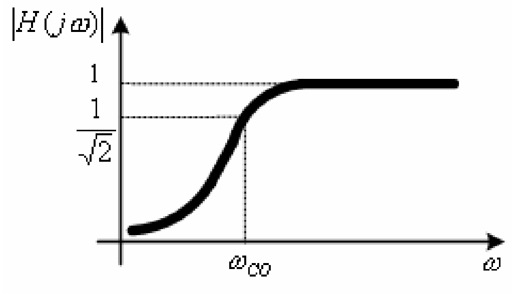
$$\angle H(j\omega) = -\tan^{-1}\left(-\frac{1}{\omega CR}\right)$$

Gambar respon frekuensi magnitude :
saat :

$$\omega = 0 \Rightarrow \left|H(j\omega)\right| = 0$$

$$\omega = \infty \Rightarrow \left|H(j\omega)\right| = 1$$

$$\omega = \frac{1}{CR} \Rightarrow \left|H(j\omega)\right| = \frac{1}{\sqrt{2}} \Rightarrow \textit{frekuensi..cut..off}$$

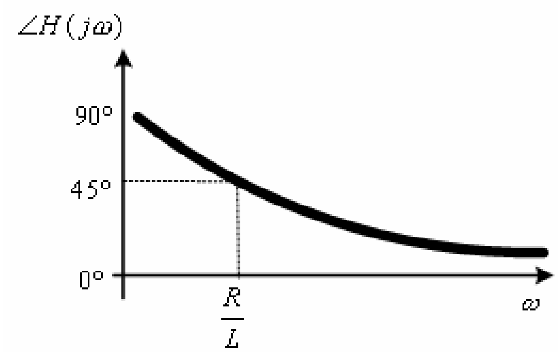


Gambar respon frekuensi phasa :
saat :

$$\omega = 0 \Rightarrow \angle H(j\omega) = 90^\circ$$

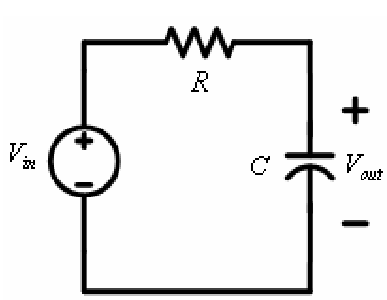
$$\omega = \infty \Rightarrow \angle H(j\omega) = 0^\circ$$

$$\omega = \frac{1}{CR} \Rightarrow \angle H(j\omega) = 45^\circ \Rightarrow frekuensi..cut..off$$



Rangkaian RC diatas sebagai *High Pass Filter (HPF)*.

Jika komponen C sebagai output :



Fungsi transfer dalam domain s :

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{1/_{sC}}{1/_{sC} + R} = \frac{1}{1 + sCR}$$

Jika $s = j\omega$, maka fungsi transfernya menjadi :

$$H(j\omega) = \frac{1}{1 + j\omega CR}$$

sehingga respon frekuensi :

$$|H(j\omega)| = \frac{1}{\sqrt{1+(\omega CR)^2}}$$

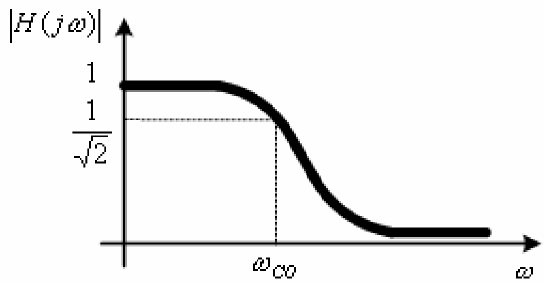
$$\angle H(j\omega) = -\tan^{-1}(\omega CR)$$

Gambar respon frekuensi magnitude :
saat :

$$\omega = 0 \Rightarrow |H(j\omega)| = 1$$

$$\omega = \infty \Rightarrow |H(j\omega)| = 0$$

$$\omega = \frac{1}{CR} \Rightarrow |H(j\omega)| = \frac{1}{\sqrt{2}} \Rightarrow \textit{frekuensi..cut..off}$$

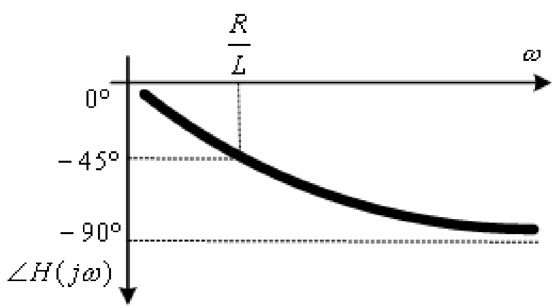


Gambar respon frekuensi phasa :
saat :

$$\omega = 0 \Rightarrow \angle H(j\omega) = 0°$$

$$\omega = \infty \Rightarrow \angle H(j\omega) = -90°$$

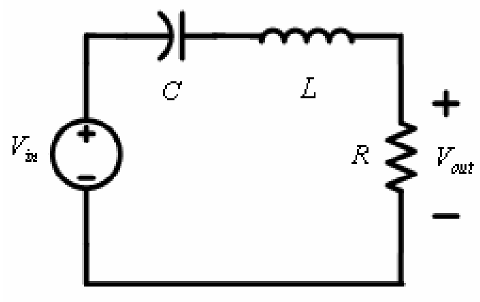
$$\omega = \frac{1}{CR} \Rightarrow \angle H(j\omega) = -45° \Rightarrow \textit{frekuensi..cut..off}$$



Rangkaian RC diatas sebagai *Low Pass Filter (LPF)*.

<u>Rangkaian RLC</u>

Jika komponen R sebagai output tegangan :



Fungsi transfer dalam domain s :

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R + sL + \frac{1}{sC}} = \frac{1}{1 + \frac{sL}{R} + \frac{1}{sCR}}$$

Jika $s = j\omega$, maka fungsi transfernya menjadi :

$$H(j\omega) = \frac{1}{1 + \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R}}$$

sehingga respon frekuensi :

$$|H(j\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)^2}}$$

$$\angle H(j\omega) = -\tan^{-1}\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

Gambar respon frekuensi magnitude :

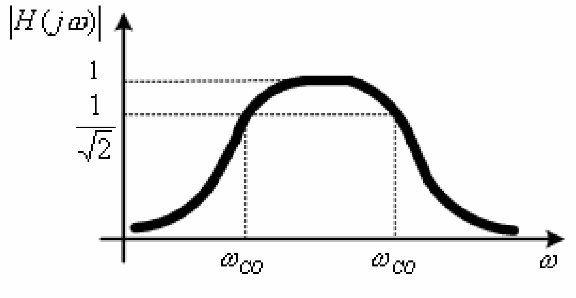
saat :

$$\omega = 0 \Rightarrow |H(j\omega)| = 0$$

$$\omega = \infty \Rightarrow |H(j\omega)| = 0$$

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow |H(j\omega)| = 1$$

$$\omega = \frac{R \pm \sqrt{R^2 + \frac{4L}{C}}}{2L} \Rightarrow |H(j\omega)| = \frac{1}{\sqrt{2}} \Rightarrow \textit{frekuensi..cut..off}$$



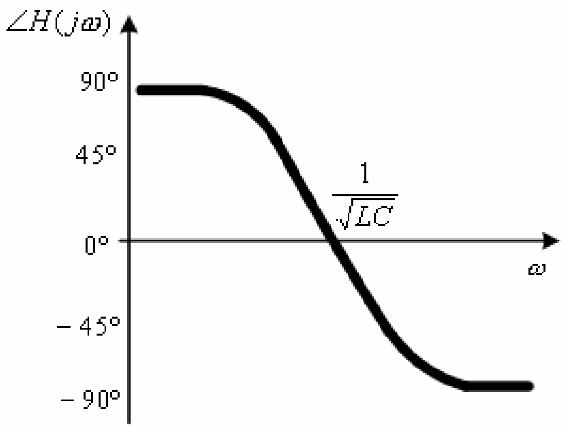
Gambar respon frekuensi phasa :
saat :

$$\omega = 0 \Rightarrow \angle H(j\omega) = 90°$$

$$\omega = \infty \Rightarrow \angle H(j\omega) = -90°$$

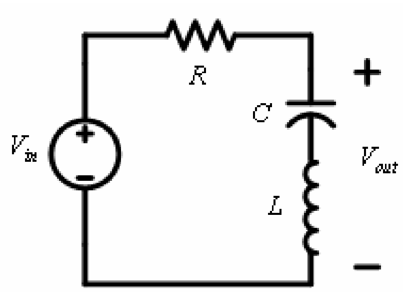
$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow \angle H(j\omega) = 0°$$

$$\omega = \frac{R \pm \sqrt{R^2 + 4L/C}}{2L} \Rightarrow \angle H(j\omega) = \pm 45° \Rightarrow frekuensi..cut..off$$



Rangkaian RLC diatas sebagai *Band Pass Filter (BPF)*.

Jika komponen LC sebagai output tegangan :



Fungsi transfer dalam domain s :

$$H(s)=\frac{V_{out}(s)}{V_{in}(s)}=\frac{sL+\frac{1}{sC}}{R+sL+\frac{1}{sC}}=\frac{1}{1+\frac{R}{\left(sL+\frac{1}{sC}\right)}}$$

Jika $s=j\omega$, maka fungsi transfernya menjadi :

$$H(j\omega)=\frac{1}{1+\frac{R}{j\left(\omega L-\frac{1}{\omega C}\right)}}=\frac{1}{1-\frac{jR}{\left(\omega L-\frac{1}{\omega C}\right)}}$$

sehingga respon frekuensi :

$$\left|H(j\omega)\right|=\frac{1}{\sqrt{1+\left(\frac{R}{\omega L-\frac{1}{\omega C}}\right)^2}}$$

$$\angle H(j\omega)=-\tan^{-1}\left(-\frac{R}{\omega L-\frac{1}{\omega C}}\right)$$

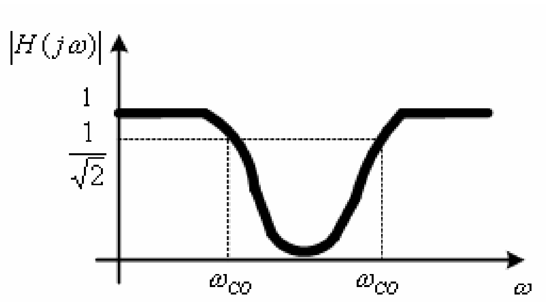
Gambar respon frekuensi magnitude :
saat :

$$\omega=0\Rightarrow\left|H(j\omega)\right|=1$$

$$\omega=\infty\Rightarrow\left|H(j\omega)\right|=1$$

$$\omega=\frac{1}{\sqrt{LC}}\Rightarrow\left|H(j\omega)\right|=0$$

$$\omega=\frac{R\pm\sqrt{R^2+\frac{4L}{C}}}{2L}\Rightarrow\left|H(j\omega)\right|=\frac{1}{\sqrt{2}}\Rightarrow \textit{frekuensi..cut..off}$$



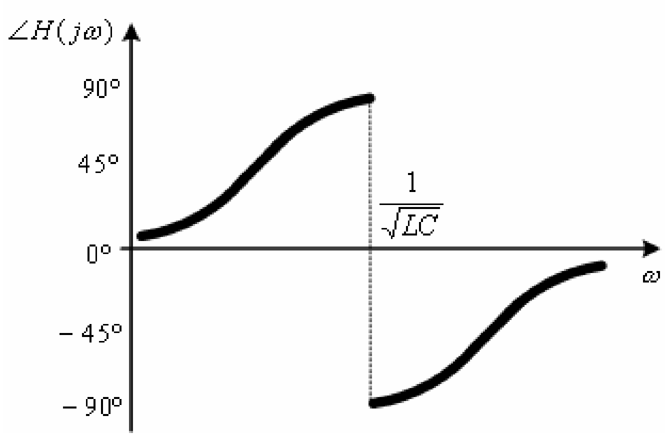
Gambar respon frekuensi phasa :
saat :

$$\omega=0\Rightarrow\angle H(j\omega)=0°$$

$$\omega=\infty\Rightarrow\angle H(j\omega)=0°$$

$$\omega=\frac{1}{\sqrt{LC}}\Rightarrow\angle H(j\omega)=90°$$

$$\omega = \frac{R \pm \sqrt{R^2 + 4L/C}}{2L} \Rightarrow \angle H(j\omega) = \pm 45° \Rightarrow \textit{frekuensi..cut..off}$$



Rangkaian RLC diatas sebagai *Band Stop Filter (BSF)*.

Resonansi

Suatu rangkaian dikatakan beresonansi ketika tegangan terpasang V dan arus yang dihasilkan I dalam kondisi satu phasa.

Misalkan :

$V = A\angle\alpha°$

$I = B\angle\beta°$

Dalam kondisi satu phasa : $\alpha° = \beta°$, sehingga :

$$Z = \frac{V}{I} = \frac{A\angle\alpha°}{B\angle\beta°} = \frac{A}{B}\angle(\alpha° - \beta°) = \frac{A}{B}\angle 0° = \frac{A}{B}$$

Terlihat bahwa ketika V dan I satu phasa, impedansi yang dihasilkan seluruhnya komponen riil atau impedansi kompleks hanya terdiri dari komponen resistor murni (R). Dengan kata lain konsep resonansi adalah menghilangkan komponen imaginer / reaktansi saling meniadakan.

Resonansi Seri

Impedansi total:

$$\mathbf{Z}_{tot} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

saat resonansi :

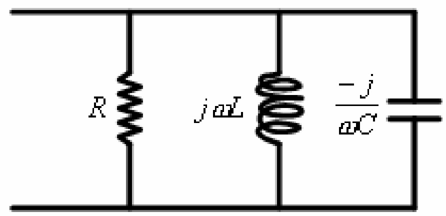
$$\omega L - \frac{1}{\omega C} = 0 \rightarrow \omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$f_o = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Pada saat resonansi impedansi **Z** minimum, sehingga arusnya maksimum.

Resonansi Paralel



Admitansi total :

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R} + \frac{1}{j\omega L} + \frac{1}{-j/\omega C} = \frac{1}{R} - \frac{j}{\omega L} + j\omega C$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

saat resonansi :

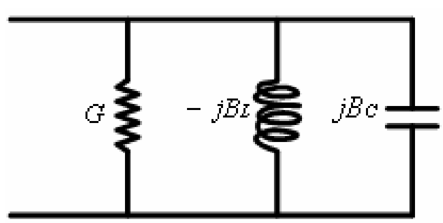
$$\omega C - \frac{1}{\omega L} = 0 \rightarrow \omega C = \frac{1}{\omega L}$$

$$\omega^2 = \frac{1}{LC}$$

$$f_o = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Pada saat resonansi impedansi **Z** maksimum, sehingga arusnya minimum.

Gambar tersebut dapat diganti notasinya :



Admitansi total :

$$Y = G + jB_C - jB_L$$

$$Y = G + j(\omega C - \frac{1}{\omega L})$$

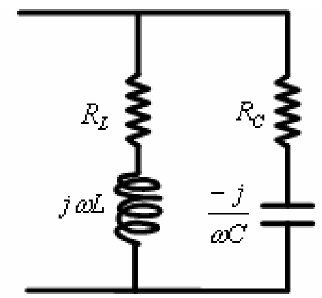
saat resonansi :

$$\omega C - \frac{1}{\omega L} = 0 \rightarrow \omega C = \frac{1}{\omega L}$$

$$\omega^2 = \frac{1}{LC}$$

$$f_o = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

Resonansi Paralel 2 Cabang



$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R_L + j\omega L} + \frac{1}{R_C - \frac{j}{\omega C}}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R_L + j\omega L}\left(\frac{R_L - j\omega L}{R_L - j\omega L}\right) + \frac{1}{R_C - \frac{j}{\omega C}}\left(\frac{R_C + \frac{j}{\omega C}}{R_C + \frac{j}{\omega C}}\right)$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R_L - j\omega L}{{R_L}^2 + (\omega L)^2} + \frac{R_C + \frac{j}{\omega C}}{{R_C}^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R_L}{{R_L}^2 + (\omega L)^2} + \frac{R_C}{{R_C}^2 + \left(\frac{1}{\omega C}\right)^2} + j\left(\frac{\frac{1}{\omega C}}{{R_C}^2 + \left(\frac{1}{\omega C}\right)^2} - \frac{\omega L}{{R_L}^2 + (\omega L)^2}\right)$$

saat resonansi:

$$\frac{\frac{1}{\omega C}}{{R_C}^2 + \left(\frac{1}{\omega C}\right)^2} - \frac{\omega L}{{R_L}^2 + (\omega L)^2} = 0$$

$$\frac{\frac{1}{\omega C}}{R_C{}^2+\left(\frac{1}{\omega C}\right)^2}=\frac{\omega L}{R_L{}^2+(\omega L)^2}$$

$$R_L{}^2+(\omega L)^2=\omega^2 LC\left(R_C{}^2+\left(\frac{1}{\omega C}\right)^2\right)$$

$$R_L{}^2+\omega^2 L^2=\omega^2 LCR_C{}^2+\frac{L}{C}$$

$$\omega^2 LCR_C{}^2-\omega^2 L^2=R_L{}^2-\frac{L}{C}$$

$$\omega^2 LC\left(R_C{}^2-\frac{L}{C}\right)=R_L{}^2-\frac{L}{C}$$

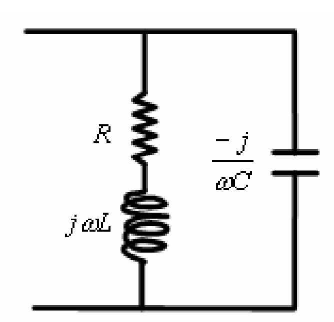
$$f_o=\frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{R_L{}^2-\frac{L}{C}}{R_C{}^2-\frac{L}{C}}}$$

Perlu diingat bahwa : $\sqrt{\frac{R_L{}^2-\frac{L}{C}}{R_C{}^2-\frac{L}{C}}}$ harus positif real sehingga syarat :

$R_l{}^2>\frac{L}{C}$ dan $R_C{}^2>\frac{L}{C}$ atau $R_L{}^2<\frac{L}{C}$ dan $R_C{}^2<\frac{L}{C}$

Ketika nilai $R_L{}^2=R_C{}^2=\frac{L}{C}$, maka rangkaian beresonansi untuk semua frekuensi.

Resonansi Kombinasi 1



$$\mathbf{Z}_1=R+j\omega L$$

$$\mathbf{Z}_2=\frac{1}{j\omega C}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} = \frac{1}{R + j\omega L} + \frac{1}{\dfrac{1}{j\omega C}} = \frac{1}{R + j\omega L} + j\omega C$$

$$\frac{1}{\mathbf{Z}_{tot}} = j\omega C + \frac{1}{R + j\omega L}\left(\frac{R - j\omega L}{R - j\omega L}\right)$$

$$\frac{1}{\mathbf{Z}_{tot}} = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

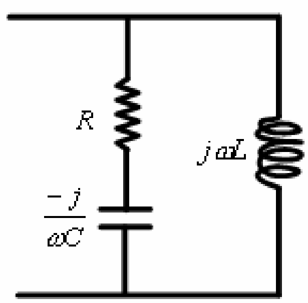
$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

saat resonansi : $\omega_C = \dfrac{\omega_L}{R^2 + \omega^2 L^2}$, sehingga :

$$R^2 + \omega^2 L^2 = \frac{L}{C} \rightarrow \omega^2 L^2 = \frac{L}{C} - R^2 \rightarrow \omega^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2} = \frac{1}{LC}\left(1 - \frac{R^2 C}{L^2}\right)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(1 - \frac{R^2 C}{L^2}\right)}$$

Resonansi Kombinasi 2



$$\mathbf{Z}_1 = R + \frac{1}{j\omega C} = R - \frac{j}{\omega C}$$

$$\mathbf{Z}_2 = j\omega L$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} = \frac{1}{R - \dfrac{j}{\omega C}} + \frac{1}{j\omega L} = \frac{1}{R - \dfrac{j}{\omega C}} - \frac{j}{\omega L}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R - \dfrac{j}{\omega C}}\left(\frac{R + \dfrac{j}{\omega C}}{R + \dfrac{j}{\omega C}}\right) - \frac{j}{\omega L} = \frac{R + \dfrac{j}{\omega C}}{R^2 + \dfrac{1}{\omega^2 C^2}} - \frac{j}{\omega L}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R}{R^2 + \dfrac{1}{\omega^2 C^2}} + j\left(\frac{\dfrac{1}{\omega C}}{R^2 + \dfrac{1}{\omega^2 C^2}} - \frac{1}{\omega L}\right)$$

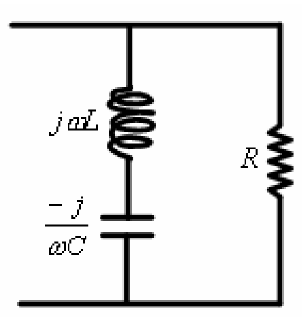
saat resonansi : $\dfrac{\dfrac{1}{\omega C}}{R^2 + \dfrac{1}{\omega^2 C^2}} = \dfrac{1}{\omega L}$, sehingga :

$$R^2 + \frac{1}{\omega^2 C^2} = \frac{L}{C} \rightarrow \frac{1}{\omega^2 C^2} = \frac{L}{C} - R^2 \rightarrow \omega^2 C^2 = \frac{1}{\dfrac{L}{C} - R^2} \rightarrow \omega^2 = \frac{1}{C^2\left(\dfrac{L}{C} - R^2\right)}$$

$$\omega^2 = \frac{1}{LC - C^2 R^2} = \frac{1}{LC\left(1 - \dfrac{CR^2}{L}\right)}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{\left(1 - \dfrac{CR^2}{L}\right)}}$$

Resonansi Kombinasi 3



$$\mathbf{Z}_1 = j\omega L + \frac{1}{j\omega C}$$

$$\mathbf{Z}_2 = R$$

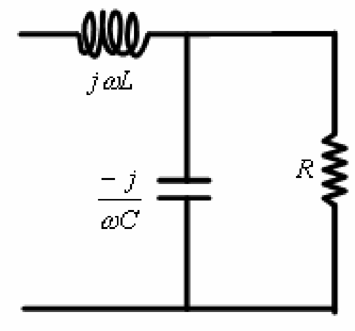
$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} = \frac{1}{j\omega L + \dfrac{1}{j\omega C}} + \frac{1}{R} = \frac{1}{R} - \frac{j\omega C}{1 - \omega^2 LC}$$

saat resonansi :

$$\frac{\omega C}{1 - \omega^2 LC} = 0$$

$$fo = 0$$

Resonansi Kombinasi 4



$$\frac{1}{\mathbf{Z}_1} = \frac{1}{R} + \frac{1}{\frac{1}{j\omega C}} = \frac{1}{R} + j\omega C \rightarrow \mathbf{Z}_1 = \frac{1}{\frac{1}{R} + j\omega C}$$

$$\mathbf{Z}_2 = j\omega L$$

$$\mathbf{Z}_{tot} = \mathbf{Z}_1 + \mathbf{Z}_2 = \frac{1}{\frac{1}{R} + j\omega C} + j\omega L$$

$$\mathbf{Z}_{tot} = j\omega L + \frac{1}{\frac{1}{R} + j\omega C}\left(\frac{\frac{1}{R} - j\omega C}{\frac{1}{R} - j\omega C}\right) = j\omega L + \frac{\frac{1}{R} - j\omega C}{\frac{1}{R^2} + \omega^2 C^2}$$

$$\mathbf{Z}_{tot} = \frac{\frac{1}{R}}{\frac{1}{R^2} + \omega^2 C^2} + j\left(\omega L - \frac{\omega C}{\frac{1}{R^2} + \omega^2 C^2}\right)$$

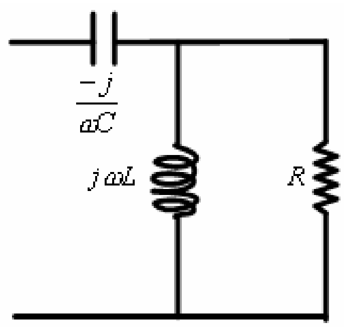
saat resonansi : $\omega L = \dfrac{\omega C}{\frac{1}{R^2} + \omega^2 C^2}$, sehingga :

$$\frac{1}{R^2} + \omega^2 C^2 = \frac{C}{L} \rightarrow \omega^2 C^2 = \frac{C}{L} - \frac{1}{R^2}$$

$$\omega^2 = \frac{1}{C^2}\left(\frac{C}{L} - \frac{1}{R^2}\right) = \frac{1}{LC} - \frac{1}{C^2 R^2} = \frac{1}{LC}\left(1 - \frac{L}{CR^2}\right)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{L}{CR^2}}$$

Resonansi Kombinasi 5



$$\frac{1}{\mathbf{Z}_1} = \frac{1}{R} + \frac{1}{j\omega L} = \frac{1}{R} - \frac{j}{\omega L} \rightarrow \mathbf{Z}_1 = \frac{1}{\frac{1}{R} - \frac{j}{\omega L}}$$

$$\mathbf{Z}_2 = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$

$$\mathbf{Z}_{tot} = \mathbf{Z}_1 + \mathbf{Z}_2 = \frac{1}{\frac{1}{R} - \frac{j}{\omega L}} - \frac{j}{\omega C} = \frac{1}{\frac{1}{R} - \frac{j}{\omega L}}\left(\frac{\frac{1}{R} + \frac{j}{\omega L}}{\frac{1}{R} + \frac{j}{\omega L}}\right) - \frac{j}{\omega C}$$

$$\mathbf{Z}_{tot} = \frac{\frac{1}{R} + \frac{j}{\omega L}}{\frac{1}{R^2} + \frac{1}{\omega^2 L^2}} - \frac{j}{\omega C} = \frac{\frac{1}{R}}{\frac{1}{R^2} + \frac{1}{\omega^2 L^2}} + j\left(\frac{\frac{1}{\omega L}}{\frac{1}{R^2} + \frac{1}{\omega^2 L^2}} - \frac{1}{\omega C}\right)$$

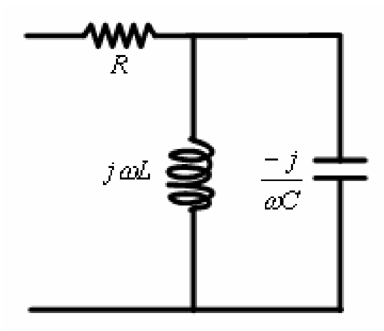
saat resonansi : $\frac{\frac{1}{\omega L}}{\frac{1}{R^2} + \frac{1}{\omega^2 L^2}} = \frac{1}{\omega C}$, sehingga :

$$\frac{1}{R^2} + \frac{1}{\omega^2 L^2} = \frac{C}{L} \rightarrow \frac{1}{\omega^2 L^2} = \frac{C}{L} - \frac{1}{R^2} \rightarrow \omega^2 L^2 = \frac{1}{\frac{C}{L} - \frac{1}{R^2}}$$

$$\omega^2 = \frac{1}{L^2\left(\frac{C}{L} - \frac{1}{R^2}\right)} = \frac{1}{LC\left(1 - \frac{L}{CR^2}\right)}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\frac{1}{1 - \frac{L}{CR^2}}}$$

Resonansi Kombinasi 6



$$\frac{1}{\mathbf{Z}_1} = \frac{1}{j\omega L} + \frac{1}{\frac{1}{j\omega C}} = \frac{1}{j\omega L} + j\omega C \rightarrow \mathbf{Z}_1 = \frac{1}{\frac{1}{j\omega L} + j\omega C}$$

$$\mathbf{Z}_2 = R$$

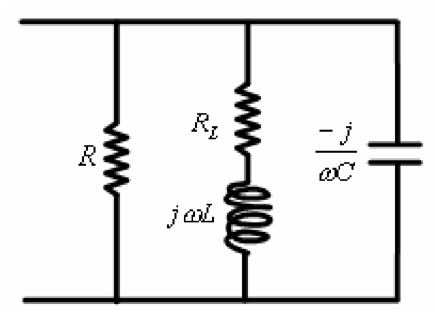
$$\mathbf{Z}_{tot} = \mathbf{Z}_1 + \mathbf{Z}_2 = \frac{1}{\frac{1}{j\omega L} + j\omega C} + R = R + \frac{j\omega L}{1-\omega^2 LC}$$

saat resonansi :

$$\frac{\omega L}{1-\omega^2 LC} = 0$$

$$fo = 0$$

Resonansi Paralel 3 Cabang



$$\mathbf{Z}_1 = R_L + j\omega L$$

$$\mathbf{Z}_2 = R$$

$$\mathbf{Z}_3 = \frac{1}{j\omega C}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \frac{1}{\mathbf{Z}_3} = \frac{1}{R_L + j\omega L} + \frac{1}{R} + \frac{1}{\frac{1}{j\omega C}}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R} + j\omega C + \frac{1}{R_L + j\omega L} = \frac{1}{R} + j\omega C + \frac{1}{R_L + j\omega L}\left(\frac{R_L - j\omega L}{R_L - j\omega L}\right)$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{1}{R} + j\omega C + \frac{R_L - j\omega L}{R_L^{\ 2} + \omega^2 L^2} = \frac{1}{R} + \frac{R_L}{R_L^{\ 2} + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R_L^{\ 2} + \omega^2 L^2}\right)$$

saat resonansi : $\omega C = \dfrac{\omega L}{R_L^{\ 2} - \omega^2 L^2}$, sehingga :

$$R_L^{\ 2} + \omega^2 L^2 = \frac{L}{C} \rightarrow \omega^2 L^2 = \frac{L}{C} - R_L^{\ 2}$$

$$\omega^2 = \frac{1}{L^2}\left(\frac{L}{C} - R_L^{\ 2}\right) = \frac{1}{LC} - \frac{R_L^{\ 2}}{L^2} = \frac{1}{LC}\left(1 - \frac{CR_L^{\ 2}}{L}\right)$$

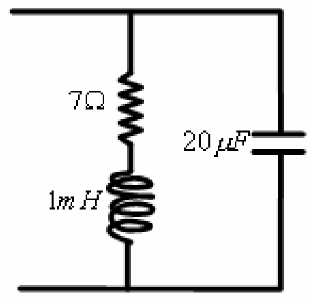
$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{CR_L^{\ 2}}{L}}$$

Contoh latihan :

1. Suatu rangkaian seri RLC dengan $R = 50\Omega, L = 0{,}05H, C = 20\mu F$ terpasang pada $V = 100\angle 0^o$ dengan frekuensi variabel. Pada frekuensi berapa tegangan induktor mencapai maksimum ? Berapakah tegangan induktor tersebut ?
   Jawaban :
   Tegangan induktor maksimum jika arus maksimum, arus maksimum jika Z minimum, Z minimum terjadi saat resonansi.
   $$f_o = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0{,}05.20.10^{-6}}} = 159{,}1Hz$$
   $$Z_{resonansi} = R \rightarrow i_{maks} = \frac{V}{Z_{res}} = \frac{100\angle 0^o}{50} = 2\angle 0^o$$
   $$V_L = i_{maks}.X_L = i_{maks}.j\omega L = 2\angle 0^o.2\pi fL\angle 90^o = 2\angle 0^o.2\pi.159{,}1.0{,}05\angle 90^o = 100\angle 90^o$$

2. Pada saat terjadi resonansi tegangan terpasang pada rangkaian seri RLC adalah $v = 70{,}7\sin(500t + 30^o)V$ menghasilkan arus sebesar $i = 2{,}83\sin(500t + 30^o)A$, jika $L = 0{,}5H$. Tentukan nilai R dan C !
   Jawaban :
   $$Z = \frac{V}{I} = \frac{70{,}7\angle 30^o}{2{,}83\angle 30^o} = 25 \rightarrow R = 25\Omega$$
   $$f_o = \frac{1}{2\pi\sqrt{LC}} \rightarrow \omega^2 = \frac{1}{LC}$$
   $$C = \frac{1}{\omega^2 L} = \frac{1}{500^2.0{,}5} = 8\mu F$$

3. Tentukan frekuensi resonansi pada gambat berikut :



Jawaban :

$$\frac{1}{\mathbf{Z}_{tot}} = j\omega C + \frac{1}{R + j\omega L}\left(\frac{R - j\omega L}{R - j\omega L}\right)$$

$$\frac{1}{\mathbf{Z}_{tot}} = j\omega C + \frac{R - j\omega L}{R^2 + \omega^2 L^2}$$

$$\frac{1}{\mathbf{Z}_{tot}} = \frac{R}{R^2 + \omega^2 L^2} + j\left(\omega C - \frac{\omega L}{R^2 + \omega^2 L^2}\right)$$

saat resonansi : $\omega_C = \dfrac{\omega_L}{R^2 + \omega^2 L^2}$, sehingga :

$$R^2 + \omega^2 L^2 = \frac{L}{C} \rightarrow \omega^2 L^2 = \frac{L}{C} - R^2 \rightarrow \omega^2 = \frac{1}{L^2}\left(\frac{L}{C} - R^2\right) = \frac{1}{LC} - \frac{R^2}{L^2} = \frac{1}{LC}\left(1 - \frac{R^2 C}{L^2}\right)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{\left(1 - \frac{R^2 C}{L^2}\right)} = \frac{1}{2\pi\sqrt{10^{-3}.20.10^{-6}}}\sqrt{\left(1 - \frac{7^2.20.10^{-6}}{10^{-3}}\right)} = 159{,}2Hz$$

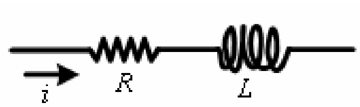
<u>Faktor Kualitas (Q)</u>

Definisi (dasar) dari Q :

$$Q = 2\pi \frac{\text{energi maksimum yang disimpan}}{\text{energi yang disipasikan tiap getaran/"percycle"}}$$

Faktor kualitas merupakan ukuran selektivitas rangkaian resonator dimana rangkaian resonator merupakan rangkaian filter BPF dengan lebar pita/*bandwidth* sempit. Semakin besar nilai Q maka semakin sempit lebar pita/*bandwidth*.

Pada Komponen RL



Misalkan : $i = I_m \sin\omega t$

<u>Pada L</u> :

$$\mathbf{V}_L(t) = L\frac{di}{dt} = I_m \omega L \cos\omega t$$

Energi :

$$\mathbf{W}_L(t) = \int_0^t \mathbf{P}_L(t)dt = \int_0^t \mathbf{V}_L(t).i(t)dt$$

$$\mathbf{W}_L(t) = \int_0^t I_m{}^2 \omega L \sin\omega t \cos\omega t dt = \int_0^t I_m{}^2 \omega L \frac{\sin 2\omega t}{2} dt = \frac{I_m{}^2 \omega L}{2} \int_0^t \sin 2\omega t dt = \frac{1}{2} I_m{}^2 L \sin^2 \omega t$$

Maksimum energi yang disimpan : $\mathbf{W}_L \max = \frac{1}{2} L I_m{}^2$

<u>Pada R</u> :
Energi :

$$\mathbf{W}_R(t) = \int_0^t \mathbf{P}_R(t)dt = \int_0^t \mathbf{V}_R(t)i(t)dt = \int_0^t R I_m{}^2 \sin^2 \omega t dt = \int_0^t R I_m{}^2 \frac{(1-\cos 2\omega t)}{2} dt$$

$$\mathbf{W}_R(t) = \frac{R I_m{}^2}{2} \int_0^t (1-\cos 2\omega t)dt = \frac{R I_m{}^2}{2}\left(t - \frac{1}{2\omega}\sin 2\omega t\right) \rightarrow T = \frac{1}{f} = \left(t - \frac{1}{2\omega}\sin 2\omega t\right)$$

Energi yang didisipasikan per cycle : $\frac{1}{2} R I_m{}^2 \frac{1}{f}$, sehingga :

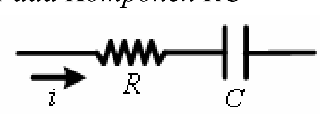
$$Q_L = 2\pi \frac{\text{energi maksimum yang disimpan}}{\text{energi yang disipasikan tiap getaran/"per cycle"}}$$

$$\mathbf{Q}_L = 2\pi \frac{\frac{1}{2} L I_m{}^2}{\frac{1}{2} R I_m{}^2 \frac{1}{f}} = 2\pi f \frac{L}{R} = \frac{\omega L}{R}$$

Jadi faktor kualitas untuk rangkaian seri RL :

$$\mathbf{Q}_L = \frac{\omega L}{R}$$

Pada Komponen RC



Misalkan : $\mathbf{V}_C = \mathbf{V}_m \sin\omega t$

<u>Pada C</u> :

$$i_c(t) = C\frac{dV_C}{dt} = C V_m \omega \cos\omega t$$

Energi :

$$\mathbf{W}_C(t) = \int_0^t \mathbf{P}_C(t)dt = \int_0^t V_C(t) i_C(t)dt$$

$$\mathbf{W}_C(t) = \int_0^t V_m{}^2 \omega C \sin\omega t \cos\omega t dt = \int_0^t V_m{}^2 \omega C \frac{\sin 2\omega t}{2} dt = \frac{V_m{}^2 \omega C}{2} \int_0^t \sin 2\omega t dt = \frac{1}{2} V_m{}^2 C \sin^2 \omega t$$

Maksimum energi yang disimpan : $\mathbf{W}_C \max = \frac{1}{2} C V_m{}^2$

<u>Pada R</u> :
Energi :

$$\mathbf{W}_R(t) = \int_0^t \mathbf{P}_R(t)dt = \int_0^t V_R(t)i_C(t)dt = \int_0^t Ri_C{}^2(t)dt = \int_0^t R(CV_m\omega)^2\cos^2\omega t dt = R(CV_m\omega)^2\int_0^t \cos^2\omega t dt$$

$$\mathbf{W}_R(t) = R(CV_m\omega)^2\int_0^t \frac{\cos 2\omega t - 1}{2}dt = \frac{R(CV_m\omega)^2}{2}\left(\frac{1}{2\omega}\sin 2\omega t - t\right) \rightarrow T = \frac{1}{f} = \frac{1}{2\omega}\sin 2\omega t - t$$

Energi yang didisipasikan per cycle : $\frac{1}{2}R(CV_m\omega)^2\frac{1}{f}$, sehingga :

$$Q_C = 2\pi\frac{\text{energi maksimum yang disimpan}}{\text{energi yang disipasikan tiap getaran/"per cycle"}}$$

$$\mathbf{Q}_C = 2\pi\frac{\frac{1}{2}CV_m{}^2}{\frac{1}{2}R(CV_m\omega)^2\frac{1}{f}} = 2\pi f\frac{1}{\omega^2 RC} = \frac{1}{\omega RC}$$

Jadi faktor kualitas untuk rangkaian seri RC :

$$\mathbf{Q}_C = \frac{1}{\omega RC}$$

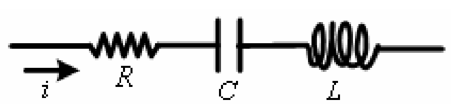
Dapat diambil kesimpulan bahwa faktor kualitas (Q) untuk rangkaian seri :

$$Q_s = \frac{X_s}{R_s}$$

Untuk rangkain seri RL : $Q_s = \frac{\omega_o L}{R}$

Untuk rangkaian seri RC : $Q_s = \frac{1}{\omega_o CR}$

Pada Komponen RLC



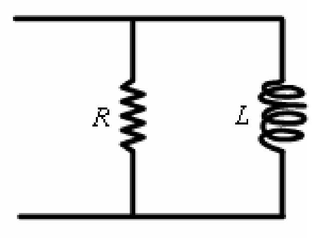
Pada saat terjadi resonansi :

$$\omega^2 = \frac{1}{LC} \rightarrow \omega L = \frac{1}{\omega C}$$

$$\mathbf{Q} = \frac{\omega_o L}{R} = \frac{1}{\omega_o CR}$$

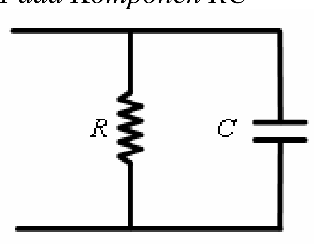
Faktor kualitas atau Q pada rangkaian paralel agak berbeda dengan Q pada rangkaian seri. Untuk harga RLC yang sama, $\mathbf{Q}_P = \frac{1}{\mathbf{Q}_S}$ atau $Q_p = \frac{R_p}{X_p}$

Pada Komponen RL



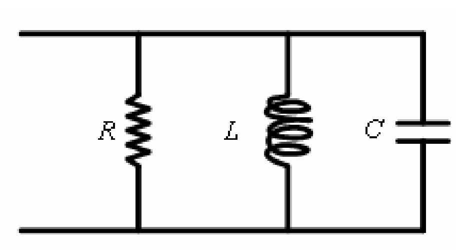
Untuk rangkaian paralel RL : $Q = \frac{R}{\omega_o L}$

Pada Komponen RC



Untuk rangkaian paralel RC : $Q = \omega_o RC$

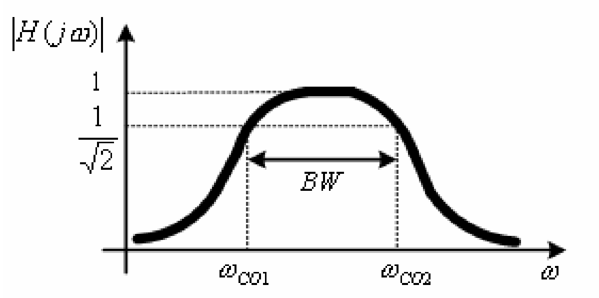
Pada Komponen RLC



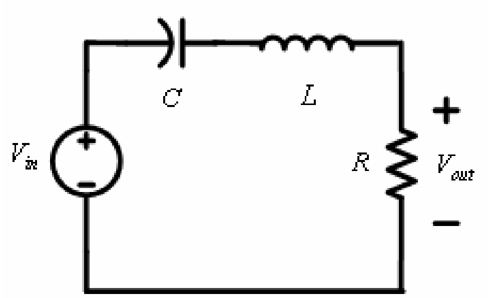
$$\mathbf{Q} = \frac{R}{\omega_o L} = \omega_o RC$$

Bandwidth (BW) 3dB

Lebar pita pada saat terjadi level dayanya adalah ½ dari daya maksimum



Perhatikan gambar rangkaian berikut :



Fungsi transfer rangkaian diatas adalah sebagai berikut :

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{1}{1 + \frac{j\left(\omega L - \frac{1}{\omega C}\right)}{R}} = \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)}$$

Jika rangkaian diatas mempunyai faktor kualitas rangkaian seri RLC dimana dinyatakan dengan :

$$Q = \frac{\omega_o L}{R} \Rightarrow \frac{L}{R} = \frac{Q}{\omega_o}$$

$$Q = \frac{1}{\omega_o CR} \Rightarrow \frac{1}{CR} = Q\omega_o$$

maka fungsi transfer diatas dapat dinyatakan dengan persamaan :

$$\frac{V_{out}(j\omega)}{V_{in}(j\omega)} = \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)} = \frac{1}{1 + j\left(\omega \frac{Q}{\omega_o} - \frac{1}{\omega} Q\omega_o\right)} = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)}$$

Respon frekuensi magnitudenya :

$$\left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = \frac{1}{\sqrt{1 + Q^2\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)^2}}$$

saat level dayanya adalah setengah dari daya maksimum atau respon frekuensi magnitudenya sebesar $\frac{1}{\sqrt{2}}$, maka :

$$\left|\frac{V_{out}(j\omega)}{V_{in}(j\omega)}\right| = \frac{1}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)^2}} = \frac{1}{\sqrt{2}}$$

$$Q^2\left(\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega}\right)^2 = 1$$

$$\frac{\omega}{\omega_o}-\frac{\omega_o}{\omega} = \frac{1}{Q}$$

sehingga :

$$\omega^2 - \frac{\omega_o}{Q}\omega - \omega_o{}^2 = 0$$

Rumus..ABC :

$$\omega_{1,2} = \frac{\frac{\omega_o}{Q} \pm \sqrt{\frac{\omega_o{}^2}{Q^2}+4\omega_o{}^2}}{2} = \frac{\omega_o}{2Q} \pm \frac{\omega_o}{2}\sqrt{\frac{1}{Q^2}+4} = \frac{\omega_o}{2Q} \pm \omega_o\sqrt{1+\left(\frac{1}{2Q}\right)^2}$$

$$\dim ana : \omega_o\sqrt{1+\left(\frac{1}{2Q}\right)^2} > \frac{\omega_o}{2Q}, maka :$$

$$\omega_1 = \omega_o\sqrt{1+\left(\frac{1}{2Q}\right)^2} - \frac{\omega_o}{2Q}$$

$$\omega_2 = \omega_o\sqrt{1+\left(\frac{1}{2Q}\right)^2} + \frac{\omega_o}{2Q}$$

Dari gambar respon frekuensi magnitude diatas didapat bahwa :

$$BW = \omega_{CO2} - \omega_{CO1} = \omega_2 - \omega_1$$

$$BW = \frac{\omega_o}{Q}$$

atau :

$$\omega_1 = \omega_o - \frac{BW}{2}$$

$$\omega_2 = \omega_o + \frac{BW}{2}$$

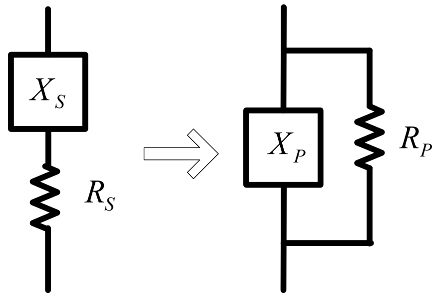
Faktor kualitas dapat dinyatakan sebagai perbandingan frekuensi resonansi terhadap bandwidth.

$$\mathbf{Q} = \frac{f_0}{f_2 - f_1} = \frac{f_0}{BW}$$

frekuensi resonansi f_0 adalah rata-rata geometri f_1 dan f_2 :

$$f_0 = \sqrt{f_1 f_2}$$

Konversi Faktor Kualitas Rangkaian Seri - Paralel



$$R_p = R_s(1+Q^2)$$

$$X_p = \frac{R_p}{Q} = \frac{R_s}{Q}\left(1+Q^2\right)$$

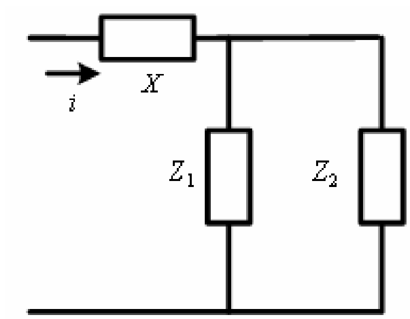
Soal – soal :

1. Rangkaian seri RLC dengan $L = 0{,}5H$ mempunyai tegangan sesaat $v = 70{,}7\sin(500t + 30^o)$ V dan arus sesaat $i = 1{,}5\sin(500t)$ A. Tentukan nilai R dan C. Berapa frekuensi resonansinya ?

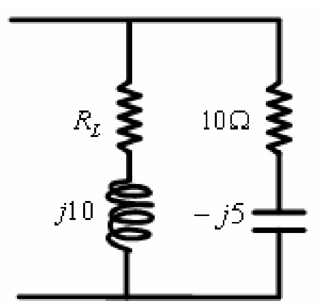
2. Suatu rangkaian seri $L = 25mH$ dan $C = 75\mu F$ mempunyai sudut phasa *lagging* 25^o pada $\omega_o = 2000 rad/s$. Berapa frekuensi sudut pada saat sudut phasa leading 25^o ?

3. Rangkaian seri RLC dengan $R = 25\Omega$ dan $L = 0{,}6H$ akan menghasilkan arus *leading* sebesar 60^o pada frekuensi 40 Hz. Tentukan frekuensi rangkaian serinya !

4. Jika $V = 480V, Z^1 = 25\angle 30^o, Z_2 = 12\angle -40^o$
   a. Tentukan nilai komponen reaktif X pada saat resonansi $f_o = 60Hz$
   b. Tentukan nilai i pada saat resonansi

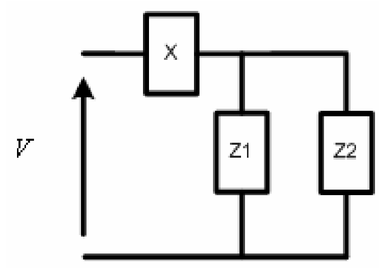


5. Tentukan komponen R_L agar terjadi resonansi !

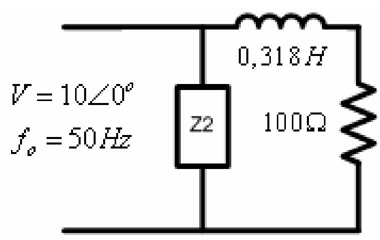


6. Suatu rangkaian seri RLC dengan $R = 50\Omega, L = 0{,}05H, C = 20\mu F$ terpasang pada $V = 100\angle 0^o$ Volt dengan frekuensi variabel. Pada frekuensi berapa tegangan induktor mencapai maksimum ? Berapakah tegangan induktor tersebut ?

7. Jika $V = 480V, Z_1 = 25\angle 30^o, Z_2 = 12\angle -40^o$
   a. Tentukan nilai komponen reaktif X saat resonansi $f_o = 60Hz$
   b. Tentukan nilai I pada saat resonansi



8. Tentukan nilai komponen reaktif X saat terjadi resonansi



9. Pada rangakain seri RLC faktor kualitas rangkain tersebut adalah 2π dengan nilai induktor 1 mH dan resistor 1 kΩ. Tentukan frekuensi resonansi dan berapa BW ?

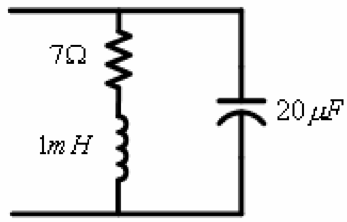
10. Pada saat terjadi resonansi tegangan terpasang pada rangkaian seri RLC adalah $v = 70{,}7\sin(500t + 30^o)$ menghasilkan arus sebesar $i = 2{,}83\sin(500t + 30^o)$ .Jika L=0,5H, tentukan nilai R dan C

11. Rangkaian seri RLC dengan R=25 dan L=0,6 H akan menghasilkan arus leaading sebesar 60 pada frekuensi 40 Hz. Tentukan frekuensi resonansai rangkauan seri tersebut.

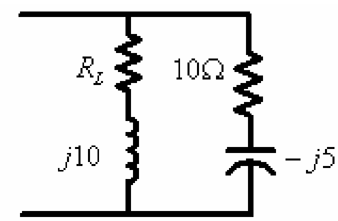
12. Suatu rangkaian seri L = 25mH dan C = 75μF mempunyai sudut phasa lagging 25^o pada $\omega_o = 2000 rad/s$. Berapa frekuensi sudut pada saat sudut phasa leading 25^o

13. Rangkaian seri RLC dengan L = 0,5H mempunyai tegangan sesaat $v = 70{,}7\sin(500t + 30^o)$ dan arus sesaat $i = 1{,}5\sin 500t$. Tentukan nilai R dan C. Berapa frekuensi resonansinya

14. Tentukan frekuensi resonansi pada gambar dibawah ini



15. Tentukan komponen RL agar terjadi resonansi

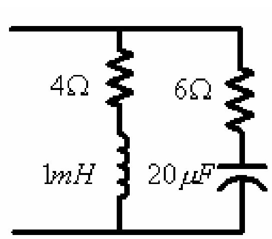


16. Rangkaian seri $R = 5\Omega, L = 20mH$ dan C variabel disuplai tegangan dengan frekuensi 1000 Hz. Tentukan C resonansi serinya

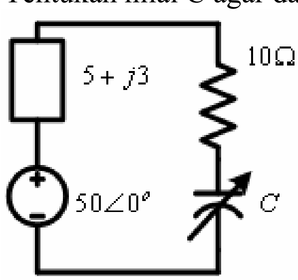
17. Rangkaian seri $R = 5\Omega, C = 20\mu F$ dan L variabel diberikan $v = 10\angle 0^o$ pada $\omega = 1000 rad/s$. L diatur-ature sampai teggangan pada R maksimum. Tentukan tegangan pada masing-masing komponen

18. Rangkaian seri RLC $R = 100\Omega, L = 0{,}5H, C = 40\mu F$. Hitung frekuensi resonansi, frekeunsi cut off bawah dan frekuensi cutt off atas

19. Tentukan frekeunsi resonansi untuk rangkaian berikut



20. Tentukan nilai C agar daya pada 10 ohm maksimum pada frekuensi 2000 Hz

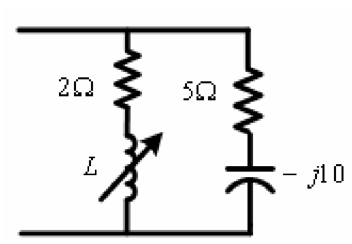


21. Tentukan daya pda resistor 10 ohm pada soal diatas

22. Rancang suatu folter LPF yang terdiri dari R dan L jika frekuensi resonasni 10 kHz dan nilai resistor 1kΩ

23. Suatu rangkaian seri RLC dengan Q = 20 dan BW = 10 kHz. Tentukan frekuensi resonasni, cut off bawah dan atas. Jika L = 2mH. Tentukan nilai R dan C

24. Hitung harga L bila rangkaian beresonansi pada $\omega = 5000 rad / s$



25. Suatu rangkaian seri RLC dengan R = 20 ohm dan L = 5mH C = 5 nF terpasang pada sumber tegangan V
    a. Hitunglah frekuensi resonansinya
    b. Saat resonansi tegangan di C = 2 V, berapakah tegangan sumber yang dipasang