

Exercise 1

b)

1) Check if erosion is compatible with superposition:

$$\text{Eros}(g_1 \vee g_2)(x, y) \stackrel{?}{=} (\text{Eros}(g_1) \vee \text{Eros}(g_2))(x, y)$$

Left:

$$\Leftrightarrow ((g_1 \vee g_2) \ominus S)(x, y)$$

$$\Leftrightarrow ((S_0 \rightarrow (g_1 \vee g_2)) \wedge \dots \wedge (S_n \rightarrow (g_1 \vee g_2)))$$

$$\stackrel{\text{short}}{\Leftrightarrow} (\neg S_0 \vee g_1 \vee g_2) \wedge (\neg S_n \vee g_1 \vee g_2) \quad \text{CNF}$$

Right:

$$\Leftrightarrow ((g_1 \ominus S)(x, y)) \vee ((g_2 \ominus S)(x, y))$$

$$\Leftrightarrow ((S_0 \rightarrow g_1) \wedge (S_n \rightarrow g_1)) \vee \dots \vee ((S_0 \rightarrow g_2) \wedge (S_n \rightarrow g_2))$$

$$\stackrel{\text{short}}{\Leftrightarrow} ((\neg S_0 \vee g_1) \wedge (\neg S_n \vee g_1)) \vee ((\neg S_0 \vee g_2) \wedge (\neg S_n \vee g_2)) \quad \text{DNF}$$

Not the same  $\rightarrow$  does not work with erosion2) Check for dilation:

$$\text{Dil}(g_1 \vee g_2)(x, y) \stackrel{?}{=} (\text{Dil}(g_1)(x, y)) \vee (\text{Dil}(g_2)(x, y))$$

Left:

$$\Leftrightarrow (g_1 \vee g_2) \oplus S(x, y)$$

$$\Leftrightarrow (S_0 \wedge (g_1 \vee g_2)) \vee \dots \vee (S_n \wedge (g_1 \vee g_2))$$

Right:

$$\Leftrightarrow (g_1 \oplus S(x, y)) \vee (g_2 \oplus S(x, y))$$

$$\Leftrightarrow ((S_0 \wedge g_1) \vee (S_n \wedge g_1)) \vee \dots \vee ((S_0 \wedge g_2) \vee (S_n \wedge g_2))$$

Left = Right

$$(S_0 \wedge (g_1 \vee g_2)) \vee \dots \vee (S_n \wedge (g_1 \vee g_2)) = ((S_0 \wedge g_1) \vee (S_n \wedge g_1)) \vee \dots \vee ((S_0 \wedge g_2) \vee (S_n \wedge g_2))$$

short  
 $\Leftrightarrow$

$$(S_0 \wedge (g_1 \vee g_2)) \vee \dots \vee (S_n \wedge (g_1 \vee g_2)) = ((S_0 \wedge g_1) \vee (S_n \wedge g_1)) \vee \dots \vee ((S_0 \wedge g_2) \vee (S_n \wedge g_2))$$

short  
 $\Leftrightarrow$

$$((S_0 \wedge g_1) \vee (S_0 \wedge g_2)) \vee ((S_n \wedge g_1) \vee (S_n \wedge g_2)) = (S_0 \wedge g_1) \vee (S_n \wedge g_1) \vee (S_0 \wedge g_2) \vee (S_n \wedge g_2)$$

$$(S_0 \wedge g_1) \vee (S_n \wedge g_1) \vee (S_0 \wedge g_2) \vee (S_n \wedge g_2) \stackrel{\vee}{=} (S_0 \wedge g_1) \vee (S_n \wedge g_1) \vee (S_0 \wedge g_2) \vee (S_n \wedge g_2) \quad \square$$

Both sides yield the same output  $\rightarrow$  dilation is compatible with superposition