Exercise 02561-11 **Geometric Modelling - NURBS Curves and Surfaces**

Readings

ANG: chap. 12 (12.12.4).
OGL-PG: From section "The GLU NURBS Interface" .

Purpose

The purpose of the exercise is to understand the underlaying theory of NURBS (Non Uniform Rational B-Splines) curves and surfaces and use them to make geometric models. Furthermore, we will consider the special cases of Bezier, uniform B-spline, and non-uniform B-spline curves and surfaces..

Part 1

The program 02561-11-2009.cpp draws the polyline P1-P7 - P1=(-10,0,0), P2=(-10,5,0), P3=(-5,5,0), P4=(0,10,0), P5=(5,5,0), P6=(10,5,0), and P7=(10,0,0).
Make a sketch of the polyline on a piece of paper with grids.
Use point P1-P7 as control points to draw a freehand sketch of each of these four types of curves.

- Bezier
- Uniform B-spline
- Non-uniform B-spline
- Non-uniform Rational B-spline (NURBS)

The order of the curves is four.
First consider for each type of curves
- the number of control point needed in determining the curves
- the number of coordinates for each controlpoint
- The relation between degree, order and number of knots

and determine the
- the controlpoint array
- the knot vector
- the type parameter

Compare the parameters for the four types of curves.

Draw the four type of 4^{th} order curves with OpenGL. (Example 12-5 in OGLPG is used in part 2 of the exercis. You may have a preview now to se how you draw Nurbs Surfaces.)

Use two calls to the function gluNurbsProperty with parameter GLU_SAMPLING_METHOD set to GLU_DOMAIN_DISTANCE and GLU_U_STEP set to 100. Try to set the value of 100 to something different. Try even small values like 1, 2, 3

Part 2

Run example 12-5 from OGL-PG to demonstrate how to draw Nurbs surfaces. Which type of surface is actually defined in the program.

Make your own surfaces by defining appropriate controlpoints.
As a start you can make two copies of the vector P1-P7 vector and change the z-coordinate to 5 and -5 respectively.

Part 3

Trim the surface to make a hole in it.

Part 4 (Optional)

Solve exercise 1 from the exame in Computer Graphics & CAD, 12 January 2001. Check your results in OpenGL whenever possible.

EXERCISE 1 (35 %) - 12 January 2001 Computer Graphics

An industrial design company, GNDESIGN, uses an extended version of OpenGL for modelling and visualization 2D and 3D models.
In a design project the points P1-P6 - see table 1- are used to define a 2D figure. The figure is a closed curve made by joining a 4th order, a 3rd order and a 2nd order Bezier curve which are defined by the control points P1-P4, P4-P6 and P6-P1, respectively.

a) Make a freehand sketch which includes the polygon P1-P6 and the three Bezier curves.
Give the order of continuity at the points where the curves are joined.
Explain which information you have used to sketch the Bezier curves.

Points on a Bezier curve are calculated from the equations

$$P(u) = \sum_{i=0}^{n} P_i B_{i,n}(u) \quad \text{and} \quad B_{i,n}(u) = \frac{n!}{i!(n-i)!} u^i (1-u)^{n-i}$$

in which $B_{i,n}$ are the blending functions and P_i are the control points.
The blending functions are examples of Bernstein polynomials.

b) Find a general expression for the blending functions for the 3rd and 2nd order Bezier curves and make a freehand sketch of the corresponding blending functions.

c) Find three points on the 3rd order Bezier curves for the parameter values u = 1/4, ½, and 3/4 and update the freehand sketch in question a) based on this information.

The extended OpenGL version is used to draw a more precise version of the Bezier curves.

d) Give the actual parameters in the three calls of the function gluNurbsCurve.

The shape of the figure is changed, as the three Bezier curves are substituted with a single 4th order non-uniform B-spline curve which starts and ends in P1.

e) Make a freehand sketch of the polygon P1-P6 and the 4th order non-uniform B-spline.
Give the order of continuity along all parts of the closed curve.

f) Give the actual parameters of the function gluNurbsCurve.

The shape of the non-uniform B-spline curve is changed so that the weight of the point P4 is doubled.

g) Give the actual parameters of the function gluNurbsCurve.

P1 = (12, 0, 6); P2 = (4, 0, 6); P3 = (4, 0, 12); P4 = (0, 0, 12); P5 = (0, 0, 0); P6= (12, 0, 0)

Table 1.