Aufgabe 1a

19 November 2021 12:45

Exercise 1

a) Show / proof that erosion and dilation are dual operators.

(1) Proof for erosion:

Short notation for erosion: $g' = g \ominus S$

To show: $\underbrace{g^* \ominus S}_{\text{Left}} = \underbrace{(g \oplus S)^*}_{\text{right}}$; with $g^*(x,y) = \neg g(x,y)$

Left: $g^* \ominus S = \Big(\underbrace{S(k+(-m), l+(-n))}_{A_{-m,-n}} \rightarrow \neg \underbrace{g(x+k, y+l)}_{B}\Big) \wedge \ldots \wedge \Big(\underbrace{S(k+m, l+n)}_{A_{m,n}} \rightarrow \neg \underbrace{g(x+k, y+l)}_{B}\Big)$

$\overset{\text{short}}{=} (A_{-m,-n} \longrightarrow \neg B) \wedge (A_{m,n} \longrightarrow \neg B)$

Right: $(g \oplus S)^* = \neg\Big[\Big(\underbrace{S(k+(-m), l+(-n))}_{C_{-m,-n}} \wedge \underbrace{g(x+k, y+l)}_{D}\Big) \vee \Big(\underbrace{S(k+m, l+n)}_{C_{m,n}} \wedge \underbrace{g(x+k, y+l)}_{D}\Big)\Big]$

$\overset{\text{short}}{=} \neg[(C_{-m,-n} \wedge D) \vee (C_{m,n} \wedge D)]$

Left = Right

$(A_{-m,-n} \longrightarrow \neg B) \wedge (A_{m,n} \longrightarrow \neg B) = \neg[(C_{-m,-n} \wedge D) \vee (C_{m,n} \wedge D)]$

$\Leftrightarrow (\neg A_{-m,-n} \vee \neg B) \wedge (\neg A_{m,n} \vee \neg B) = \neg(C_{-m,-n} \wedge D) \wedge \neg(C_{m,n} \wedge D)$

$\Leftrightarrow (\neg A_{-m,-n} \vee \neg B) \wedge (\neg A_{m,n} \vee \neg B) = (\neg C_{-m,-n} \vee \neg D) \wedge (\neg C_{m,n} \vee D)$ $\square$

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(2) Proof for dilation:

Short notation for dilation: $g' = g \oplus S$

To show: $g^* \oplus S = (g \ominus S)^*$; with $g^*(x,y) = \neg g(x,y)$

Left: $g^* \oplus S = \Big(\underbrace{S(k+(-m), l+(-n))}_{A_{-m,-n}} \wedge \neg \underbrace{g(x+k, y+l)}_{B}\Big) \vee \ldots \vee \Big(\underbrace{S(k+m, l+n)}_{A_{m,n}} \wedge \neg \underbrace{g(x+k, y+l)}_{B}\Big)$

$\overset{\text{short}}{=} (A_{-m,-n} \wedge \neg B) \vee (A_{m,n} \wedge \neg B)$

Right: $(g \ominus S)^* = \neg\Big[\Big(\underbrace{S(k+(-m), l+(-n))}_{C_{-m,-n}} \rightarrow \underbrace{g(x+k, y+l)}_{D}\Big) \wedge \ldots \wedge \Big(\underbrace{S(k+m, l+n)}_{C_{m,n}} \rightarrow \underbrace{g(x+k, y+l)}_{D}\Big)\Big]$

$\overset{\text{short}}{=} \neg[(C_{-m,-n} \longrightarrow D) \wedge (C_{m,n} \longrightarrow D)]$

Left = Right

$$(A_{-m,-n} \land \neg B) \lor (A_{m,n} \land \neg B) = \neg\left[(C_{-m,-n} \to D) \land (C_{m,n} \to D)\right]$$

$$\Leftrightarrow \qquad = \neg(C_{-m,-n} \to D) \lor \neg(C_{m,n} \to D)$$

$$(A_{-m,-n} \land \neg B) \lor (A_{m,n} \land \neg B) = (C_{-m,-n} \land \neg D) \lor (C_{m,n} \land \neg D) \qquad \square$$