
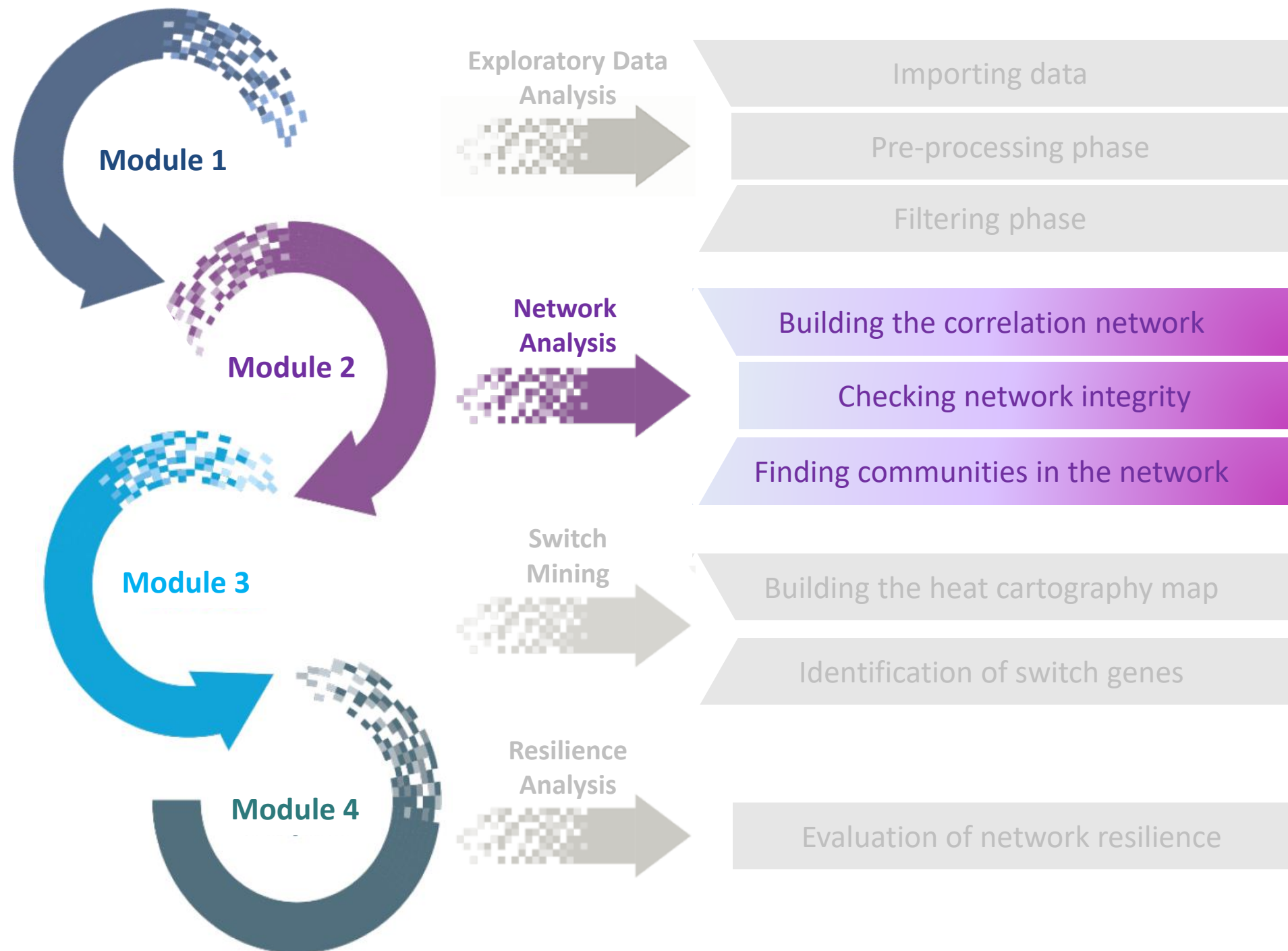
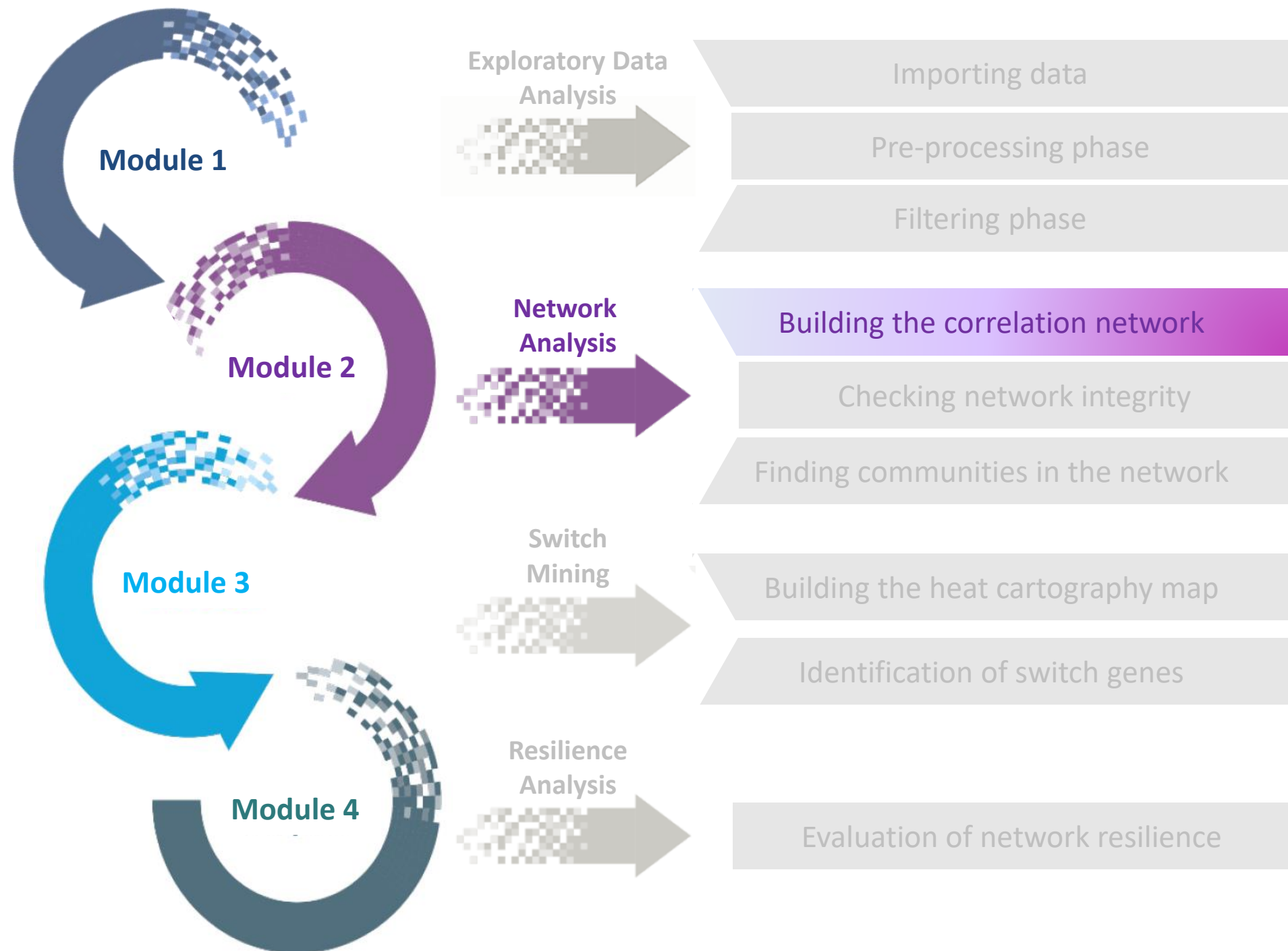


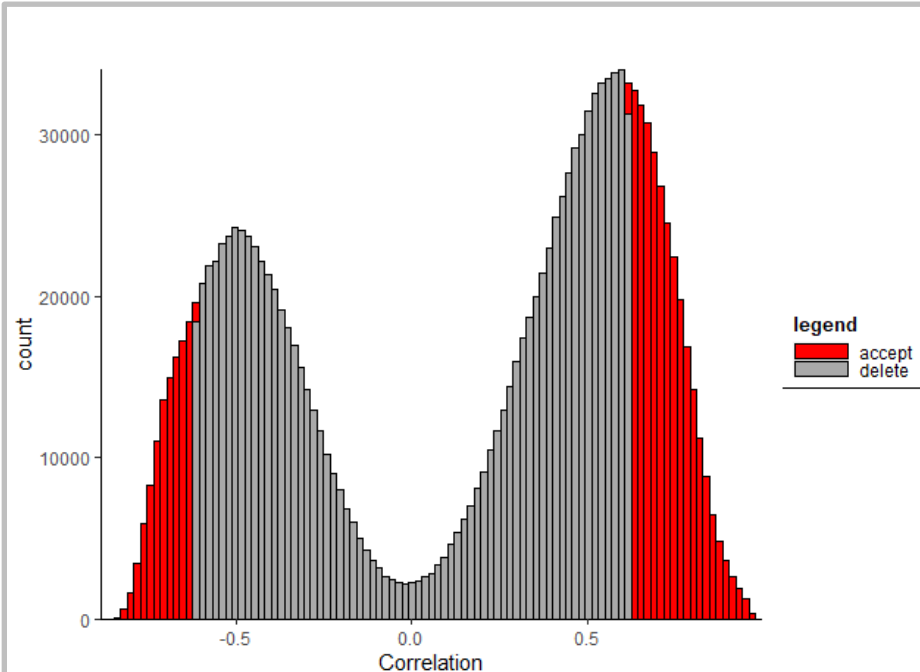
Module 2: Network Analysis







Pearson correlation coefficient distribution



- x-axis represents the Pearson correlation coefficient between the expression profiles of all pairs of genes
- y-axis represents the frequency
- red bars are the selected highly correlated pairs
- grey bars are the deleted poorly correlated pairs

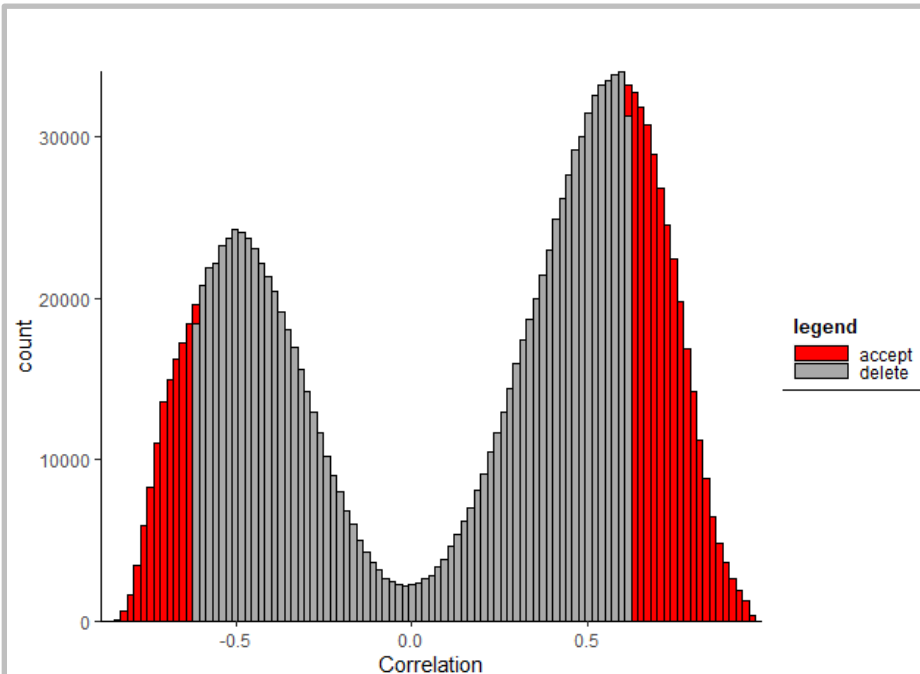
Correlation between two variables

- The existence of a linear relationship between two normally distributed continuous variables (e.g., gene expression values of gene X and gene Y) can be expressed by the **Pearson correlation coefficient ρ**

$$\rho_{xy} = \frac{\sum_{i=1}^{col} (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{col} (x_i - \bar{x})^2 \sum_{i=1}^{col} (y_i - \bar{y})^2}}$$

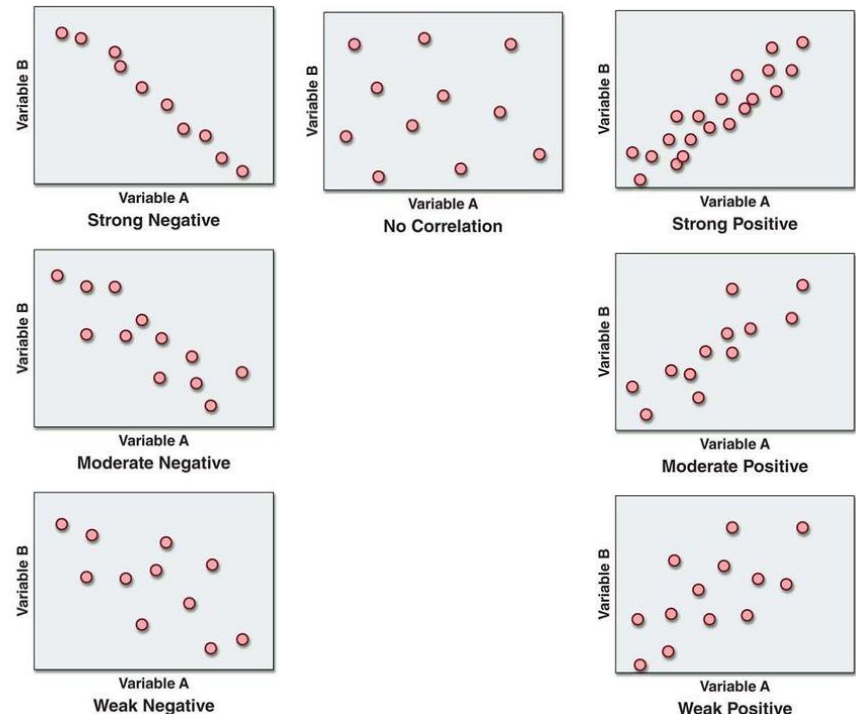
- ρ coefficient varies between -1 and 1 and shows strength (value) and direction (sign) of correlation

Pearson correlation coefficient distribution

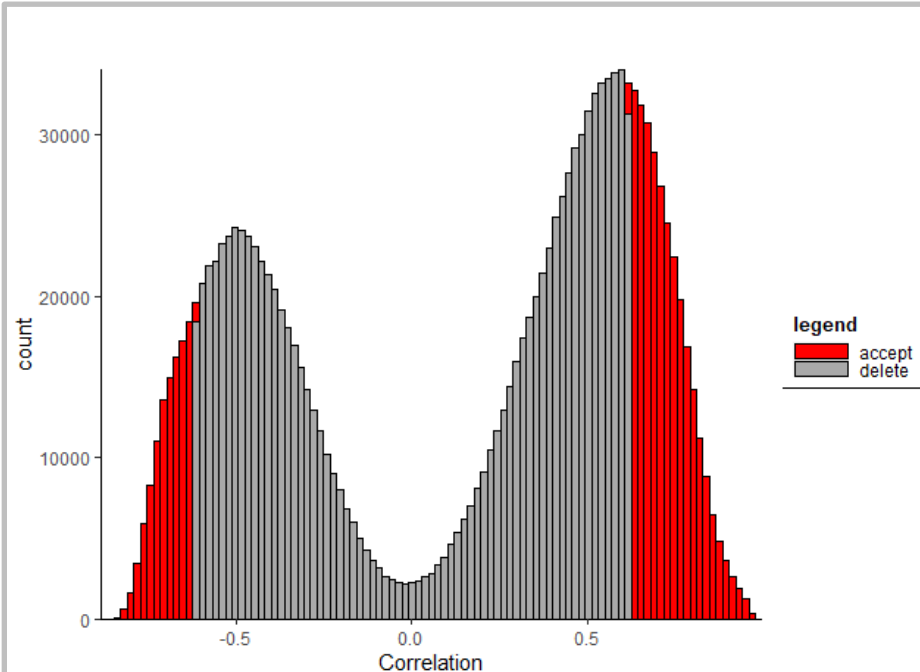


- x-axis represents the Pearson correlation coefficient between the expression profiles of all pairs of genes
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Correlation between two variables



Pearson correlation coefficient distribution



- x-axis represents the Pearson correlation coefficient between the expression profiles of all pairs of genes
- y-axis represents the frequency
- red bars are the selected highly correlated pairs
- grey bars are the deleted poorly correlated pairs

Correlation between two variables

```
computeCorrelation <- function(data,type,method){
  res <- rcorr(t(data), type = type)
  rho <- res$r
  pval <- res$p

  pval_adj <- p.adjust(pval, method = method)

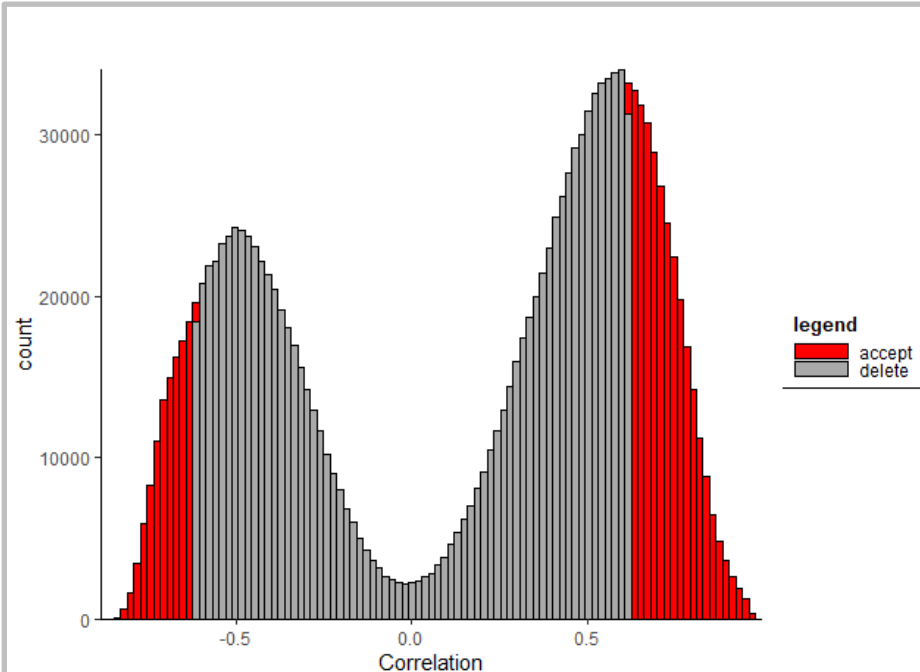
  pval_adj <- matrix(pval_adj, ncol = ncol(pval),
                    dimnames = list(row = row.names(rho), col = colnames(rho)))

  ut <- upper.tri(rho)

  df <- data.frame(source = rownames(rho)[row(rho)[ut]],
                  target = rownames(rho)[col(rho)[ut]],
                  correlation = rho[ut],
                  pval = pval[ut],
                  pval_adj = pval_adj[ut])

  return(df)
}
```

Pearson correlation coefficient distribution



- x-axis represents the Pearson correlation coefficient between the expression profiles of all pairs of genes
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- red bars are the selected highly correlated pairs
- grey bars are the deleted poorly correlated pairs

Correlation between two variables

```
computeCorrelation <- function(data,type,method){
  res <- rcorr(t(data), type = type)

  rho <- res$r
  pval <- res$p

  pval_adj <- p.adjust(pval, method = method)

  pval_adj <- matrix(pval_adj, ncol = ncol(pval),
    dimnames = list(row = row.names(rho), col = colnames(rho)))

  ut <- upper.tri(rho)

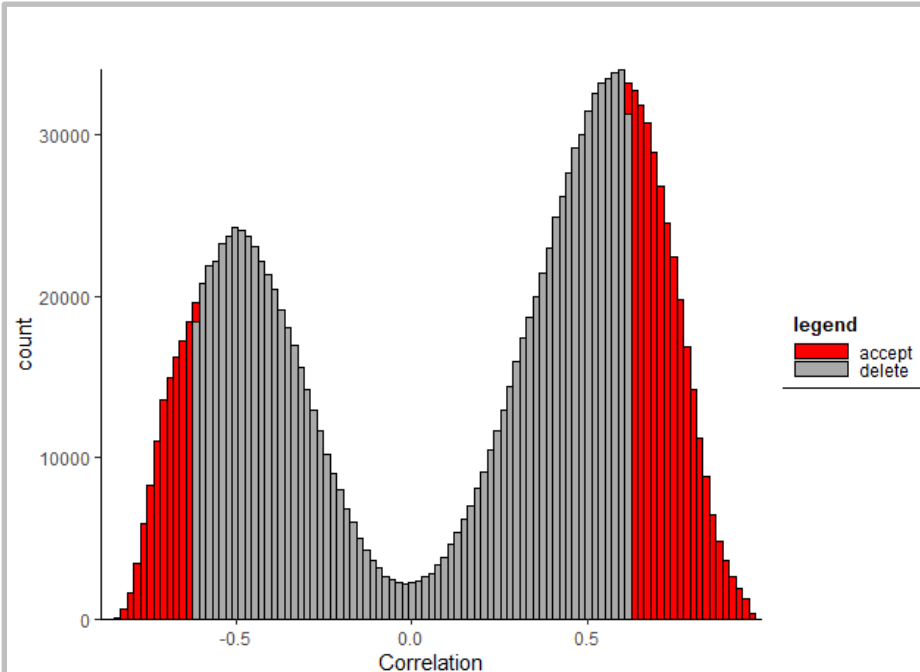
  df <- data.frame(source = rownames(rho)[row(rho)[ut]],
    target = rownames(rho)[col(rho)[ut]],
    correlation = rho[ut],
    pval = pval[ut],
    pval_adj = pval_adj[ut])

  return(df)
}
```



Caveat: the user can choose even other correlation methods (e.g. type = Spearman)

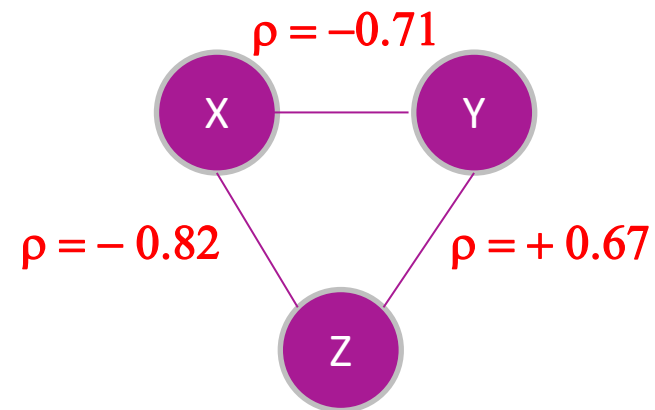
Pearson correlation coefficient distribution



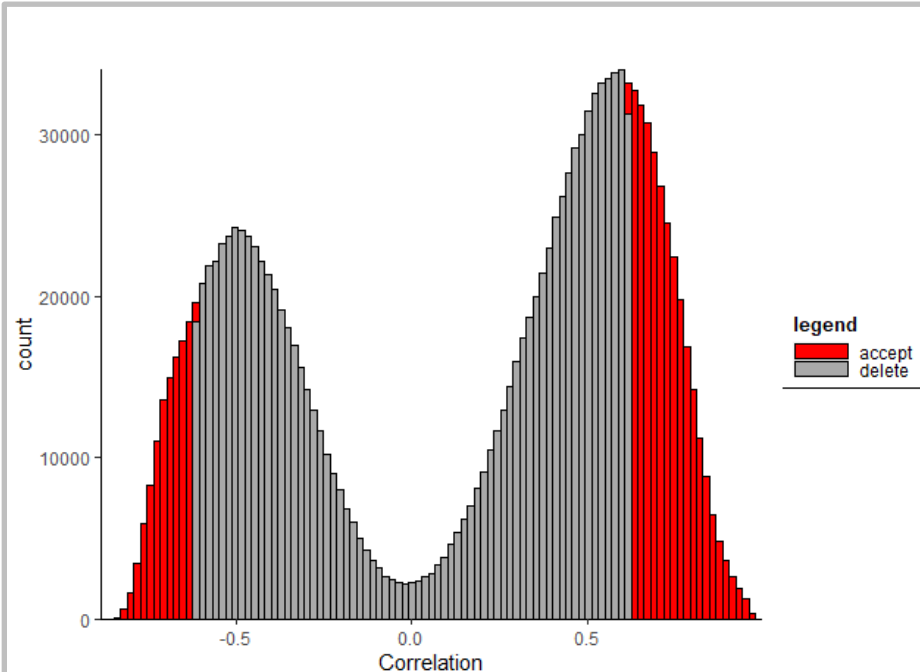
- x-axis represents the Pearson correlation coefficient between the expression profiles of all pairs of genes
- y-axis represents the frequency
- red bars are the selected highly correlated pairs
- grey bars are the deleted poorly correlated pairs

Correlation network

Nodes are genes (including RNAs and miRNAs if available) and a link occurs between two nodes if the absolute value of the Pearson correlation is greater than the selected threshold (e.g., 80th percentile \rightarrow 0.63)



Pearson correlation coefficient distribution



- x-axis represents the Pearson correlation coefficient between the expression profiles of all pairs of genes
- y-axis represents the frequency
- red bars are the selected highly correlated pairs
- grey bars are the deleted poorly correlated pairs

Correlation network

```
buildCorrelationNetwork <- function(network, threshold_corr, threshold_pval_adj,
                                     output_file_CorrelationNetwork,
                                     output_file_CorrelationNetwork_R, save=F){

  ind <- which((abs(network$correlation) >= threshold_corr) &
              (network$pval_adj <= threshold_pval_adj))

  network <- network[ind,]

  if(save){

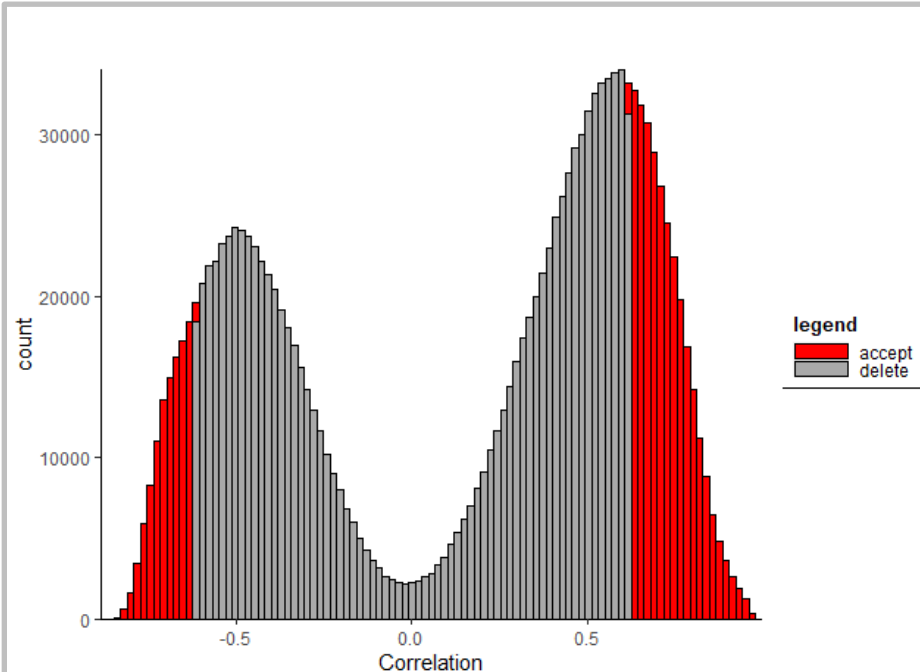
    write.table(network, output_file_CorrelationNetwork, sep="\t",
                row.names = F, col.names = T, quote = F)

    save(network, file = output_file_CorrelationNetwork_R)

  }

  return(network)
}
```

Pearson correlation coefficient distribution



- x-axis represents the Pearson correlation coefficient between the expression profiles of all pairs of genes
- y-axis represents the frequency
- red bars are the selected highly correlated pairs
- grey bars are the deleted poorly correlated pairs

Histogram of Pearson correlation

```
getHistogram <- function(x, threshold, title, xlabel){
  df <- data.frame(variable = x)

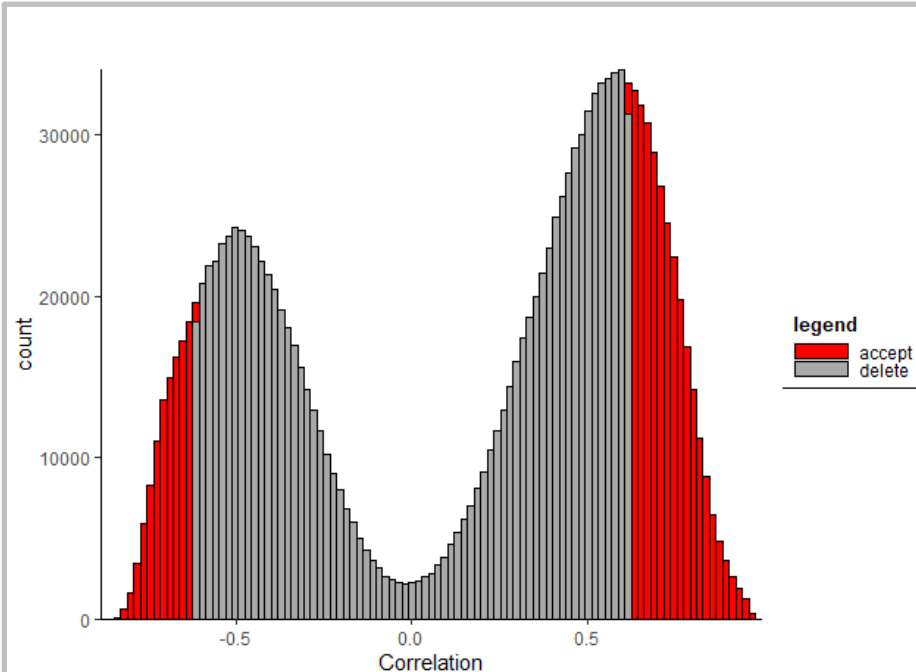
  df$legend <- ifelse( (abs(x) <= threshold), "delete", "accept")

  w <- (max(x) - min(x)) / 100

  p = ggplot(df, aes(variable, fill = legend)) + geom_histogram
    (binwidth = w, colour='black') +
    scale_x_continuous(expand = c(0, 0)) + scale_y_continuous(expand
    = c(0, 0)) +
    scale_fill_manual(values = c("delete" = "darkgrey", "accept" =
    "red")) +
    theme(panel.grid.major = element_blank(), panel.grid.minor =
    element_blank(),
    panel.background = element_blank(), axis.line = element_li
    ne(colour = "black"),
    plot.title = element_text(hjust = 0.5, face = "bold"),
    legend.title = element_text(colour = "black", size=10,
    face="bold"),
    legend.key.height = unit(0.2, "cm"), legend.key.width =
    unit(1, "cm"),
    legend.box.background = element_rect(colour = "black")) +
    labs(title = title, x = xlabel)

  print(p)
}
```

Pearson correlation coefficient distribution



- x-axis represents the Pearson correlation coefficient between the expression profiles of all pairs of genes
- y-axis represents the frequency
- red bars represent pairs with correlation coefficient greater than the selected threshold
- grey bars represent pairs with correlation coefficient less than the selected threshold



Caveat: Only the highly correlated pairs (red bars) according to the selected threshold will be used to build the correlation network

Histogram of Pearson correlation

```
getHistogram <- function(x, threshold, title, xlabel){
  df <- data.frame(variable = x)

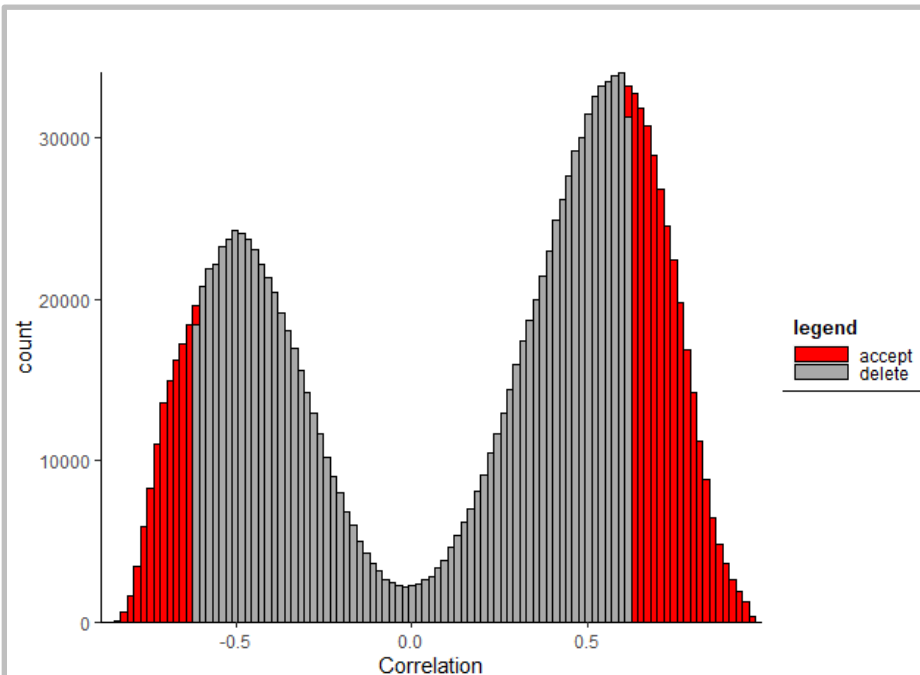
  df$legend <- ifelse( (abs(x) <= threshold), "delete", "accept")

  w <- (max(x) - min(x)) / 100

  p = ggplot(df, aes(variable, fill = legend)) + geom_histogram
    (binwidth = w, colour='black') +
    scale_x_continuous(expand = c(0, 0)) + scale_y_continuous(expand
    = c(0, 0)) +
    scale_fill_manual(values = c("delete" = "darkgrey", "accept" =
    "red")) +
    theme(panel.grid.major = element_blank(), panel.grid.minor =
    element_blank(),
          panel.background = element_blank(), axis.line = element_li
    ne(colour = "black"),
          plot.title = element_text(hjust = 0.5, face = "bold"),
          legend.title = element_text(colour = "black", size=10,
          face="bold"),
          legend.key.height = unit(0.2, "cm"), legend.key.width =
          unit(1, "cm"),
          legend.box.background = element_rect(colour = "black")) +
    labs(title = title, x = xlabel)

  print(p)
```

Pearson correlation coefficient distribution



Histogram of Pearson correlation

```
getHistogram <- function(x, threshold, title, xlabel){
  df <- data.frame(variable = x)

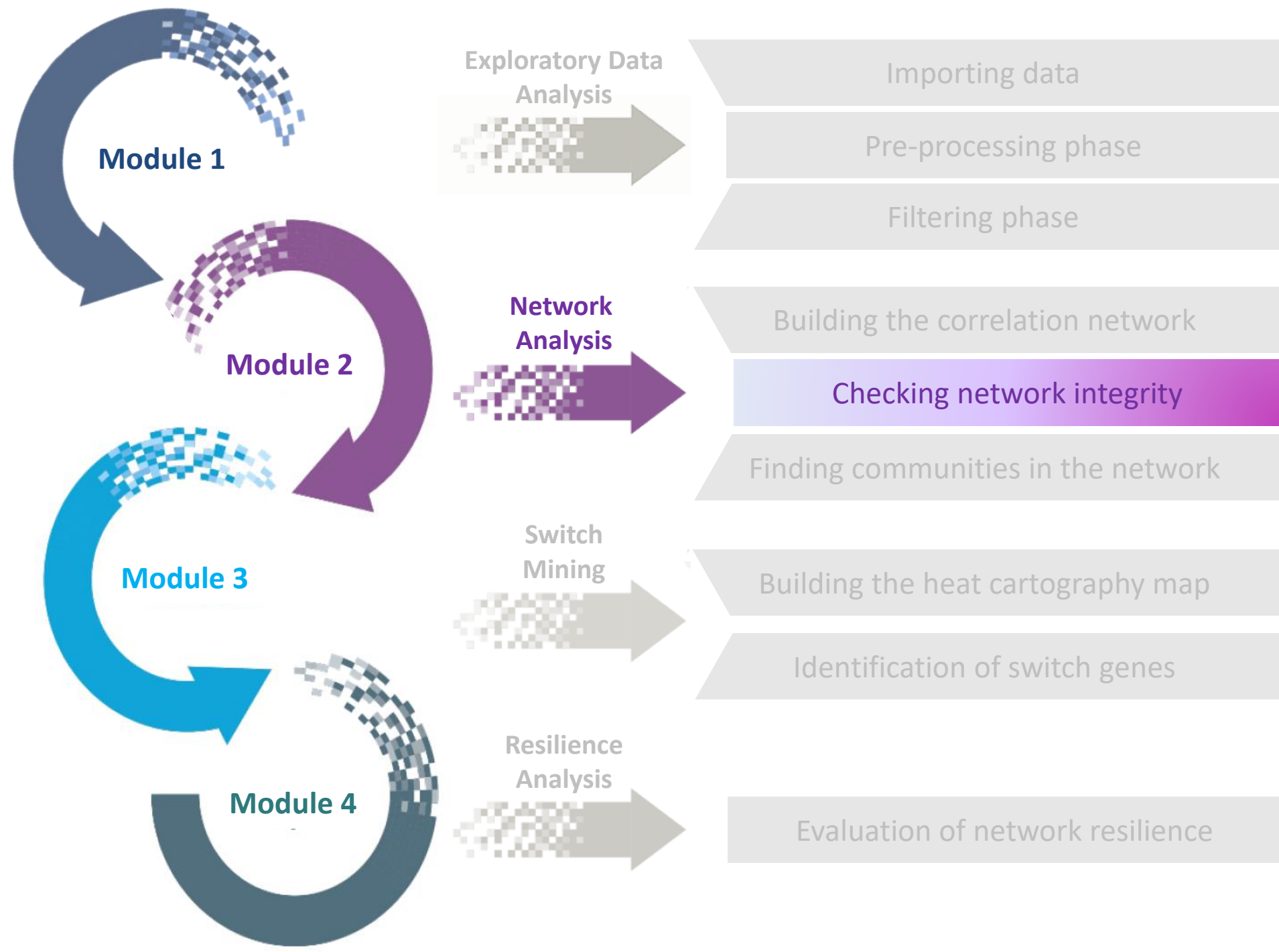
  df$legend <- ifelse( (abs(x) <= threshold), "delete", "accept")

  w <- (max(x) - min(x)) / 100

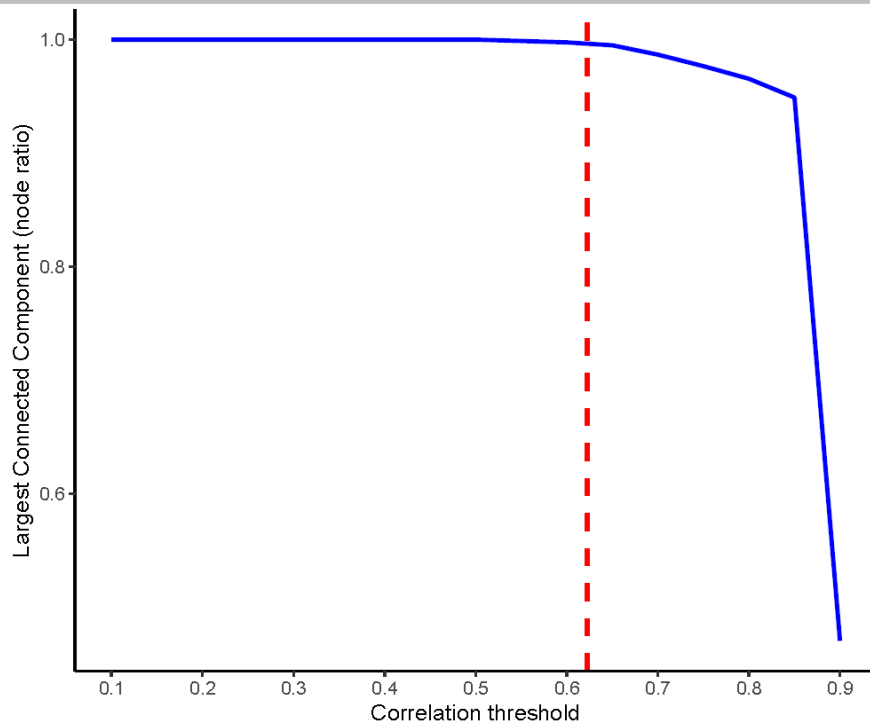
  p = ggplot(df, aes(variable, fill = legend)) + geom_histogram
    (binwidth = w, colour='black') +
    scale_x_continuous(expand = c(0, 0)) + scale_y_continuous(expand
    = c(0, 0)) +
    scale_fill_manual(values = c("delete" = "darkgrey", "accept" =
    "red")) +
    theme(panel.grid.major = element_blank(), panel.grid.minor =
    element_blank(),
          panel.background = element_blank(), axis.line = element_li
    ne(colour = "black"),
          plot.title = element_text(hjust = 0.5, face = "bold"),
```



Correlation threshold should reflect a right balance between the number of edges and the number of connected components of the network: the number of **edges** should be **as small as possible** in order to have a manageable network (**high** threshold) and the number of **connected components** should be **as small as possible** in order to preserve the integrity of the network (**small** threshold).



Network integrity plot



- x-axis represents the Pearson correlation threshold varying in the chosen range
- y-axis represents the fraction of nodes populating the largest connected component
- dashed red line correspond to the selected threshold

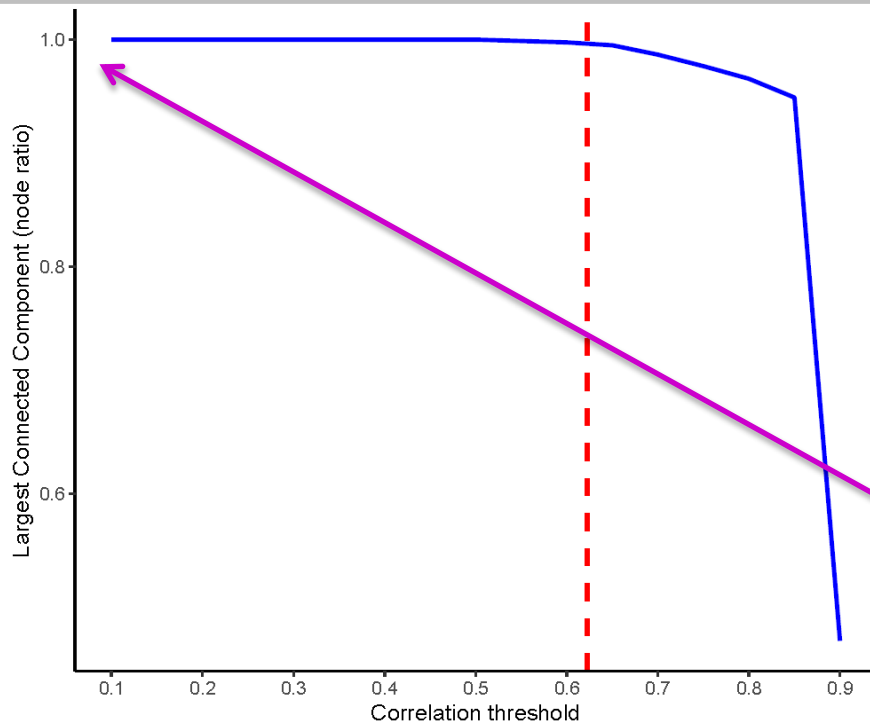
Network integrity

- This step evaluates the effect on the **network connectivity** of varying the Pearson correlation threshold
- It is **optional**: set “T” on main.R

```
network <- NetworkAnalysis(data, checkNetIntegrity = T,  
screepPlot = T)
```



Network integrity plot



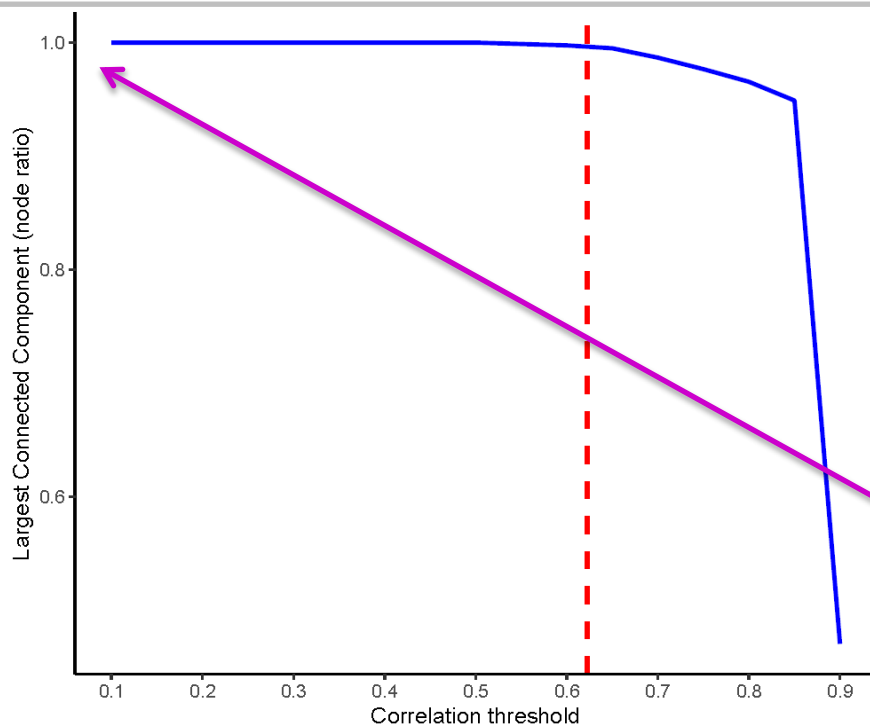
Network integrity

- This step evaluates the effect on the **network connectivity** of varying the Pearson correlation threshold
- It is **optional**: set “T” on main.R

y=1 means that all nodes fall in the largest component and thus the network is **fully connected**; otherwise more components exist

- x-axis represents the Pearson correlation threshold varying in the chosen range
- y-axis represents the fraction of nodes populating the largest connected component
- dashed red line correspond to the selected threshold

Network integrity plot



Network integrity

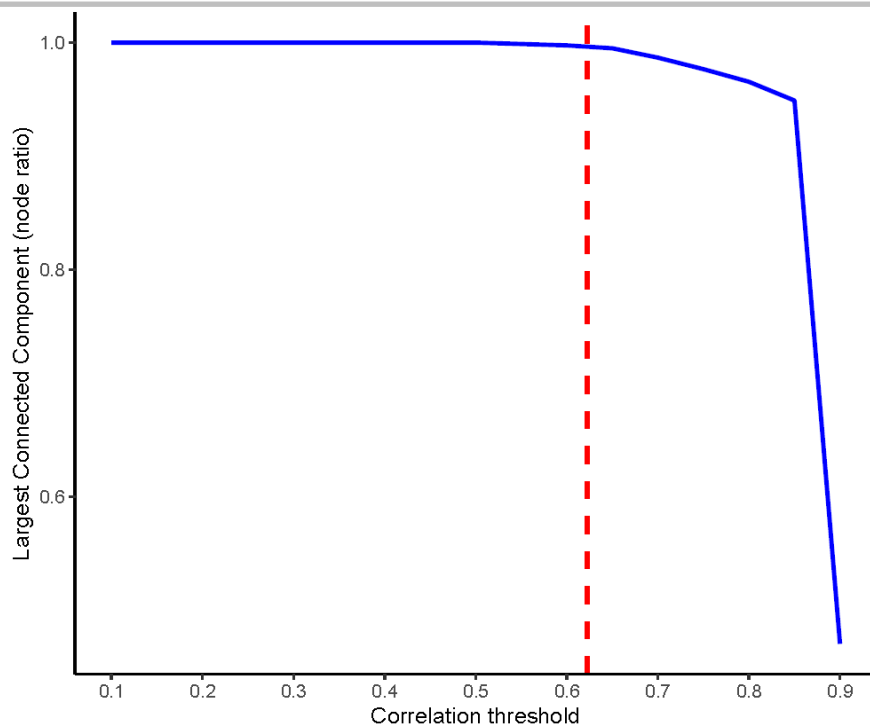
- This step evaluates the effect on the **network connectivity** of varying the Pearson correlation threshold
- It is **optional**: set “T” on main.R

y=1 means that all nodes fall in the largest component and thus the network is **fully connected**; otherwise more components exist



A reasonable choice for the correlation threshold should be the largest one (corresponding to the smallest number of edges) for which the fraction of nodes of the largest connected component is equal to 1. Here, we could have increased the threshold up to 0.7.

Network integrity plot



- x-axis represents the Pearson correlation threshold varying in the chosen range
- y-axis represents the fraction of nodes populating the largest component
- dashed red line correspond to the selected threshold

Network integrity plot

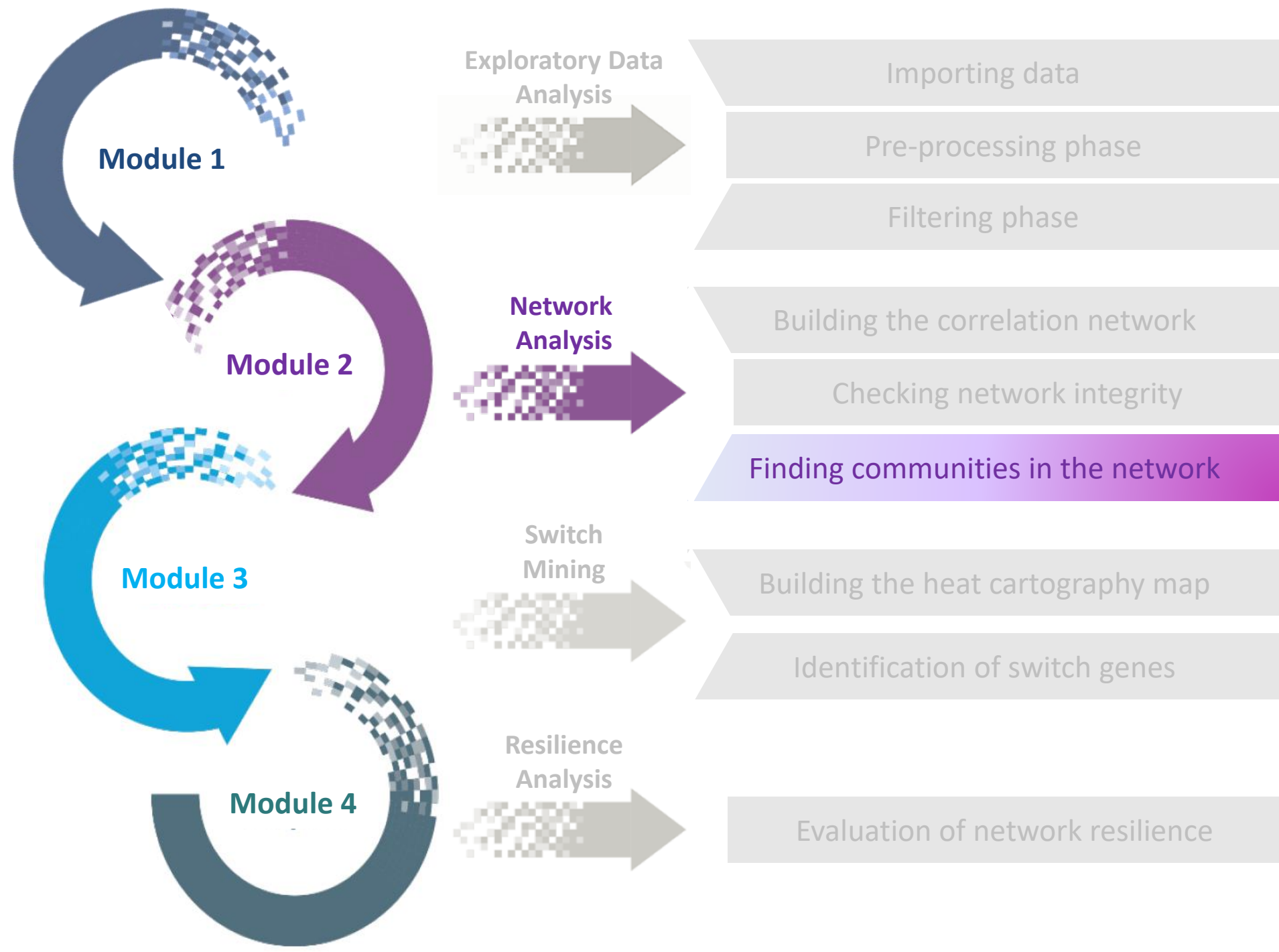
```
getNetworkIntegrityPlot <- function(df,min_rho,max_rho
,threshold_corr,output_file){

  p <- ggplot(df, aes(x = threshold_corr, y = frac_node_LCC))
  +
    geom_line(color = "blue", size = 1) + scale_x_continuous
    (breaks=seq(min_rho,max_rho,0.1)) +
    theme(panel.grid.major = element_blank(), panel.grid.minor
    = element_blank(),|
          panel.background = element_blank(), #axis.title =
    element_text(face = "bold"),
          axis.line = element_line(colour = "black")) +
    labs(x = "Correlation threshold", y = "Largest Connected
    Component (node ratio)") +
    geom_vline(xintercept = threshold_corr, linetype =
    "dashed", color = "red", size = 1)

  print(p)

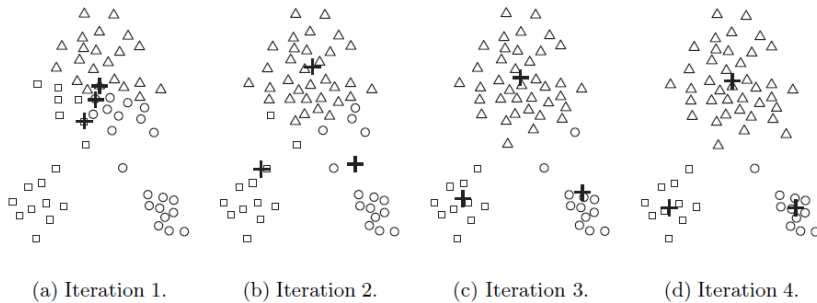
  savePDF(p,output_file)

}
```



K-means

- 1: Select K points as initial centroids.
- 2: **repeat**
- 3: Form K clusters by assigning each point to its closest centroid.
- 4: Recompute the centroid of each cluster.
- 5: **until** Centroids do not change.

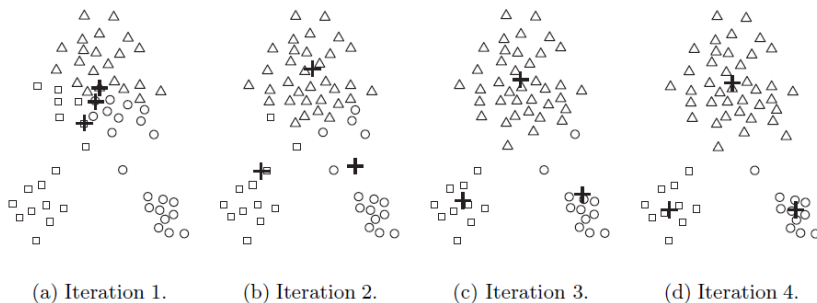


K-means algorithm

- This step searches for communities in the correlation network by a **k-means algorithm**
- **k-means** is a clustering algorithm whose aim is to partition a set of n objects in N groups (clusters) so that each object belongs to the cluster with the nearest centroid

K-means

- 1: Select K points as initial centroids.
- 2: **repeat**
- 3: Form K clusters by assigning each point to its closest centroid.
- 4: Recompute the centroid of each cluster.
- 5: **until** Centroids do not change.

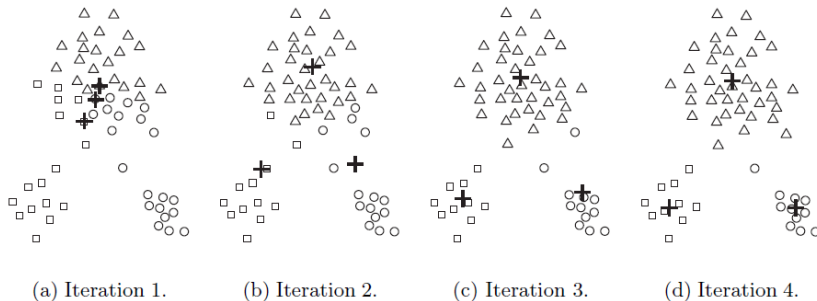


K-means algorithm

- It requires to know the **number of clusters** in advance
- The k-means algorithm performs **iterations** until the minimum of the Sum of the Squared Error (SSE) function is reached
- SWIMmer repeats the clustering many times (**replicates**), each with a new set of initial cluster centroid positions, randomly chosen, and selects the clusters configuration corresponding the minimum of SSE among all replicates

K-means

- 1: Select K points as initial centroids.
- 2: **repeat**
- 3: Form K clusters by assigning each point to its closest centroid.
- 4: Recompute the centroid of each cluster.
- 5: **until** Centroids do not change.



K-means algorithm

- The **Sum of the Squared Error** function is

$$SSE = \sum_{i=1}^K \sum_{x \in C_i} dist^2(c_i, x)$$

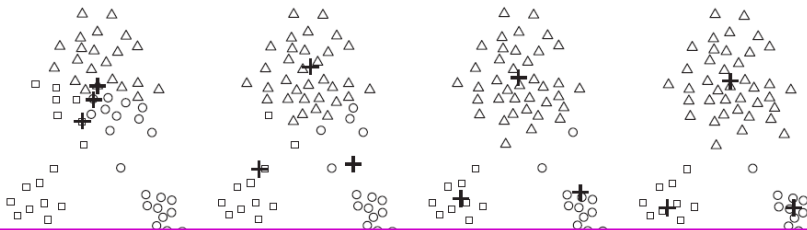
- where $dist$ is the Euclidean distance, K is the number of the clusters, C_i is the i^{th} cluster, x is a node in the i^{th} cluster, c_i is the centroid of the i^{th} cluster given by:

$$c_i = \frac{1}{m_i} \sum_{x \in C_i} x_i$$

- where m_i is the number of nodes in the i^{th} cluster. There are as many centroids as the number of the clusters.

K-means

- 1: Select K points as initial centroids.
- 2: **repeat**
- 3: Form K clusters by assigning each point to its closest centroid.
- 4: Recompute the centroid of each cluster.
- 5: **until** Centroids do not change.



K-means algorithm

- The **Sum of the Squared Error** function is

$$SSE = \sum_{i=1}^K \sum_{x \in C_i} dist^2(c_i, x)$$

- where $dist$ is the Euclidean distance, K is the number of the clusters, C_i is the i^{th} cluster, x is a node in the i^{th} cluster, c_i is the centroid of the i^{th} cluster given by:



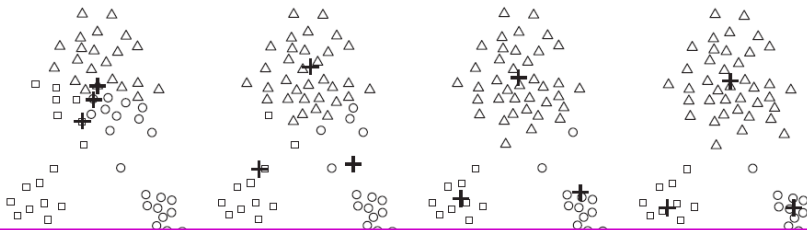
The choice of the Euclidean distance is intended to emphasize the differences between correlated and anti-correlated genes.

$$dist^2(A, B) = (x_1^A - x_1^B)^2 + (x_2^A - x_2^B)^2 + \dots + (x_N^A - x_N^B)^2$$

$x_1^A = a_{A,1}$ where $a_{N,N}$ is the weighted adjacency matrix and the weights are the Pearson correlation coefficients

K-means

- 1: Select K points as initial centroids.
- 2: **repeat**
- 3: Form K clusters by assigning each point to its closest centroid.
- 4: Recompute the centroid of each cluster.
- 5: **until** Centroids do not change.



K-means algorithm

- The **Sum of the Squared Error** function is

$$SSE = \sum_{i=1}^K \sum_{x \in C_i} dist^2(c_i, x)$$

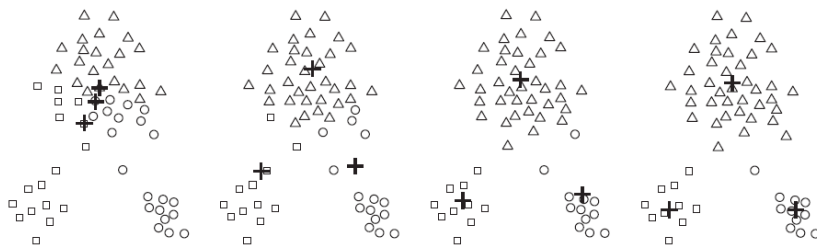
- where $dist$ is the Euclidean distance, K is the number of the clusters, C_i is the i^{th} cluster, x is a node in the i^{th} cluster, c_i is the centroid of the i^{th} cluster given by:



Thus, two nodes A, B are close in the network ($dist = 0$) if they are highly correlated ($\rho = 1$) and they are far apart in the network ($dist = dist_{max}$) if they are highly anti-correlated ($\rho = -1$).

K-means

- 1: Select K points as initial centroids.
- 2: **repeat**
- 3: Form K clusters by assigning each point to its closest centroid.
- 4: Recompute the centroid of each cluster.
- 5: **until** Centroids do not change.



(a) Iteration 1.

(b) Iteration 2.

(c) Iteration 3.

(d) Iteration 4.

K-means algorithm

weighted
adjacency matrix

parameters

```
getClustering <- function(w_adj,num_clusters,iter_max,num_repeats,output_file_idx){
#####
model <- kmeans(w_adj, centers = num_clusters, iter.max = iter_max, nstart = num_repeats)

WSS <- model$withinss      # Within-cluster Sum of Square (i.e. the compactness of the each cluster)
TWSS <- model$tot.withinss # Total Within-cluster Sum of Square, i.e. SSE = TWSS = sum(WSS)
TSS <- model$totss         # the Total Sum of Squares (i.e. the compactness of the whole network)
BSS <- model$betweenss     # the Between Sum of Squares, i.e. BSS = TSS - TWSS
size <- model$size
idx <- data.frame(cluster = as.factor(model$cluster))
#####

cluster_connectivity <- computeClusterConnectivity(w_adj,idx)

cluster_compactness <- (1 - WSS/TSS) * 100
cluster_compactness <- signif(cluster_compactness,3)

partition_quality <- BSS / TSS * 100 # (TSS - TWSS)/TSS * 100 = (1 - TWSS/TSS) * 100
partition_quality <- signif(partition_quality,3)

# The higher the percentage, the better the quality of the k-means partition
# (i.e. BSS is large and/or TWSS is small)
# The smaller TWSS, the more compact the clusters (close to the centroids)

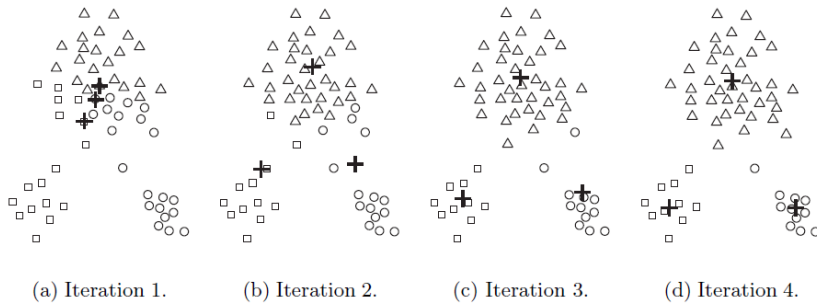
write.table(idx, output_file_idx, sep="\t", row.names = T, col.names = NA, quote = F)

res <- list(size = size, idx = idx,
            cluster_connectivity = cluster_connectivity,
            cluster_compactness = cluster_compactness,
            partition_quality = partition_quality)

return(res)
}
```


K-means

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- 4: Recompute the centroid of each cluster.
- 5: **until** Centroids do not change.



K-means algorithm

```
computeClusterConnectivity <- function(adj,idx){

#####
adj[adj != 0] <- 1
diag(adj) <- 0

node <- rownames(idx)
#####

cluster <- split(idx,idx$cluster)

cluster_connectivity <- sapply(names(cluster), function(x){

  member <- rownames(cluster[[x]])
  not_member <- setdiff(node,member)

  adj_in <- adj[member,member]
  adj_out <- adj[member,not_member]

  deg_in <- sum(adj_in)/2
  deg_out <- sum(adj_out)

  deg_tot <- deg_in + deg_out

  P <- length(member) / length(node)

  CC <- deg_in / (deg_tot + P) * 100
  CC <- signif(CC,3)

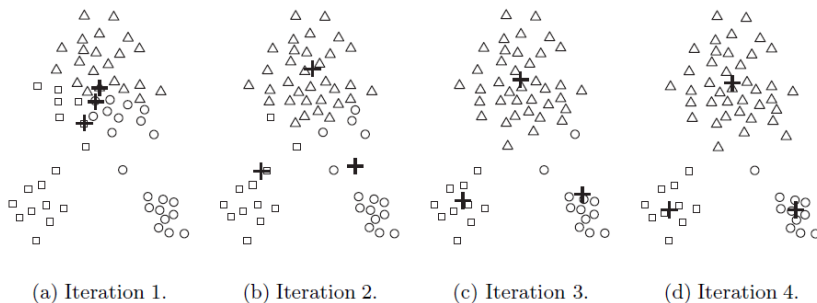
})

return(cluster_connectivity)

}
```

K-means

- 1: Select K points as initial centroids.
- 2: **repeat**
- 3: Form K clusters by assigning each point to its closest centroid.
- 4: Recompute the centroid of each cluster.
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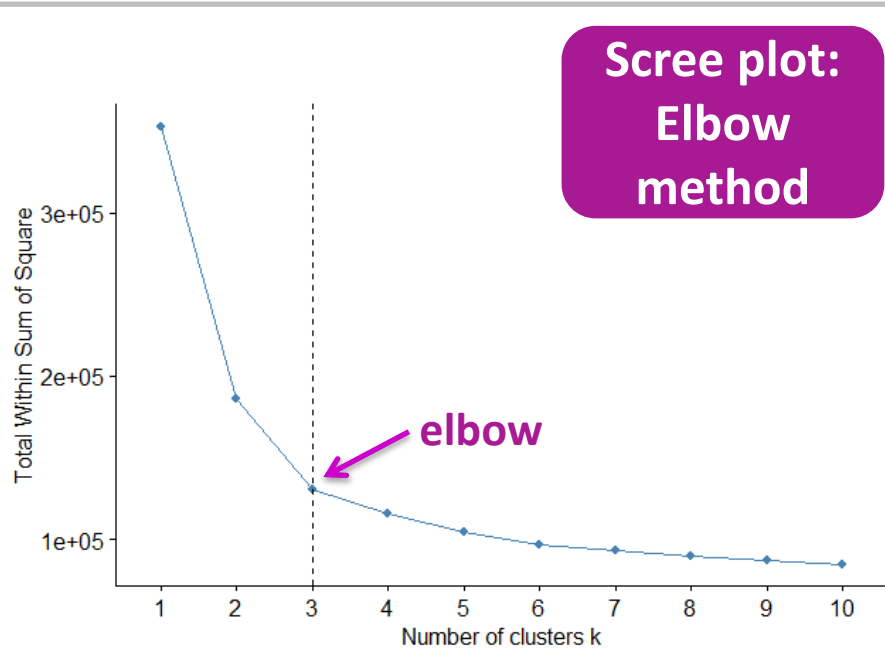
K-means parameters

- the **number of clusters**
- the maximum number of **iterations** allowed for each replicate of k-means
- the number of **replicates** for a given number of clusters

! **Caveat:** maximum number of **iterations** should be large enough to guarantee convergence

! **Caveat:** more **replicates** get you more confident to be far from a local minimum

Scree plot



- x-axis refers to the number of clusters
- y-axis refers to the value of the error function corresponding to the best clusters configuration among all replicates for that number of clusters

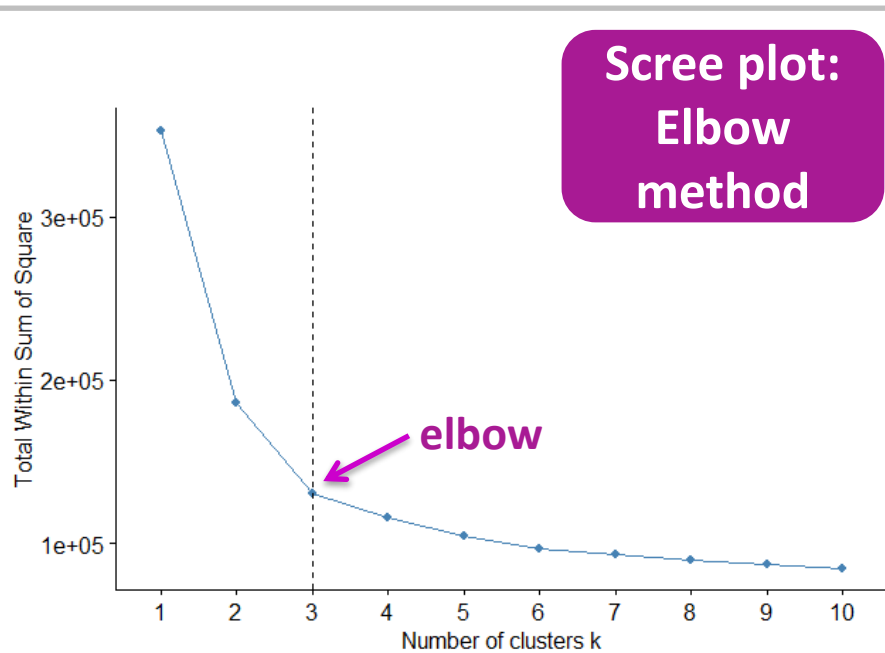
Scree plot computation

- Scree plot will be produced to suggest the best choice for the number of **clusters**
- It is **optional**: set “**T**” on main.R

```
network <- NetworkAnalysis(data, checkNetIntegrity = T,  
  screePlot = T)
```

- It shows the value of the error function (**SSE**) for each number of clusters
- a good choice for the optimal number of clusters is in correspondence of the **elbow**

Scree plot



- x-axis refers to the number of clusters
- y-axis refers to the value of the error function corresponding to the best clusters configuration among all replicates for that number of clusters


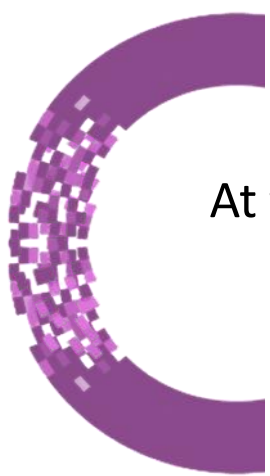
Scree plot computation

```
getScreePlot <- function(w_adj){
  #####
  # input parameters

  num_clusters <- input_parameter$num_clusters
  iter_max <- input_parameter$iter_max
  num_repeats <- input_parameter$num_repeats
  #####

  # Elbow method
  p <- fviz_nbclust(w_adj, kmeans, method = "wss", iter
    .max = iter_max, nstart = num_repeats) +
    geom_vline(xintercept = num_clusters, linetype = 2) +
  # add line for better visualisation
    labs(subtitle = "Elbow method") # add subtitle

  print(p)
}
```



At the end of module 2, you will obtain the
correlation network