

Aufgabe 1c

19 November 2021 14:10

$$\begin{array}{lll} (g \oplus S_1) \oplus S_2 & = g \oplus (S_1 \oplus S_2) & = (g \oplus S_2) \oplus S_1 \\ (g \ominus S_1) \ominus S_2 & = g \ominus (S_1 \ominus S_2) & = (g \ominus S_2) \ominus S_1 \end{array}$$

Exercise 1

c)

Proof for dilation

$$\text{To show: } \underbrace{(g \oplus S_1)}_{T_1} \oplus S_2 = \underbrace{g \oplus (S_1 \oplus S_2)}_{T_2} = \underbrace{(g \oplus S_2) \oplus S_1}_{T_3}$$

$$\begin{aligned} \textcircled{T_1} \quad (g \oplus S_1) \oplus S_2 &\hat{=} (S_1(k+m, l+n) \wedge g(x+k, y+l)) \wedge S_2(k+m, l+n) \\ &\hat{=} S_1(k+m, l+n) \wedge g(x+k, y+l) \wedge S_2(k+m, l+n) \end{aligned}$$

$$\begin{aligned} \textcircled{T_2} \quad g \oplus (S_1 \oplus S_2) &\hat{=} g(x+k, y+l) \wedge (S_1(k+m, l+n) \wedge S_2(k+m, l+n)) \\ &\hat{=} T_1 \end{aligned}$$

$$\begin{aligned} \textcircled{T_3} \quad (g \oplus S_2) \oplus S_1 &\hat{=} (S_2 \wedge g) \wedge S_1 \\ &\hat{=} S_2 \wedge g \wedge S_1 \\ &\hat{=} S_1 \wedge g \wedge S_2 \\ &\hat{=} T_1 \end{aligned}$$

Proof for erosion

$$\text{To show: } \underbrace{(g \ominus S_1)}_{T_1} \ominus S_2 = \underbrace{g \ominus (S_1 \ominus S_2)}_{T_2} = \underbrace{(g \ominus S_2) \ominus S_1}_{T_3}$$

$$\begin{aligned} \textcircled{T_1} \quad (g \ominus S_1) \ominus S_2 &\hat{=} (S_1 \rightarrow g) \rightarrow S_2 \\ &\hat{=} \neg(S_1 \rightarrow g) \vee S_2 \\ &\hat{=} (S_1 \wedge \neg g) \vee S_2 \end{aligned}$$

$$\begin{aligned} \textcircled{T_2} \quad g \ominus (S_1 \ominus S_2) &\hat{=} g \rightarrow (S_1 \rightarrow S_2) \\ &\hat{=} \neg g \vee (S_1 \rightarrow S_2) \end{aligned}$$

$$\equiv \neg g \vee (\neg S_1 \vee S_2)$$

$$\equiv \neg g \vee \neg S_1 \vee S_2 \quad \hookrightarrow \quad T_1 \neq \overline{T_2}$$