Aufgabe 1c

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$(g \oplus S_1) \oplus S_2 \quad = g \oplus (S_1 \oplus S_2) \quad = (g \oplus S_2) \oplus S_1$
$(g \ominus S_1) \ominus S_2 \quad = g \ominus (S_1 \ominus S_2) \quad = (g \ominus S_2) \ominus S_1$

Exercise 1

c)

Proof for dilation

To show: $\underbrace{(g \oplus S_1) \oplus S_2}_{T_1} = \underbrace{g \oplus (S_1 \oplus S_2)}_{T_2} = \underbrace{(g \oplus S_2) + S_1}_{T_3}$

T_1 $(g \oplus S_1) \oplus S_2 \hat{=} (S_1(k+m, l+n) \wedge g(x+k, y+l)) \wedge S_2(k+m, l+n)$

$\hat{=} S_1(k+m, l+n) \wedge g(x+k, y+l) \wedge S_2(k+m, l+n)$

T_2 $g \oplus (S_1 \oplus S_2) \hat{=} g(x+k, y+l) \wedge (S_1(k+m, l+n) \wedge S_2(k+m, l+n))$

$\hat{=} T_1$

T_3 $(g \oplus S_2) \oplus S_1 \hat{=} (S_2 \wedge g) \wedge S_1$

$\hat{=} S_2 \wedge g \wedge S_1$

$\hat{=} S_1 \wedge g \wedge S_2$

$\hat{=} T_1$

Proof for erosion

To show: $\underbrace{(g \ominus S_1) \ominus S_2}_{T_1} = \underbrace{g \ominus (S_1 \ominus S_2)}_{T_2} = \underbrace{(g \ominus S_2) \ominus S_1}_{T_3}$

T_1 $(g \ominus S_1) \ominus S_2 \hat{=} (S_1 \to g) \to S_2$

$\hat{=} \neg(S_1 \to g) \vee S_2$

$\hat{=} (S_1 \wedge \neg g) \vee S_2$

T_2 $g \ominus (S_1 \ominus S_2) \hat{=} g \to (S_1 \to S_2)$

$\hat{=} \neg g \vee (S_1 \to S_2)$

$\hat{\equiv} \neg g \vee (\neg S_1 \vee S_2)$

$\hat{\equiv} \neg g \vee \neg S_1 \vee S_2$ ↯ $T_1 \neq T_2$