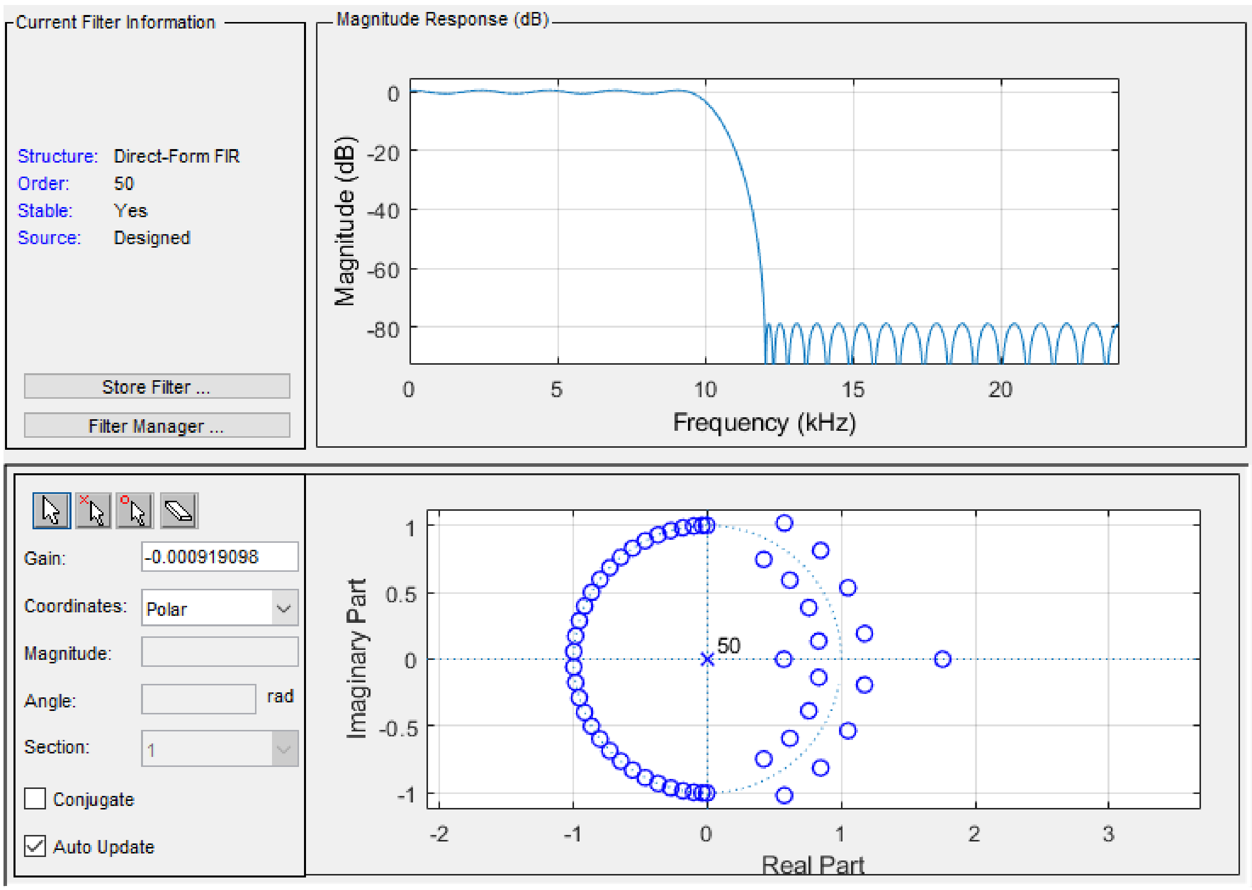
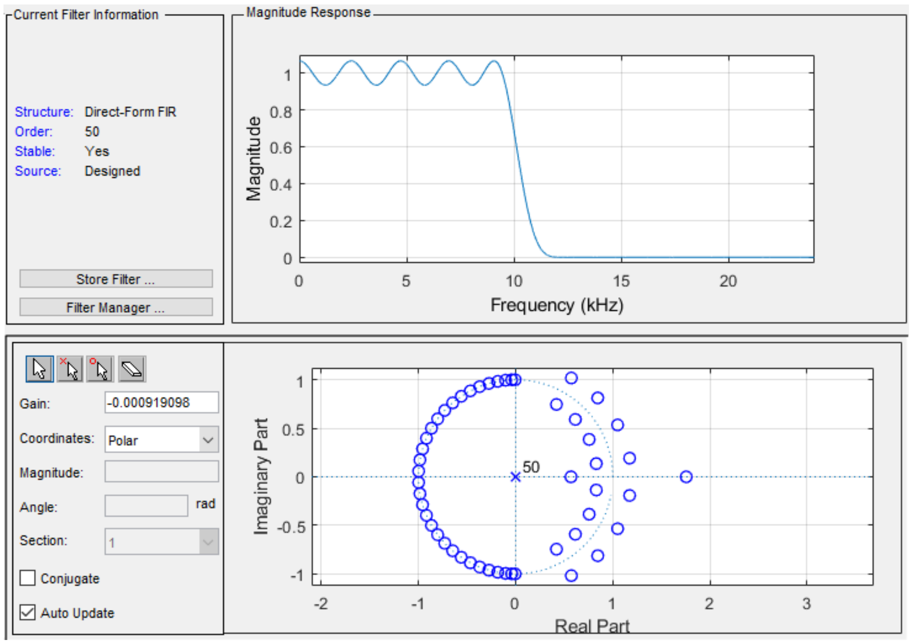
Justin Miller SID# 026494154
CECS - 463
Dr. Mehrnia
2 December 2021

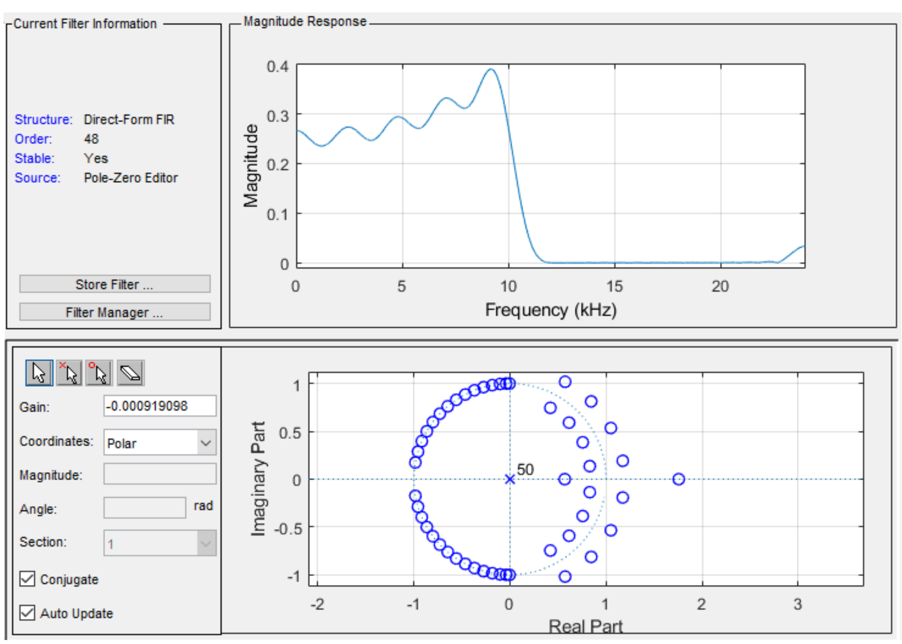
CECS 463 Final Project

1. The default filter we use is a FIR filter because the poles of the filter are found only at z=0. This means the impulse of the response for the filter settles at 0 in a finite amount of time.

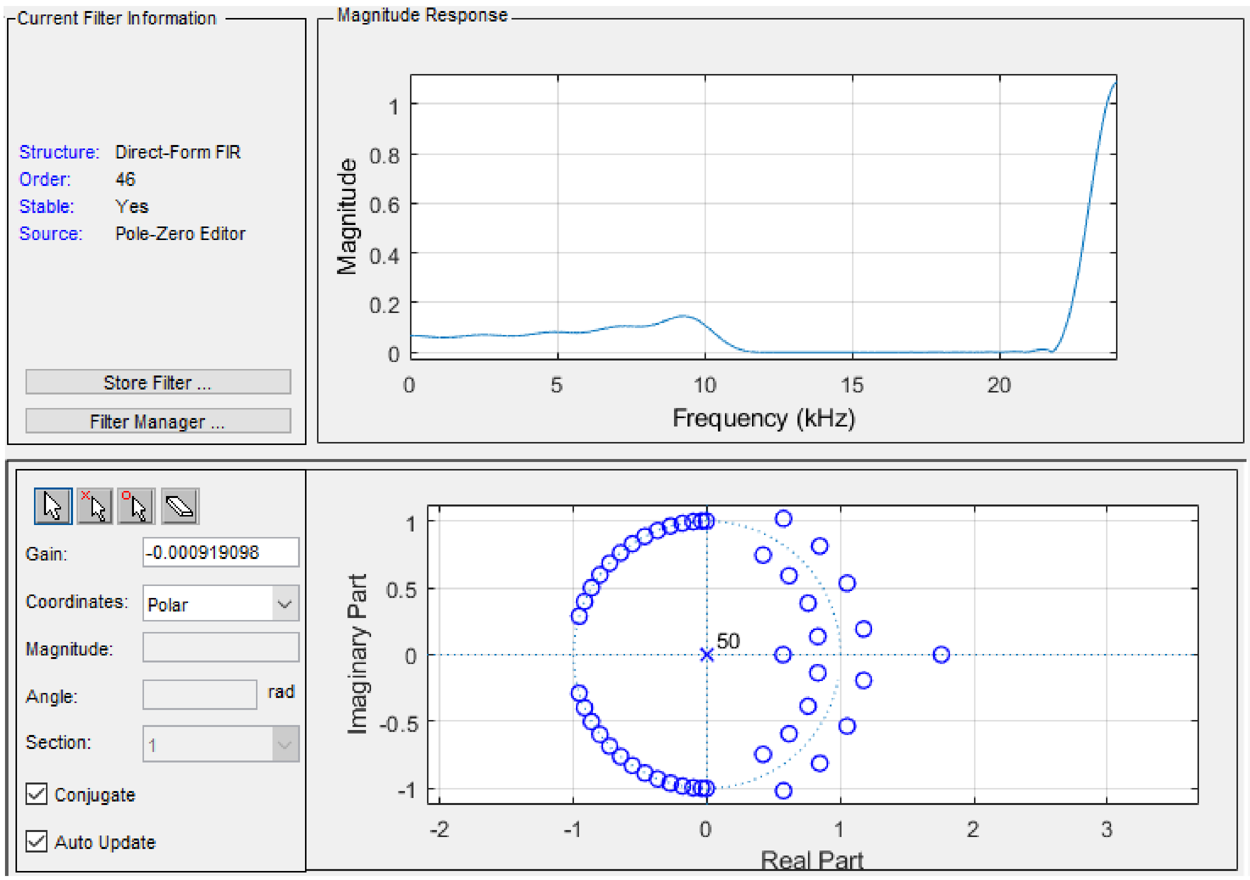


2. The zeros are deleted, then the magnitude response is changed so the response from 0 kHz to around 9.16 kHz no longer oscillates at ~1, but it starts at a magnitude of 0.266 and oscillates with an increasing average value up to 0.39. The magnitude between about 11.836 kHz and 22.699 kHz remains at 0 like before, but after this region now begins slowly increasing logarithmically. These changes occur because in the equation for the frequency response, the zeros are found in the numerator. When a zero is removed, the numerator value is lowered, also lowering the frequency response, explaining the drop at the beginning of the plot. As well, removing the zero takes away a point at which the response's magnitude would be at 0, causing the increase from zero seen at the end of the plot.

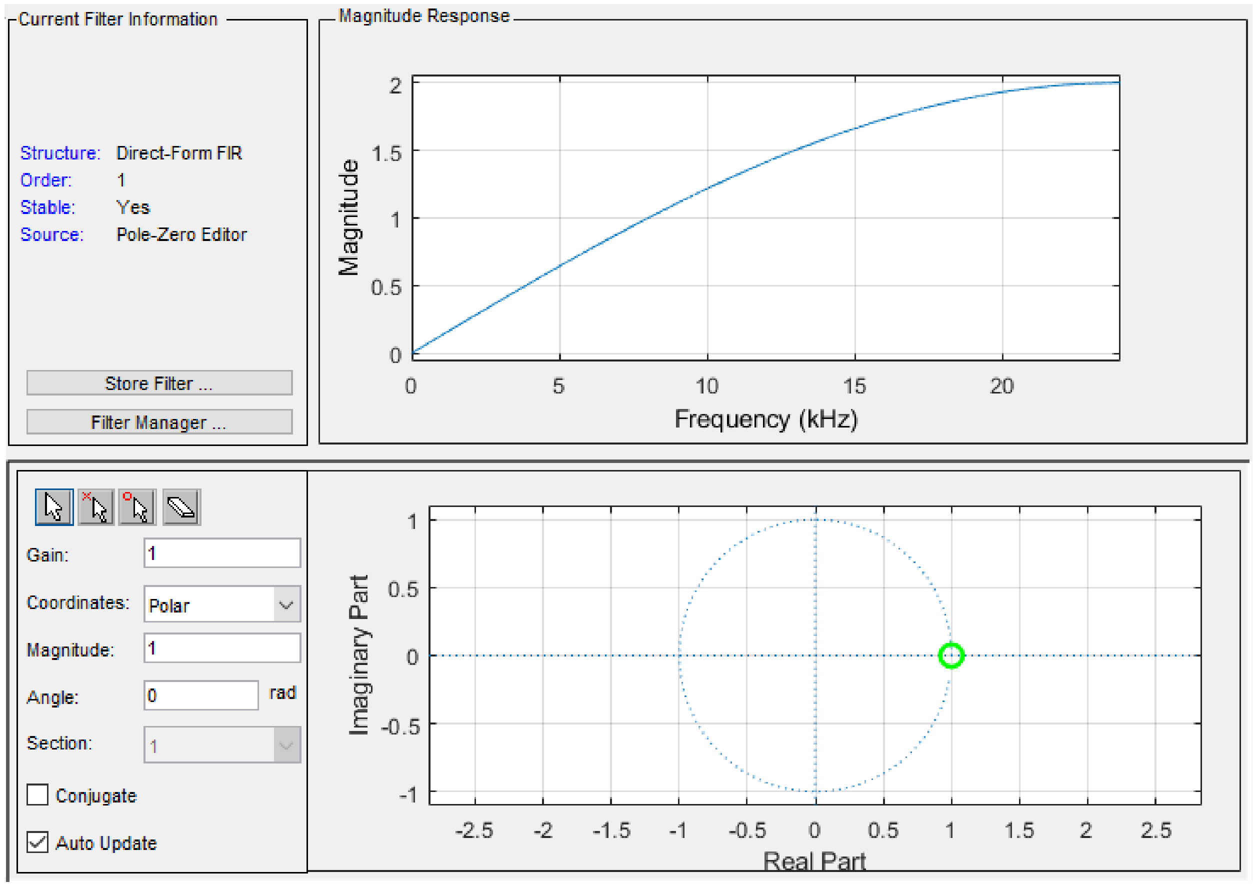




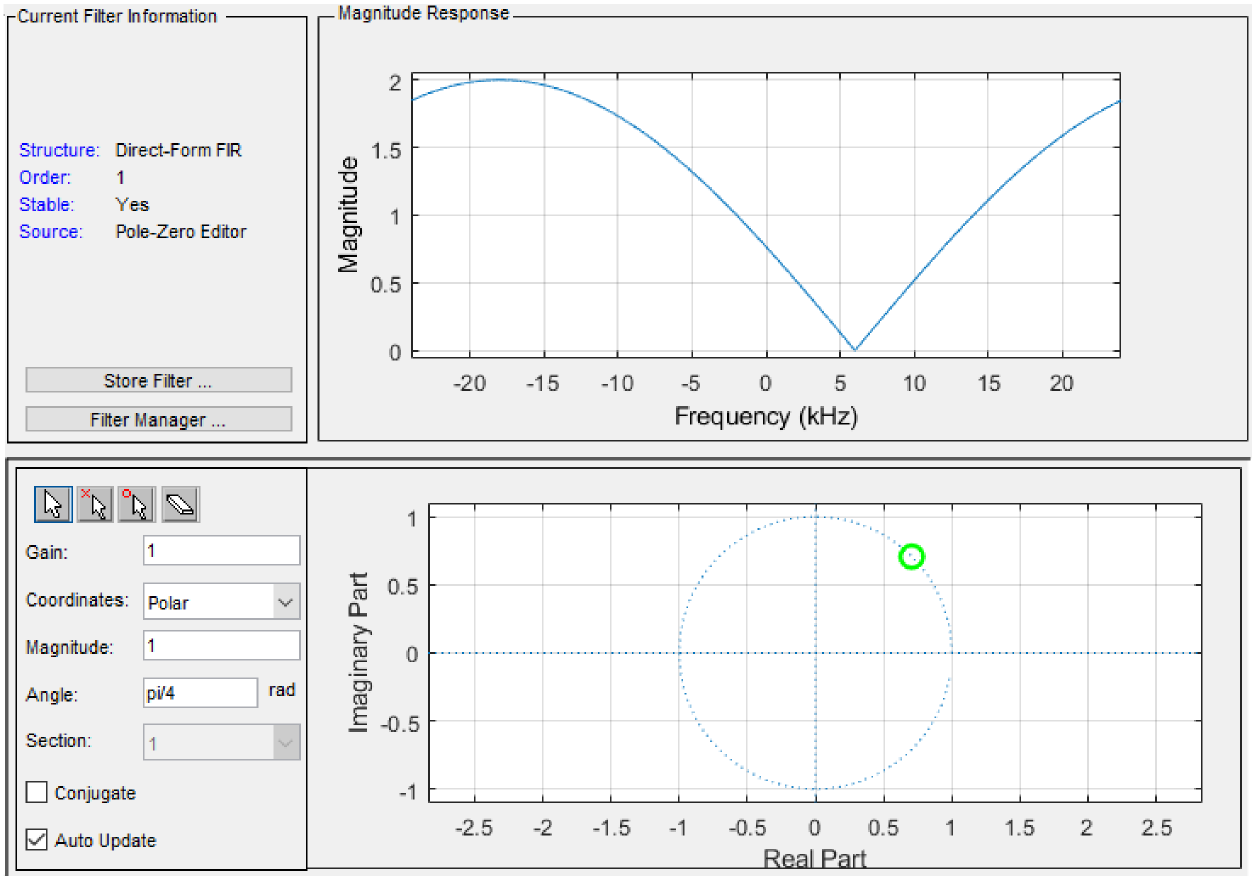
3. By deleting the zeros, the magnitude response becomes significantly flatter in the beginning. The frequencies at which there are increases and decreases are similar, but the starting magnitude is much lower at around 0.067, with less oscillation in the increase to only about 0.1455. The frequencies with a magnitude of zero remain unchanged, but the spike following around 22.699 kHz now jumps to a much larger magnitude — greater than 1 — when compared to the relatively small spike it had previously.



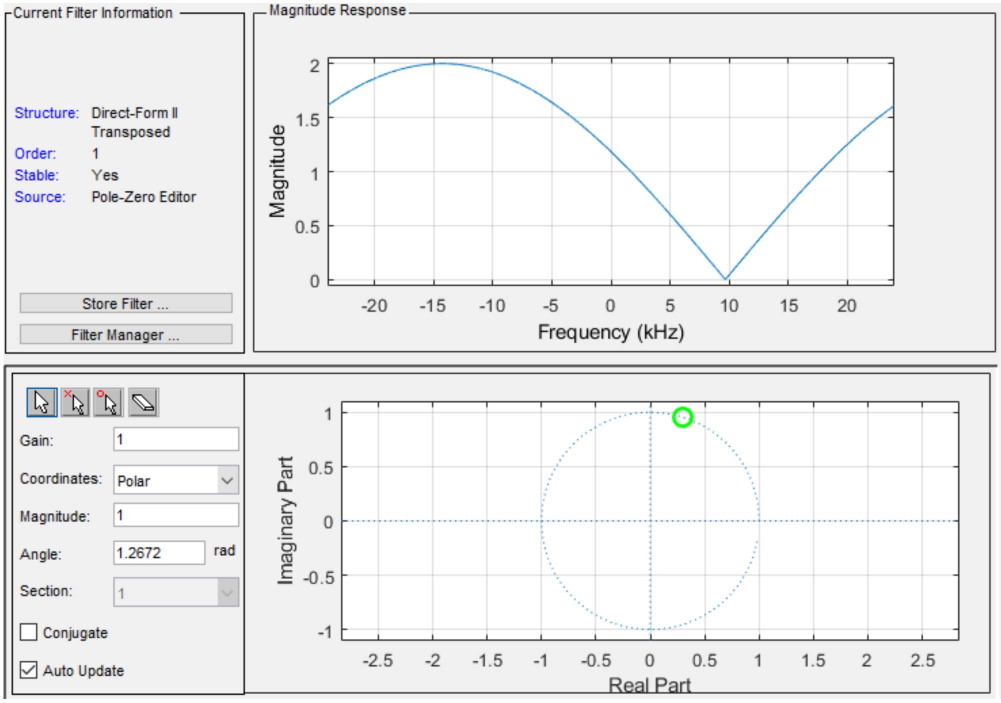
4. The filter created by a zero at z=1 is a high pass filter. This makes sense with the zero at the location of z=1, as the magnitude begins with 0 at 0 Hz and increases logarithmically as the frequency increases up to 24 Hz.

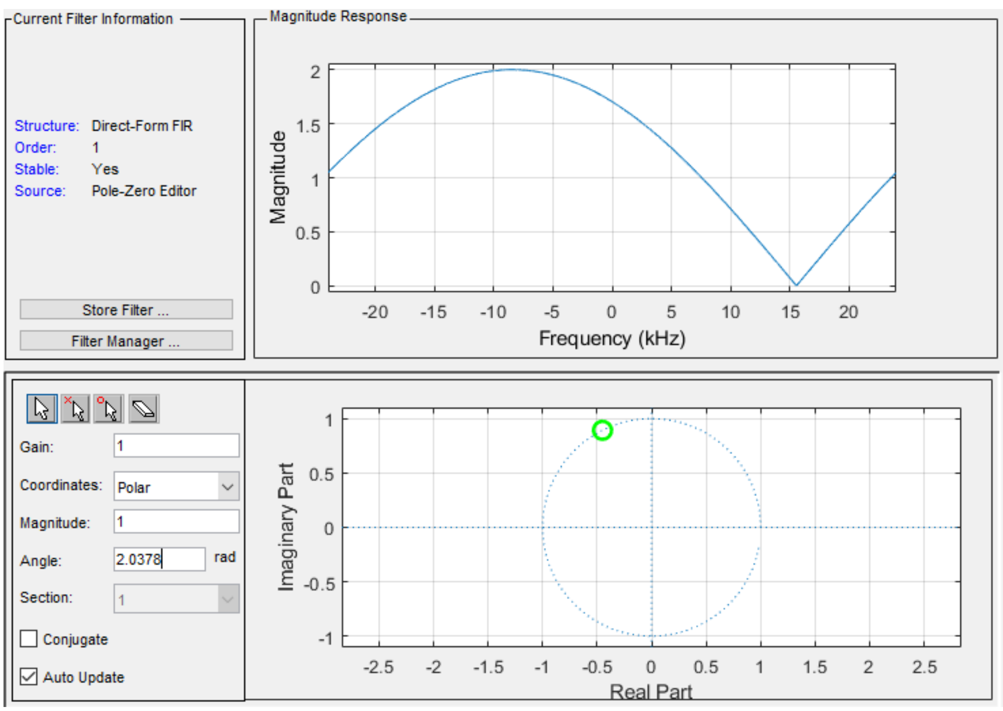


5. When the zero is moved from $z=1$ to $z=e^{jp/4}$, the plot of the magnitude response extends the x-axis to show the frequency from -24 kHz to 24 kHz, and the zero is now found at 6 kHz. Since this filter's magnitude response is symmetrical about its zero, when the zero sits at $z=1$, only the positive frequencies are shown, as it would be mirrored along the negative portion of the axis. However, since the origin is no longer centered at 0, but the edges of the plot are still limited between -24 kHz and 24 kHz due to the sampling frequency, the plot extends the x-axis to show the periodicity and symmetrical behavior is still present throughout the range of frequencies.

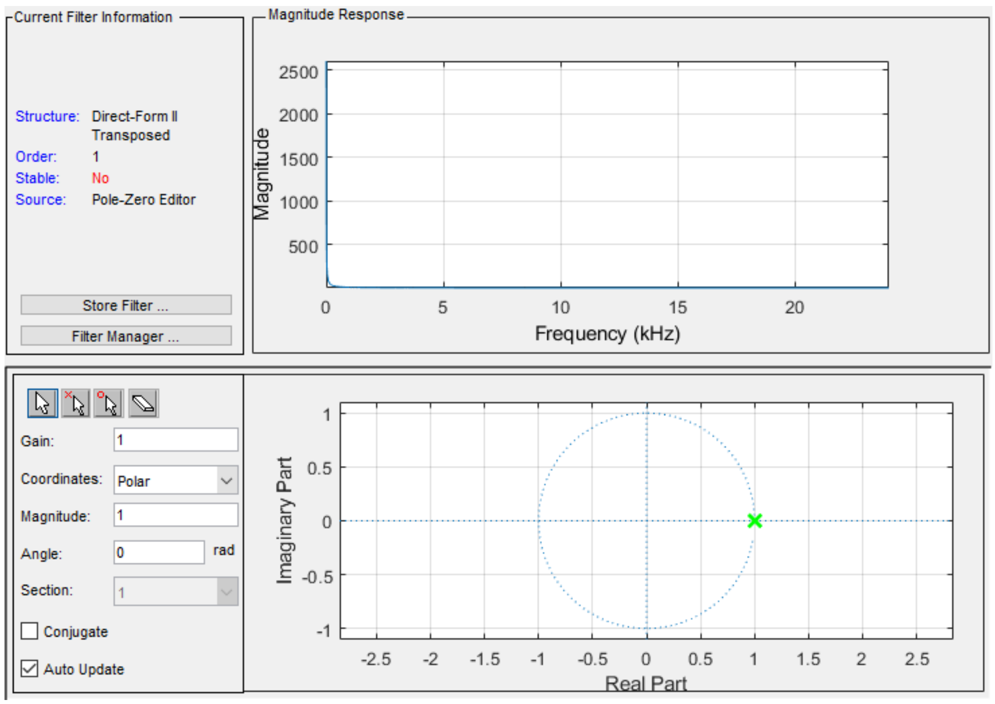


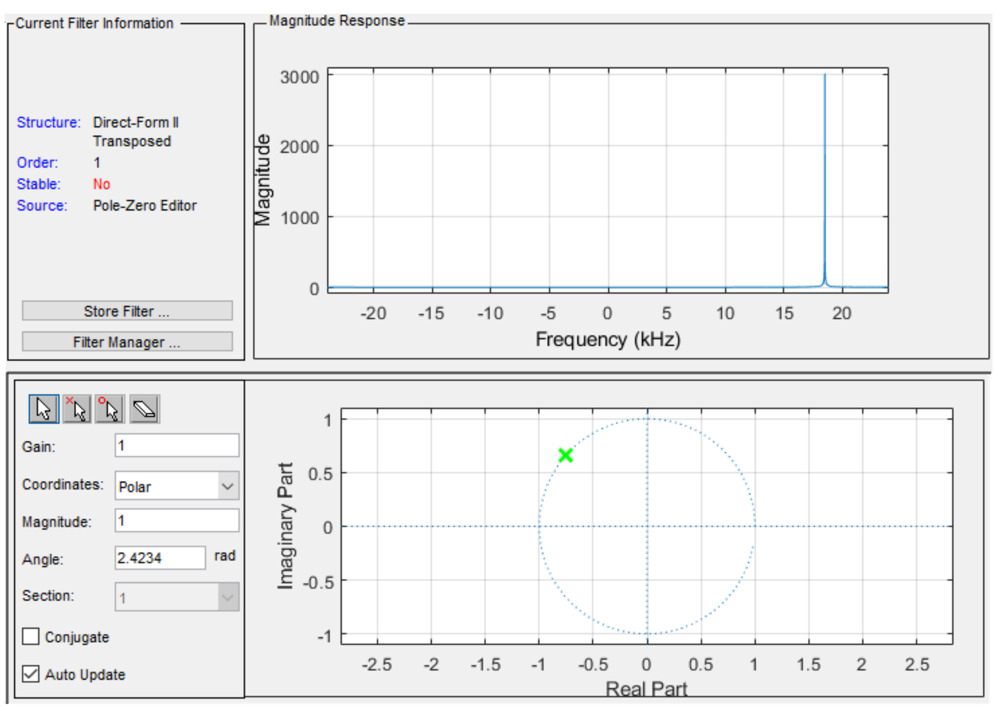
6. By moving the zero along the unit circle, the frequency at which the frequency response has a magnitude of 0 changes. This matches the expected behavior, as the frequencies of the magnitude response correspond to the points on a unit circle. The corresponding frequencies can be found by multiplying the sampling frequency, which in this case is 48 kHz, by the angle of the point in radians, and dividing that value by 2π. This correlation makes it so moving the zero throughout the circle will cycle it through the frequencies found in the range between -24 kHz and 24 kHz.



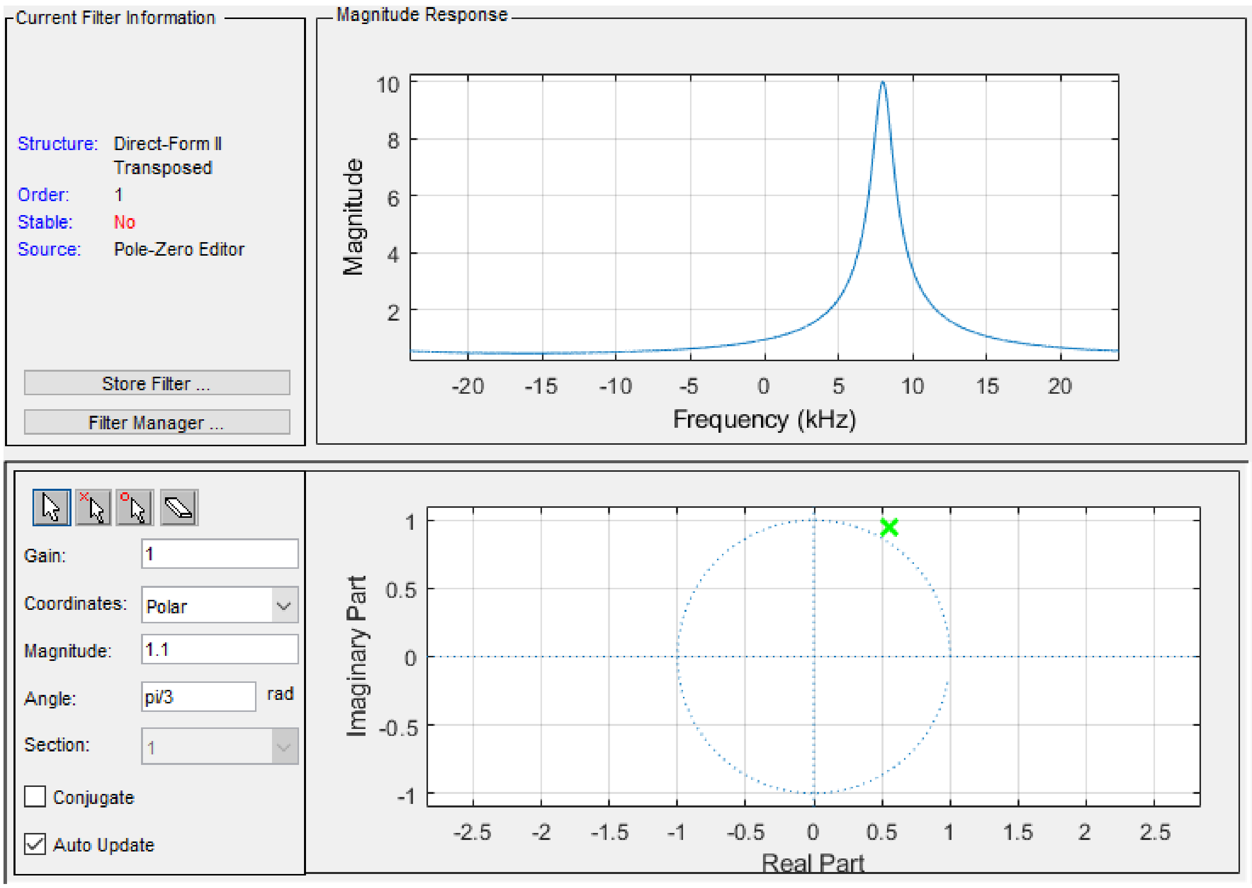


7. With a pole at z=1, the magnitude response displays a spike to infinity at 0 kHz. A filter is considered stable when its impulse response approaches 0 as time goes to infinity. This only occurs when the poles of the filter are within the unit circle. Because the single pole is on the unit circle, the impulse response has an exponentially increasing amplitude, meaning the filter is unstable. Similarly to moving the zero around the unit circle, moving the pole around the unit circle moves the spike to the frequency associated with the angle.

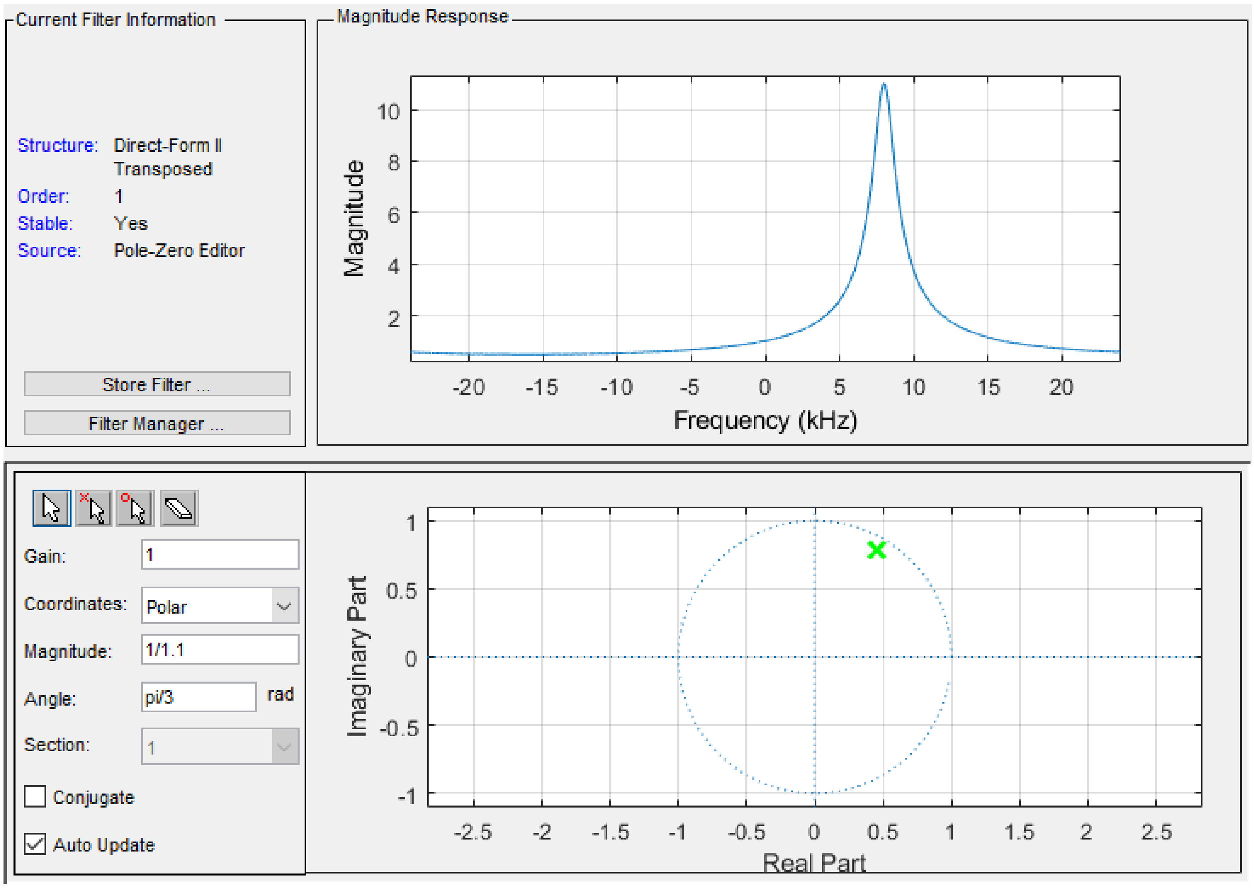




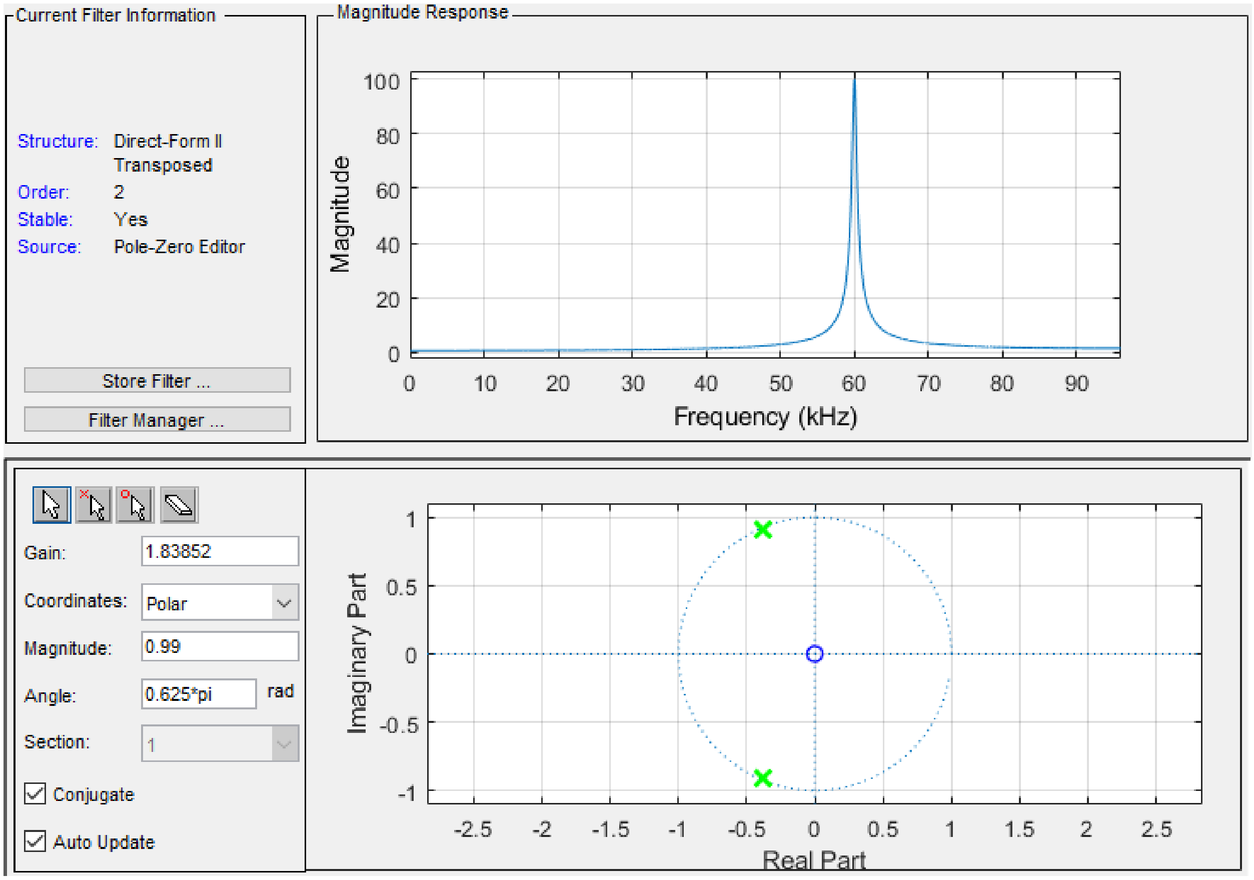
8. We know this filter is unstable for the same reason the previous one was. The single pole of this filter is found outside the unit circle, meaning that the impulse response has an exponentially increasing component. By looking only at the frequency response, you cannot tell that the filter is unstable, as it appears extremely similar to the filter if the pole's location was within the unit circle.



9. Since the pole of this filter is within the unit circle, the impulse response will decay towards 0 as time approaches infinity, meaning the filter is stable. By looking solely at the frequency response, we would not be able to tell if the filter is stable or not.



10.



Zeros: z=0, z=0
Poles: z=-0.37886+0.91464i, z=-0.37886-0.91464i

$$H(z) = 1.83852 \frac{z^2}{\big(z - (-0.37886 + 0.91464\mathrm{i})\big)\big(z - (-0.37886 - 0.91464\mathrm{i})\big)}$$

$$= \frac{1.83852z^2}{(z + 0.37886 - 0.91464\mathrm{i})(z + 0.37886 + 0.91464\mathrm{i})}$$

$$= \frac{1.83852z^2}{z^2 + 0.37886z + 0.37886z + (0.37886 - 0.91464i) * (0.37886 + 0.91464i)}$$

$$= \frac{1.83852z^2}{z^2 + 0.7577z + 0.9801}$$

11. The coefficients exported from the filter designer match the values calculated using the coordinates of the poles and zeros. This proves that the calculation for H(z) provides the correct coefficients for the numerator and denominator.

```
Num =

    1.8385         0         0


Den =

    1.0000    0.7577    0.9801
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