# MathOptInterface

The JuMP core developers and contributors

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# Part I Introduction

# **Chapter 1**

# Introduction

Welcome to the documentation for MathOptInterface.

#### Note

This documentation is also available in PDF format: MathOptInterface.pdf.

# 1.1 What is MathOptInterface?

MathOptInterface.jl (MOI) is an abstraction layer designed to provide a unified interface to mathematical optimization solvers so that users do not need to understand multiple solver-specific APIs.

## Tip

This documentation is aimed at developers writing software interfaces to solvers and modeling languages using the MathOptInterface API. If you are a user interested in solving optimization problems, we encourage you instead to use MOI through a higher-level modeling interface like JuMP or Convex.jl.

#### 1.2 How the documentation is structured

Having a high-level overview of how this documentation is structured will help you know where to look for certain things.

- The **Tutorials** section contains articles on how to use and implement the MathOptInteraface API. Look here if you want to write a model in MOI, or write an interface to a new solver.
- The Manual contains short code-snippets that explain how to use the MOI API. Look here for more details on particular areas of MOI.
- The **Background** section contains articles on the theory behind MathOptInterface. Look here if you want to understand why, rather than how.
- The **API Reference** contains a complete list of functions and types that comprise the MOI API. Look here if you want to know how to use (or implement) a particular function.
- The **Submodules** section contains stand-alone documentation for each of the submodules within MOI. These submodules are not required to interface a solver with MOI, but they make the job much easier.

# 1.3 Citing MathOptInterface

If you find MathOptInterface useful in your work, we kindly request that you cite the following paper:

```
@article{legat2021mathoptinterface,
    title={{MathOptInterface}: a data structure for mathematical optimization problems},
    author={Legat, Beno{\^\i}t and Dowson, Oscar and Garcia, Joaquim Dias and Lubin, Miles},
    journal={INFORMS Journal on Computing},
    year={2021},
    doi={10.1287/ijoc.2021.1067},
    publisher={INFORMS}
}
```

A preprint of this paper is freely available.

# **Chapter 2**

# **Motivation**

MathOptInterface (MOI) is a replacement for MathProgBase, the first-generation abstraction layer for mathematical optimization previously used by JuMP and Convex.jl.

To address a number of limitations of MathProgBase, MOI is designed to:

- · Be simple and extensible
  - unifying linear, quadratic, and conic optimization,
  - seamlessly facilitating extensions to essentially arbitrary constraints and functions (for example, indicator constraints, complementarity constraints, and piecewise-linear functions)
- Be fast
  - by allowing access to a solver's in-memory representation of a problem without writing intermediate files (when possible)
  - by using multiple dispatch and avoiding requiring containers of non-concrete types
- Allow a solver to return multiple results (for example, a pool of solutions)
- Allow a solver to return extra arbitrary information via attributes (for example, variable- and constraintwise membership in an irreducible inconsistent subset for infeasibility analysis)
- Provide a greatly expanded set of status codes explaining what happened during the optimization procedure
- Enable a solver to more precisely specify which problem classes it supports
- Enable both primal and dual warm starts
- Enable adding and removing both variables and constraints by indices that are not required to be consecutive
- Enable any modification that the solver supports to an existing model
- · Avoid requiring the solver wrapper to store an additional copy of the problem data

Part II

**Tutorials** 

# **Chapter 3**

# Solving a problem using MathOptInterface

In this tutorial we demonstrate how to use MathOptInterface to solve the binary-constrained knapsack problem:

$$\max c^{\top} x$$
$$s.t. \ w^{\top} x \le C$$
$$x_i \in \{0, 1\}, \quad \forall i = 1, \dots, n$$

# 3.1 Required packages

Load the MathOptInterface module and define the shorthand MOI:

```
import MathOptInterface as MOI
```

As an optimizer, we choose GLPK:

```
using GLPK
optimizer = GLPK.Optimizer()
```

# 3.2 Define the data

We first define the constants of the problem:

```
julia> c = [1.0, 2.0, 3.0]
3-element Vector{Float64}:
1.0
2.0
3.0

julia> w = [0.3, 0.5, 1.0]
3-element Vector{Float64}:
0.3
0.5
1.0

julia> C = 3.2
3.2
```

#### 3.3 Add the variables

```
julia> x = MOI.add_variables(optimizer, length(c));
```

# 3.4 Set the objective

```
Tip

MOI.ScalarAffineTerm.(c, x) is a shortcut for [MOI.ScalarAffineTerm(c[i], x[i]) for i = 1:3]. This is Julia's broadcast syntax in action, and is used quite often throughout MOI.
```

#### 3.5 Add the constraints

We add the knapsack constraint and integrality constraints:

Add integrality constraints:

# 3.6 Optimize the model

```
julia> MOI.optimize!(optimizer)
```

## 3.7 Understand why the solver stopped

The first thing to check after optimization is why the solver stopped, for example, did it stop because of a time limit or did it stop because it found the optimal solution?

```
julia> MOI.get(optimizer, MOI.TerminationStatus())
OPTIMAL::TerminationStatusCode = 1
```

Looks like we found an optimal solution.

#### 3.8 Understand what solution was returned

```
julia> MOI.get(optimizer, MOI.ResultCount())

julia> MOI.get(optimizer, MOI.PrimalStatus())
FEASIBLE_POINT::ResultStatusCode = 1

julia> MOI.get(optimizer, MOI.DualStatus())
NO_SOLUTION::ResultStatusCode = 0
```

# 3.9 Query the objective

What is its objective value?

```
julia> MOI.get(optimizer, MOI.ObjectiveValue())
6.0
```

# 3.10 Query the primal solution

And what is the value of the variables x?

```
julia> MOI.get(optimizer, MOI.VariablePrimal(), x)
3-element Vector{Float64}:
1.0
1.0
1.0
```

# **Chapter 4**

# Implementing a solver interface

This guide outlines the basic steps to implement an interface to MathOptInterface for a new solver.

#### **Danger**

Implementing an interface to MathOptInterface for a new solver is a lot of work. Before starting, we recommend that you join the Developer chatroom and explain a little bit about the solver you are wrapping. If you have questions that are not answered by this guide, please ask them in the Developer chatroom so we can improve this guide.

## 4.1 A note on the API

The API of MathOptInterface is large and varied. In order to support the diversity of solvers and use-cases, we make heavy use of duck-typing. That is, solvers are not expected to implement the full API, nor is there a well-defined minimal subset of what must be implemented. Instead, you should implement the API as necessary to make the solver function as you require.

The main reason for using duck-typing is that solvers work in different ways and target different use-cases.

For example:

- Some solvers support incremental problem construction, support modification after a solve, and have native support for things like variable names.
- Other solvers are "one-shot" solvers that require all of the problem data to construct and solve the problem in a single function call. They do not support modification or things like variable names.
- Other "solvers" are not solvers at all, but things like file readers. These may only support functions like read\_from\_file, and may not even support the ability to add variables or constraints directly.
- Finally, some "solvers" are layers which take a problem as input, transform it according to some rules, and pass the transformed problem to an inner solver.

#### 4.2 Preliminaries

Before starting on your wrapper, you should do some background research and make the solver accessible via Julia.

# Decide if MathOptInterface is right for you

The first step in writing a wrapper is to decide whether implementing an interface is the right thing to do.

MathOptInterface is an abstraction layer for unifying constrained mathematical optimization solvers. If your solver doesn't fit in the category, for example, it implements a derivative-free algorithm for unconstrained objective functions, MathOptInterface may not be the right tool for the job.

#### Tip

If you're not sure whether you should write an interface, ask in the Developer chatroom.

## Find a similar solver already wrapped

The next step is to find (if possible) a similar solver that is already wrapped. Although not strictly necessary, this will be a good place to look for inspiration when implementing your wrapper.

The JuMP documentation has a good list of solvers, along with the problem classes they support.

#### Tip

If you're not sure which solver is most similar, ask in the Developer chatroom.

#### Create a low-level interface

Before writing a MathOptInterface wrapper, you first need to be able to call the solver from Julia.

#### Wrapping solvers written in Julia

If your solver is written in Julia, there's nothing to do here. Go to the next section.

#### Wrapping solvers written in C

Julia is well suited to wrapping solvers written in C.

#### Info

This is not true for C++. If you have a solver written in C++, first write a C interface, then wrap the C interface.

Before writing a MathOptInterface wrapper, there are a few extra steps.

## Create a JLL

If the C code is publicly available under an open source license, create a JLL package via Yggdrasil. The easiest way to do this is to copy an existing solver. Good examples to follow are the COIN-OR solvers.

#### Warning

Building the solver via Yggdrasil is non-trivial. Please ask the Developer chatroom for help.

If the code is commercial or not publicly available, the user will need to manually install the solver. See Gurobi.jl or CPLEX.jl for examples of how to structure this.

## Use Clang.jl to wrap the C API

The next step is to use Clang.jl to automatically wrap the C API. The easiest way to do this is to follow an example. Good examples to follow are Cbc.jl and HiGHS.jl.

Sometimes, you will need to make manual modifications to the resulting files.

## Solvers written in other languages

Ask the Developer chatroom for advice. You may be able to use one of the JuliaInterop packages to call out to the solver.

For example, SeDuMi.jl uses MATLAB.jl to call the SeDuMi solver written in MATLAB.

# 4.3 Structuring the package

Structure your wrapper as a Julia package. Consult the Julia documentation if you haven't done this before.

MOI solver interfaces may be in the same package as the solver itself (either the C wrapper if the solver is accessible through C, or the Julia code if the solver is written in Julia, for example), or in a separate package which depends on the solver package.

#### Note

The JuMP core contributors request that you do not use "JuMP" in the name of your package without prior consent.

Your package should have the following structure:

```
/.github
    /workflows
        ci.yml
        format_check.yml
        TagBot.yml
/gen
   gen.jl # Code to wrap the C API
/src
   NewSolver.jl
   /aen
        libnewsolver_api.jl
        libnewsolver_common.jl
    /MOI wrapper
       MOI_wrapper.jl
        other files.jl
/test
   runtests.jl
   /MOI_wrapper
       MOI_wrapper.jl
.gitignore
.JuliaFormatter.toml
README.md
LICENSE.md
Project.toml
```

• The /.github folder contains the scripts for GitHub actions. The easiest way to write these is to copy the ones from an existing solver.

- The /gen and /src/gen folders are only needed if you are wrapping a solver written in C.
- The /src/MOI\_wrapper folder contains the Julia code for the MOI wrapper.
- The /test folder contains code for testing your package. See Setup tests for more information.
- The .JuliaFormatter.toml and .github/workflows/format\_check.yml enforce code formatting using JuliaFormatter.jl. Check existing solvers or JuMP.jl for details.

#### **Documentation**

Your package must include documentation explaining how to use the package. The easiest approach is to include documentation in your README.md. A more involved option is to use Documenter.jl.

Examples of packages with README-based documentation include:

- Cbc.jl
- HiGHS.jl
- SCS.jl

Examples of packages with Documenter-based documentation include:

- Alpine.jl
- COSMO.jl
- Juniper.jl

# Setup tests

The best way to implement an interface to MathOptInterface is via test-driven development.

The MOI. Test submodule contains a large test suite to help check that you have implemented things correctly. Follow the guide How to test a solver to set up the tests for your package.

## Tip

Run the tests frequently when developing. However, at the start there is going to be a lot of errors. Start by excluding large classes of tests (for example, exclude = ["test\_basic\_", "test\_model\_"], implement any missing methods until the tests pass, then remove an exclusion and repeat.

# 4.4 Initial code

By this point, you should have a package setup with tests, formatting, and access to the underlying solver. Now it's time to start writing the wrapper.

# The Optimizer object

The first object to create is a subtype of AbstractOptimizer. This type is going to store everything related to the problem.

By convention, these optimizers should not be exported and should be named PackageName.Optimizer.

```
import MathOptInterface as MOI

struct Optimizer <: MOI.AbstractOptimizer
    # Fields go here
end</pre>
```

#### Optimizer objects for C solvers

#### Warning

This section is important if you wrap a solver written in C.

Wrapping a solver written in C will require the use of pointers, and for you to manually free the solver's memory when the Optimizer is garbage collected by Julia.

#### Never pass a pointer directly to a Julia ccall function.

Instead, store the pointer as a field in your Optimizer, and implement Base.cconvert and Base.unsafe\_convert. Then you can pass Optimizer to any ccall function that expects the pointer.

In addition, make sure you implement a finalizer for each model you create.

If newsolver\_createProblem() is the low-level function that creates the problem pointer in C, and newsolver\_freeProblem(::Pt is the low-level function that frees memory associated with the pointer, your Optimizer() function should look like this:

```
struct Optimizer <: MOI.AbstractOptimizer
   ptr::Ptr{Cvoid}

function Optimizer()
   ptr = newsolver_createProblem()
   model = Optimizer(ptr)
   finalizer(model) do m
        newsolver_freeProblem(m)
        return
   end
   return model
end

Base.cconvert(::Type{Ptr{Cvoid}}, model::Optimizer) = model
Base.unsafe_convert(::Type{Ptr{Cvoid}}, model::Optimizer) = model.ptr</pre>
```

## Implement methods for Optimizer

All Optimizers must implement the following methods:

```
• empty!
```

• is\_empty

Other methods, detailed below, are optional or depend on how you implement the interface.

#### Tip

For this and all future methods, read the docstrings to understand what each method does, what it expects as input, and what it produces as output. If it isn't clear, let us know and we will improve the docstrings. It is also very helpful to look at an existing wrapper for a similar solver.

You should also implement Base.summary(::I0, ::Optimizer) to print a nice string when someone shows your model. For example

```
function Base.summary(io::IO, model::Optimizer)
    return print(io, "NewSolver with the pointer $(model.ptr)")
end
```

# Implement attributes

MathOptInterface uses attributes to manage different aspects of the problem.

For each attribute

- get gets the current value of the attribute
- set sets a new value of the attribute. Not all attributes can be set. For example, the user can't modify the SolverName.
- supports returns a Bool indicating whether the solver supports the attribute.

#### Info

Use attribute\_value\_type to check the value expected by a given attribute. You should make sure that your get function correctly infers to this type (or a subtype of it).

Each column in the table indicates whether you need to implement the particular method for each attribute.

For example:

```
function MOI.get(model::Optimizer, ::MOI.Silent)
    return # true if MOI.Silent is set
end

function MOI.set(model::Optimizer, ::MOI.Silent, v::Bool)
    if v
        # Set a parameter to turn off printing
    else
        # Restore the default printing
    end
    return
end

MOI.supports(::Optimizer, ::MOI.Silent) = true
```

Attribute	get	set	supports
SolverName	Yes	No	No
SolverVersion	Yes	No	No
RawSolver	Yes	No	No
Name	Yes	Yes	Yes
Silent	Yes	Yes	Yes
TimeLimitSec	Yes	Yes	Yes
ObjectiveLimit	Yes	Yes	Yes
SolutionLimit	Yes	Yes	Yes
NodeLimit	Yes	Yes	Yes
RawOptimizerAttribute	Yes	Yes	Yes
NumberOfThreads	Yes	Yes	Yes
AbsoluteGapTolerance	Yes	Yes	Yes
RelativeGapTolerance	Yes	Yes	Yes

# **Define** supports\_constraint

The next step is to define which constraints and objective functions you plan to support.

For each function-set constraint pair, define supports\_constraint:

```
function MOI.supports_constraint(
    ::Optimizer,
    ::Type{MOI.VariableIndex},
    ::Type{MOI.ZeroOne},
)
    return true
end
```

To make this easier, you may want to use Unions:

```
function MOI.supports_constraint(
    ::Optimizer,
    ::Type{MOI.VariableIndex},
    ::Type{<:Union{MOI.LessThan,MOI.GreaterThan,MOI.EqualTo}},
)
    return true
end</pre>
```

```
Tip
```

Only support a constraint if your solver has native support for it.

# 4.5 The big decision: incremental modification?

Now you need to decide whether to support incremental modification or not.

Incremental modification means that the user can add variables and constraints one-by-one without needing to rebuild the entire problem, and they can modify the problem data after an optimize! call. Supporting incremental modification means implementing functions like add\_variable and add\_constraint.

The alternative is to accept the problem data in a single optimize! or copy\_to function call. Because these functions see all of the data at once, it can typically call a more efficient function to load data into the underlying solver.

Good examples of solvers supporting incremental modification are MILP solvers like GLPK.jl and Gurobi.jl. Examples of non-incremental solvers are AmplNLWriter.jl and SCS.jl

It is possible for a solver to implement both approaches, but you should probably start with one for simplicity.

#### Tip

Only support incremental modification if your solver has native support for it.

In general, supporting incremental modification is more work, and it usually requires some extra book-keeping. However, it provides a more efficient interface to the solver if the problem is going to be resolved multiple times with small modifications. Moreover, once you've implemented incremental modification, it's usually not much extra work to add a copy\_to interface. The converse is not true.

#### Tip

If this is your first time writing an interface, start with the one-shot optimize!.

#### The non-incremental interface

There are two ways to implement the non-incremental interface. The first uses a two-argument version of optimize!, the second implements copy to followed by the one-argument version of optimize!.

If your solver does not support modification, and requires all data to solve the problem in a single function call, you should implement the "one-shot" optimize!.

```
optimize!(::ModelLike, ::ModelLike)
```

If your solver separates data loading and the actual optimization into separate steps, implement the copy\_to interface.

```
• copy_to(::ModelLike, ::ModelLike)
```

## The incremental interface

• optimize!(::ModelLike)

#### Warning

Writing this interface is a lot of work. The easiest way is to consult the source code of a similar solver.

To implement the incremental interface, implement the following functions:

- add variable
- add\_variables
- add\_constraint

- add\_constraints
- is\_valid
- delete
- optimize!(::ModelLike)

#### Info

Solvers do not have to support AbstractScalarFunction in GreaterThan, LessThan, EqualTo, or Interval with a nonzero constant in the function. Throw ScalarFunctionConstantNotZero if the function constant is not zero.

In addition, you should implement the following model attributes:

Attribute	get	set	supports
ListOfModelAttributesSet	Yes	No	No
<pre>ObjectiveFunctionType</pre>	Yes	No	No
<b>ObjectiveFunction</b>	Yes	Yes	Yes
ObjectiveSense	Yes	Yes	Yes
Name	Yes	Yes	Yes

#### Variable-related attributes:

Attribute	get	set	supports
ListOfVariableAttributesSet	Yes	No	No
ListOfVariablesWithAttributeSet	Yes	No	No
NumberOfVariables	Yes	No	No
ListOfVariableIndices	Yes	No	No

#### Constraint-related attributes:

Attribute	get	set	supports
ListOfConstraintAttributesSet	Yes	No	No
ListOfConstraintsWithAttributeSet	Yes	No	No
NumberOfConstraints	Yes	No	No
ListOfConstraintTypesPresent	Yes	No	No
ConstraintFunction	Yes	Yes	No
ConstraintSet	Yes	Yes	No

#### **Modifications**

If your solver supports modifying data in-place, implement modify for the following AbstractModifications:

- ScalarConstantChange
- ScalarCoefficientChange
- ScalarQuadraticCoefficientChange

- VectorConstantChange
- MultirowChange

#### Variables constrained on creation

Some solvers require variables be associated with a set when they are created. This conflicts with the incremental modification approach, since you cannot first add a free variable and then constrain it to the set.

If this is the case, implement:

- add\_constrained\_variable
- add\_constrained\_variables
- supports add constrained variables

By default, MathOptInterface assumes solvers support free variables. If your solver does not support free variables, define:

```
MOI.supports_add_constrained_variables(::Optimizer, ::Type{Reals}) = false
```

# Incremental and copy\_to

If you implement the incremental interface, you have the option of also implementing copy\_to.

If you don't want to implement copy\_to, for example, because the solver has no API for building the problem in a single function call, define the following fallback:

```
MOI.supports_incremental_interface(::Optimizer) = true

function MOI.copy_to(dest::Optimizer, src::MOI.ModelLike)
    return MOI.Utilities.default_copy_to(dest, src)
end
```

#### 4.6 Names

Regardless of which interface you implement, you have the option of implementing the Name attribute for variables and constraints:

Attribute	get	set	supports
VariableName	Yes	Yes	Yes
ConstraintName	Yes	Yes	Yes

If you implement names, you must also implement the following three methods:

```
function MOI.get(model::Optimizer, ::Type{MOI.VariableIndex}, name::String)
    return # The variable named `name`.
end

function MOI.get(model::Optimizer, ::Type{MOI.ConstraintIndex}, name::String)
```

```
return # The constraint any type named `name`.
end

function MOI.get(
    model::Optimizer,
    ::Type{MOI.ConstraintIndex{F,S}},
    name::String,
) where {F,S}
    return # The constraint of type F-in-S named `name`.
end
```

These methods have the following rules:

- If there is no variable or constraint with the name, return nothing
- If there is a single variable or constraint with that name, return the variable or constraint
- If there are multiple variables or constraints with the name, throw an error.

#### Warning

You should not implement ConstraintName for VariableIndex constraints. If you implement ConstraintName for other constraints, you can add the following two methods to disable ConstraintName for VariableIndex constraints.

```
function MOI.supports(
    ::Optimizer,
    ::MOI.ConstraintName,
    ::Type{<:MOI.ConstraintIndex{MOI.VariableIndex,<:MOI.AbstractScalarSet}},
)
    return throw(MOI.VariableIndexConstraintNameError())
end
function MOI.set(
    ::Optimizer,
    ::MOI.ConstraintName,
    ::MOI.ConstraintIndex{MOI.VariableIndex,<:MOI.AbstractScalarSet},
    ::String,
)
    return throw(MOI.VariableIndexConstraintNameError())
end</pre>
```

# 4.7 Solutions

Implement optimize! to solve the model:

• optimize!

All Optimizers must implement the following attributes:

• DualStatus

- PrimalStatus
- RawStatusString
- ResultCount
- TerminationStatus

#### Info

You only need to implement get for solution attributes. Don't implement set or supports.

#### Note

Solver wrappers should document how the low-level statuses map to the MOI statuses. Statuses like NEARLY\_FEASIBLE\_POINT and INFEASIBLE\_POINT, are designed to be used when the solver explicitly indicates that relaxed tolerances are satisfied or the returned point is infeasible, respectively.

You should also implement the following attributes:

- ObjectiveValue
- SolveTimeSec
- VariablePrimal

#### Tip

Attributes like VariablePrimal and ObjectiveValue are indexed by the result count. Use MOI.check\_result\_index\_bounds(model, attr) to throw an error if the attribute is not available.

If your solver returns dual solutions, implement:

- ConstraintDual
- DualObjectiveValue

For integer solvers, implement:

- ObjectiveBound
- RelativeGap

If applicable, implement:

- SimplexIterations
- BarrierIterations
- NodeCount

If your solver uses the Simplex method, implement:

• ConstraintBasisStatus

If your solver accepts primal or dual warm-starts, implement:

- VariablePrimalStart
- ConstraintDualStart

# 4.8 Other tips

Here are some other points to be aware of when writing your wrapper.

# Unsupported constraints at runtime

In some cases, your solver may support a particular type of constraint (for example, quadratic constraints), but only if the data meets some condition (for example, it is convex).

In this case, declare that you support the constraint, and throw AddConstraintNotAllowed.

## Dealing with multiple variable bounds

MathOptInterface uses VariableIndex constraints to represent variable bounds. Defining multiple variable bounds on a single variable is not allowed.

Throw LowerBoundAlreadySet or UpperBoundAlreadySet if the user adds a constraint that results in multiple bounds.

Only throw if the constraints conflict. It is okay to add VariableIndex-in-GreaterThan and then VariableIndex-in-LessThan, but not VariableIndex-in-Interval and then VariableIndex-in-LessThan,

#### **Expect duplicate coefficients**

Solvers must expect that functions such as ScalarAffineFunction and VectorQuadraticFunction may contain duplicate coefficients.

For example, ScalarAffineFunction([ScalarAffineTerm(x, 1), ScalarAffineTerm(x, 1)], 0.0).

Use Utilities.canonical to return a new function with the duplicate coefficients aggregated together.

## Don't modify user-data

All data passed to the solver must be copied immediately to internal data structures. Solvers may not modify any input vectors and must assume that input vectors will not be modified by users in the future.

This applies, for example, to the terms vector in ScalarAffineFunction. Vectors returned to the user, for example, via ObjectiveFunction or ConstraintFunction attributes, must not be modified by the solver afterwards. The in-place version of get! can be used by users to avoid extra copies in this case.

#### **Column Generation**

There is no special interface for column generation. If the solver has a special API for setting coefficients in existing constraints when adding a new variable, it is possible to queue modifications and new variables and then call the solver's API once all of the new coefficients are known.

# **Solver-specific attributes**

You don't need to restrict yourself to the attributes defined in the MathOptInterface.jl package.

Solver-specific attributes should be specified by creating an appropriate subtype of AbstractModelAttribute, AbstractOptimizerAttribute, AbstractVariableAttribute, or AbstractConstraintAttribute.

For example, Gurobi.jl adds attributes for multiobjective optimization by defining:

```
struct NumberOfObjectives <: MOI.AbstractModelAttribute end

function MOI.set(model::Optimizer, ::NumberOfObjectives, n::Integer)
    # Code to set NumberOfObjectives
    return
end

function MOI.get(model::Optimizer, ::NumberOfObjectives)
    n = # Code to get NumberOfObjectives
    return n
end</pre>
```

Then, the user can write:

```
model = Gurobi.Optimizer()
MOI.set(model, Gurobi.NumberofObjectives(), 3)
```

# **Chapter 5**

# **Transitioning from MathProgBase**

MathOptInterface is a replacement for MathProgBase.jl. However, it is not a direct replacement.

# 5.1 Transitioning a solver interface

MathOptInterface is more extensive than MathProgBase which may make its implementation seem daunting at first. There are however numerous utilities in MathOptInterface that the simplify implementation process.

For more information, read Implementing a solver interface.

# 5.2 Transitioning the high-level functions

MathOptInterface doesn't provide replacements for the high-level interfaces in MathProgBase. We recommend you use JuMP as a modeling interface instead.

Tip

If you haven't used JuMP before, start with the tutorial Getting started with JuMP

#### linprog

Here is one way of transitioning from linprog:

```
using JuMP

function linprog(c, A, sense, b, l, u, solver)
    N = length(c)
    model = Model(solver)
    @variable(model, l[i] <= x[i=1:N] <= u[i])
    @objective(model, Min, c' * x)
    eq_rows, ge_rows, le_rows = sense .== '=', sense .== '>', sense .== '<'
    @constraint(model, A[eq_rows, :] * x .== b[eq_rows])
    @constraint(model, A[ge_rows, :] * x .>= b[ge_rows])
    @constraint(model, A[le_rows, :] * x .<= b[le_rows])
    optimize!(model)
    return (
        status = termination_status(model),
        objval = objective_value(model),
        sol = value.(x)</pre>
```

```
)
end
```

# mixintprog

Here is one way of transitioning from mixintprog:

```
using JuMP
function mixintprog(c, A, rowlb, rowub, vartypes, lb, ub, solver)
    N = length(c)
    model = Model(solver)
    @variable(model, lb[i] <= x[i=1:N] <= ub[i])</pre>
    for i in 1:N
        if vartypes[i] == :Bin
            set_binary(x[i])
        elseif vartypes[i] == :Int
            set_integer(x[i])
        end
    end
    @objective(model, Min, c' * x)
    @constraint(model, rowlb .<= A * x .<= rowub)</pre>
    optimize!(model)
    return (
        status = termination_status(model),
        objval = objective_value(model),
        sol = value.(x)
end
```

# quadprog

Here is one way of transitioning from quadprog:

# **Chapter 6**

# Implementing a constraint bridge

This guide outlines the basic steps to create a new bridge from a constraint expressed in the formalism Function-in-Set.

#### 6.1 Preliminaries

First, decide on the set you want to bridge. Then, study its properties: the most important one is whether the set is scalar or vector, which impacts the dimensionality of the functions that can be used with the set.

- A scalar function only has one dimension. MOI defines three types of scalar functions: a variable (VariableIndex), an affine function (ScalarAffineFunction), or a quadratic function (ScalarQuadraticFunction).
- A vector function has several dimensions (at least one). MOI defines three types of vector functions: several variables (VectorOfVariables), an affine function (VectorAffineFunction), or a quadratic function (VectorQuadraticFunction). The main difference with scalar functions is that the order of dimensions can be very important: for instance, in an indicator constraint (Indicator), the first dimension indicates whether the constraint about the second dimension is active.

To explain how to implement a bridge, we present the example of Bridges.Constraint.FlipSignBridge. This bridge maps <= (LessThan) constraints to >= (GreaterThan) constraints. This corresponds to reversing the sign of the inequality. We focus on scalar affine functions (we disregard the cases of a single variable or of quadratic functions). This example is a simplified version of the code included in MOI.

# 6.2 Four mandatory parts in a constraint bridge

The first part of a constraint bridge is a new concrete subtype of Bridges. Constraint. AbstractBridge. This type must have fields to store all the new variables and constraints that the bridge will add. Typically, these types are parametrized by the type of the coefficients in the model.

Then, three sets of functions must be defined:

- 1. Bridges.Constraint.bridge\_constraint: this function implements the bridge and creates the required variables and constraints.
- 2. supports\_constraint: these functions must return true when the combination of function and set is supported by the bridge. By default, the base implementation always returns false and the bridge does not have to provide this implementation.

3. Bridges.added\_constrained\_variable\_types and Bridges.added\_constraint\_types: these functions return the types of variables and constraints that this bridge adds. They are used to compute the set of other bridges that are required to use the one you are defining, if need be.

More functions can be implemented, for instance to retrieve properties from the bridge or deleting a bridged constraint.

# 1. Structure for the bridge

A typical struct behind a bridge depends on the type of the coefficients that are used for the model (typically Float64, but coefficients might also be integers or complex numbers).

This structure must hold a reference to all the variables and the constraints that are created as part of the bridge.

The type of this structure is used throughout MOI as an identifier for the bridge. It is passed as argument to most functions related to bridges.

The best practice is to have the name of this type end with Bridge.

In our example, the bridge maps any ScalarAffineFunction{T}-in-LessThan{T} constraint to a single ScalarAffineFunction{T in-GreaterThan{T} constraint. The affine function has coefficients of type T. The bridge is parametrized with T, so that the constraint that the bridge creates also has coefficients of type T.

```
struct SignBridge{T<:Number} <: Bridges.Constraint.AbstractBridge
  constraint::ConstraintIndex{ScalarAffineFunction{T}, GreaterThan{T}}
end</pre>
```

# 2. Bridge creation

The function <code>Bridges.Constraint.bridge\_constraint</code> is called whenever the bridge is instantiated for a specific model, with the given function and set. The arguments to <code>bridge\_constraint</code> are similar to <code>add\_constraint</code>, with the exception of the first argument: it is the Type of the struct defined in the first step (for our example, <code>Type{SignBridge{T}})</code>.

bridge\_constraint returns an instance of the struct defined in the first step. the first step.

In our example, the bridge constraint could be defined as:

```
function Bridges.Constraint.bridge_constraint(
    ::Type{SignBridge{T}}, # Bridge to use.
    model::ModelLike, # Model to which the constraint is being added.
    f::ScalarAffineFunction{T}, # Function to rewrite.
    s::LessThan{T}, # Set to rewrite.
) where {T}
    # Create the variables and constraints required for the bridge.
    con = add_constraint(model, -f, GreaterThan(-s.upper))

# Return an instance of the bridge type with a reference to all the
    # variables and constraints that were created in this function.
    return SignBridge(con)
end
```

# 3. Supported constraint types

The function supports\_constraint determines whether the bridge type supports a given combination of function and set.

This function must closely match bridge\_constraint, because it will not be called if supports\_constraint returns false.

```
function supports_constraint(
    ::Type{SignBridge{T}}, # Bridge to use.
    ::Type{ScalarAffineFunction{T}}, # Function to rewrite.
    ::Type{LessThan{T}}, # Set to rewrite.
) where {T}
    # Do some computation to ensure that the constraint is supported.
    # Typically, you can directly return true.
    return true
end
```

## 4. Metadata about the bridge

To determine whether a bridge can be used, MOI uses a shortest-path algorithm that uses the variable types and the constraints that the bridge can create. This information is communicated from the bridge to MOI using the functions <code>Bridges.added\_constrained\_variable\_types</code> and <code>Bridges.added\_constraint\_types</code>. Both return lists of tuples: either a list of 1-tuples containing the variable types (typically, ZeroOne or Integer) or a list of 2-tuples contained the functions and sets (like ScalarAffineFunction{T}-GreaterThan).

For our example, the bridge does not create any constrained variables, and only  $ScalarAffineFunction\{T\}-in-GreaterThan\{T\}$  constraints:

```
function Bridges.added_constrained_variable_types(::Type{SignBridge{T}}) where {T}
    # The bridge does not create variables, return an empty list of tuples:
    return Tuple{Type}[]
end

function Bridges.added_constraint_types(::Type{SignBridge{T}}) where {T}
    return Tuple{Type, Type}[
          # One element per F-in-S the bridge creates.
          (ScalarAffineFunction{T}, GreaterThan{T}),
    ]
end
```

A bridge that creates binary variables would rather have this definition of added\_constrained\_variable\_types:

```
function Bridges.added_constrained_variable_types(::Type{SomeBridge{T}}) where {T}
    # The bridge only creates binary variables:
    return Tuple{Type}[(ZeroOne,)]
end
```

#### Warning

If you declare the creation of constrained variables in added\_constrained\_variable\_types, the corresponding constraint type VariableIndex must not be indicated in added\_constraint\_types. This would restrict the use of the bridge to solvers that can add such a constraint after the variable is created.

More concretely, if you declare in added\_constrained\_variable\_types that your bridge creates binary variables (ZeroOne), and if you never add such a constraint afterward (you do not call add\_constraint(model, var, ZeroOne())), then you must not list (VariableIndex, ZeroOne) in added\_constraint\_types.

Typically, the function <a href="mailto:Bridges.Constraint.concrete\_bridge\_type">Bridges.Constraint.concrete\_bridge\_type</a> does not have to be defined for most bridges.

# 6.3 Bridge registration

For a bridge to be used by MOI, it must be known by MOI.

#### SingleBridgeOptimizer

The first way to do so is to create a single-bridge optimizer. This type of optimizer wraps another optimizer and adds the possibility to use only one bridge. It is especially useful when unit testing bridges.

It is common practice to use the same name as the type defined for the bridge (SignBridge, in our example) without the suffix Bridge.

```
const Sign{T,OT<: ModelLike} =
   SingleBridgeOptimizer{SignBridge{T}, OT}</pre>
```

In the context of unit tests, this bridge is used in conjunction with a Utilities.MockOptimizer:

```
mock = Utilities.MockOptimizer(
   Utilities.UniversalFallback(Utilities.Model{Float64}()),
)
bridged_mock = Sign{Float64}(mock)
```

# New bridge for a LazyBridgeOptimizer

Typical user-facing models for MOI are based on Bridges.LazyBridgeOptimizer. For instance, this type of model is returned by Bridges.full\_bridge\_optimizer. These models can be added more bridges by using Bridges.add\_bridge:

```
inner_optimizer = Utilities.Model{Float64}()
optimizer = Bridges.full_bridge_optimizer(inner_optimizer, Float64)
Bridges.add_bridge(optimizer, SignBridge{Float64})
```

# 6.4 Bridge improvements

#### **Attribute retrieval**

Like models, bridges have attributes that can be retrieved using get and set. The most important ones are the number of variables and constraints, but also the lists of variables and constraints.

In our example, we only have one constraint and only have to implement the NumberOfConstraints and ListOfConstraintIndices attributes:

```
function get(
    ::SignBridge{T},
    ::NumberOfConstraints{
        ScalarAffineFunction{T},
        GreaterThan{T},
   },
) where {T}
    return 1
function get(
   bridge::SignBridge{T},
    ::ListOfConstraintIndices{
        ScalarAffineFunction{T},
        GreaterThan{T},
   },
) where {T}
    return [bridge.constraint]
end
```

You must implement one such pair of functions for each type of constraint the bridge adds to the model.

#### Warning

Avoid returning a list from the bridge object without copying it. Users must be able to change the contents of the returned list without altering the bridge object.

For variables, the situation is simpler. If your bridge creates new variables, you must implement the NumberOfVariables and ListOfVariableIndices attributes. However, these attributes do not have parameters, unlike their constraint counterparts. Only two functions suffice:

```
function get(
    ::SignBridge{T},
    ::NumberOfVariables,
) where {T}
    return 0
end

function get(
    ::SignBridge{T},
    ::ListOfVariableIndices,
) where {T}
    return VariableIndex[]
end
```

In order for the user to be able to access the function and set of the original constraint, the bridge needs to implement getters for the ConstraintFunction and ConstraintSet attributes:

```
function get(
    model::MOI.ModelLike,
    attr::MOI.ConstraintFunction,
    bridge::SignBridge,
)
    return -MOI.get(model, attr, bridge.constraint)
end

function get(
    model::MOI.ModelLike,
    attr::MOI.ConstraintSet,
    bridge::SignBridge,
)
    set = MOI.get(model, attr, bridge.constraint)
    return MOI.LessThan(-set.lower)
end
```

#### Warning

Alternatively, one could store the original function and set in SignBridge during <code>Bridges.Constraint.bridge\_constraint</code> to make these getters simpler and more efficient. On the other hand, this will increase the memory footprint of the bridges as the garbage collector won't be able to delete that object. The convention is to not store the function in the bridge and not care too much about the efficiency of these getters. If the user needs efficient getters for <code>ConstraintFunction</code> then they should use a <code>Utilities.CachingOptimizer</code>.

#### **Model modifications**

To avoid copying the model when the user request to change a constraint, MOI provides modify. Bridges can also implement this API to allow certain changes, such as coefficient changes.

In our case, a modification of a coefficient in the original constraint (for example, replacing the value of the coefficient of a variable in the affine function) must be transmitted to the constraint created by the bridge, but with a sign change.

```
function modify(
    model::ModelLike,
    bridge::SignBridge,
    change::ScalarCoefficientChange,
)
    modify(
        model,
        bridge.constraint,
        ScalarCoefficientChange(change.variable, -change.new_coefficient),
    )
    return
end
```

# **Bridge deletion**

When a bridge is deleted, the constraints it added must be deleted too.

```
function delete(model::ModelLike, bridge::SignBridge)
   delete(model, bridge.constraint)
   return
end
```

# **Manipulating expressions**

This guide highlights a syntactically appealing way to build expressions at the MOI level, but also to look at their contents. It may be especially useful when writing models or bridge code.

# 7.1 Creating functions

This section details the ways to create functions with MathOptInterface.

## Creating scalar affine functions

The simplest scalar function is simply a variable:

```
julia> x = MOI.add_variable(model) # Create the variable x
MOI.VariableIndex(1)
```

This type of function is extremely simple; to express more complex functions, other types must be used. For instance, a ScalarAffineFunction is a sum of linear terms (a factor times a variable) and a constant. Such an object can be built using the standard constructor:

```
julia> f = MOI.ScalarAffineFunction([MOI.ScalarAffineTerm(1, x)], 2) # x + 2
(2) + (1) MOI.VariableIndex(1)
```

However, you can also use operators to build the same scalar function:

```
julia> f = x + 2
(2) + (1) MOI.VariableIndex(1)
```

# Creating scalar quadratic functions

Scalar quadratic functions are stored in ScalarQuadraticFunction objects, in a way that is highly similar to scalar affine functions. You can obtain a quadratic function as a product of affine functions:

```
julia> 1 * x * x
(0) + 1.0 MOI.VariableIndex(1)²

julia> f * f # (x + 2)²
```

```
(4) + (2) MOI.VariableIndex(1) + (2) MOI.VariableIndex(1) + 1.0 MOI.VariableIndex(1)<sup>2</sup>

julia> f^2 # (x + 2)<sup>2</sup> too
(4) + (2) MOI.VariableIndex(1) + (2) MOI.VariableIndex(1) + 1.0 MOI.VariableIndex(1)<sup>2</sup>
```

#### **Creating vector functions**

A vector function is a function with several values, irrespective of the number of input variables. Similarly to scalar functions, there are three main types of vector functions: VectorOfVariables, VectorAffineFunction, and VectorQuadraticFunction.

The easiest way to create a vector function is to stack several scalar functions using Utilities.vectorize. It takes a vector as input, and the generated vector function (of the most appropriate type) has each dimension corresponding to a dimension of the vector.

#### Warning

Utilities.vectorize only takes a vector of similar scalar functions: you cannot mix VariableIndex and ScalarAffineFunction, for instance. In practice, it means that Utilities.vectorize([x, f]) does not work; you should rather use Utilities.vectorize([1 \* x, f]) instead to only have ScalarAffineFunction objects.

# 7.2 Canonicalizing functions

In more advanced use cases, you might need to ensure that a function is "canonical." Functions are stored as an array of terms, but there is no check that these terms are redundant: a ScalarAffineFunction object might have two terms with the same variable, like x + x + 1. These terms could be merged without changing the semantics of the function: 2x + 1.

Working with these objects might be cumbersome. Canonicalization helps maintain redundancy to zero.

Utilities.is\_canonical checks whether a function is already in its canonical form:

```
julia> MOI.Utilities.is_canonical(f + f) # (x + 2) + (x + 2) is stored as x + x + 4
false
```

Utilities.canonical returns the equivalent canonical version of the function:

```
julia> MOI.Utilities.canonical(f + f) # Returns 2x + 4
(4) + (2) MOI.VariableIndex(1)
```

### 7.3 Exploring functions

At some point, you might need to dig into a function, for instance to map it into solver constructs.

# **Vector functions**

Utilities.scalarize returns a vector of scalar functions from a vector function:

```
julia> MOI.Utilities.scalarize(g) # Returns a vector [f, 2 * f].
2-element Vector{MathOptInterface.ScalarAffineFunction{Int64}}:
    (2) + (1) MOI.VariableIndex(1)
    (4) + (2) MOI.VariableIndex(1)
```

#### Note

Utilities.eachscalar returns an iterator on the dimensions, which serves the same purpose as Utilities.scalarize.

output\_dimension returns the number of dimensions of the output of a function:

```
julia> MOI.output_dimension(g)
2
```

# Latency

MathOptInterface suffers the "time-to-first-solve" problem of start-up latency.

This hurts both the user- and developer-experience of MathOptInterface. In the first case, because simple models have a multi-second delay before solving, and in the latter, because our tests take so long to run.

This page contains some advice on profiling and fixing latency-related problems in the MathOptInterface.jl repository.

# 8.1 Background

Before reading this part of the documentation, you should familiarize yourself with the reasons for latency in Julia and how to fix them.

- Read the blogposts on julialang.org on precompilation and SnoopCompile
- Read the SnoopCompile documentation.
- Watch Tim Holy's talk at JuliaCon 2021
- Watch the package development workshop at JuliaCon 2021

## 8.2 Causes

There are three main causes of latency in MathOptInterface:

- 1. A large number of types
- 2. Lack of method ownership
- 3. Type-instability in the bridge layer

#### A large number of types

Julia is very good at specializing method calls based on the input type. Each specialization has a compilation cost, but the benefit of faster run-time performance.

The best-case scenario is for a method to be called a large number of times with a single set of argument types. The worst-case scenario is for a method to be called a single time for a large set of argument types.

Because of MathOptInterface's function-in-set formulation, we fall into the worst-case situation.

This is a fundamental limitation of Julia, so there isn't much we can do about it. However, if we can precompile MathOptInterface, much of the cost can be shifted from start-up latency to the time it takes to precompile a package on installation.

However, there are two things which make MathOptInterface hard to precompile.

## Lack of method ownership

Lack of method ownership happens when a call is made using a mix of structs and methods from different modules. Because of this, no single module "owns" the method that is being dispatched, and so it cannot be precompiled.

#### Tip

This is a slightly simplified explanation. Read the precompilation tutorial for a more in-depth discussion on back-edges.

Unfortunately, the design of MOI means that this is a frequent occurrence: we have a bunch of types in MOI.Utilities that wrap types defined in external packages (for example, the Optimizers), which implement methods of functions defined in MOI (for example, add\_variable, add\_constraint).

Here's a simple example of method-ownership in practice:

The result is that there was one method that required type inference. If we visualize tinf:

```
using ProfileView
ProfileView.view(flamegraph(tinf))
```

we see a flamegraph with a large red-bar indicating that the method MyMOI.optimize(MyMOI.Wrapper{MyOptimizer.Optimizer} cannot be precompiled.

To fix this, we need to designate a module to "own" that method (that is, create a back-edge). The easiest way to do this is for MyOptimizer to call MyMOI.optimize(MyMOI.Wrapper{MyOptimizer.Optimizer}) during using MyOptimizer. Let's see that in practice:

```
module MyMOI
struct Wrapper{T}
    inner::T
optimize(x::Wrapper) = optimize(x.inner)
end # MyMOI
module MyOptimizer
using ..MyMOI
struct Optimizer end
MyMOI.optimize(x::Optimizer) = 1
# The syntax of this let-while loop is very particular:
# * `let ... end` keeps everything local to avoid polluting the MyOptimizer
   namespace
\# * `while true \dots break end` runs the code once, and forces Julia to compile
    the inner loop, rather than interpret it.
let
   while true
        model = MyMOI.Wrapper(Optimizer())
        MyMOI.optimize(model)
        break
    end
end
end # MyOptimizer
using SnoopCompile
model = MyMOI.Wrapper(MyOptimizer.Optimizer())
julia> tinf = @snoopi deep MyMOI.optimize(model)
InferenceTimingNode: 0.006822/0.006822 on InferenceFrameInfo for Core.Compiler.Timings.ROOT() with
\hookrightarrow 0 direct children
```

There are now 0 direct children that required type inference because the method was already stored in MyOptimizer!

Unfortunately, this trick only works if the call-chain is fully inferrable. If there are breaks (due to type instability), then the benefit of doing this is reduced. And unfortunately for us, the design of MathOptInterface has a lot of type instabilities.

# Type instability in the bridge layer

Most of MathOptInterface is pretty good at ensuring type-stability. However, a key component is not type stable, and that is the bridging layer.

In particular, the bridging layer defines Bridges.LazyBridgeOptimizer, which has fields like:

```
struct LazyBridgeOptimizer
  constraint_bridge_types::Vector{Any}
  constraint_node::Dict{Tuple{Type,Type}},ConstraintNode}
  constraint_types::Vector{Tuple{Type,Type}}
end
```

This is because the LazyBridgeOptimizer needs to be able to deal with any function-in-set type passed to it, and we also allow users to pass additional bridges that they defined in external packages.

So to recap, MathOptInterface suffers package latency because:

- 1. there are a large number of types and functions
- 2. and these are split between multiple modules, including external packages
- 3. and there are type-instabilities like those in the bridging layer.

#### 8.3 Resolutions

There are no magic solutions to reduce latency. Issue #1313 tracks progress on reducing latency in MathOpt-Interface.

A useful script is the following (replace GLPK as needed):

```
import GLPK
import MathOptInterface as MOI
function example_diet(optimizer, bridge)
   category_data = [
       1800.0 2200.0;
         91.0 Inf;
          0.0 65.0;
          0.0 1779.0
   ]
   cost = [2.49, 2.89, 1.50, 1.89, 2.09, 1.99, 2.49, 0.89, 1.59]
   food_data = [
       410 24 26 730;
       420 32 10 1190;
       560 20 32 1800;
       380 4 19 270;
       320 12 10 930;
       320 15 12 820;
       320 31 12 1230;
       100 8 2.5 125;
       330 8 10 180
   bridge_model = if bridge
       MOI.instantiate(optimizer; with_bridge_type=Float64)
       MOI.instantiate(optimizer)
   model = MOI.Utilities.CachingOptimizer(
       MOI.Utilities.UniversalFallback(MOI.Utilities.Model{Float64}()),
       MOI.Utilities.AUTOMATIC,
   MOI.Utilities.reset_optimizer(model, bridge_model)
   MOI.set(model, MOI.Silent(), true)
   nutrition = MOI.add_variables(model, size(category_data, 1))
   for (i, v) in enumerate(nutrition)
       MOI.add_constraint(model, v, MOI.GreaterThan(category_data[i, 1]))
       MOI.add_constraint(model, v, MOI.LessThan(category_data[i, 2]))
   end
   buy = MOI.add_variables(model, size(food_data, 1))
   MOI.add_constraint.(model, buy, MOI.GreaterThan(0.0))
   MOI.set(model, MOI.ObjectiveSense(), MOI.MIN_SENSE)
   f = MOI.ScalarAffineFunction(MOI.ScalarAffineTerm.(cost, buy), 0.0)
   MOI.set(model, MOI.ObjectiveFunction{typeof(f)}(), f)
```

```
for (j, n) in enumerate(nutrition)
        f = MOI.ScalarAffineFunction(
            MOI.ScalarAffineTerm.(food_data[:, j], buy),
            0.0,
        push!(f.terms, MOI.ScalarAffineTerm(-1.0, n))
        {\tt MOI.add\_constraint(model, f, MOI.EqualTo(0.0))}
    end
    MOI.optimize!(model)
    term_status = MOI.get(model, MOI.TerminationStatus())
    @assert term_status == MOI.OPTIMAL
    MOI.add constraint(
        model.
        MOI.ScalarAffineFunction(
            \label{eq:moisses} \mbox{MOI.ScalarAffineTerm.} (1.0, \mbox{ [buy[end-1], buy[end]]),}
        ),
        MOI.LessThan(6.0),
    MOI.optimize!(model)
     @assert \ MOI.get(model, \ MOI.TerminationStatus()) \ == \ MOI.INFEASIBLE \\
end
if length(ARGS) > 0
    bridge = get(ARGS, 2, "") != "--no-bridge"
    println("Running: $(ARGS[1]) $(get(ARGS, 2, ""))")
    @time example_diet(GLPK.Optimizer, bridge)
    @time example_diet(GLPK.Optimizer, bridge)
    exit(0)
end
```

You can create a flame-graph via

```
using SnoopCompile
tinf = @snoopi_deep example_diet(GLPK.Optimizer, true)
using ProfileView
ProfileView.view(flamegraph(tinf))
```

Here's how things looked in mid-August 2021:

There are a few opportunities for improvement (non-red flames, particularly on the right). But the main problem is a large red (non-precompilable due to method ownership) flame.



Figure 8.1: flamegraph

Part III

Manual

# Standard form problem

MathOptInterface represents optimization problems in the standard form:

$$\min_{x \in \mathbb{R}^n} \qquad \qquad f_0(x) \tag{9.1}$$
 s.t. 
$$f_i(x) \in \mathcal{S}_i \qquad \qquad i = 1 \dots m \tag{9.2}$$

 $i = 1 \dots m$ s.t. (9.2)

where:

- ullet the functions  $f_0, f_1, \dots, f_m$  are specified by <code>AbstractFunction</code> objects
- the sets  $\mathcal{S}_1,\dots,\mathcal{S}_m$  are specified by <code>AbstractSet</code> objects

### Tip

For more information on this standard form, read our paper.

MOI defines some commonly used functions and sets, but the interface is extensible to other sets recognized by the solver.

# 9.1 Functions

The function types implemented in MathOptInterface.jl are:

Function	Description
VariableIndex	$x_j$ , the projection onto a single coordinate defined by a variable index $j$ .
VectorOfVariables	The projection onto multiple coordinates (that is, extracting a sub-vector).
ScalarAffineFunction	$a^Tx+b$ , where $a$ is a vector and $b$ scalar.
ScalarNonlinearFunctio	nf(x), where $f$ is a nonlinear function.
VectorAffineFunction	Ax + b, where $A$ is a matrix and $b$ is a vector.
ScalarQuadraticFunctio	$\frac{1}{2}x^TQx + a^Tx + b$ , where $Q$ is a symmetric matrix, $a$ is a vector, and $b$ is a
	constant.
VectorQuadraticFunctio	n A vector of scalar-valued quadratic functions.
VectorNonlinearFunctio	n $f(x)$ , where $f$ is a vector-valued nonlinear function.

Extensions for nonlinear programming are present but not yet well documented.

# 9.2 One-dimensional sets

The one-dimensional set types implemented in MathOptInterface.jl are:

Set	Description
LessThan(u)	$(-\infty, u]$
<pre>GreaterThan(l)</pre>	$[l,\infty)$
EqualTo(v)	$\{v\}$
<pre>Interval(l, u)</pre>	[l,u]
<pre>Integer()</pre>	$\mathbb{Z}$
ZeroOne()	$\{0, 1\}$
<pre>Semicontinuous(l, u)</pre>	$\{0\} \cup [l,u]$
<pre>Semiinteger(l, u)</pre>	$\{0\} \cup \{l, l+1, \dots, u-1, u\}$

#### 9.3 Vector cones

The vector-valued set types implemented in MathOptInterface.jl are:

Set	Description
Reals(d)	$\mathbb{R}^d$
Zeros(d)	$0^d$
Nonnegatives(d)	$\{x \in \mathbb{R}^d : x \ge 0\}$
Nonpositives(d)	$\{x \in \mathbb{R}^d : x \le 0\}$
SecondOrderCone(d)	$\{(t,x)\in\mathbb{R}^d:t\geq\ x\ _2\}$
RotatedSecondOrderCone(d)	$\{(t, u, x) \in \mathbb{R}^d : 2tu \ge   x  _2^2, t \ge 0, u \ge 0\}$
<pre>ExponentialCone()</pre>	$\{(x,y,z) \in \mathbb{R}^3 : y \exp(x/y) \le z, y > 0\}$
<pre>DualExponentialCone()</pre>	$\{(u, v, w) \in \mathbb{R}^3 : -u \exp(v/u) \le \exp(1)w, u < 0\}$
<pre>GeometricMeanCone(d)</pre>	$\{(t,x)\in\mathbb{R}^{1+n}:x\geq 0,t\leq\sqrt[n]{x_1x_2\cdots x_n}\}$ where $n$ is $d-1$
PowerCone(α)	$\{(x, y, z) \in \mathbb{R}^3 : x^{\alpha} y^{1-\alpha} \ge  z , x \ge 0, y \ge 0\}$
$DualPowerCone(\alpha)$	$\{(u, v, w) \in \mathbb{R}^3 : \left(\frac{u}{\alpha}\right)^{\alpha} \left(\frac{v}{1-\alpha}\right)^{1-\alpha} \ge  w , u, v \ge 0\}$
NormOneCone(d)	$\{(t,x) \in \mathbb{R}^d : t \ge \sum_i  x_i  \}$
NormInfinityCone(d)	$\{(t,x) \in \mathbb{R}^d : t \ge \max_i  x_i \}$
RelativeEntropyCone(d)	$\{(u, v, w) \in \mathbb{R}^d : u \ge \sum_i w_i \log(\frac{w_i}{v_i}), v_i \ge 0, w_i \ge 0\}$
<pre>HyperRectangle(l, u)</pre>	$\{x \in \mathbb{R}^d : x_i \in [l_i, u_i] \forall i = 1, \dots, d\}$
NormCone(p, d)	$\{(t,x) \in \mathbb{R}^d : t \ge \left(\sum_i  x_i ^p\right)^{\frac{1}{p}}\}$

# 9.4 Matrix cones

The matrix-valued set types implemented in MathOptInterface.jl are:

Some of these cones can take two forms: XXXConeTriangle and XXXConeSquare.

In XXXConeTriangle sets, the matrix is assumed to be symmetric, and the elements are provided by a vector, in which the entries of the upper-right triangular part of the matrix are given column by column (or equivalently, the entries of the lower-left triangular part are given row by row).

In XXXConeSquare sets, the entries of the matrix are given column by column (or equivalently, row by row), and the matrix is constrained to be symmetric. As an example, given a 2-by-2 matrix of variables X and a one-

Set	Description
RootDetConeTriangle(d)	$\{(t,X) \in \mathbb{R}^{1+d(1+d)/2} : t \le 1\}$
	$\det(X)^{1/d}, X$ is the upper triangle of a PSD matrix $\}$
RootDetConeSquare(d)	$\{(t,X)\in\mathbb{R}^{1+d^2}:t\leq \det(X)^{1/d},X$ is a PSD matrix $\}$
PositiveSemidefiniteConeTriangl	e ( $\{X \in \mathbb{R}^{d(d+1)/2} : X$ is the upper triangle of a PSD matrix $\}$
PositiveSemidefiniteConeSquare(	d) $\{X \in \mathbb{R}^{d^2}: X  ext{ is a PSD matrix}\}$
LogDetConeTriangle(d)	$\{(t, u, X) \in \mathbb{R}^{2+d(1+d)/2} : t \le 1\}$
	$u\log(\det(X/u)), X$ is the upper triangle of a PSD matrix, $u>0\}$
LogDetConeSquare(d)	$\{(t, u, X) \in \mathbb{R}^{2+d^2} : t \le$
	$u\log(\det(X/u)), X$ is a PSD matrix, $u>0\}$
NormSpectralCone(r, c)	$\{(t,X)\in\mathbb{R}^{1+r imes c}:t\geq\sigma_1(X),X  ext{ is a } r imes c  ext{ matrix}\}$
NormNuclearCone(r, c)	$\{(t,X) \in \mathbb{R}^{1+r  imes c} : t \geq \sum_i \sigma_i(X), X  ext{ is a } r  imes c  ext{ matrix} \}$
$Hermitian Positive Semi definite College \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ $	oneThe cogeofalHermitian positive semidefinite matrices, with
$\verb side_dimension  rows and columns.\\$	
Scaled(S)	The set S scaled so that Utilities.set_dot corresponds to
	LinearAlgebra.dot

dimensional variable t, we can specify a root-det constraint as [t, X11, X12, X22]  $\in$  RootDetConeTriangle or [t, X11, X12, X21, X22]  $\in$  RootDetConeSquare.

We provide both forms to enable flexibility for solvers who may natively support one or the other. Transformations between XXXConeTriangle and XXXConeSquare are handled by bridges, which removes the chance of conversion mistakes by users or solver developers.

# 9.5 Multi-dimensional sets with combinatorial structure

Other sets are vector-valued, with a particular combinatorial structure. Read their docstrings for more information on how to interpret them.

Set	Description
SOS1	A Special Ordered Set (SOS) of Type I
S0S2	A Special Ordered Set (SOS) of Type II
Indicator	A set to specify an indicator constraint
Complements	A set to specify a mixed complementarity constraint
AllDifferent	The all_different global constraint
BinPacking	The bin_packing global constraint
Circuit	The circuit global constraint
CountAtLeast	The at_least global constraint
CountBelongs	The nvalue global constraint
CountDistinct	The distinct global constraint
CountGreaterThan	The count_gt global constraint
Cumulative	The cumulative global constraint
Path	The path global constraint
Table	The table global constraint

# **Models**

The most significant part of MOI is the definition of the **model API** that is used to specify an instance of an optimization problem (for example, by adding variables and constraints). Objects that implement the model API must inherit from the ModelLike abstract type.

Notably missing from the model API is the method to solve an optimization problem. ModelLike objects may store an instance (for example, in memory or backed by a file format) without being linked to a particular solver. In addition to the model API, MOI defines AbstractOptimizer and provides methods to solve the model and interact with solutions. See the Solutions section for more details.

#### Info

Throughout the rest of the manual, model is used as a generic ModelLike, and optimizer is used as a generic AbstractOptimizer.

#### Tip

MOI does not export functions, but for brevity we often omit qualifying names with the MOI module. Best practice is to have

import MathOptInterface as MOI

and prefix all MOI methods with MOI. in user code. If a name is also available in base Julia, we always explicitly use the module prefix, for example, with MOI.get.

## 10.1 Attributes

Attributes are properties of the model that can be queried and modified. These include constants such as the number of variables in a model NumberOfVariables), and properties of variables and constraints such as the name of a variable (VariableName).

There are four types of attributes:

- Model attributes (subtypes of AbstractModelAttribute) refer to properties of a model.
- Optimizer attributes (subtypes of AbstractOptimizerAttribute) refer to properties of an optimizer.

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 Constraint attributes (subtypes of AbstractConstraintAttribute) refer to properties of an individual constraint.

Variable attributes (subtypes of AbstractVariableAttribute) refer to properties of an individual variable.

Some attributes are values that can be queried by the user but not modified, while other attributes can be modified by the user.

All interactions with attributes occur through the get and set functions.

Consult the docstrings of each attribute for information on what it represents.

### 10.2 ModelLike API

The following attributes are available:

- ListOfConstraintAttributesSet
- ListOfConstraintIndices
- ListOfConstraintTypesPresent
- ListOfConstraintsWithAttributeSet
- ListOfModelAttributesSet
- ListOfVariableAttributesSet
- ListOfVariableIndices
- ListOfVariablesWithAttributeSet
- NumberOfConstraints
- NumberOfVariables
- Name
- ObjectiveFunction
- ObjectiveFunctionType
- ObjectiveSense

# 10.3 AbstractOptimizer API

The following attributes are available:

- DualStatus
- PrimalStatus
- RawStatusString
- ResultCount
- TerminationStatus

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- BarrierIterations
- DualObjectiveValue
- NodeCount
- NumberOfThreads
- ObjectiveBound
- ObjectiveValue
- RelativeGap
- RawOptimizerAttribute
- RawSolver
- Silent
- SimplexIterations
- SolverName
- SolverVersion
- SolveTimeSec
- TimeLimitSec
- ObjectiveLimit
- SolutionLimit
- NodeLimit
- AutomaticDifferentiationBackend

# **Variables**

# 11.1 Add a variable

Use add\_variable to add a single variable.

```
julia> x = MOI.add_variable(model)
MOI.VariableIndex(1)
```

add\_variable returns a VariableIndex type, which is used to refer to the added variable in other calls.

Check if a VariableIndex is valid using is\_valid.

```
julia> MOI.is_valid(model, x)
true
```

Use add\_variables to add a number of variables.

```
julia> y = MOI.add_variables(model, 2)
2-element Vector{MathOptInterface.VariableIndex}:
    MOI.VariableIndex(2)
    MOI.VariableIndex(3)
```

#### Warning

The integer does not necessarily correspond to the column inside an optimizer.

## 11.2 Delete a variable

Delete a variable using delete.

```
julia> MOI.delete(model, x)

julia> MOI.is_valid(model, x)
false
```

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#### Warning

Not all ModelLike models support deleting variables. A DeleteNotAllowed error is thrown if this is not supported.

# 11.3 Variable attributes

The following attributes are available for variables:

- VariableName
- VariablePrimalStart
- VariablePrimal

Get and set these attributes using get and set.

```
julia> MOI.set(model, MOI.VariableName(), x, "var_x")

julia> MOI.get(model, MOI.VariableName(), x)
"var_x"
```

# **Constraints**

### 12.1 Add a constraint

Use add\_constraint to add a single constraint.

add\_constraint returns a ConstraintIndex type, which is used to refer to the added constraint in other calls.

Check if a ConstraintIndex is valid using is\_valid.

```
julia> MOI.is_valid(model, c)
true
```

Use add\_constraints to add a number of constraints of the same type.

This time, a vector of ConstraintIndex are returned.

Use supports\_constraint to check if the model supports adding a constraint type.

```
)
true
```

#### 12.2 Delete a constraint

Use delete to delete a constraint.

```
julia> MOI.delete(model, c)

julia> MOI.is_valid(model, c)
false
```

### 12.3 Constraint attributes

The following attributes are available for constraints:

- ConstraintName
- ConstraintPrimalStart
- ConstraintDualStart
- ConstraintPrimal
- ConstraintDual
- ConstraintBasisStatus
- ConstraintFunction
- CanonicalConstraintFunction
- ConstraintSet

Get and set these attributes using get and set.

```
julia> MOI.set(model, MOI.ConstraintName(), c, "con_c")

julia> MOI.get(model, MOI.ConstraintName(), c)
"con_c"
```

# 12.4 Constraints by function-set pairs

Below is a list of common constraint types and how they are represented as function-set pairs in MOI. In the notation below, x is a vector of decision variables,  $x_i$  is a scalar decision variable,  $\alpha, \beta$  are scalar constants, a, b are constant vectors, A is a constant matrix and  $\mathbb{R}_+$  (resp.  $\mathbb{R}_-$ ) is the set of non-negative (resp. non-positive) real numbers.

Mathematical Constraint	MOI Function	MOI Set
$a^T x \le \beta$	ScalarAffineFunction	LessThan
$a^T x \ge \alpha$	ScalarAffineFunction	GreaterThan
$a^T x = \beta$	ScalarAffineFunction	EqualTo
$\alpha \le a^T x \le \beta$	ScalarAffineFunction	Interval
$x_i \leq \beta$	VariableIndex	LessThan
$x_i \ge \alpha$	VariableIndex	GreaterThan
$x_i = \beta$	VariableIndex	EqualTo
$\alpha \le x_i \le \beta$	VariableIndex	Interval
$Ax + b \in \mathbb{R}^n_+$	VectorAffineFunction	Nonnegatives
$Ax + b \in \mathbb{R}^n$	VectorAffineFunction	Nonpositives
Ax + b = 0	VectorAffineFunction	Zeros

#### **Linear constraints**

By convention, solvers are not expected to support nonzero constant terms in the ScalarAffineFunctions the first four rows of the preceding table because they are redundant with the parameters of the sets. For example, encode  $2x+1\leq 2$  as  $2x\leq 1$ .

Constraints with VariableIndex in LessThan, GreaterThan, EqualTo, or Interval sets have a natural interpretation as variable bounds. As such, it is typically not natural to impose multiple lower- or upper-bounds on the same variable, and the solver interfaces will throw respectively LowerBoundAlreadySet or UpperBoundAlreadySet.

Moreover, adding two VariableIndex constraints on the same variable with the same set is impossible because they share the same index as it is the index of the variable, see ConstraintIndex.

It is natural, however, to impose upper- and lower-bounds separately as two different constraints on a single variable. The difference between imposing bounds by using a single Interval constraint and by using separate LessThan and GreaterThan constraints is that the latter will allow the solver to return separate dual multipliers for the two bounds, while the former will allow the solver to return only a single dual for the interval constraint.

#### **Conic constraints**

Mathematical Constraint	MOI Function	MOI Set
$  Ax + b  _2 \le c^T x + d$	VectorAffineFunction	SecondOrderCone
$y \ge   x  _2$	VectorOfVariables	SecondOrderCone
$2yz \ge   x  _2^2, y, z \ge 0$	VectorOfVariables	RotatedSecondOrderCone
$(a_1^T x + b_1, a_2^T x + b_2, a_3^T x + b_3) \in \mathcal{E}$	VectorAffineFunction	ExponentialCone
$A(x) \in \mathcal{S}_+$	VectorAffineFunction	PositiveSemidefiniteConeTriangle
$B(x) \in \mathcal{S}_+$	VectorAffineFunction	PositiveSemidefiniteConeSquare
$x \in \mathcal{S}_+$	VectorOfVariables	${\tt Positive Semidefinite Cone Triangle}$
$x \in \mathcal{S}_+$	VectorOfVariables	${\tt Positive Semidefinite Cone Square}$

where  $\mathcal{E}$  is the exponential cone (see ExponentialCone),  $\mathcal{S}_+$  is the set of positive semidefinite symmetric matrices, A is an affine map that outputs symmetric matrices and B is an affine map that outputs square matrices.

Mathematical Constraint	MOI Function	MOI Set
$\frac{1}{2}x^TQx + a^Tx + b \ge 0$	ScalarQuadraticFunction	GreaterThan
$\frac{1}{2}x^TQx + a^Tx + b \le 0$	ScalarQuadraticFunction	LessThan
$\frac{1}{2}x^TQx + a^Tx + b = 0$	ScalarQuadraticFunction	EqualTo
Bilinear matrix inequality	VectorQuadraticFunction	PositiveSemidefiniteCone

## **Quadratic constraints**

#### Note

For more details on the internal format of the quadratic functions see ScalarQuadraticFunction or VectorQuadraticFunction.

### Discrete and logical constraints

Mathematical Constraint	MOI Function	MOI Set
$x_i \in \mathbb{Z}$	VariableIndex	Integer
$x_i \in \{0, 1\}$	VariableIndex	Zero0ne
$x_i \in \{0\} \cup [l, u]$	VariableIndex	Semicontinuous
$x_i \in \{0\} \cup \{l, l+1, \dots, u-1, u\}$	VariableIndex	Semiinteger
At most one component of $\boldsymbol{x}$ can be nonzero	VectorOfVariables	S0S1
At most two components of $\boldsymbol{x}$ can be nonzero, and if so they must be	VectorOfVariables	S0S2
adjacent components		
$y = 1 \implies a^T x \in S$	VectorAffineFunct	ionIndicator

# 12.5 JuMP mapping

The following bullet points show examples of how JuMP constraints are translated into MOI function-set pairs:

- @constraint(m, 2x + y <= 10) becomes ScalarAffineFunction-in-LessThan
- @constraint(m,  $2x + y \ge 10$ ) becomes ScalarAffineFunction-in-GreaterThan
- @constraint(m, 2x + y == 10) becomes ScalarAffineFunction-in-EqualTo
- @constraint(m,  $0 \le 2x + y \le 10$ ) becomes ScalarAffineFunction-in-Interval
- @constraint(m, 2x + y in ArbitrarySet()) becomes ScalarAffineFunction-in-ArbitrarySet.

Variable bounds are handled in a similar fashion:

- @variable(m, x <= 1) becomes VariableIndex-in-LessThan
- @variable(m, x >= 1) becomes VariableIndex-in-GreaterThan

One notable difference is that a variable with an upper and lower bound is translated into two constraints, rather than an interval, that is:

• @variable(m,  $\theta \le x \le 1$ ) becomes VariableIndex-in-LessThan and VariableIndex-in-GreaterThan.

# **Solutions**

# 13.1 Solving and retrieving the results

Once an optimizer is loaded with the objective function and all of the constraints, we can ask the solver to solve the model by calling optimize!.

```
MOI.optimize!(optimizer)
```

# 13.2 Why did the solver stop?

The optimization procedure may stop for a number of reasons. The TerminationStatus attribute of the optimizer returns a TerminationStatusCode object which explains why the solver stopped.

The termination statuses distinguish between proofs of optimality, infeasibility, local convergence, limits, and termination because of something unexpected like invalid problem data or failure to converge.

A typical usage of the TerminationStatus attribute is as follows:

After checking the TerminationStatus, check ResultCount. This attribute returns the number of results that the solver has available to return. A result is defined as a primal-dual pair, but either the primal or the dual may be missing from the result. While the OPTIMAL termination status normally implies that at least one result is available, other statuses do not. For example, in the case of infeasibility, a solver may return no result or a proof of infeasibility. The ResultCount attribute distinguishes between these two cases.

## 13.3 Primal solutions

Use the PrimalStatus optimizer attribute to return a ResultStatusCode describing the status of the primal solution.

Common returns are described below in the Common status situations section.

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Query the primal solution using the VariablePrimal and ConstraintPrimal attributes.

Query the objective function value using the ObjectiveValue attribute.

# 13.4 Dual solutions

#### Warning

See Duality for a discussion of the MOI conventions for primal-dual pairs and certificates.

Use the DualStatus optimizer attribute to return a ResultStatusCode describing the status of the dual solution

Query the dual solution using the ConstraintDual attribute.

Query the dual objective function value using the DualObjectiveValue attribute.

#### 13.5 Common status situations

The sections below describe how to interpret typical or interesting status cases for three common classes of solvers. The example cases are illustrative, not comprehensive. Solver wrappers may provide additional information on how the solver's statuses map to MOI statuses.

#### Info

\* in the tables indicate that multiple different values are possible.

## **Primal-dual convex solver**

Linear programming and conic optimization solvers fall into this category.

What happened?	TerminationSta <b>Res</b>	ultCoun	t PrimalStatus	DualStatus	
Proved optimality	OPTIMAL	1	FEASIBLE_POINT	FEASIBLE_POINT	
Proved infeasible	INFEASIBLE	1	NO_SOLUTION	INFEASIBILITY_CERTI	FICATE
Optimal within relaxed	ALMOST_OPTIMAL	1	FEASIBLE_POINT	FEASIBLE_POINT	
tolerances					
Optimal within relaxed	ALMOST_OPTIMAL	1	ALMOST_FEASIBLE_P01	NALMOST_FEASIBLE_POI	NT
tolerances					
Detected an unbounded ray	DUAL_INFEASIBLE	1	INFEASIBILITY_CERTI	FICATE NO_SOLUTION	
of the primal					
Stall	SLOW_PROGRESS	1	*	*	

#### Global branch-and-bound solvers

Mixed-integer programming solvers fall into this category.

### Info

CPXMIP\_OPTIMAL\_INFEAS is a CPLEX status that indicates that a preprocessed problem was solved to optimality, but the solver was unable to recover a feasible solution to the original problem. Handling this status was one of the motivating drivers behind the design of MOI.

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What happened?	TerminationStatus	ResultCoun	t PrimalStatus	DualStatus
Proved optimality	OPTIMAL	1	FEASIBLE_POINT	NO_SOLUTION
Presolve detected infeasibility or	INFEASIBLE_OR_UNBOUN	DED 0	NO_SOLUTION	NO_SOLUTION
unboundedness				
Proved infeasibility	INFEASIBLE	0	NO_SOLUTION	NO_SOLUTION
Timed out (no solution)	TIME_LIMIT	0	NO_SOLUTION	NO_SOLUTION
Timed out (with a solution)	TIME_LIMIT	1	FEASIBLE_POINT	NO_SOLUTION
CPXMIP_OPTIMAL_INFEAS	ALMOST_OPTIMAL	1	INFEASIBLE_POI	NNTO_SOLUTION

#### Local search solvers

Nonlinear programming solvers fall into this category. It also includes non-global tree search solvers like Juniper.

What happened?	TerminationStatus	ResultCou	ın₱rimalStatus	DualStatus
Converged to a stationary point	LOCALLY_SOLVED	1	FEASIBLE_P0I	NTFEASIBLE_POIN
Completed a non-global tree search (with a solution)	LOCALLY_SOLVED	1	FEASIBLE_POI	NTFEASIBLE_POIN
Converged to an infeasible point	LOCALLY_INFEASIBLE	1	INFEASIBLE_P	OINT *
Completed a non-global tree search (no solution found)	LOCALLY_INFEASIBLE	0	NO_SOLUTION	NO_SOLUTION
Iteration limit	ITERATION_LIMIT	1	*	*
Diverging iterates	NORM_LIMIT or OBJECTIVE_LIMIT	1	*	*

## 13.6 Querying solution attributes

Some solvers will not implement every solution attribute. Therefore, a call like MOI.get(model, MOI.SolveTimeSec()) may throw an UnsupportedAttribute error.

If you need to write code that is agnostic to the solver (for example, you are writing a library that an end-user passes their choice of solver to), you can work-around this problem using a try-catch:

```
function get_solve_time(model)
    try
        return MOI.get(model, MOI.SolveTimeSec())
    catch err
        if err isa MOI.UnsupportedAttribute
            return NaN # Solver doesn't support. Return a placeholder value.
        end
        rethrow(err) # Something else went wrong. Rethrow the error
    end
end
```

If, after careful profiling, you find that the try-catch is taking a significant portion of your runtime, you can improve performance by caching the result of the try-catch:

```
mutable struct CachedSolveTime{M}
  model::M
  supports_solve_time::Bool
  CachedSolveTime(model::M) where {M} = new(model, true)
```

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```
end

function get_solve_time(model::CachedSolveTime)
   if !model.supports_solve_time
        return NaN
   end
   try
       return MOI.get(model, MOI.SolveTimeSec())
   catch err
       if err isa MOI.UnsupportedAttribute
            model.supports_solve_time = false
            return NaN
        end
        rethrow(err) # Something else went wrong. Rethrow the error
   end
end
```

# **Problem modification**

In addition to adding and deleting constraints and variables, MathOptInterface supports modifying, in-place, coefficients in the constraints and the objective function of a model.

These modifications can be grouped into two categories:

- · modifications which replace the set of function of a constraint with a new set or function
- modifications which change, in-place, a component of a function

#### Warning

Some ModelLike objects do not support problem modification.

# 14.1 Modify the set of a constraint

Use set and ConstraintSet to modify the set of a constraint by replacing it with a new instance of the same type.

However, the following will fail as the new set is of a different type to the original set:

```
julia> MOI.set(model, MOI.ConstraintSet(), c, MOI.GreaterThan(2.0))
ERROR: [...]
```

#### Special cases: set transforms

If our constraint is an affine inequality, then this corresponds to modifying the right-hand side of a constraint in linear programming.

In some special cases, solvers may support efficiently changing the set of a constraint (for example, from LessThan to GreaterThan). For these cases, MathOptInterface provides the transform method.

The transform function returns a new constraint index, and the old constraint index (that is, c) is no longer valid.

### Note

transform cannot be called with a set of the same type. Use set instead.

## 14.2 Modify the function of a constraint

Use set and ConstraintFunction to modify the function of a constraint by replacing it with a new instance of the same type.

However, the following will fail as the new function is of a different type to the original function:

```
julia> MOI.set(model, MOI.ConstraintFunction(), c, x)
ERROR: [...]
```

# 14.3 Modify constant term in a scalar function

 $Use \ modify \ and \ Scalar Constant Change \ to \ modify \ the \ constant \ term \ in \ a \ Scalar Affine Function \ or \ Scalar Quadratic Function.$ 

ScalarConstantChange can also be used to modify the objective function by passing an instance of ObjectiveFunction:

## 14.4 Modify constant terms in a vector function

Use modify and VectorConstantChange to modify the constant vector in a VectorAffineFunction or VectorQuadraticFunction

# 14.5 Modify affine coefficients in a scalar function

Use modify and ScalarCoefficientChange to modify the affine coefficient of a ScalarAffineFunction or ScalarQuadraticFunction.

ScalarCoefficientChange can also be used to modify the objective function by passing an instance of ObjectiveFunction.

## 14.6 Modify quadratic coefficients in a scalar function

 $Use \ modify\ and\ Scalar Quadratic Coefficient Change\ to\ modify\ the\ quadratic\ coefficient\ of\ a\ Scalar Quadratic\ Function.$ 

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variables(model, 2);
julia> c = MOI.add_constraint(
```

ScalarQuadraticCoefficientChange can also be used to modify the objective function by passing an instance of ObjectiveFunction.

# 14.7 Modify affine coefficients in a vector function

Use modify and MultirowChange to modify a vector of affine coefficients in a VectorAffineFunction or a VectorQuadraticFunction.

```
julia> c = MOI.add_constraint(
           model.
           MOI.VectorAffineFunction([
                   MOI.VectorAffineTerm(1, MOI.ScalarAffineTerm(1.0, x)),
                  MOI.VectorAffineTerm(2, MOI.ScalarAffineTerm(2.0, x)),
               ],
               [0.0, 0.0],
           ),
           MOI.Nonnegatives(2),
MathOptInterface.ConstraintIndex{MathOptInterface.VectorAffineFunction{Float64},
→ MathOptInterface.Nonnegatives}(1)
julia> MOI.modify(model, c, MOI.MultirowChange(x, [(1, 3.0), (2, 4.0)]));
julia> new_f = MOI.VectorAffineFunction(
       MOI.VectorAffineTerm(1, MOI.ScalarAffineTerm(3.0, x)),
       MOI. VectorAffineTerm(2, MOI. ScalarAffineTerm(4.0, x)),
           1.
           [0.0, 0.0],
      );
```

```
julia> MOI.get(model, MOI.ConstraintFunction(), c) \approx new_f true
```

# Part IV

# **Background**

# **Duality**

Conic duality is the starting point for MOI's duality conventions. When all functions are affine (or coordinate projections), and all constraint sets are closed convex cones, the model may be called a conic optimization problem.

For a minimization problem in geometric conic form, the primal is:

$$\min_{\mathbf{a}_0^T x + b_0 \tag{15.1}$$

s.t. 
$$A_i x + b_i \in \mathcal{C}_i$$
  $i = 1 \dots m$  (15.2)

and the dual is a maximization problem in standard conic form:

$$\max_{y_1, \dots, y_m} -\sum_{i=1}^m b_i^T y_i + b_0 \tag{15.3}$$

s.t. 
$$a_0 - \sum_{i=1}^m A_i^T y_i = 0$$
 (15.4)

$$y_i \in \mathcal{C}_i^* \qquad \qquad i = 1 \dots m \tag{15.5}$$

where each  $\mathcal{C}_i$  is a closed convex cone and  $\mathcal{C}_i^*$  is its dual cone.

For a maximization problem in geometric conic form, the primal is:

$$\max_{a_0^T x + b_0} \qquad (15.6)$$

s.t. 
$$A_i x + b_i \in \mathcal{C}_i$$
  $i = 1 \dots m$  (15.7)

and the dual is a minimization problem in standard conic form:

$$\min_{y_1, \dots, y_m} \sum_{i=1}^m b_i^T y_i + b_0$$
(15.8)

s.t. 
$$a_0 + \sum_{i=1}^m A_i^T y_i = 0 ag{15.9}$$

$$y_i \in \mathcal{C}_i^* \qquad \qquad i = 1 \dots m \tag{15.10}$$

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A linear inequality constraint  $a^Tx+b\geq c$  is equivalent to  $a^Tx+b-c\in\mathbb{R}_+$ , and  $a^Tx+b\leq c$  is equivalent to  $a^Tx+b-c\in\mathbb{R}_+$ . Variable-wise constraints are affine constraints with the appropriate identity mapping in place of  $A_i$ .

For the special case of minimization LPs, the MOI primal form can be stated as:

$$\min_{x \in \mathbb{R}^n} \qquad \qquad a_0^T x + b_0 \tag{15.11}$$

s.t. 
$$A_1 x \ge b_1$$
 (15.12)

$$A_2 x \le b_2 \tag{15.13}$$

$$A_3 x = b_3 (15.14)$$

By applying the stated transformations to conic form, taking the dual, and transforming back into linear inequality form, one obtains the following dual:

$$\max_{y_1, y_2, y_3} b_1^T y_1 + b_2^T y_2 + b_3^T y_3 + b_0$$
 (15.15)

s.t. 
$$A_1^T y_1 + A_2^T y_2 + A_3^T y_3 = a_0$$
 (15.16)

$$y_1 \ge 0$$
 (15.17)

$$y_2 \le 0$$
 (15.18)

For maximization LPs, the MOI primal form can be stated as:

$$\max_{x \in \mathbb{R}^n} a_0^T x + b_0 \tag{15.19}$$

s.t. 
$$A_1 x \ge b_1$$
 (15.20)

$$A_2 x \le b_2$$
 (15.21)

$$A_3 x = b_3 (15.22)$$

and similarly, the dual is:

$$\min_{y_1, y_2, y_3} -b_1^T y_1 - b_2^T y_2 - b_3^T y_3 + b_0$$
(15.23)

s.t. 
$$A_1^T y_1 + A_2^T y_2 + A_3^T y_3 = -a_0$$
 (15.24)

$$y_1 \ge 0$$
 (15.25)

$$y_2 \le 0$$
 (15.26)

#### Warning

For the LP case, the signs of the feasible dual variables depend only on the sense of the corresponding primal inequality and not on the objective sense.

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# 15.1 Duality and scalar product

The scalar product is different from the canonical one for the sets PositiveSemidefiniteConeTriangle, LogDetConeTriangle, RootDetConeTriangle.

If the set  $C_i$  of the section Duality is one of these three cones, then the rows of the matrix  $A_i$  corresponding to off-diagonal entries are twice the value of the coefficients field in the VectorAffineFunction for the corresponding rows. See PositiveSemidefiniteConeTriangle for details.

# 15.2 Dual for problems with quadratic functions

### **Quadratic Programs (QPs)**

For quadratic programs with only affine conic constraints,

$$\min_{x\in\mathbb{R}^n} \qquad \qquad \frac{1}{2}x^TQ_0x+a_0^Tx+b_0$$
 s.t. 
$$A_ix+b_i\in\mathcal{C}_i \qquad \qquad i=1\dots m.$$

with cones  $\mathcal{C}_i \subseteq \mathbb{R}^{m_i}$  for  $i=1,\ldots,m$ , consider the Lagrangian function

$$L(x,y) = \frac{1}{2}x^{T}Q_{0}x + a_{0}^{T}x + b_{0} - \sum_{i=1}^{m} y_{i}^{T}(A_{i}x + b_{i}).$$

Let z(y) denote  $\sum_{i=1}^m A_i^T y_i - a_0$ , the Lagrangian can be rewritten as

$$L(x,y) = \frac{1}{2}x^{T}Q_{0}x - z(y)^{T}x + b_{0} - \sum_{i=1}^{m} y_{i}^{T}b_{i}.$$

The condition  $\nabla_x L(x,y) = 0$  gives

$$0 = \nabla_x L(x, y) = Q_0 x + a_0 - \sum_{i=1}^m y_i^T b_i$$

which gives  $Q_0x=z(y)$ . This allows to obtain that

$$\min_{x \in \mathbb{R}^n} L(x, y) = -\frac{1}{2} x^T Q_0 x + b_0 - \sum_{i=1}^m y_i^T b_i$$

so the dual problem is

$$\max_{y_i \in \mathcal{C}_i^*} \min_{x \in \mathbb{R}^n} -\frac{1}{2} x^T Q_0 x + b_0 - \sum_{i=1}^m y_i^T b_i.$$

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If  $Q_0$  is invertible, we have  $x = Q_0^{-1}z(y)$  hence

$$\min_{x \in \mathbb{R}^n} L(x, y) = -\frac{1}{2} z(y)^T Q_0^{-1} z(y) + b_0 - \sum_{i=1}^m y_i^T b_i$$

so the dual problem is

$$\max_{y_i \in \mathcal{C}_i^*} -\frac{1}{2} z(y)^T Q_0^{-1} z(y) + b_0 - \sum_{i=1}^m y_i^T b_i.$$

# **Quadratically Constrained Quadratic Programs (QCQPs)**

Given a problem with both quadratic function and quadratic objectives:

$$\min_{x \in \mathbb{R}^n} \qquad \qquad \frac{1}{2} x^T Q_0 x + a_0^T x + b_0$$
 s.t. 
$$\frac{1}{2} x^T Q_i x + a_i^T x + b_i \in \mathcal{C}_i \qquad \qquad i = 1 \dots m.$$

with cones  $C_i \subseteq \mathbb{R}$  for  $i=1\dots m$ , consider the Lagrangian function

$$L(x,y) = \frac{1}{2}x^{T}Q_{0}x + a_{0}^{T}x + b_{0} - \sum_{i=1}^{m} y_{i}(\frac{1}{2}x^{T}Q_{i}x + a_{i}^{T}x + b_{i})$$

A pair of primal-dual variables  $(x^\star, y^\star)$  is optimal if

•  $x^*$  is a minimizer of

$$\min_{x \in \mathbb{R}^n} L(x, y^*).$$

That is,

$$0 = \nabla_x L(x, y^*) = Q_0 x + a_0 - \sum_{i=1}^m y_i^* (Q_i x + a_i).$$

• and  $y^{\star}$  is a maximizer of

$$\max_{y_i \in \mathcal{C}_i^*} L(x^*, y).$$

That is, for all  $i=1,\ldots,m$ ,  $\frac{1}{2}x^TQ_ix+a_i^Tx+b_i$  is either zero or in the normal cone of  $\mathcal{C}_i^*$  at  $y^*$ . For instance, if  $\mathcal{C}_i$  is  $\{z\in\mathbb{R}:z\leq 0\}$ , this means that if  $\frac{1}{2}x^TQ_ix+a_i^Tx+b_i$  is nonzero at  $x^*$  then  $y_i^*=0$ . This is the classical complementary slackness condition.

If  $C_i$  is a vector set, the discussion remains valid with  $y_i(\frac{1}{2}x^TQ_ix + a_i^Tx + b_i)$  replaced with the scalar product between  $y_i$  and the vector of scalar-valued quadratic functions.

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# 15.3 Dual for square semidefinite matrices

The set PositiveSemidefiniteConeTriangle is a self-dual. That is, querying ConstraintDual of a PositiveSemidefiniteConeTriangle constraint returns a vector that is itself a member of PositiveSemidefiniteConeTriangle.

However, the dual of PositiveSemidefiniteConeSquare is not so straight forward. This section explains the duality convention we use, and how it is derived.

#### Info

If you have a PositiveSemidefiniteConeSquare constraint, the result matrix A from ConstraintDual is not positive semidefinite. However,  $A+A^{\top}$  is positive semidefinite.

Let  $S_+$  be the cone of symmetric semidefinite matrices in the  $\frac{n(n+1)}{2}$  dimensional space of symmetric  $\mathbb{R}^{n\times n}$  matrices. That is,  $S_+$  is the set PositiveSemidefiniteConeTriangle. It is well known that  $S_+$  is a self-dual proper cone.

Let  $\mathcal{P}_+$  be the cone of symmetric semidefinite matrices in the  $n^2$  dimensional space of  $\mathbb{R}^{n\times n}$  matrices. That is  $\mathcal{P}_+$  is the set PositiveSemidefiniteConeSquare.

In addition, let  $\mathcal{D}_+$  be the cone of matrices A such that  $A + A^{\top} \in \mathcal{P}_+$ .

 $\mathcal{P}_+$  is not proper because it is not solid (it is not  $n^2$  dimensional), so it is not necessarily true that  $\mathcal{P}_+^{**}=\mathcal{P}_+$ .

However, this is the case, because we will show that  $\mathcal{P}_+^*=\mathcal{D}_+$  and  $\mathcal{D}_+^*=\mathcal{P}_+.$ 

First, let us see why  $\mathcal{P}_+^* = \mathcal{D}_+.$ 

If B is symmetric, then

$$\langle A, B \rangle = \langle A^{\top}, B^{\top} \rangle = \langle A^{\top}, B \rangle$$

so

$$2\langle A, B \rangle = \langle A, B \rangle + \langle A^{\top}, B \rangle = \langle A + A^{\top}, B \rangle.$$

Therefore,  $\langle A,B\rangle \geq 0$  for all  $B\in \mathcal{P}_+$  if and only if  $\langle A+A^\top,B\rangle \geq 0$  for all  $B\in \mathcal{P}_+$ . Since  $A+A^\top$  is symmetric, and we know that  $\mathcal{S}_+$  is self-dual, we have shown that  $\mathcal{P}_+^*$  is the set of matrices A such that  $A+A^\top\in \mathcal{P}_+$ .

Second, let us see why  $\mathcal{D}_+^* = \mathcal{P}_+$ .

Since  $A \in \mathcal{D}_+$  implies that  $A^{\top} \in \mathcal{D}_+$ ,  $B \in \mathcal{D}_+^*$  means that  $\langle A + A^{\top}, B \rangle \geq 0$  for all  $A \in \mathcal{D}_+$ , and hence  $B \in \mathcal{D}_+$ 

 $mathcalP_{+}$ .

To see why it should be symmetric, simply notice that if  $B_{i,j} < B_{j,i}$ , then  $\langle A,B \rangle$  can be made arbitrarily small by setting  $A_{i,j} = A_{i,j} + s$  and  $A_{j,i} = A_{j,i} - s$ , with s arbitrarily large, and A stays in  $\mathcal{D}_+$  because  $A + A^\top$  does not change.

Typically, the primal/dual pair for semidefinite programs is presented as:

$$\min\langle C, X \rangle \tag{15.27}$$

s.t. 
$$\langle A_k, X \rangle = b_k \forall k$$
 (15.28)

$$X \in \mathcal{S}_{+} \tag{15.29}$$

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with the dual

$$\max \sum_{k} b_k y_k \tag{15.30}$$

s.t. 
$$C - \sum A_k y_k \in \mathcal{S}_+$$
 (15.31)

If we allow  ${\cal A}_k$  to be non-symmetric, we should instead use:

$$\min\langle C, X \rangle$$
 (15.32)

s.t. 
$$\langle A_k, X \rangle = b_k \forall k$$
 (15.33)

$$X \in \mathcal{D}_{+} \tag{15.34}$$

with the dual

$$\max \sum b_k y_k \tag{15.35}$$

s.t. 
$$C - \sum A_k y_k \in \mathcal{P}_+$$
 (15.36)

This is implemented as:

$$\min\langle C, Z \rangle + \langle C - C^{\top}, S \rangle \tag{15.37}$$

s.t. 
$$\langle A_k, Z \rangle + \langle A_k - A_k^\top, S \rangle = b_k \forall k$$
 (15.38)

$$Z \in \mathcal{S}_{+} \tag{15.39}$$

with the dual

$$\max \sum b_k y_k \tag{15.40}$$

s.t. 
$$C + C^{\top} - \sum (A_k + A_k^{\top}) y_k \in \mathcal{S}_+$$
 (15.41)

$$C - C^{\top} - \sum_{k=0}^{\infty} (A_k - A_k^{\top}) y_k = 0$$
 (15.42)

and we recover  $Z = X + X^{\top}$ .

# **Chapter 16**

# Infeasibility certificates

When given a conic problem that is infeasible or unbounded, some solvers can produce a certificate of infeasibility. This page explains what a certificate of infeasibility is, and the related conventions that MathOptInterface adopts.

# 16.1 Conic duality

MathOptInterface uses conic duality to define infeasibility certificates. A full explanation is given in the section Duality, but here is a brief overview.

# **Minimization problems**

For a minimization problem in geometric conic form, the primal is:

$$\min_{x \in \mathbb{R}^n} \qquad \qquad a_0^\top x + b_0 \tag{16.1}$$

s.t. 
$$A_i x + b_i \in \mathcal{C}_i$$
  $i = 1 \dots m,$  (16.2)

and the dual is a maximization problem in standard conic form:

$$\max_{y_1, \dots, y_m} \qquad -\sum_{i=1}^m b_i^\top y_i + b_0 \tag{16.3}$$

s.t. 
$$a_0 - \sum_{i=1}^m A_i^\top y_i = 0 \tag{16.4}$$

$$y_i \in \mathcal{C}_i^* \qquad \qquad i = 1 \dots m, \tag{16.5}$$

where each  $\mathcal{C}_i$  is a closed convex cone and  $\mathcal{C}_i^*$  is its dual cone.

#### **Maximization problems**

For a maximization problem in geometric conic form, the primal is:

$$\max_{a_0^\top x + b_0} \qquad \qquad a_0^\top x + b_0 \tag{16.6}$$

s.t. 
$$A_i x + b_i \in \mathcal{C}_i$$
  $i = 1 \dots m,$  (16.7)

and the dual is a minimization problem in standard conic form:

$$\min_{y_1, \dots, y_m} \qquad \sum_{i=1}^m b_i^\top y_i + b_0 \tag{16.8}$$

s.t. 
$$a_0 + \sum_{i=1}^m A_i^\top y_i = 0 \tag{16.9}$$

$$y_i \in \mathcal{C}_i^* \qquad \qquad i = 1 \dots m. \tag{16.10}$$

# 16.2 Unbounded problems

A problem is unbounded if and only if:

- 1. there exists a feasible primal solution
- 2. the dual is infeasible.

A feasible primal solution—if one exists—can be obtained by setting <code>ObjectiveSense</code> to <code>FEASIBILITY\_SENSE</code> before optimizing. Therefore, most solvers stop after they prove the dual is infeasible via a certificate of dual infeasibility, but before they have found a feasible primal solution. This is also the reason that <code>MathOptInterface</code> defines the <code>DUAL\_INFEASIBLE</code> status instead of <code>UNBOUNDED</code>.

A certificate of dual infeasibility is an improving ray of the primal problem. That is, there exists some vector d such that for all  $\eta > 0$ :

$$A_i(x + \eta d) + b_i \in \mathcal{C}_i, \quad i = 1 \dots m,$$

and (for minimization problems):

$$a_0^{\top}(x + \eta d) + b_0 < a_0^{\top}x + b_0,$$

for any feasible point x. The latter simplifies to  $a_0^\top d < 0$ . For maximization problems, the inequality is reversed, so that  $a_0^\top d > 0$ .

If the solver has found a certificate of dual infeasibility:

- TerminationStatus must be DUAL INFEASIBLE
- PrimalStatus must be INFEASIBILITY\_CERTIFICATE
- ullet VariablePrimal must be the corresponding value of d
- ConstraintPrimal must be the corresponding value of  $A_id$
- ObjectiveValue must be the value  $a_0^\top d$ . Note that this is the value of the objective function at d, ignoring the constant b\_0.

#### Note

The choice of whether to scale the ray d to have magnitude 1 is left to the solver.

# 16.3 Infeasible problems

A certificate of primal infeasibility is an improving ray of the dual problem. However, because infeasibility is independent of the objective function, we first homogenize the primal problem by removing its objective.

For a minimization problem, a dual improving ray is some vector d such that for all  $\eta > 0$ :

$$-\sum_{i=1}^{m} A_i^{\top}(y_i + \eta d_i) = 0$$
 (16.11)

$$(y_i + \eta d_i) \in \mathcal{C}_i^* \qquad \qquad i = 1 \dots m, \tag{16.12}$$

and:

$$-\sum_{i=1}^{m} b_{i}^{\top}(y_{i} + \eta d_{i}) > -\sum_{i=1}^{m} b_{i}^{\top} y_{i},$$

for any feasible dual solution y. The latter simplifies to  $-\sum_{i=1}^m b_i^\top d_i > 0$ . For a maximization problem, the inequality is  $\sum_{i=1}^m b_i^\top d_i < 0$ . (Note that these are the same inequality, modulo a - sign.)

If the solver has found a certificate of primal infeasibility:

- TerminationStatus must be INFEASIBLE
- DualStatus must be INFEASIBILITY\_CERTIFICATE
- ullet ConstraintDual must be the corresponding value of d
- DualObjectiveValue must be the value  $-\sum_{i=1}^m b_i^\top d_i$  for minimization problems and  $\sum_{i=1}^m b_i^\top d_i$  for maximization problems.

# Note

The choice of whether to scale the ray d to have magnitude 1 is left to the solver.

#### Infeasibility certificates of variable bounds

Many linear solvers (for example, Gurobi) do not provide explicit access to the primal infeasibility certificate of a variable bound. However, given a set of linear constraints:

$$l_A \le Ax \le u_A \tag{16.13}$$

$$l_x < x < u_x, \tag{16.14}$$

the primal certificate of the variable bounds can be computed using the primal certificate associated with the affine constraints, d. (Note that d will have one element for each row of the A matrix, and that some or all of the elements in the vectors  $l_A$  and  $u_A$  may be  $\pm\infty$ . If both  $l_A$  and  $u_A$  are finite for some row, the corresponding element in 'd must be 0.)

Given d, compute  $\bar{d} = d^{\top}A$ . If the bound is finite, a certificate for the lower variable bound of  $x_i$  is  $\max\{\bar{d}_i,0\}$ , and a certificate for the upper variable bound is  $\min\{\bar{d}_i,0\}$ .

# **Chapter 17**

# **Naming conventions**

MOI follows several conventions for naming functions and structures. These should also be followed by packages extending MOI.

#### 17.1 Sets

Sets encode the structure of constraints. Their names should follow the following conventions:

- Abstract types in the set hierarchy should begin with Abstract and end in Set, for example, AbstractScalarSet, AbstractVectorSet.
- Vector-valued conic sets should end with Cone, for example, NormInfinityCone, SecondOrderCone.
- Vector-valued Cartesian products should be plural and not end in Cone, for example, Nonnegatives, not NonnegativeCone.
- Matrix-valued conic sets should provide two representations: ConeSquare and ConeTriangle, for example, RootDetConeTriangle and RootDetConeSquare. See Matrix cones for more details.
- Scalar sets should be singular, not plural, for example, Integer, not Integers.
- As much as possible, the names should follow established conventions in the domain where this set is used: for instance, convex sets should have names close to those of CVX, and constraint-programming sets should follow MiniZinc's constraints.

# Part V API Reference

# **Chapter 18**

# Standard form

# 18.1 Functions

MathOptInterface.AbstractFunction - Type.

AbstractFunction

Abstract supertype for function objects.

#### **Required methods**

All functions must implement:

- Base.copy
- Base.isapprox
- constant

Abstract subtypes of AbstractFunction may require additional methods to be implemented.

source

MathOptInterface.output\_dimension - Function.

```
output_dimension(f::AbstractFunction)
```

 $Return \ 1\ if\ f\ is\ an\ Abstract Scalar Function, or\ the\ number\ of\ output\ components\ if\ f\ is\ an\ Abstract Vector Function.$  source

MathOptInterface.constant - Function.

```
constant(f::AbstractFunction[, ::Type{T}]) where {T}
```

Returns the constant term of a scalar-valued function, or the constant vector of a vector-valued function.

If f is untyped and T is provided, returns zero(T).

source

```
constant(set::Union{EqualTo,GreaterThan,LessThan,Parameter})
```

Returns the constant term of the set set.

#### **Example**

```
julia> MOI.constant(MOI.GreaterThan(1.0))
1.0

julia> MOI.constant(MOI.LessThan(2.5))
2.5

julia> MOI.constant(MOI.EqualTo(3))
3

julia> MOI.constant(MOI.Parameter(4.5))
4.5
```

source

# 18.2 Scalar functions

MathOptInterface.AbstractScalarFunction - Type.

```
abstract type AbstractScalarFunction <: AbstractFunction</pre>
```

Abstract supertype for scalar-valued AbstractFunctions.

source

MathOptInterface.VariableIndex - Type.

```
VariableIndex
```

A type-safe wrapper for Int64 for use in referencing variables in a model. To allow for deletion, indices need not be consecutive.

source

MathOptInterface.ScalarAffineTerm - Type.

```
ScalarAffineTerm{T}(coefficient::T, variable::VariableIndex) where {T}
```

Represents the scalar-valued term coefficient \* variable.

```
julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)

julia> MOI.ScalarAffineTerm(2.0, x)
MathOptInterface.ScalarAffineTerm{Float64}(2.0, MOI.VariableIndex(1))
```

 ${\tt MathOptInterface.ScalarAffineFunction-Type.}$ 

```
ScalarAffineFunction{T}(
   terms::Vector{ScalarAffineTerm{T}},
   constant::T,
) where {T}
```

Represents the scalar-valued affine function  $a^{T}x + b$ , where:

- $a^{T}x$  is represented by the vector of ScalarAffineTerms
- b is a scalar constant::T

#### **Duplicates**

Duplicate variable indices in terms are accepted, and the corresponding coefficients are summed together.

# **Example**

```
julia> x = MOI.VariableIndex(1)

julia> terms = [MOI.ScalarAffineTerm(2.0, x), MOI.ScalarAffineTerm(3.0, x)]
2-element Vector{MathOptInterface.ScalarAffineTerm{Float64}}:
    MathOptInterface.ScalarAffineTerm{Float64}(2.0, MOI.VariableIndex(1))
    MathOptInterface.ScalarAffineTerm{Float64}(3.0, MOI.VariableIndex(1))

julia> f = MOI.ScalarAffineFunction(terms, 4.0)
4.0 + 2.0 MOI.VariableIndex(1) + 3.0 MOI.VariableIndex(1)
```

source

MathOptInterface.ScalarQuadraticTerm - Type.

```
ScalarQuadraticTerm{T}(
    coefficient::T,
    variable_1::VariableIndex,
    variable_2::VariableIndex,
) where {T}
```

Represents the scalar-valued term  $cx_ix_i$  where c is coefficient,  $x_i$  is variable\_1 and  $x_i$  is variable\_2.

```
julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)

julia> MOI.ScalarQuadraticTerm(2.0, x, x)
MathOptInterface.ScalarQuadraticTerm{Float64}(2.0, MOI.VariableIndex(1), MOI.VariableIndex(1))
```

 ${\tt MathOptInterface.ScalarQuadraticFunction-Type}.$ 

```
ScalarQuadraticFunction{T}(
    quadratic_terms::Vector{ScalarQuadraticTerm{T}},
    affine_terms::Vector{ScalarAffineTerm{T}},
    constant::T,
) wher {T}
```

The scalar-valued quadratic function  $\frac{1}{2}x^{T}Qx + a^{T}x + b$ , where:

- ullet Q is the symmetric matrix given by the vector of  ${\it ScalarQuadraticTerms}$
- $a^{T}x$  is a sparse vector given by the vector of ScalarAffineTerms
- b is the scalar constant::T.

#### **Duplicates**

Duplicate indices in quadratic\_terms or affine\_terms are accepted, and the corresponding coefficients are summed together.

In quadratic\_terms, "mirrored" indices, (q, r) and (r, q) where r and q are VariableIndexes, are considered duplicates; only one needs to be specified.

#### The 0.5 factor

Coupled with the interpretation of mirrored indices, the 0.5 factor in front of the  ${\it Q}$  matrix is a common source of bugs.

As a rule, to represent  $a * x^2 + b * x * y$ :

- ullet The coefficient a in front of squared variables (diagonal elements in Q) must be doubled when creating a ScalarQuadraticTerm
- The coefficient b in front of off-diagonal elements in Q should be left as b, be cause the mirrored index b\*y\*x will be implicitly added.

#### Example

To represent the function  $f(x,y) = 2 * x^2 + 3 * x * y + 4 * x + 5$ , do:

```
julia> x = MOI.VariableIndex(1);
julia> y = MOI.VariableIndex(2);
julia> constant = 5.0;
```

MathOptInterface.ScalarNonlinearFunction - Type.

```
ScalarNonlinearFunction(head::Symbol, args::Vector{Any})
```

The scalar-valued nonlinear function head(args...), represented as a symbolic expression tree, with the call operator head and ordered arguments in args.

#### head

The head::Symbol must be an operator supported by the model.

The default list of supported univariate operators is given by:

• Nonlinear.DEFAULT UNIVARIATE OPERATORS

and the default list of supported multivariate operators is given by:

• Nonlinear.DEFAULT MULTIVARIATE OPERATORS

Additional operators can be registered by setting a UserDefinedFunction attribute.

 $See the full list of operators \, supported \, by \, a \, {\tt ModelLike} \, by \, querying \, {\tt ListOfSupportedNonlinearOperators}.$ 

# args

The vector args contains the arguments to the nonlinear function. If the operator is univariate, it must contain one element. Otherwise, it may contain multiple elements.

Each element must be one of the following:

- A constant value of type T<:Real
- A VariableIndex
- A ScalarAffineFunction
- A ScalarQuadraticFunction
- A ScalarNonlinearFunction

# **Unsupported operators**

If the optimizer does not support head, an UnsupportedNonlinearOperator error will be thrown.

There is no guarantee about when this error will be thrown; it may be thrown when the function is first added to the model, or it may be thrown when optimize! is called.

#### **Example**

To represent the function  $f(x) = sin(x)^2$ , do:

source

# 18.3 Vector functions

MathOptInterface.AbstractVectorFunction - Type.

```
abstract type AbstractVectorFunction <: AbstractFunction</pre>
```

Abstract supertype for vector-valued AbstractFunctions.

#### **Required methods**

All subtypes of AbstractVectorFunction must implement:

• output\_dimension

source

MathOptInterface.VectorOfVariables - Type.

```
VectorOfVariables(variables::Vector{VariableIndex}) <: AbstractVectorFunction</pre>
```

The vector-valued function f(x) = variables, where variables is a subset of VariableIndexes in the model.

The list of variables may contain duplicates.

```
julia> x = MOI.VariableIndex.(1:2)
2-element Vector{MathOptInterface.VariableIndex}:
MOI.VariableIndex(1)
```

MathOptInterface.VectorAffineTerm - Type.

```
VectorAffineTerm{T}(
   output_index::Int64,
   scalar_term::ScalarAffineTerm{T},
) where {T}
```

 $A \ Vector Affine Term is a \ scalar\_term \ that \ appears \ in \ the \ output\_index \ row \ of the \ vector-valued \ Vector Affine Function \ or \ Vector Quadratic Function.$ 

# **Example**

```
julia> x = MOI.VariableIndex(1);

julia> MOI.VectorAffineTerm(Int64(2), MOI.ScalarAffineTerm(3.0, x))

MathOptInterface.VectorAffineTerm{Float64}(2, MathOptInterface.ScalarAffineTerm{Float64}(3.0, → MOI.VariableIndex(1)))
```

source

 ${\tt MathOptInterface.VectorAffineFunction-Type.}$ 

```
VectorAffineFunction{T}(
    terms::Vector{VectorAffineTerm{T}},
    constants::Vector{T},
) where {T}
```

The vector-valued affine function Ax + b, where:

- ullet Ax is the sparse matrix given by the vector of VectorAffineTerms
- $oldsymbol{\cdot}$  b is the vector constants

# **Duplicates**

Duplicate indices in the A are accepted, and the corresponding coefficients are summed together.

 ${\tt MathOptInterface.VectorQuadraticTerm-Type.}$ 

```
VectorQuadraticTerm{T}(
   output_index::Int64,
   scalar_term::ScalarQuadraticTerm{T},
) where {T}
```

A VectorQuadraticTerm is a ScalarQuadraticTerm scalar\_term that appears in the output\_index row of the vector-valued VectorQuadraticFunction.

# Example

source

MathOptInterface.VectorQuadraticFunction - Type.

```
VectorQuadraticFunction{T}(
   quadratic_terms::Vector{VectorQuadraticTerm{T}},
   affine_terms::Vector{VectorAffineTerm{T}},
   constants::Vector{T},
) where {T}
```

The vector-valued quadratic function with ith component ("output index") defined as  $\frac{1}{2}x^{\top}Q_ix + a_i^{\top}x + b_i$ , where:

- $\frac{1}{2}x^{\top}Q_ix$  is the symmetric matrix given by the VectorQuadraticTerm elements in quadratic\_terms with output\_index == i
- $a_i^{\top} x$  is the sparse vector given by the VectorAffineTerm elements in affine\_terms with output\_index == i
- $b_i$  is a scalar given by constants[i]

#### **Duplicates**

Duplicate indices in quadratic\_terms and affine\_terms with the same output\_index are handled in the same manner as duplicates in ScalarQuadraticFunction.

#### **Example**

source

MathOptInterface.VectorNonlinearFunction - Type.

```
VectorNonlinearFunction(args::Vector{ScalarNonlinearFunction})
```

The vector-valued nonlinear function composed of a vector of ScalarNonlinearFunction.

#### args

The vector args contains the scalar elements of the nonlinear function. Each element must be a ScalarNonlinearFunction, but if you pass a Vector{Any}, the elements can be automatically converted from one of the following:

- A constant value of type T<:Real
- A VariableIndex

- A ScalarAffineFunction
- A ScalarQuadraticFunction
- A ScalarNonlinearFunction

#### **Example**

To represent the function  $f(x) = [sin(x)^2, x]$ , do:

Note the automatic conversion from x to +(x).

source

# 18.4 Sets

MathOptInterface.AbstractSet - Type.

```
AbstractSet
```

Abstract supertype for set objects used to encode constraints.

# **Required methods**

For sets of type S with isbitstype(S) == false, you must implement:

```
Base.copy(set::S)Base.:(==)(x::S, y::S)
```

Subtypes of AbstractSet such as AbstractScalarSet and AbstractVectorSet may prescribe additional required methods.

#### **Optional methods**

You may optionally implement:

- dual set
- dual\_set\_type

# Note for developers

When creating a new set, the set struct must not contain any VariableIndex or ConstraintIndex objects.

source

 ${\tt MathOptInterface.AbstractScalarSet-Type}.$ 

```
AbstractScalarSet
```

Abstract supertype for subsets of  $\mathbb{R}$ .

source

MathOptInterface.AbstractVectorSet - Type.

```
AbstractVectorSet
```

Abstract supertype for subsets of  $\mathbb{R}^n$  for some n.

# **Required methods**

All AbstractVectorSets of type S must implement:

- dimension, unless the dimension is stored in the set.dimension field
- Utilities.set\_dot, unless the dot product between two vectors in the set is equivalent to LinearAlgebra.dot.

source

#### **Utilities**

MathOptInterface.dimension - Function.

```
dimension(set::AbstractSet)
```

Return the output\_dimension that an AbstractFunction should have to be used with the set set.

# **Example**

```
julia> MOI.dimension(MOI.Reals(4))
4

julia> MOI.dimension(MOI.LessThan(3.0))
1

julia> MOI.dimension(MOI.PositiveSemidefiniteConeTriangle(2))
3
```

source

MathOptInterface.dual\_set - Function.

```
dual_set(set::AbstractSet)
```

Return the dual set of set, that is the dual cone of the set. This follows the definition of duality discussed in Duality.

See Dual cone for more information.

If the dual cone is not defined it returns an error.

# **Example**

```
julia> MOI.dual_set(MOI.Reals(4))
MathOptInterface.Zeros(4)

julia> MOI.dual_set(MOI.SecondOrderCone(5))
MathOptInterface.SecondOrderCone(5)

julia> MOI.dual_set(MOI.ExponentialCone())
MathOptInterface.DualExponentialCone()
```

source

MathOptInterface.dual\_set\_type - Function.

```
dual_set_type(S::Type{<:AbstractSet})</pre>
```

Return the type of dual set of sets of type S, as returned by dual\_set. If the dual cone is not defined it returns an error.

# Example

```
julia> MOI.dual_set_type(MOI.Reals)
MathOptInterface.Zeros

julia> MOI.dual_set_type(MOI.SecondOrderCone)
MathOptInterface.SecondOrderCone

julia> MOI.dual_set_type(MOI.ExponentialCone)
MathOptInterface.DualExponentialCone
```

source

 ${\tt MathOptInterface.constant-Method}.$ 

```
constant(set::Union{EqualTo,GreaterThan,LessThan,Parameter})
```

Returns the constant term of the set set.

```
julia> MOI.constant(MOI.GreaterThan(1.0))
1.0

julia> MOI.constant(MOI.LessThan(2.5))
2.5

julia> MOI.constant(MOI.EqualTo(3))
3

julia> MOI.constant(MOI.Parameter(4.5))
4.5
```

 ${\tt MathOptInterface.supports\_dimension\_update-Function}.$ 

```
supports_dimension_update(S::Type{<:MOI.AbstractVectorSet})</pre>
```

Return a Bool indicating whether the elimination of any dimension of n-dimensional sets of type S give an n-1-dimensional set S. By default, this function returns false so it should only be implemented for sets that supports dimension update.

For instance, supports\_dimension\_update(MOI.Nonnegatives) is true because the elimination of any dimension of the n-dimensional nonnegative orthant gives the n-1-dimensional nonnegative orthant. However supports\_dimension\_update(MOI.ExponentialCone) is false.

source

 ${\tt MathOptInterface.update\_dimension-Function}.$ 

```
update_dimension(s::AbstractVectorSet, new_dim::Int)
```

Returns a set with the dimension modified to new\_dim.

source

### 18.5 Scalar sets

List of recognized scalar sets.

 ${\tt MathOptInterface.GreaterThan-Type}.$ 

```
GreaterThan{T<:Real}(lower::T)</pre>
```

The set  $[lower, \infty) \subseteq \mathbb{R}$ .

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> MOI.add_constraint(model, x, MOI.GreaterThan(0.0))

MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,

→ MathOptInterface.GreaterThan{Float64}}(1)
```

MathOptInterface.LessThan - Type.

```
LessThan{T<: Real} (upper::T)
```

The set  $(-\infty, upper] \subseteq \mathbb{R}$ .

# **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> MOI.add_constraint(model, x, MOI.LessThan(2.0))

MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,

MathOptInterface.LessThan{Float64}}(1)
```

source

MathOptInterface.EqualTo - Type.

```
EqualTo{T<: Number} (value::T)</pre>
```

The set containing the single point  $\{value\} \subseteq \mathbb{R}$ .

# Example

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> MOI.add_constraint(model, x, MOI.EqualTo(2.0))

MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,

→ MathOptInterface.EqualTo{Float64}}(1)
```

source

MathOptInterface.Interval - Type.

```
Interval{T<:Real}(lower::T, upper::T)</pre>
```

The interval  $[lower, upper] \subseteq \mathbb{R} \cup \{-\infty, +\infty\}.$ 

If lower or upper is -Inf or Inf, respectively, the set is interpreted as a one-sided interval.

#### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> MOI.add_constraint(model, x, MOI.Interval(1.0, 2.0))

MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,

MathOptInterface.Interval{Float64}}(1)
```

source

MathOptInterface.Integer - Type.

```
Integer()
```

The set of integers,  $\mathbb{Z}$ .

# **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> MOI.add_constraint(model, x, MOI.Integer())

MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.Integer}(1)
```

source

MathOptInterface.ZeroOne - Type.

```
ZeroOne()
```

The set  $\{0, 1\}$ .

Variables belonging to the Zero0ne set are also known as "binary" variables.

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> MOI.add_constraint(model, x, MOI.ZeroOne())

MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.ZeroOne}(1)
```

MathOptInterface.Semicontinuous - Type.

```
Semicontinuous{T<:Real}(lower::T, upper::T)
```

The set  $\{0\} \cup [lower, upper]$ .

#### Example

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> MOI.add_constraint(model, x, MOI.Semicontinuous(2.0, 3.0))

MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,

→ MathOptInterface.Semicontinuous{Float64}}(1)
```

source

MathOptInterface.Semiinteger - Type.

```
Semiinteger{T<:Real}(lower::T, upper::T)</pre>
```

```
The set \{0\} \cup \{lower, lower + 1, \dots, upper - 1, upper\}.
```

Note that if lower and upper are not equivalent to an integer, then the solver may throw an error, or it may round up lower and round down upper to the nearest integers.

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> MOI.add_constraint(model, x, MOI.Semiinteger(2.0, 3.0))

MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,

→ MathOptInterface.Semiinteger{Float64}}(1)
```

MathOptInterface.Parameter - Type.

```
Parameter{T<:Number}(value::T)
```

The set containing the single point  $\{value\} \subseteq \mathbb{R}$ .

The Parameter set is conceptually similar to the EqualTo set, except that a variable constrained to the Parameter set cannot have other constraints added to it, and the Parameter set can never be deleted. Thus, solvers are free to treat the variable as a constant, and they need not add it as a decision variable to the model.

Because of this behavior, you must add parameters using add\_constrained\_variable, and solvers should declare supports\_add\_constrained\_variable and not supports\_constraint for the Parameter set.

# **Example**

source

# 18.6 Vector sets

List of recognized vector sets.

MathOptInterface.Reals - Type.

```
Reals(dimension::Int)
```

The set  $\mathbb{R}^{dimension}$  (containing all points) of non-negative dimension dimension.

#### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variables(model, 3);
julia> MOI.add_constraint(model, MOI.VectorOfVariables(x), MOI.Reals(3))
MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables, MathOptInterface.Reals}(1)
```

source

MathOptInterface.Zeros - Type.

```
Zeros(dimension::Int)
```

The set  $\{0\}^{dimension}$  (containing only the origin) of non-negative dimension dimension.

#### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variables(model, 3);

julia> MOI.add_constraint(model, MOI.VectorOfVariables(x), MOI.Zeros(3))
MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables, MathOptInterface.Zeros}(1)
```

source

MathOptInterface.Nonnegatives - Type.

```
Nonnegatives(dimension::Int)
```

The nonnegative orthant  $\{x \in \mathbb{R}^{dimension} : x \geq 0\}$  of non-negative dimension dimension.

## **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variables(model, 3);

julia> MOI.add_constraint(model, MOI.VectorOfVariables(x), MOI.Nonnegatives(3))

MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,

MathOptInterface.Nonnegatives}(1)
```

source

MathOptInterface.Nonpositives - Type.

```
Nonpositives(dimension::Int)
```

The nonpositive orthant  $\{x \in \mathbb{R}^{dimension} : x \leq 0\}$  of non-negative dimension dimension.

```
julia> model = MoI.Utilities.Model{Float64}();

julia> x = MoI.add_variables(model, 3);

julia> MoI.add_constraint(model, MoI.VectorOfVariables(x), MoI.Nonpositives(3))
MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,

MathOptInterface.Nonpositives}(1)
```

MathOptInterface.NormInfinityCone - Type.

```
NormInfinityCone(dimension::Int)
```

The  $\ell_{\infty}$ -norm cone  $\{(t,x) \in \mathbb{R}^{dimension} : t \geq ||x||_{\infty} = \max_{i} |x_{i}|\}$  of dimension dimension.

The dimension must be at least 1.

# **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> t = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> x = MOI.add_variables(model, 3);

julia> MOI.add_constraint(model, MOI.VectorOfVariables([t; x]), MOI.NormInfinityCone(4))

MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,

→ MathOptInterface.NormInfinityCone}(1)
```

source

MathOptInterface.NormOneCone - Type.

```
NormOneCone(dimension::Int)
```

The  $\ell_1$ -norm cone  $\{(t,x)\in\mathbb{R}^{dimension}:t\geq \|x\|_1=\sum_i |x_i|\}$  of dimension dimension.

The dimension must be at least 1.

### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> t = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> x = MOI.add_variables(model, 3);

julia> MOI.add_constraint(model, MOI.VectorOfVariables([t; x]), MOI.NormOneCone(4))

MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,

→ MathOptInterface.NormOneCone}(1)
```

source

MathOptInterface.NormCone - Type.

```
NormCone(p::Float64, dimension::Int)
```

The  $\ell_p$  -norm cone  $\{(t,x)\in\mathbb{R}^{dimension}:t\geq\left(\sum\limits_i|x_i|^p\right)^{\frac{1}{p}}\}$  of dimension dimension.

The dimension must be at least 1.

# **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> t = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> x = MOI.add_variables(model, 3);

julia> MOI.add_constraint(model, MOI.VectorOfVariables([t; x]), MOI.NormCone(3, 4))

MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,

MathOptInterface.NormCone}(1)
```

source

MathOptInterface.SecondOrderCone - Type.

```
SecondOrderCone(dimension::Int)
```

The second-order cone (or Lorenz cone or  $\ell_2$ -norm cone)  $\{(t,x)\in\mathbb{R}^{dimension}:t\geq\|x\|_2\}$  of dimension dimension.

The dimension must be at least 1.

#### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> t = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> x = MOI.add_variables(model, 3);

julia> MOI.add_constraint(model, MOI.VectorOfVariables([t; x]), MOI.SecondOrderCone(4))

MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,

MathOptInterface.SecondOrderCone}(1)
```

source

 ${\tt MathOptInterface.RotatedSecondOrderCone-Type.}$ 

```
RotatedSecondOrderCone(dimension::Int)
```

The rotated second-order cone  $\{(t,u,x)\in\mathbb{R}^{dimension}: 2tu\geq \|x\|_2^2, t,u\geq 0\}$  of dimension dimension. The dimension must be at least 2.

#### **Example**

source

 ${\tt MathOptInterface.GeometricMeanCone-Type}.$ 

```
GeometricMeanCone(dimension::Int)
```

The geometric mean cone  $\{(t,x)\in\mathbb{R}^{n+1}:x\geq 0,t\leq\sqrt[n]{x_1x_2\cdots x_n}\}$ , where dimension = n + 1 >= 2.

# **Duality note**

The dual of the geometric mean cone is  $\{(u,v)\in\mathbb{R}^{n+1}:u\leq 0,v\geq 0,-u\leq n\sqrt[n]{\prod_i v_i}\}$ , where dimension = n + 1 >= 2.

MathOptInterface.ExponentialCone - Type.

```
ExponentialCone()
```

The 3-dimensional exponential cone  $\{(x,y,z)\in\mathbb{R}^3:y\exp(x/y)\leq z,y>0\}.$ 

# **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variables(model, 3);

julia> MOI.add_constraint(model, MOI.VectorOfVariables(x), MOI.ExponentialCone())
MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,

MathOptInterface.ExponentialCone}(1)
```

source

MathOptInterface.DualExponentialCone - Type.

```
DualExponentialCone()
```

The 3-dimensional dual exponential cone  $\{(u,v,w)\in\mathbb{R}^3: -u\exp(v/u)\leq \exp(1)w, u<0\}.$ 

# Example

source

MathOptInterface.PowerCone - Type.

```
PowerCone{T<: Real}(exponent::T)
```

The 3-dimensional power cone  $\{(x,y,z)\in\mathbb{R}^3:x^{exponent}y^{1-exponent}\geq |z|,x\geq 0,y\geq 0\}$  with parameter exponent.

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variables(model, 3);

julia> MOI.add_constraint(model, MOI.VectorOfVariables(x), MOI.PowerCone(0.5))
MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,

→ MathOptInterface.PowerCone{Float64}}(1)
```

MathOptInterface.DualPowerCone - Type.

```
DualPowerCone{T<:Real}(exponent::T)</pre>
```

The 3-dimensional power cone  $\{(u,v,w)\in\mathbb{R}^3: (\frac{u}{exponent})^{exponent}(\frac{v}{1-exponent})^{1-exponent}\geq |w|, u\geq 0, v\geq 0\}$  with parameter exponent.

#### **Example**

source

MathOptInterface.RelativeEntropyCone - Type.

```
RelativeEntropyCone(dimension::Int)
```

The relative entropy cone  $\{(u,v,w)\in\mathbb{R}^{1+2n}:u\geq\sum_{i=1}^nw_i\log(\frac{w_i}{v_i}),v_i\geq0,w_i\geq0\}$ , where dimension = 2n + 1 >= 1.

# **Duality note**

The dual of the relative entropy cone is  $\{(u,v,w)\in\mathbb{R}^{1+2n}: \forall i,w_i\geq u(\log(\frac{u}{v_i})-1),v_i\geq 0,u>0\}$  of dimension dimension =2n+1.

```
julia> model = MOI.Utilities.Model{Float64}();
julia> u = MOI.add_variable(model);
julia> v = MOI.add_variables(model, 3);
julia> w = MOI.add_variables(model, 3);
```

MathOptInterface.NormSpectralCone - Type.

```
NormSpectralCone(row_dim::Int, column_dim::Int)
```

The epigraph of the matrix spectral norm (maximum singular value function)  $\{(t,X)\in\mathbb{R}^{1+row_dim\times column_dim}:t\geq\sigma_1(X)\}$ , where  $\sigma_i$  is the ith singular value of the matrix X of non-negative row dimension row\_dim and column dimension column dim.

The matrix X is vectorized by stacking the columns, matching the behavior of Julia's vec function.

#### **Example**

source

MathOptInterface.NormNuclearCone - Type.

```
NormNuclearCone(row_dim::Int, column_dim::Int)
```

The epigraph of the matrix nuclear norm (sum of singular values function)  $\{(t,X)\in\mathbb{R}^{1+row_dim imes column_dim}:t\geq\sum_i\sigma_i(X)\}$ , where  $\sigma_i$  is the ith singular value of the matrix X of non-negative row dimension row\_dim and column dimension column\_dim.

The matrix X is vectorized by stacking the columns, matching the behavior of Julia's vec function.

# **Example**

source

MathOptInterface.SOS1 - Type.

```
SOS1{T<:Real}(weights::Vector{T})
```

The set corresponding to the Special Ordered Set (SOS) constraint of Type I.

Of the variables in the set, at most one can be nonzero.

The weights induce an ordering of the variables such that the kth element in the set corresponds to the kth weight in weights. Solvers may use these weights to improve the efficiency of the solution process, but the ordering does not change the set of feasible solutions.

#### **Example**

source

```
SOS2{T<:Real}(weights::Vector{T})
```

The set corresponding to the Special Ordered Set (SOS) constraint of Type II.

The weights induce an ordering of the variables such that the kth element in the set corresponds to the kth weight in weights. Therefore, the weights must be unique.

Of the variables in the set, at most two can be nonzero, and if two are nonzero, they must be adjacent in the ordering of the set.

#### **Example**

source

MathOptInterface.Indicator - Type.

```
Indicator{ACTIVATE_ON_ZERO}(set::AbstractScalarSet)
Indicator{ACTIVATE_ON_ONE}(set::AbstractScalarSet)
```

The set corresponding to an indicator constraint.

The type parameter must be an ActivationCondition.

When the type parameter is ACTIVATE\_ON\_ZERO, this means:

```
\{(y,x) \in \{0,1\} \times \mathbb{R} : y = 0 \implies x \in set\}
```

When the type parameter is ACTIVATE\_ON\_ONE, this means:

$$\{(y,x) \in \{0,1\} \times \mathbb{R} : y = 1 \implies x \in set\}$$

#### Notes

Most solvers expect that the first row of the function is interpretable as a VariableIndex (for example, 1.0 \* x + 0.0), and that the variable is constrained to the Zero0ne set. An error will be thrown if this is not the case.

```
The constraint \{(y,x)\in\{0,1\}\times\mathbb{R}^2:y=1\implies x_1+x_2\leq 9\} is defined as
```

The constraint  $\{(y,x)\in\{0,1\}\times\mathbb{R}:y=0\implies x=0\}$  is defined as

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model);

julia> y, _ = MOI.add_constrained_variable(model, MOI.ZeroOne());

julia> f = MOI.VectorOfVariables([y, x]);

julia> s = MOI.Indicator{MOI.ACTIVATE_ON_ZERO}(MOI.EqualTo(0.0))

MathOptInterface.Indicator{MathOptInterface.ACTIVATE_ON_ZERO,

→ MathOptInterface.EqualTo{Float64}}(MathOptInterface.EqualTo{Float64}(0.0))

julia> MOI.add_constraint(model, f, s)

MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,

→ MathOptInterface.Indicator{MathOptInterface.ACTIVATE_ON_ZERO,

→ MathOptInterface.Indicator{MathOptInterface.ACTIVATE_ON_ZERO,

→ MathOptInterface.EqualTo{Float64}}}(1)
```

source

MathOptInterface.ActivationCondition - Type.

```
ActivationCondition
```

Activation condition for an indicator constraint.

The enum value is used as first type parameter of Indicator{A,S}.

#### **Values**

```
ACTIVATE_ON_ZERO
```

The indicator constraint holds when the binary variable is zero.

ACTIVATE\_ON\_ONE

The indicator constraint holds when the binary variable is one.

source

MathOptInterface.ACTIVATE\_ON\_ZERO - Constant.

```
{\tt ACTIVATE\_ON\_ZER0::ActivationCondition}
```

An instance of the ActivationCondition enum.

#### **About**

The indicator constraint holds when the binary variable is zero.

source

MathOptInterface.ACTIVATE\_ON\_ONE - Constant.

```
ACTIVATE_ON_ONE::ActivationCondition
```

An instance of the ActivationCondition enum.

#### **About**

The indicator constraint holds when the binary variable is one.

source

MathOptInterface.Complements - Type.

```
Complements(dimension::Base.Integer)
```

The set corresponding to a mixed complementarity constraint.

Complementarity constraints should be specified with an AbstractVectorFunction-in-Complements (dimension) constraint.

The dimension of the vector-valued function F must be dimension. This defines a complementarity constraint between the scalar function F[i] and the variable in F[i + dimension/2]. Thus, F[i + dimension/2] must be interpretable as a single variable  $x_i$  (for example, 1.0 \*  $x_i$  + 0.0), and dimension must be even.

The mixed complementarity problem consists of finding  $x_i$  in the interval [lb, ub] (that is, in the set Interval(lb, ub)), such that the following holds:

```
1. F_i(x) == 0 \text{ if } lb_i < x_i < ub_i
```

2. 
$$F_i(x) >= 0 \text{ if } lb_i == x_i$$

3. 
$$F_i(x) \le 0 \text{ if } x_i = ub_i$$

Classically, the bounding set for  $x_i$  is Interval(0, Inf), which recovers:  $0 <= F_i(x) \perp x_i >= 0$ , where the  $\bot$  operator implies  $F_i(x) * x_i = 0$ .

#### **Example**

The problem:

```
x -in- Interval(-1, 1)
[-4 * x - 3, x] -in- Complements(2)
```

defines the mixed complementarity problem where the following holds:

```
1. -4 * x - 3 == 0 \text{ if } -1 < x < 1
2. -4 * x - 3 >= 0 \text{ if } x == -1
```

3. 
$$-4 * x - 3 \le 0 \text{ if } x == 1$$

There are three solutions:

```
1. x = -3/4 with F(x) = 0
```

2. 
$$x = -1$$
 with  $F(x) = 1$ 

3. 
$$x = 1$$
 with  $F(x) = -7$ 

The function F can also be defined in terms of single variables. For example, the problem:

```
[x_3, x_4] -in- Nonnegatives(2)
[x_1, x_2, x_3, x_4] -in- Complements(4)
```

defines the complementarity problem where 0 <=  $x_1 \perp x_3 >= 0$  and 0 <=  $x_2 \perp x_4 >= 0$ .

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variables(model, 4);

julia> MOI.add_constraint(model, MOI.VectorOfVariables(x[3:4]), MOI.Nonnegatives(2))

MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,

MathOptInterface.Nonnegatives}(1)

julia> MOI.add_constraint(
```

source

MathOptInterface.HyperRectangle - Type.

```
HyperRectangle(lower::Vector{T}, upper::Vector{T}) where {T}
```

The set  $\{x \in \mathbb{R}^d : x_i \in [lower_i, upper_i] \forall i = 1, \dots, d\}$ .

## **Example**

source

MathOptInterface.Scaled - Type.

```
struct Scaled{S<:AbstractVectorSet} <: AbstractVectorSet
    set::S
end</pre>
```

Given a vector  $a \in \mathbb{R}^d$  and a set representing the set  $\mathcal{S} \in \mathbb{R}^d$  such that Utilities.set\_dot for  $x \in \mathcal{S}$  and  $y \in \mathcal{S}^*$  is

$$\sum_{i=1}^{d} a_i x_i y_i$$

$$\{(\sqrt{a_1}x_1, \sqrt{a_2}x_2, \dots, \sqrt{a_d}x_d) : x \in S\}$$

## **Example**

This can be used to scale a vector of numbers

It can be also used to scale a vector of function

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variables(model, 3);
julia> func = MOI.VectorOfVariables(x)

[MOI.VariableIndex(1) |
MOI.VariableIndex(2) |
MOI.VariableIndex(3) |

julia> set = MOI.PositiveSemidefiniteConeTriangle(2)
MathOptInterface.PositiveSemidefiniteConeTriangle(2)

julia> MOI.Utilities.operate(*, Float64, Diagonal(a), func)

[0.0 + 1.0 MOI.VariableIndex(1) |
0.0 + 1.4142135623730951 MOI.VariableIndex(2) |
0.0 + 1.0 MOI.VariableIndex(3) |
]
```

source

## 18.7 Constraint programming sets

MathOptInterface.AllDifferent - Type.

```
AllDifferent(dimension::Int)
```

The set  $\{x \in \mathbb{Z}^d\}$  such that no two elements in x take the same value and dimension = d.

#### Also known as

This constraint is called all\_different in MiniZinc, and is sometimes also called distinct.

## Example

```
To enforce x[1] != x[2] AND x[1] != x[3] AND x[2] != x[3]:
```

source

 ${\tt MathOptInterface.BinPacking-Type.}$ 

```
BinPacking(c::T, w::Vector{T}) where {T}
```

The set  $\{x \in \mathbb{Z}^d\}$  where d = length(w), such that each item i in 1:d of weight w[i] is put into bin x[i], and the total weight of each bin does not exceed c.

There are additional assumptions that the capacity, c, and the weights, w, must all be non-negative.

The bin numbers depend on the bounds of x, so they may be something other than the integers 1:d.

#### Also known as

This constraint is called bin\_packing in MiniZinc.

```
julia> model = MOI.Utilities.Model{Float64}();

julia> bins = MOI.add_variables(model, 5)
5-element Vector{MathOptInterface.VariableIndex}:
MOI.VariableIndex(1)
MOI.VariableIndex(2)
MOI.VariableIndex(3)
MOI.VariableIndex(4)
MOI.VariableIndex(5)
julia> weights = Float64[1, 1, 2, 2, 3]
```

```
5-element Vector{Float64}:
1.0
1.0
2.0
 2.0
3.0
julia> MOI.add_constraint.(model, bins, MOI.Integer())
5-element\ \ Vector\{Math0ptInterface.ConstraintIndex\{Math0ptInterface.VariableIndex, Albertander(Math0ptInterface)\} \\
→ MathOptInterface.Integer}}:
MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.Integer}(1)
MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.Integer}(2)
{\tt MathOptInterface.ConstraintIndex\{MathOptInterface.VariableIndex,\ MathOptInterface.Integer\}(3)}
{\tt MathOptInterface.ConstraintIndex\{MathOptInterface.VariableIndex,\ MathOptInterface.Integer\}(4)}
MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.Integer}(5)
julia> MOI.add constraint.(model, bins, MOI.Interval(4.0, 6.0))
5-element Vector{MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,
→ MathOptInterface.Interval{Float64}}}:
MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,
→ MathOptInterface.Interval{Float64}}(1)
MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,
→ MathOptInterface.Interval{Float64}}(2)
MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,
→ MathOptInterface.Interval{Float64}}(3)
MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,
→ MathOptInterface.Interval{Float64}}(4)
MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,
→ MathOptInterface.Interval{Float64}}(5)
julia> MOI.add constraint(model, MOI.VectorOfVariables(bins), MOI.BinPacking(3.0, weights))
MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,
→ MathOptInterface.BinPacking{Float64}}(1)
```

source

MathOptInterface.Circuit - Type.

```
Circuit(dimension::Int)
```

The set  $\{x \in \{1..d\}^d\}$  that constraints x to be a circuit, such that  $x_i = j$  means that j is the successor of i, and dimension = d.

Graphs with multiple independent circuits, such as [2, 1, 3] and [2, 1, 4, 3], are not valid.

#### Also known as

This constraint is called circuit in MiniZinc, and it is equivalent to forming a (potentially sub-optimal) tour in the travelling salesperson problem.

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = [MOI.add_constrained_variable(model, MOI.Integer())[1] for _ in 1:3]
3-element Vector{MathOptInterface.VariableIndex}:
    MOI.VariableIndex(1)
    MOI.VariableIndex(2)
    MOI.VariableIndex(3)

julia> MOI.add_constraint(model, MOI.VectorOfVariables(x), MOI.Circuit(3))
MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,
    MathOptInterface.Circuit}(1)
```

source

MathOptInterface.CountAtLeast - Type.

```
CountAtLeast(n::Int, d::Vector{Int}, set::Set{Int})
```

The set  $\{x \in \mathbb{Z}^{d_1+d_2+\dots d_N}\}$ , where x is partitioned into N subsets ( $\{x_1,\dots,x_{d_1}\}$ ,  $\{x_{d_1+1},\dots,x_{d_1+d_2}\}$  and so on), and at least n elements of each subset take one of the values in set.

#### Also known as

This constraint is called at\_least in MiniZinc.

#### **Example**

To ensure that 3 appears at least once in each of the subsets {a, b} and {b, c}:

```
julia> model = MOI.Utilities.Model{Float64}();
 julia> a, _ = MOI.add_constrained_variable(model, MOI.Integer())
 (\texttt{MOI.VariableIndex}(1), \ \texttt{MathOptInterface.ConstraintIndex} \\ (\texttt{MathOptInterface.VariableIndex}, \\ (\texttt{Moi.VariableIndex}(1), \ \texttt{MathOptInterface.VariableIndex}) \\ (\texttt{Moi.VariableIndex}(1), \ \texttt{Moi.VariableIndex}) \\ (\texttt{Moi.VariableIndex}(1), \ \texttt
 \hookrightarrow MathOptInterface.Integer\}(1))
 julia> b, _ = MOI.add_constrained_variable(model, MOI.Integer())
 (\texttt{MOI.VariableIndex}(2), \ \texttt{MathOptInterface.ConstraintIndex}(\texttt{MathOptInterface.VariableIndex}, \\
 → MathOptInterface.Integer}(2))
 julia> c, _ = MOI.add_constrained_variable(model, MOI.Integer())
 (MOI.VariableIndex (3), \ MathOptInterface. ConstraintIndex \\ \{MathOptInterface. VariableIndex, and a substitution of the property of the pr
 → MathOptInterface.Integer}(3))
 julia> x, d, set = [a, b, b, c], [2, 2], [3]
 ({\tt MathOptInterface.VariableIndex[MOI.VariableIndex(1),\ MOI.VariableIndex(2),\ MOI.Vari
 → MOI.VariableIndex(2), MOI.VariableIndex(3)], [2, 2], [3])
 julia> MOI.add_constraint(model, MOI.VectorOfVariables(x), MOI.CountAtLeast(1, d, Set(set)))
 MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,
 → MathOptInterface.CountAtLeast}(1)
```

source

```
CountBelongs(dimenson::Int, set::Set{Int})
```

The set  $\{(n,x)\in\mathbb{Z}^{1+d}\}$ , such that n elements of the vector x take on of the values in set and dimension = 1 + d.

#### Also known as

This constraint is called among by MiniZinc.

#### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();
julia> n, _ = MOI.add_constrained_variable(model, MOI.Integer())
(\texttt{MOI.VariableIndex}(1), \ \texttt{MathOptInterface.ConstraintIndex}(\texttt{MathOptInterface.VariableIndex}, \texttt{MathOptInterface.VariableIndex})) \\
→ MathOptInterface.Integer}(1))
julia> x = [MOI.add_constrained_variable(model, MOI.Integer())[1] for _ in 1:3]
3-element Vector{MathOptInterface.VariableIndex}:
 MOI.VariableIndex(2)
 MOI.VariableIndex(3)
 MOI.VariableIndex(4)
julia> set = Set([3, 4, 5])
Set{Int64} with 3 elements:
  5
  4
  3
julia> MOI.add_constraint(model, MOI.VectorOfVariables([n; x]), MOI.CountBelongs(4, set))
MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,
→ MathOptInterface.CountBelongs}(1)
```

source

MathOptInterface.CountDistinct - Type.

```
CountDistinct(dimension::Int)
```

The set  $\{(n,x)\in\mathbb{Z}^{1+d}\}$ , such that the number of distinct values in x is n and dimension = 1 + d.

#### Also known as

This constraint is called nvalues in MiniZinc.

#### **Example**

To model:

```
• if n == 1, then x[1] == x[2] == x[3]
```

$$-x[1] == x[2] != x[3] or$$

```
- x[1] != x[2] == x[3] or

- x[1] == x[3] != x[2]

• if n == 3, then x[1] != x[2], x[2] != x[3] and x[3] != x[1]
```

#### **Relationship to AllDifferent**

When the first element is d, CountDistinct is equivalent to an AllDifferent constraint.

source

MathOptInterface.CountGreaterThan - Type.

```
CountGreaterThan(dimension::Int)
```

The set  $\{(c,y,x)\in\mathbb{Z}^{1+1+d}\}$ , such that c is strictly greater than the number of occurences of y in x and dimension = 1 + 1 + d.

#### Also known as

This constraint is called count gt in MiniZinc.

source

MathOptInterface.Cumulative - Type.

```
Cumulative(dimension::Int)
```

The set  $\{(s,d,r,b)\in\mathbb{Z}^{3n+1}\}$ , representing the cumulative global constraint, where n == length(s) == length(r) == length(b) and dimension = 3n + 1.

Cumulative requires that a set of tasks given by start times s, durations d, and resource requirements r, never requires more than the global resource bound b at any one time.

#### Also known as

This constraint is called cumulative in MiniZinc.

```
julia> model = MOI.Utilities.Model{Float64}();
julia> s = [MOI.add_constrained_variable(model, MOI.Integer())[1] for _ in 1:3]
3-element Vector{MathOptInterface.VariableIndex}:
 MOI.VariableIndex(1)
 MOI.VariableIndex(2)
 MOI.VariableIndex(3)
julia> d = [MOI.add_constrained_variable(model, MOI.Integer())[1] for _ in 1:3]
3-element Vector{MathOptInterface.VariableIndex}:
MOI.VariableIndex(4)
 MOI.VariableIndex(5)
MOI.VariableIndex(6)
julia> r = [MOI.add_constrained_variable(model, MOI.Integer())[1] for _ in 1:3]
3-element Vector{MathOptInterface.VariableIndex}:
 MOI.VariableIndex(7)
 MOI.VariableIndex(8)
 MOI.VariableIndex(9)
julia> b, _ = MOI.add_constrained_variable(model, MOI.Integer())
(\texttt{MOI.VariableIndex}(10), \ \texttt{MathOptInterface.ConstraintIndex}(\texttt{MathOptInterface.VariableIndex}, \texttt{MothOptInterface.VariableIndex}))
→ MathOptInterface.Integer}(10))
julia> MOI.add_constraint(model, MOI.VectorOfVariables([s; d; r; b]), MOI.Cumulative(10))
MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables,
→ MathOptInterface.Cumulative}(1)
```

MathOptInterface.Path - Type.

```
Path(from::Vector{Int}, to::Vector{Int})
```

Given a graph comprised of a set of nodes 1..N and a set of arcs 1..E represented by an edge from node from[i] to node to[i], Path constrains the set  $(s,t,ns,es) \in (1..N) \times (1..E) \times \{0,1\}^N \times \{0,1\}^E$ , to form subgraph that is a path from node s to node t, where node n is in the path if ns[n] is 1, and edge e is in the path if es[e] is 1.

The path must be acyclic, and it must traverse all nodes n for which ns[n] is 1, and all edges e for which es[e] is 1.

#### Also known as

This constraint is called path in MiniZinc.

```
julia> model = MOI.Utilities.Model{Float64}();
julia> N, E = 4, 5
 (4, 5)
 julia> from = [1, 1, 2, 2, 3]
 5-element Vector{Int64}:
     1
     2
     2
     3
 julia> to = [2, 3, 3, 4, 4]
 5-element Vector{Int64}:
    2
     3
     3
     4
     4
 julia> s, _ = MOI.add_constrained_variable(model, MOI.Integer())
 (\texttt{MOI.VariableIndex}(1), \ \texttt{MathOptInterface.ConstraintIndex} \\ (\texttt{MathOptInterface.VariableIndex}, \\ (\texttt{Moi.VariableIndex}(1), \ \texttt{MathOptInterface.VariableIndex}) \\ (\texttt{Moi.VariableIndex}(1), \ \texttt{Moi.VariableIndex}) \\ (\texttt{Moi.VariableIndex}(1), \ \texttt
 → MathOptInterface.Integer}(1))
julia> t, _ = MOI.add_constrained_variable(model, MOI.Integer())
 (\texttt{MOI.VariableIndex}(2), \ \texttt{MathOptInterface.ConstraintIndex}(\texttt{MathOptInterface.VariableIndex}, \\
 → MathOptInterface.Integer}(2))
 julia> ns = MOI.add_variables(model, N)
 4-element Vector{MathOptInterface.VariableIndex}:
    MOI.VariableIndex(3)
     MOI.VariableIndex(4)
     MOI.VariableIndex(5)
    MOI.VariableIndex(6)
 julia> MOI.add_constraint.(model, ns, MOI.ZeroOne())
```

```
4\text{-element Vector} \{ \texttt{MathOptInterface}. ConstraintIndex \\ \{ \texttt{MathOptInterface}. \texttt{VariableIndex}, \texttt{VariableIndex}
→ MathOptInterface.ZeroOne}}:
  MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.ZeroOne}(3)
  MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.ZeroOne}(4)
  MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.ZeroOne}(5)
  MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.ZeroOne}(6)
julia> es = MOI.add_variables(model, E)
5-element Vector{MathOptInterface.VariableIndex}:
  MOI.VariableIndex(7)
  MOI. VariableIndex(8)
  MOI.VariableIndex(9)
  MOI.VariableIndex(10)
  MOI.VariableIndex(11)
julia> MOI.add_constraint.(model, es, MOI.ZeroOne())
5-element Vector{MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,
→ MathOptInterface.ZeroOne}}:
 MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.ZeroOne}(7)
  MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.ZeroOne}(8)
  {\tt MathOptInterface.ConstraintIndex\{MathOptInterface.VariableIndex,\ MathOptInterface.ZeroOne\}(9)}
  MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.ZeroOne}(10)
  MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex, MathOptInterface.ZeroOne}(11)
julia> MOI.add constraint(model, MOI.VectorOfVariables([s; t; ns; es]), MOI.Path(from, to))
MathOptInterface.ConstraintIndex{MathOptInterface.VectorOfVariables, MathOptInterface.Path}(1)
```

source

MathOptInterface.Reified - Type.

```
Reified(set::AbstractSet)
```

The constraint  $[z; f(x)] \in Reified(S)$  ensures that  $f(x) \in S$  if and only if z == 1, where  $z \in \{0, 1\}$ .

MathOptInterface.Table - Type.

```
Table(table::Matrix{T}) where {T}
```

The set  $\{x \in \mathbb{R}^d\}$  where d = size(table, 2), such that x belongs to one row of table. That is, there exists some j in 1:size(table, 1), such that x[i] = table[j, i] for all i=1:size(table, 2).

#### Also known as

This constraint is called table in MiniZinc.

#### **Example**

source

## 18.8 Matrix sets

Matrix sets are vectorized to be subtypes of AbstractVectorSet.

For sets of symmetric matrices, storing both the (i, j) and (j, i) elements is redundant. Use the AbstractSymmetricMatrixSe set to represent only the vectorization of the upper triangular part of the matrix.

When the matrix of expressions constrained to be in the set is not symmetric, and hence additional constraints are needed to force the equality of the (i, j) and (j, i) elements, use the AbstractSymmetricMatrixSetSquare set.

The Bridges.Constraint.SquareBridge can transform a set from the square form to the triangular\_form by adding appropriate constraints if the (i, j) and (j, i) expressions are different.

 ${\tt MathOptInterface.AbstractSymmetricMatrixSetTriangle-Type.}$ 

```
abstract type AbstractSymmetricMatrixSetTriangle <: AbstractVectorSet end</pre>
```

Abstract supertype for subsets of the (vectorized) cone of symmetric matrices, with side\_dimension rows and columns. The entries of the upper-right triangular part of the matrix are given column by column (or equivalently, the entries of the lower-left triangular part are given row by row). A vectorized cone of dimension n corresponds to a square matrix with side dimension  $\sqrt{1/4+2n}-1/2$ . (Because a  $d\times d$  matrix has d(d+1)/2 elements in the upper or lower triangle.)

## **Example**

The matrix

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 3 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

has side dimension 3 and vectorization (1, 2, 3, 4, 5, 6).

#### Note

Two packed storage formats exist for symmetric matrices, the respective orders of the entries are:

- upper triangular column by column (or lower triangular row by row);
- lower triangular column by column (or upper triangular row by row).

The advantage of the first format is the mapping between the (i, j) matrix indices and the k index of the vectorized form. It is simpler and does not depend on the side dimension of the matrix. Indeed,

- the entry of matrix indices (i, j) has vectorized index k = div((j 1) \* j, 2) + i if  $i \le j$  and k = div((i 1) \* i, 2) + j if j < i;
- and the entry with vectorized index k has matrix indices i = div(1 + isqrt(8k 7), 2) and j = k div((i 1) \* i, 2) or j = div(1 + isqrt(8k 7), 2) and i = k div((j 1) \* j, 2).

## **Duality note**

The scalar product for the symmetric matrix in its vectorized form is the sum of the pairwise product of the diagonal entries plus twice the sum of the pairwise product of the upper diagonal entries; see [p. 634, 1]. This has important consequence for duality.

 $Consider for example the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and the following problem \cite{ConeTriangle} is a subtype of Abstract Symmetric Matrix and$ 

$$\max_{x \in \mathbb{R}} \qquad \qquad x$$
 s.t. 
$$(1,-x,1) \in \mathsf{PositiveSemidefiniteConeTriangle}(2).$$

The dual is the following problem

$$\min_{x\in\mathbb{R}^3} \qquad y_1+y_3$$
 s.t. 
$$2y_2=1$$
 
$$y\in \mathsf{PositiveSemidefiniteConeTriangle}(2).$$

Why do we use  $2y_2$  in the dual constraint instead of  $y_2$ ? The reason is that  $2y_2$  is the scalar product between y and the symmetric matrix whose vectorized form is (0,1,0). Indeed, with our modified scalar products we have

$$\langle (0,1,0), (y_1,y_2,y_3) \rangle = \operatorname{trace} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} y_1 & y_2 \\ y_2 & y_3 \end{pmatrix} = 2y_2.$$

#### References

[1] Boyd, S. and Vandenberghe, L.. Convex optimization. Cambridge university press, 2004.

source

MathOptInterface.AbstractSymmetricMatrixSetSquare - Type.

```
abstract type AbstractSymmetricMatrixSetSquare <: AbstractVectorSet end</pre>
```

Abstract supertype for subsets of the (vectorized) cone of symmetric matrices, with  ${\sf side\_dimension}$  rows and columns. The entries of the matrix are given column by column (or equivalently, row by row). The matrix is both constrained to be symmetric and to have its  ${\sf triangular\_form}$  belong to the corresponding set. That is, if the functions in entries (i,j) and (j,i) are different, then a constraint will be added to make sure that the entries are equal.

#### Example

 $Positive Semidefinite Cone Square \ is \ a \ subtype \ of \ Abstract Symmetric Matrix Set Square \ and \ constraining \ the \ matrix$ 

$$\begin{bmatrix} 1 & -y \\ -z & 0 \end{bmatrix}$$

to be symmetric positive semidefinite can be achieved by constraining the vector (1,-z,-y,0) (or (1,-y,-z,0)) to belong to the PositiveSemidefiniteConeSquare(2). It both constrains y=z and (1,-y,0) (or (1,-z,0)) to be in PositiveSemidefiniteConeTriangle(2), since triangular\_form(PositiveSemidefiniteConeSquare) is PositiveSemidefiniteConeTriangle.

source

MathOptInterface.side\_dimension - Function.

```
side_dimension(
    set::Union{
        AbstractSymmetricMatrixSetTriangle,
        AbstractSymmetricMatrixSetSquare,
        HermitianPositiveSemidefiniteConeTriangle,
    },
)
```

Side dimension of the matrices in set.

#### Convention

By convention, the side dimension should be stored in the side\_dimension field. If this is not the case for a subtype of AbstractSymmetricMatrixSetTriangle, or AbstractSymmetricMatrixSetSquare you must implement this method.

source

MathOptInterface.triangular\_form - Function.

```
triangular_form(S::Type{<:AbstractSymmetricMatrixSetSquare})
triangular_form(set::AbstractSymmetricMatrixSetSquare)</pre>
```

Return the AbstractSymmetricMatrixSetTriangle corresponding to the vectorization of the upper triangular part of matrices in the AbstractSymmetricMatrixSetSquare set.

source

List of recognized matrix sets.

MathOptInterface.PositiveSemidefiniteConeTriangle - Type.

```
PositiveSemidefiniteConeTriangle(side dimension:: Int) <: AbstractSymmetricMatrixSetTriangle
```

The (vectorized) cone of symmetric positive semidefinite matrices, with non-negative side\_dimension rows and columns.

See AbstractSymmetricMatrixSetTriangle for more details on the vectorized form.

source

 ${\tt MathOptInterface.PositiveSemidefiniteConeSquare-Type}.$ 

```
PositiveSemidefiniteConeSquare(side_dimension::Int) <: AbstractSymmetricMatrixSetSquare
```

The cone of symmetric positive semidefinite matrices, with non-negative side length side\_dimension.

See AbstractSymmetricMatrixSetSquare for more details on the vectorized form.

The entries of the matrix are given column by column (or equivalently, row by row).

The matrix is both constrained to be symmetric and to be positive semidefinite. That is, if the functions in entries (i,j) and (j,i) are different, then a constraint will be added to make sure that the entries are equal.

## **Example**

Constraining the matrix

$$\begin{bmatrix} 1 & -y \\ -z & 0 \end{bmatrix}$$

to be symmetric positive semidefinite can be achieved by constraining the vector (1, -z, -y, 0) (or (1, -y, -z, 0)) to belong to the PositiveSemidefiniteConeSquare(2).

It both constrains y=z and (1,-y,0) (or (1,-z,0)) to be in PositiveSemidefiniteConeTriangle(2).

source

 ${\tt MathOptInterface.HermitianPositiveSemidefiniteConeTriangle-Type.}$ 

```
HermitianPositiveSemidefiniteConeTriangle(side_dimension::Int) <: AbstractVectorSet
```

The (vectorized) cone of Hermitian positive semidefinite matrices, with non-negative side\_dimension rows and columns.

Becaue the matrix is Hermitian, the diagonal elements are real, and the complex-valued lower triangular entries are obtained as the conjugate of corresponding upper triangular entries.

#### **Vectorization format**

The vectorized form starts with real part of the entries of the upper triangular part of the matrix, given column by column as explained in AbstractSymmetricMatrixSetSquare.

It is then followed by the imaginary part of the off-diagonal entries of the upper triangular part, also given column by column.

For example, the matrix

$$\begin{bmatrix} 1 & 2+7im & 4+8im \\ 2-7im & 3 & 5+9im \\ 4-8im & 5-9im & 6 \end{bmatrix}$$

has side\_dimension 3 and is represented as the vector [1, 2, 3, 4, 5, 6, 7, 8, 9].

source

MathOptInterface.LogDetConeTriangle - Type.

```
LogDetConeTriangle(side_dimension::Int)
```

The log-determinant cone  $\{(t,u,X)\in\mathbb{R}^{2+d(d+1)/2}:t\leq u\log(\det(X/u)),u>0\}$ , where the matrix X is represented in the same symmetric packed format as in the PositiveSemidefiniteConeTriangle.

The non-negative argument side\_dimension is the side dimension of the matrix X, that is, its number of rows or columns.

source

 ${\tt MathOptInterface.LogDetConeSquare-Type}.$ 

```
LogDetConeSquare(side_dimension::Int)
```

The log-determinant cone  $\{(t,u,X)\in\mathbb{R}^{2+d^2}:t\leq u\log(\det(X/u)),X \text{ symmetric},u>0\}$ , where the matrix X is represented in the same format as in the PositiveSemidefiniteConeSquare.

Similarly to PositiveSemidefiniteConeSquare, constraints are added to ensure that X is symmetric.

The non-negative argument side\_dimension is the side dimension of the matrix X, that is, its number of rows or columns.

#### **Example**

source

MathOptInterface.RootDetConeTriangle - Type.

```
RootDetConeTriangle(side_dimension::Int)
```

The root-determinant cone  $\{(t,X)\in\mathbb{R}^{1+d(d+1)/2}:t\leq \det(X)^{1/d}\}$ , where the matrix X is represented in the same symmetric packed format as in the PositiveSemidefiniteConeTriangle.

The non-negative argument side\_dimension is the side dimension of the matrix X, that is, its number of rows or columns.

```
julia> model = MOI.Utilities.Model{Float64}();

julia> t = MOI.add_variable(model)

MOI.VariableIndex(1)
```

source

 ${\tt MathOptInterface.RootDetConeSquare-Type.}$ 

```
RootDetConeSquare(side_dimension::Int)
```

The root-determinant cone  $\{(t,X)\in\mathbb{R}^{1+d^2}:t\leq \det(X)^{1/d},X \text{ symmetric}\}$ , where the matrix X is represented in the same format as PositiveSemidefiniteConeSquare.

Similarly to PositiveSemidefiniteConeSquare, constraints are added to ensure that X is symmetric.

The non-negative argument side\_dimension is the side dimension of the matrix X, that is, its number of rows or columns.

## **Example**

source

# **Chapter 19**

# **Models**

## 19.1 Attribute interface

MathOptInterface.is\_set\_by\_optimize - Function.

```
is_set_by_optimize(::AnyAttribute)
```

Return a Bool indicating whether the value of the attribute is set during an optimize! call, that is, the attribute is used to guery the result of the optimization.

If an attribute can be set by the user, define is\_copyable instead.

An attribute cannot be both is\_copyable and is\_set\_by\_optimize.

#### **Default fallback**

This function returns false by default so it should be implemented for attributes that are set by optimize!.

# **Undefined behavior**

Querying the value of the attribute that is\_set\_by\_optimize before a call to optimize! is undefined and depends on solver-specific behavior.

#### **Example**

```
julia> MOI.is_set_by_optimize(MOI.ObjectiveValue())
true

julia> MOI.is_set_by_optimize(MOI.VariableName())
false
```

source

MathOptInterface.is\_copyable - Function.

```
is_copyable(::AnyAttribute)
```

Return a Bool indicating whether the value of the attribute may be copied during copy\_to using set.

If an attribute is\_copyable, then it cannot be modified by the optimizer, and get must always return the value that was set by the user.

If an attribute is the result of an optimization, define is\_set\_by\_optimize instead.

An attribute cannot be both is\_set\_by\_optimize and is\_copyable.

## **Default fallback**

By default is\_copyable(attr) returns !is\_set\_by\_optimize(attr), which is most probably true.

If an attribute should not be copied, define is\_copyable(::MyAttribute) = false.

source

MathOptInterface.get - Function.

```
MOI.get(b::AbstractBridge, ::MOI.NumberOfVariables)::Int64
```

Return the number of variables created by the bridge b in the model.

See also MOI.NumberOfConstraints.

#### Implementation notes

• There is a default fallback, so you need only implement this if the bridge adds new variables.

source

```
MOI.get(b::AbstractBridge, ::MOI.ListOfVariableIndices)
```

Return the list of variables created by the bridge b.

See also MOI.ListOfVariableIndices.

#### Implementation notes

• There is a default fallback, so you need only implement this if the bridge adds new variables.

source

```
\label{eq:MOI.get} MOI.get(b::AbstractBridge, ::MOI.NumberOfConstraints\{F,S\}):: \\ \underline{\textbf{Int64}} \ \ \text{where} \ \ \{F,S\}
```

Return the number of constraints of the type F-in-S created by the bridge b.

See also  ${\tt MOI.Number0fConstraints}.$ 

#### Implementation notes

 There is a default fallback, so you need only implement this for the constraint types returned by added constraint types.

source

```
\label{eq:MOI.get} MOI.get (b::AbstractBridge, ::MOI.ListOfConstraintIndices\{F,S\}) \ \ where \ \{F,S\}
```

Return a  $Vector{ConstraintIndex{F,S}}$  with indices of all constraints of type F-in-S created by the bride h

See also MOI.ListOfConstraintIndices.

#### Implementation notes

• There is a default fallback, so you need only implement this for the constraint types returned by added\_constraint\_types.

source

```
function MOI.get(
    model::MOI.ModelLike,
    attr::MOI.AbstractConstraintAttribute,
    bridge::AbstractBridge,
)
```

Return the value of the attribute attr of the model model for the constraint bridged by bridge.

source

```
get(optimizer::AbstractOptimizer, attr::AbstractOptimizerAttribute)
```

Return an attribute attr of the optimizer optimizer.

```
get(model::ModelLike, attr::AbstractModelAttribute)
```

Return an attribute attr of the model model.

```
get(model::ModelLike, attr::AbstractVariableAttribute, v::VariableIndex)
```

If the attribute attr is set for the variable v in the model model, return its value, return nothing otherwise. If the attribute attr is not supported by model then an error should be thrown instead of returning nothing.

```
\texttt{get}(\texttt{model}::\texttt{ModelLike}, \ \texttt{attr}::\texttt{AbstractVariableAttribute}, \ \texttt{v}::\textcolor{red}{\textbf{Vector}}\{\texttt{VariableIndex}\})
```

Return a vector of attributes corresponding to each variable in the collection v in the model model.

```
get(model::ModelLike, attr::AbstractConstraintAttribute, c::ConstraintIndex)
```

If the attribute attr is set for the constraint c in the model model, return its value, return nothing otherwise. If the attribute attr is not supported by model then an error should be thrown instead of returning nothing.

```
get(
   model::ModelLike,
   attr::AbstractConstraintAttribute,
   c::Vector{ConstraintIndex{F,S}},
) where {F,S}
```

Return a vector of attributes corresponding to each constraint in the collection c in the model model.

```
get(model::ModelLike, ::Type{VariableIndex}, name::String)
```

If a variable with name name exists in the model model, return the corresponding index, otherwise return nothing. Errors if two variables have the same name.

```
get(
    model::ModelLike,
    ::Type{ConstraintIndex{F,S}},
    name::String,
) where {F,S}
```

If an F-in-S constraint with name name exists in the model model, return the corresponding index, otherwise return nothing. Errors if two constraints have the same name.

```
get(model::ModelLike, ::Type{ConstraintIndex}, name::String)
```

If any constraint with name name exists in the model model, return the corresponding index, otherwise return nothing. This version is available for convenience but may incur a performance penalty because it is not type stable. Errors if two constraints have the same name.

source

 ${\tt MathOptInterface.get!-Function}.$ 

```
get!(output, model::ModelLike, args...)
```

An in-place version of get.

The signature matches that of get except that the result is placed in the vector output.

source

MathOptInterface.set - Function.

```
function MOI.set(
    model::MOI.ModelLike,
    attr::MOI.AbstractConstraintAttribute,
    bridge::AbstractBridge,
    value,
)
```

Set the value of the attribute attr of the model model for the constraint bridged by bridge.

source

```
set(optimizer::AbstractOptimizer, attr::AbstractOptimizerAttribute, value)
```

Assign value to the attribute attr of the optimizer optimizer.

```
set(model::ModelLike, attr::AbstractModelAttribute, value)
```

Assign value to the attribute attr of the model model.

```
set(model::ModelLike, attr::AbstractVariableAttribute, v::VariableIndex, value)
```

Assign value to the attribute attr of variable v in model model.

```
set(
    model::ModelLike,
    attr::AbstractVariableAttribute,
    v::Vector{VariableIndex},
    vector_of_values,
)
```

Assign a value respectively to the attribute attr of each variable in the collection v in model model.

```
set(
    model::ModelLike,
    attr::AbstractConstraintAttribute,
    c::ConstraintIndex,
    value,
)
```

Assign a value to the attribute attr of constraint c in model model.

```
set(
    model::ModelLike,
    attr::AbstractConstraintAttribute,
    c::Vector{ConstraintIndex{F,S}},
    vector_of_values,
) where {F,S}
```

Assign a value respectively to the attribute attr of each constraint in the collection c in model model.

An UnsupportedAttribute error is thrown if model does not support the attribute attr (see supports) and a SetAttributeNotAllowed error is thrown if it supports the attribute attr but it cannot be set.

```
set(
   model::ModelLike,
   ::ConstraintSet,
   c::ConstraintIndex{F,S},
   set::S,
) where {F,S}
```

Change the set of constraint c to the new set set which should be of the same type as the original set.

```
set(
   model::ModelLike,
   ::ConstraintFunction,
   c::ConstraintIndex{F,S},
   func::F,
) where {F,S}
```

Replace the function in constraint c with func. F must match the original function type used to define the constraint.

#### Note

Setting the constraint function is not allowed if F is VariableIndex; a SettingVariableIndexNotAllowed error is thrown instead. This is because, it would require changing the index c since the index of VariableIndex constraints must be the same as the index of the variable.

source

MathOptInterface.supports - Function.

```
MOI.supports(
    model::MOI.ModelLike,
    attr::MOI.AbstractConstraintAttribute,
    BT::Type{<:AbstractBridge},
)</pre>
```

Return a Bool indicating whether BT supports setting attr to model.

source

```
supports(model::ModelLike, sub::AbstractSubmittable)::Bool
```

Return a Bool indicating whether model supports the submittable sub.

```
supports(model::ModelLike, attr::AbstractOptimizerAttribute)::Bool
```

Return a Bool indicating whether model supports the optimizer attribute attr. That is, it returns false if copy\_to(model, src) shows a warning in case attr is in the ListOfOptimizerAttributesSet of src; see copy\_to for more details on how unsupported optimizer attributes are handled in copy.

```
supports(model::ModelLike, attr::AbstractModelAttribute)::Bool
```

Return a Bool indicating whether model supports the model attribute attr. That is, it returns false if copy to(model, src) cannot be performed in case attr is in the ListOfModelAttributesSet of src.

```
supports(
   model::ModelLike,
   attr::AbstractVariableAttribute,
   ::Type{VariableIndex},
)::Bool
```

Return a Bool indicating whether model supports the variable attribute attr. That is, it returns false if copy\_to(model, src) cannot be performed in case attr is in the ListOfVariableAttributesSet of src.

```
supports(
   model::ModelLike,
   attr::AbstractConstraintAttribute,
   ::Type{ConstraintIndex{F,S}},
)::Bool where {F,S}
```

Return a Bool indicating whether model supports the constraint attribute attr applied to an F-in-S constraint. That is, it returns false if copy\_to(model, src) cannot be performed in case attr is in the ListOfConstraintAttributesSet of src.

For all five methods, if the attribute is only not supported in specific circumstances, it should still return true.

Note that supports is only defined for attributes for which is\_copyable returns true as other attributes do not appear in the list of attributes set obtained by ListOfXXXAttributesSet.

source

MathOptInterface.attribute\_value\_type - Function.

```
attribute_value_type(attr::AnyAttribute)
```

Given an attribute attr, return the type of value expected by get, or returned by set.

#### Notes

• Only implement this if it make sense to do so. If un-implemented, the default is Any.

source

## 19.2 Model interface

MathOptInterface.ModelLike - Type.

```
ModelLike
```

Abstract supertype for objects that implement the "Model" interface for defining an optimization problem.

source

MathOptInterface.is\_empty - Function.

```
is_empty(model::ModelLike)
```

Returns false if the model has any model attribute set or has any variables or constraints.

Note that an empty model can have optimizer attributes set.

source

MathOptInterface.empty! - Function.

```
empty!(model::ModelLike)
```

Empty the model, that is, remove all variables, constraints and model attributes but not optimizer attributes.

source

MathOptInterface.write\_to\_file - Function.

```
write_to_file(model::ModelLike, filename::String; kwargs...)
```

Write the current model to the file at filename.

Supported file types depend on the model type.

Additional keyword arguments are passed to the Model constructor of the relevant file format. See, for example, FileFormats.LP.Model and FileFormats.MPS.Model.

```
julia> model = MOI.Utilities.Model{Int}();
julia> x, _ = MOI.add_constrained_variable(model, MOI.Interval(2, 3));
julia> MOI.set(model, MOI.VariableName(), x, "x");
julia> filename = joinpath(tempdir(), "model.lp");
julia> MOI.write_to_file(model, filename; coefficient_type = Int);
julia> print(read(filename, String))
minimize
obj:
```

```
subject to
Bounds
2 <= x <= 3
End</pre>
```

source

MathOptInterface.read\_from\_file - Function.

```
read_from_file(model::ModelLike, filename::String; kwargs...)
```

Read the file filename into the model model. If model is non-empty, this may throw an error.

Supported file types depend on the model type.

Additional keyword arguments are passed to the Model constructor of the relevant file format. See, for example, FileFormats.LP.Model and FileFormats.MPS.Model.

#### Note

Once the contents of the file are loaded into the model, you can query the variables via MOI.get(model, MOI.ListOfVariableIndices()). However, some filetypes, such as LP files, do not maintain an explicit ordering of the variables. Therefore, the returned list may be in an arbitrary order.

To avoid depending on the order of the indices, look up each variable index by name using MOI.get(model, MOI.VariableIndex, "name").

```
julia> model = MOI.Utilities.Model{Int}();
julia> x, _ = MOI.add_constrained_variable(model, MOI.GreaterThan(2));
julia> MOI.set(model, MOI.VariableName(), x, "x");
julia> filename = joinpath(tempdir(), "model.lp");
julia> MOI.write_to_file(model, filename; coefficient_type = Int);
julia> new_model = MOI.Utilities.Model{Int}();
julia> mOI.read_from_file(new_model, filename; coefficient_type = Int);
julia> print(new_model)
Minimize ScalarAffineFunction{Int64}:
    (0)
Subject to:

VariableIndex-in-GreaterThan{Int64}
    x >= (2)
julia> MOI.get(new_model, MOI.VariableIndex, "x")
MOI.VariableIndex(1)
```

source

MathOptInterface.supports incremental interface - Function.

```
supports_incremental_interface(model::ModelLike)
```

Return a Bool indicating whether model supports building incrementally via add variable and add constraint.

The main purpose of this function is to determine whether a model can be loaded into model incrementally or whether it should be cached and copied at once instead.

source

MathOptInterface.copy\_to - Function.

```
copy_to(dest::ModelLike, src::ModelLike)::IndexMap
```

Copy the model from src into dest.

The target dest is emptied, and all previous indices to variables and constraints in dest are invalidated.

Returns an IndexMap object that translates variable and constraint indices from the src model to the corresponding indices in the dest model.

#### Notes

- If a constraint that in src is not supported by dest, then an UnsupportedConstraint error is thrown.
- If an AbstractModelAttribute, AbstractVariableAttribute, or AbstractConstraintAttribute is set in src but not supported by dest, then an UnsupportedAttribute error is thrown.

AbstractOptimizerAttributes are not copied to the dest model.

## IndexMap

Implementations of copy\_to must return an IndexMap. For technical reasons, this type is defined in the Utilities submodule as MOI.Utilities.IndexMap. However, since it is an integral part of the MOI API, we provide MOI.IndexMap as an alias.

## Example

```
# Given empty `ModelLike` objects `src` and `dest`.

x = add_variable(src)

is_valid(src, x)  # true
is_valid(dest, x)  # false (`dest` has no variables)

index_map = copy_to(dest, src)
is_valid(dest, x)  # false (unless index_map[x] == x)
is_valid(dest, index_map[x])  # true
```

source

MathOptInterface.IndexMap - Type.

```
IndexMap()
```

The dictionary-like object returned by copy\_to.

#### IndexMap

Implementations of copy\_to must return an IndexMap. For technical reasons, the IndexMap type is defined in the Utilities submodule as MOI.Utilities.IndexMap. However, since it is an integral part of the MOI API, we provide this MOI.IndexMap as an alias.

source

#### 19.3 Model attributes

MathOptInterface.AbstractModelAttribute - Type.

```
AbstractModelAttribute
```

Abstract supertype for attribute objects that can be used to set or get attributes (properties) of the model.

source

MathOptInterface.Name - Type.

```
Name()
```

An AbstractModelAttribute for the string identifying the model.

It has a default value of "" if not set.

# Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.Name)::String
MOI.set(::Optimizer, ::MOI.Name, ::String)::Nothing
MOI.supports(::Optimizer, ::MOI.Name)::Bool
```

source

MathOptInterface.ObjectiveFunction - Type.

```
ObjectiveFunction{F<:AbstractScalarFunction}()</pre>
```

An AbstractModelAttribute for the objective function which has a type F<: AbstractScalarFunction.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(
     ::Optimizer,
     ::MOI.ObjectiveFunction{F},
)::F where {F<:MOI.AbstractFunction}
MOI.set(
     ::Optimizer,
     ::MOI.ObjectiveFunction{F},
     ::F,
)::F where {F<:MOI.AbstractFunction}
MOI.supports(::Optimizer, ::MOI.ObjectiveFunction{<:MOI.AbstractFunction})::Bool</pre>
```

When implementing get, F may to be equivalent but not necessarily identical to the function type set by the user. If the objective function cannot be converted to F, an InexactError must be thrown.

source

MathOptInterface.ObjectiveFunctionType - Type.

```
ObjectiveFunctionType()
```

An AbstractModelAttribute for the type F of the objective function set using the 0bjectiveFunction $\{F\}$  attribute.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ObjectiveFunctionType,
)::Type{<:MOI.AbstractFunction}</pre>
```

They should not implement set or supports.

#### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model)

MOI.VariableIndex(1)

julia> MOI.set(model, MOI.ObjectiveFunction{MOI.VariableIndex}(), x)

julia> MOI.get(model, MOI.ObjectiveFunctionType())

MathOptInterface.VariableIndex
```

source

MathOptInterface.ObjectiveSense - Type.

```
ObjectiveSense()
```

An AbstractModelAttribute for the objective sense of the objective function, which must be an OptimizationSense.

The default is FEASIBILITY\_SENSE.

## Interaction with ObjectiveFunction

Setting the sense to FEASIBILITY\_SENSE unsets the ObjectiveFunction attribute. That is, if you first set ObjectiveFunction and then set ObjectiveSense to be FEASIBILITY\_SENSE, no objective function will be passed to the solver.

In addition, some reformulations of ObjectiveFunction via bridges rely on the value of ObjectiveSense. Therefore, you should set ObjectiveSense before setting ObjectiveFunction.

#### Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.ObjectiveSense)::MOI.OptimizationSense
MOI.set(::Optimizer, ::MOI.ObjectiveSense, ::MOI.OptimizationSense)::Nothing
MOI.supports(::Optimizer, ::MOI.ObjectiveSense)::Bool
```

source

MathOptInterface.OptimizationSense - Type.

```
OptimizationSense
```

An Enum for the value of the ObjectiveSense attribute.

## **Values**

MIN\_SENSE

The goal is to minimize the objective function.

**MAX SENSE** 

The goal is to maximize the objective function.

FEASIBILITY\_SENSE

The model does not have an objective function.

source

 ${\tt MathOptInterface.MIN\_SENSE-Constant}.$ 

```
MIN_SENSE::OptimizationSense
```

An instance of the OptimizationSense enum.

#### **About**

The goal is to minimize the objective function.

source

MathOptInterface.MAX\_SENSE - Constant.

```
MAX_SENSE::OptimizationSense
```

An instance of the OptimizationSense enum.

#### **About**

The goal is to maximize the objective function.

source

MathOptInterface.FEASIBILITY\_SENSE - Constant.

```
FEASIBILITY_SENSE::OptimizationSense
```

An instance of the OptimizationSense enum.

## **About**

The model does not have an objective function.

source

MathOptInterface.NumberOfVariables - Type.

```
NumberOfVariables()
```

An AbstractModelAttribute for the number of variables in the model.

## Implementation

Optimizers should implement the following methods:

```
\label{eq:MOI.get} \texttt{MOI.get(::Optimizer, ::MOI.NumberOfVariables)::} \\ \underline{\textbf{Int64}}
```

They should not implement set or supports.

source

MathOptInterface.ListOfVariableIndices - Type.

```
ListOfVariableIndices()
```

An AbstractModelAttribute for querying the Vector{MOI.VariableIndex} of all [MOI.VariableIndex] present in the model.

#### Order

The variables must be returned in the order in which they were added to the model.

#### Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.ListOfVariableIndices)::Vector{MOI.VariableIndex}
```

They should not implement set or supports.

source

MathOptInterface.ListOfConstraintTypesPresent - Type.

```
ListOfConstraintTypesPresent()
```

An AbstractModelAttribute for the list of tuples of the form (F, S), indicating that the attribute NumberOfConstraints{F, S} has a value greater than zero.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ListOfConstraintTypesPresent,
)::Vector{Tuple{Type, Type}}
```

They should not implement set or supports.

source

MathOptInterface.NumberOfConstraints - Type.

```
NumberOfConstraints{F,S}()
```

An AbstractModelAttribute for querying the number of constraints of the type F-in-S present in the model.

#### Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.NumberOfConstraints{F,S},
)::Int64 where {F<:MOI.AbstractFunction,MOI.AbstractSet}</pre>
```

They should not implement set or supports.

source

MathOptInterface.ListOfConstraintIndices - Type.

```
ListOfConstraintIndices{F,S}()
```

An AbstractModelAttribute for the Vector  $\{MOI.ConstraintIndex\{F,S\}\}\$  of all constraint indices of type F-in-S in the model.

#### Order

The constraints must be returned in the order in which they were added to the model.

#### Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ListOfConstraintIndices{F,S},
)::Vector{MOI.ConstraintIndex{F,S}} where {F<:MOI.AbstractFunction,MOI.AbstractSet}</pre>
```

They should not implement set or supports.

source

MathOptInterface.ListOfOptimizerAttributesSet - Type.

```
ListOfOptimizerAttributesSet()
```

 $An \ Abstract 0 ptimizer Attribute for the \ Vector \{Abstract 0 ptimizer Attribute\} \ of all \ optimizer \ attributes that \ were \ set.$ 

## Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ListOfOptimizerAttributesSet,
)::Vector{MOI.AbstractOptimizerAttribute}
```

They should not implement set or supports.

source

MathOptInterface.ListOfModelAttributesSet - Type.

```
ListOfModelAttributesSet()
```

 $An \ Abstract \texttt{ModelAttribute} \ for the \ Vector \{Abstract \texttt{ModelAttribute}\} \ of \ all \ model \ attributes \ attr such that:$ 

- 1. is\_copyable(attr) returns true, and
- 2. the attribute was set to the model

source

MathOptInterface.ListOfVariableAttributesSet - Type.

```
ListOfVariableAttributesSet()
```

An AbstractModelAttribute for the Vector{AbstractVariableAttribute} of all variable attributes attr such that:

- is\_copyable(attr) returns true
- 2. the attribute was set for at least one variable in the model

## Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ListOfVariableAttributesSet,
)::Vector{MOI.AbstractVariableAttribute}
```

They should not implement set or supports.

source

MathOptInterface.ListOfVariablesWithAttributeSet - Type.

```
ListOfVariablesWithAttributeSet(attr::AbstractVariableAttribute)
```

An AbstractModelAttribute for the Vector{MOI.VariableIndex} of all variables with the attribute attr set.

The returned list may not be minimal, so some elements may have their default value set.

#### Note

This is an optional attribute to implement. The default fallback is to get ListOfVariableIndices.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ListOfVariablesWithAttributeSet{<:MOI.AbstractVariableAttribute},
)::Vector{MOI.VarialbeIndex}</pre>
```

They should not implement set or supports.

source

MathOptInterface.ListOfConstraintAttributesSet - Type.

```
ListOfConstraintAttributesSet{F, S}()
```

An AbstractModelAttribute for the Vector{AbstractConstraintAttribute} of all constraint attributes attr such that:

- 1. is\_copyable(attr) returns true and
- 2. the attribute was set to F-in-S constraints.

#### Note

The attributes ConstraintFunction and ConstraintSet should not be included in the list even if then have been set with set.

source

MathOptInterface.ListOfConstraintsWithAttributeSet - Type.

```
ListOfConstraintsWithAttributeSet{F,S}(attr:AbstractConstraintAttribute)
```

An AbstractModelAttribute for the  $Vector\{ConstraintIndex\{F,S\}\}$  of all constraints with the attribute attr set.

The returned list may not be minimal, so some elements may have their default value set.

#### Note

This is an optional attribute to implement. The default fallback is to get ListOfConstraintIndices.

source

MathOptInterface.UserDefinedFunction - Type.

```
UserDefinedFunction(name::Symbol, arity::Int) <: AbstractModelAttribute</pre>
```

Set this attribute to register a user-defined function by the name of name with arity arguments.

Once registered, name will appear in ListOfSupportedNonlinearOperators.

You cannot register multiple UserDefinedFunctions with the same name but different arity.

# Value type

The value to be set is a tuple containing one, two, or three functions to evaluate the function, the first-order derivative, and the second-order derivative respectively. Both derivatives are optional, but if you pass the second-order derivative you must also pass the first-order derivative.

For univariate functions with arity == 1, the functions in the tuple must have the form:

- f(x::T)::T: returns the value of the function at x
- $\nabla f(x::T)::T$ : returns the first-order derivative of f with respect to x
- $\nabla^2 f(x::T)::T$ : returns the second-order derivative of f with respect to x.

For multivariate functions with arity > 1, the functions in the tuple must have the form:

• f(x::T...)::T: returns the value of the function at x

 Vf(g::AbstractVector{T}, x::T...)::Nothing: fills the components of g, with g[i] being the first-order partial derivative of f with respect to x[i]

∇²f(H::AbstractMatrix{T}, x::T...)::Nothing: fills the non-zero components of H, with H[i, j] being the second-order partial derivative of f with respect to x[i] and then x[j]. H is initialized to the zero matrix, so you do not need to set any zero elements.

## **Example**

```
julia> f(x, y) = x^2 + y^2
f (generic function with 1 method)
julia> function \nabla f(g, x, y)
           g = 2 * x, 2 * y
∇f (generic function with 1 method)
julia> function \nabla^2 f(H, x...)
           H[1, 1] = H[2, 2] = 2.0
           return
\nabla^2 f (generic function with 1 method)
julia> model = MOI.Utilities.UniversalFallback(MOI.Utilities.Model{Float64}());
julia> MOI.set(model, MOI.UserDefinedFunction(:f, 2), (f,))
julia> MOI.set(model, MOI.UserDefinedFunction(:g, 2), (f, ∇f))
julia> MOI.set(model, MOI.UserDefinedFunction(:h, 2), (f, \nablaf, \nabla<sup>2</sup>f))
julia> x = MOI.add_variables(model, 2)
2-element Vector{MathOptInterface.VariableIndex}:
MOI.VariableIndex(1)
MOI.VariableIndex(2)
julia> MOI.set(model, MOI.ObjectiveSense(), MOI.MIN_SENSE)
julia> obj_f = MOI.ScalarNonlinearFunction(:f, Any[x[1], x[2]])
f(MOI.VariableIndex(1), MOI.VariableIndex(2))
julia> MOI.set(model, MOI.ObjectiveFunction{typeof(obj_f)}(), obj_f)
julia> print(model)
Minimize ScalarNonlinearFunction:
f(v[1], v[2])
Subject to:
```

source

MathOptInterface.ListOfSupportedNonlinearOperators - Type.

```
ListOfSupportedNonlinearOperators() <: AbstractModelAttribute
```

When queried with get, return a Vector{Symbol} listing the operators supported by the model. These operators may appear in the head field of ScalarNonlinearFunction.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ListOfSupportedNonlinearOperators,
)::Vector{Symbol}
```

They should not implement set or supports.

source

## 19.4 Optimizer interface

MathOptInterface.AbstractOptimizer - Type.

```
AbstractOptimizer <: ModelLike
```

Abstract supertype for objects representing an instance of an optimization problem tied to a particular solver. This is typically a solver's in-memory representation. In addition to ModelLike, AbstractOptimizer objects let you solve the model and query the solution.

source

MathOptInterface.OptimizerWithAttributes - Type.

```
struct OptimizerWithAttributes
   optimizer_constructor
   params::Vector{Pair{AbstractOptimizerAttribute,<:Any}}
end</pre>
```

Object grouping an optimizer constructor and a list of optimizer attributes. Instances are created with instantiate.

source

MathOptInterface.optimize! - Function.

```
optimize!(optimizer::AbstractOptimizer)
```

Optimize the problem contained in optimizer.

Before calling optimize!, the problem should first be constructed using the incremental interface (see supports\_incremental\_interface) or copy\_to.

Source

MathOptInterface.optimize! - Method.

```
optimize!(dest::AbstractOptimizer, src::ModelLike)::Tuple{IndexMap,Bool}
```

A "one-shot" call that copies the problem from src into dest and then uses dest to optimize the problem. Returns a tuple of an IndexMap and a Bool copied.

- The IndexMap object translates variable and constraint indices from the src model to the corresponding indices in the dest optimizer. See <a href="mailto:copy\_to">copy\_to</a> for details.
- If copied == true, src was copied to dest and then cached, allowing incremental modification if supported by the solver.
- If copied == false, a cache of the model was not kept in dest. Therefore, only the solution information (attributes for which is\_set\_by\_optimize is true) is available to query.

```
Note

The main purpose of optimize! method with two arguments is for use in Utilities.CachingOptimizer.
```

## Relationship to the single-argument optimize!

The default fallback of optimize! (dest::AbstractOptimizer, src::ModelLike) is

```
function optimize!(dest::AbstractOptimizer, src::ModelLike)
  index_map = copy_to(dest, src)
  optimize!(dest)
  return index_map, true
end
```

Therefore, subtypes of AbstractOptimizer should either implement this two-argument method, or implement both copy\_to(::Optimizer, ::ModelLike) and optimize!(::Optimizer).

source

MathOptInterface.instantiate - Function.

```
instantiate(
   optimizer_constructor,
   with_cache_type::Union{Nothing, Type} = nothing,
   with_bridge_type::Union{Nothing, Type} = nothing,
)
```

Create an instance of an optimizer by either:

• calling optimizer\_constructor.optimizer\_constructor() and setting the parameters in optimizer\_constructor.p if optimizer\_constructor is a OptimizerWithAttributes

• calling optimizer\_constructor() if optimizer\_constructor is callable.

## withcachetype

If with\_cache\_type is not nothing, then the optimizer is wrapped in a Utilities.CachingOptimizer to store a cache of the model. This is most useful if the optimizer you are constructing does not support the incremental interface (see supports\_incremental\_interface).

## withbridgetype

If with\_bridge\_type is not nothing, the optimizer is wrapped in a Bridges.full\_bridge\_optimizer, enabling all the bridges defined in the MOI.Bridges submodule with coefficient type with\_bridge\_type.

In addition, if the optimizer created by optimizer\_constructor does not support the incremental interface (see supports\_incremental\_interface), then, irrespective of with\_cache\_type, the optimizer is wrapped in a Utilities.CachingOptimizer to store a cache of the bridged model.

If with\_cache\_type and with\_bridge\_type are both not nothing, then they must be the same type.

source

MathOptInterface.default\_cache - Function.

```
default_cache(optimizer::ModelLike, ::Type{T}) where {T}
```

Return a new instance of the default model type to be used as cache for optimizer in a Utilities. CachingOptimizer for holding constraints of coefficient type T. By default, this returns Utilities. UniversalFallback(Utilities.Model{T}()) If copying from a instance of a given model type is faster for optimizer then a new method returning an instance of this model type should be defined.

source

## 19.5 Optimizer attributes

MathOptInterface.AbstractOptimizerAttribute - Type.

```
AbstractOptimizerAttribute
```

Abstract supertype for attribute objects that can be used to set or get attributes (properties) of the optimizer.

## Notes

The difference between AbstractOptimizerAttribute and AbstractModelAttribute lies in the behavior of is\_empty, empty! and copy\_to. Typically optimizer attributes affect only how the model is solved.

source

MathOptInterface.SolverName - Type.

```
SolverName()
```

An AbstractOptimizerAttribute for the string identifying the solver.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.SolverName)::String
```

They should not implement set or supports.

source

MathOptInterface.SolverVersion - Type.

```
SolverVersion()
```

An AbstractOptimizerAttribute for the string identifying the version of the solver.

## Versioning systems

For solvers supporting semantic versioning, the SolverVersion should be a string of the form "vMAJOR.MINOR.PATCH", so that it can be converted to a Julia VersionNumber (for example, 'VersionNumber("v1.2.3")).

We do not require Semantic Versioning because some solvers use alternate versioning systems. For example, CPLEX uses Calendar Versioning, so SolverVersion will return a string like "202001".

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.SolverVersion)::String
```

They should not implement set or supports.

source

MathOptInterface.Silent - Type.

```
Silent()
```

An AbstractOptimizerAttribute for silencing the output of an optimizer.

When set to true, this attribute takes precedence over any other attribute controlling verbosity and requires the optimizer to produce no output.

The default value is false which has no effect. In this case the verbosity is controlled by other optimizer-specific attributes.

## Value and default

The provided value must be a Bool.

The default value is false.

#### Note

Every optimizer should have verbosity on by default. For instance, if a solver has a solver-specific log level attribute, the MOI implementation should set it to 1 by default. If the user sets Silent to true, then the log level should be set to  $\theta$ , even if the user specifically sets a value of log level. If the value of Silent is false then the log level set to the solver is the value given by the user for this solver-specific parameter or 1 if none is given.

#### Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.Silent)::Bool
MOI.set(::Optimizer, ::MOI.Silent, ::Bool)::Nothing
MOI.supports(::Optimizer, ::MOI.Silent)::Bool
```

source

MathOptInterface.TimeLimitSec - Type.

```
TimeLimitSec()
```

An AbstractOptimizerAttribute for setting a time limit (in seconds) for a call to optimize!.

## Value and default

The provided limit must be a Union{Nothing, Real}.

When set to nothing, it deactivates the time limit.

The default value is nothing.

## **TerminationStatus**

The optimizer may stop when the SolveTimeSec is larger than the TimeLimitSec. If stopped because of this limit, the TerminationStatus must be TIME\_LIMIT.

Note that most optimizers do not strictly respect a time limit. Instead, they terminate at the first convenient time after the time limit has been exceeded. Thus, you may find that the SolveTimeSec exceeds the TimeLimitSec by a few seconds.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.TimeLimitSec)::Union{Nothing,Float64}
MOI.set(::Optimizer, ::MOI.TimeLimitSec, ::Union{Nothing,Real})::Nothing
MOI.supports(::Optimizer, ::MOI.TimeLimitSec)::Bool
```

source

MathOptInterface.ObjectiveLimit - Type.

```
ObjectiveLimit()
```

An AbstractOptimizerAttribute for setting a limit on the objective value.

#### Value and default

The provided limit must be a Union{Nothing, Real}.

When set to nothing, the limit reverts to the solver's default.

The default value is nothing.

#### **TerminationStatus**

The solver may stop when the ObjectiveValue is better (lower for minimization, higher for maximization) than the ObjectiveLimit. If stopped, because of this limit, the TerminationStatus should be OBJECTIVE LIMIT.

#### Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.ObjectiveLimit)::Union{Nothing,Float64}
MOI.set(::Optimizer, ::MOI.ObjectiveLimit, ::Union{Nothing,Real})::Nothing
MOI.supports(::Optimizer, ::MOI.ObjectiveLimit)::Bool
```

source

MathOptInterface.SolutionLimit - Type.

```
SolutionLimit()
```

An AbstractOptimizerAttribute for setting a limit on the number of available feasible solutions.

### Value and default

The provided limit must be a Union{Nothing, Int}.

When set to nothing, the limit reverts to the solver's default.

The default value is nothing.

## **Termination criteria**

The solver may stop when the ResultCount is larger than or equal to the SolutionLimit. If stopped because of this attribute, the TerminationStatus must be SOLUTION LIMIT.

## **Solution quality**

The quality of the available solutions is solver-dependent. The set of resulting solutions is not guaranteed to contain an optimal solution.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.SolutionLimit)::Union{Nothing,Int}
MOI.set(::Optimizer, ::MOI.SolutionLimit, ::Union{Nothing,Int})::Nothing
MOI.supports(::Optimizer, ::MOI.SolutionLimit)::Bool
```

source

MathOptInterface.NodeLimit - Type.

```
NodeLimit()
```

An AbstractOptimizerAttribute for setting a limit on the number of branch-and-bound nodes explored by a mixed-integer program (MIP) solver.

#### Value and default

The provided limit must be a Union{Nothing, Int}.

When set to nothing, the limit reverts to the solver's default.

The default value is nothing.

## **Termination criteria**

The solver may stop when the NodeCount is larger than or equal to the NodeLimit. If stopped because of this attribute, the TerminationStatus must be NODE LIMIT.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.NodeLimit)::Union{Nothing,Int}
MOI.set(::Optimizer, ::MOI.NodeLimit, ::Union{Nothing,Int})::Nothing
MOI.supports(::Optimizer, ::MOI.NodeLimit)::Bool
```

source

MathOptInterface.RawOptimizerAttribute - Type.

```
RawOptimizerAttribute(name::String)
```

An AbstractOptimizerAttribute for the solver-specific parameter identified by name.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.RawOptimizerAttribute)::Any
MOI.set(::Optimizer, ::MOI.RawOptimizerAttribute, ::Any)::Nothing
MOI.supports(::Optimizer, ::MOI.RawOptimizerAttribute)::Bool
```

source

MathOptInterface.NumberOfThreads - Type.

```
NumberOfThreads()
```

An AbstractOptimizerAttribute for setting the number of threads used for an optimization.

#### Value and default

The provided value must be nothing or a positive Int.

When set to nothing, the value reverts to the solver's default.

The default value is nothing.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.NumberOfThreads)::Union{Nothing,Int}
MOI.set(::Optimizer, ::MOI.NumberOfThreads, ::Union{Nothing,Int})::Nothing
MOI.supports(::Optimizer, ::MOI.NumberOfThreads)::Bool
```

source

MathOptInterface.RawSolver - Type.

```
RawSolver()
```

An AbstractModelAttribute for the object that may be used to access a solver-specific API for this optimizer.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.RawSolver)::Any
```

They should not implement set or supports.

source

 ${\tt MathOptInterface.AbsoluteGapTolerance-Type.}$ 

```
AbsoluteGapTolerance()
```

An AbstractOptimizerAttribute for setting the absolute gap tolerance for an optimization.

## Definition

The mathematical definition of "absolute gap" and its allowed range are solver-dependent. However, most solvers that implement this attribute will stop once  $|f-b|g_{abs}$ , where b is the best bound, f is the best feasible objective value, and  $g_{abs}$  is the absolute gap.

## Value and default

The provided value must be a Union{Nothing,Float64}.

When set to nothing, the limit reverts to the solver's default.

#### **TerminationStatus**

The optimizer may stop when the absolute difference between <code>ObjectiveValue</code> and <code>ObjectiveBound</code> is smaller than the <code>AbsoluteGapTolerance</code>. If stopped because of this limit, the <code>TerminationStatus</code> may be <code>OPTIMAL</code>.

#### Implementation

Optimizers should implement the following methods:

source

MathOptInterface.RelativeGapTolerance - Type.

```
RelativeGapTolerance()
```

An AbstractOptimizerAttribute for setting the relative gap tolerance for an optimization.

#### **Definition**

The mathematical definition of "relative gap" and its allowed range are solver-dependent. Typically, solvers expect a value between 0.0 and 1.0.

## Value and default

The provided value must be a Union{Nothing, Float64}.

When set to nothing, the limit reverts to the solver's default.

## **TerminationStatus**

The optimizer may stop when the RelativeGap is smaller than the RelativeGapTolerance. If stopped because of this limit, the TerminationStatus may be OPTIMAL.

## Implementation

Optimizers should implement the following methods:

source

 ${\tt MathOptInterface.AutomaticDifferentiationBackend-Type.}\\$ 

```
AutomaticDifferentiationBackend() <: AbstractOptimizerAttribute
```

An AbstractOptimizerAttribute for setting the automatic differentiation backend used by the solver.

The value must be a subtype of Nonlinear. AbstractAutomaticDifferentiation.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(
     ::Optimizer,
     ::MOI.AutomaticDifferentiationBackend,
)::MOI.Nonlinear.AbstractAutomaticDifferentiation
MOI.set(
     ::Optimizer,
     ::MOI.AutomaticDifferentiationBackend,
     ::MOI.Nonlinear.AbstractAutomaticDifferentiation,
)::Nothing
MOI.supports(::Optimizer, ::MOI.AutomaticDifferentiationBackend)::Bool
```

source

List of attributes useful for optimizers

 ${\tt MathOptInterface.TerminationStatus-Type}.$ 

```
TerminationStatus()
```

 $\label{thm:condition} An \ Abstract \texttt{ModelAttribute} \ for \ the \ \texttt{TerminationStatusCode} \ explaining \ why \ the \ optimizer \ stopped.$ 

## Implementation

Optimizers should implement the following methods:

```
\verb"MOI.get" (:: Optimizer, :: MOI.TerminationStatus) :: MOI.TerminationStatus Code" (:: Optimizer, :: MOI.TerminationStatus) :: MOI.TerminationStatus (:: Optimizer, :: Optim
```

They should not implement set or supports.

source

MathOptInterface.TerminationStatusCode - Type.

```
TerminationStatusCode
```

An Enum of possible values for the TerminationStatus attribute.

This attribute explains why the optimizer stopped executing in the most recent call to optimize!.

#### **Values**

## **OPTIMIZE NOT CALLED**

The algorithm has not started.

#### **OPTIMAL**

The algorithm found a globally optimal solution.

#### **INFEASIBLE**

The algorithm proved that no primal feasible solution exists.

## **DUAL INFEASIBLE**

The algorithm proved that no dual feasible solution exists.

To check if the primal problem is feasible, set the objective sense to FEASIBILITY\_SENSE and re-solve the problem.

If a primal feasible point does not exist, the original problem is both primal and dual infeasible.

If a primal feasible solution exists, this status typically implies that the problem is unbounded, with some technical exceptions (for example, if the problem is a conic optimization problem in which strong duality does not hold).

The technical exceptions do not apply to linear programs. The combination of DUAL\_INFEASIBLE and a primal feasible point means that the primal linear program is unbounded.

## LOCALLY\_SOLVED

The algorithm converged to a stationary point, local optimal solution, could not find directions for improvement, or otherwise completed its search without global guarantees.

## **LOCALLY INFEASIBLE**

The algorithm converged to an infeasible point or otherwise completed its search without finding a feasible solution, without guarantees that no feasible solution exists.

If you know a primal feasible solution exists, use VariablePrimalStart to provide a feasible starting point to the solver.

## INFEASIBLE\_OR\_UNBOUNDED

The algorithm stopped because it proved that the problem is infeasible or unbounded, without distinguishing between the two cases.

To distinguish between the two cases, set the objective sense to FEASIBILITY\_SENSE and re-solve the problem. If a primal feasible point exists, the original problem is unbounded. If a primal feasible point does not exist, the original problem is infeasible.

## ALMOST\_OPTIMAL

The algorithm found a globally optimal solution to relaxed tolerances.

## ALMOST\_INFEASIBLE

The algorithm concluded that no feasible solution exists within relaxed tolerances.

## ALMOST\_DUAL\_INFEASIBLE

The algorithm concluded that no dual bound exists for the problem within relaxed tolerances.

## ALMOST\_LOCALLY\_SOLVED

The algorithm converged to a stationary point, local optimal solution, or could not find directions for improvement within relaxed tolerances.

### **ITERATION LIMIT**

An iterative algorithm stopped after conducting the maximum number of iterations.

## TIME\_LIMIT

The algorithm stopped after a user-specified computation time.

This status may be returned in relation to the TimeLimitSec attribute, or some other solver-specific attribute.

## NODE LIMIT

A branch-and-bound algorithm stopped because it explored a maximum number of nodes in the branch-and-bound tree.

This status may be returned in relation to the NodeLimit attribute, or some other solver-specific attribute.

## **SOLUTION LIMIT**

The algorithm stopped because it found the required number of solutions. This is often used in MIPs to get the solver to return the first feasible solution it encounters.

This status may be returned in relation to the SolutionLimit attribute, or some other solver-specific attribute.

## MEMORY\_LIMIT

The algorithm stopped because it ran out of memory.

## **OBJECTIVE LIMIT**

The algorithm stopped because it found a solution better than a minimum limit set by the user.

This status may be returned in relation to the <code>ObjectiveLimit</code> attribute, or some other solver-specific attribute.

## NORM LIMIT

The algorithm stopped because the norm of an iterate became too large.

This typically means that the primal problem is unbounded, but that the solver could not prove so.

#### OTHER LIMIT

The algorithm stopped due to a limit not covered by one of the \_LIMIT\_ statuses above.

## SLOW\_PROGRESS

The algorithm stopped because it was unable to continue making progress towards the solution.

## **NUMERICAL ERROR**

The algorithm stopped because it encountered unrecoverable numerical error.

## INVALID\_MODEL

The algorithm stopped because the model is invalid.

The reason for this return code is solver-specific, but common causes are that the problem has zero variables or constraints, or that the problem data contains an invalid number such as NaN.

## INVALID\_OPTION

The algorithm stopped because it was provided an invalid option.

#### **INTERRUPTED**

The algorithm stopped because of an interrupt signal.

This typically means that the solver was interrupted by the user with CTRL+C.

## OTHER ERROR

The algorithm stopped because of an error not covered by one of the statuses defined above. Check the solver log for further details.

source

MathOptInterface.OPTIMIZE\_NOT\_CALLED - Constant.

```
OPTIMIZE_NOT_CALLED::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

## **About**

The algorithm has not started.

source

MathOptInterface.OPTIMAL - Constant.

```
OPTIMAL::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

## **About**

The algorithm found a globally optimal solution.

source

MathOptInterface.INFEASIBLE - Constant.

```
INFEASIBLE::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

## **About**

The algorithm proved that no primal feasible solution exists.

source

MathOptInterface.DUAL\_INFEASIBLE - Constant.

```
{\tt DUAL\_INFEASIBLE::} TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

#### **About**

The algorithm proved that no dual feasible solution exists.

To check if the primal problem is feasible, set the objective sense to FEASIBILITY\_SENSE and re-solve the problem.

If a primal feasible point does not exist, the original problem is both primal and dual infeasible.

If a primal feasible solution exists, this status typically implies that the problem is unbounded, with some technical exceptions (for example, if the problem is a conic optimization problem in which strong duality does not hold).

The technical exceptions do not apply to linear programs. The combination of DUAL\_INFEASIBLE and a primal feasible point means that the primal linear program is unbounded.

source

MathOptInterface.LOCALLY\_SOLVED - Constant.

```
LOCALLY_SOLVED::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

## **About**

The algorithm converged to a stationary point, local optimal solution, could not find directions for improvement, or otherwise completed its search without global guarantees.

source

MathOptInterface.LOCALLY\_INFEASIBLE - Constant.

```
\verb|LOCALLY_INFEASIBLE:: TerminationStatusCode|\\
```

An instance of the TerminationStatusCode enum.

## **About**

The algorithm converged to an infeasible point or otherwise completed its search without finding a feasible solution, without guarantees that no feasible solution exists.

If you know a primal feasible solution exists, use VariablePrimalStart to provide a feasible starting point to the solver.

source

 ${\tt MathOptInterface.INFEASIBLE\_OR\_UNBOUNDED-Constant}.$ 

```
{\tt INFEASIBLE\_OR\_UNBOUNDED::TerminationStatusCode}
```

An instance of the TerminationStatusCode enum.

#### About

The algorithm stopped because it proved that the problem is infeasible or unbounded, without distinguishing between the two cases.

To distinguish between the two cases, set the objective sense to FEASIBILITY\_SENSE and re-solve the problem. If a primal feasible point exists, the original problem is unbounded. If a primal feasible point does not exist, the original problem is infeasible.

source

 ${\tt MathOptInterface.ALMOST\_OPTIMAL-Constant}.$ 

```
{\tt ALMOST\_OPTIMAL::TerminationStatusCode}
```

An instance of the TerminationStatusCode enum.

## **About**

The algorithm found a globally optimal solution to relaxed tolerances.

source

 ${\tt MathOptInterface.ALMOST\_INFEASIBLE-Constant}.$ 

```
ALMOST_INFEASIBLE::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

#### **About**

The algorithm concluded that no feasible solution exists within relaxed tolerances.

source

MathOptInterface.ALMOST\_DUAL\_INFEASIBLE - Constant.

```
ALMOST_DUAL_INFEASIBLE::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

## About

The algorithm concluded that no dual bound exists for the problem within relaxed tolerances.

source

MathOptInterface.ALMOST\_LOCALLY\_SOLVED - Constant.

```
{\tt ALMOST\_LOCALLY\_SOLVED::TerminationStatusCode}
```

An instance of the TerminationStatusCode enum.

#### **About**

The algorithm converged to a stationary point, local optimal solution, or could not find directions for improvement within relaxed tolerances.

source

 ${\tt MathOptInterface.ITERATION\_LIMIT-Constant}.$ 

```
ITERATION_LIMIT::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

## **About**

An iterative algorithm stopped after conducting the maximum number of iterations.

source

MathOptInterface.TIME\_LIMIT - Constant.

```
TIME_LIMIT::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

## **About**

The algorithm stopped after a user-specified computation time.

This status may be returned in relation to the TimeLimitSec attribute, or some other solver-specific attribute.

source

MathOptInterface.NODE LIMIT - Constant.

```
NODE_LIMIT::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

## **About**

A branch-and-bound algorithm stopped because it explored a maximum number of nodes in the branch-and-bound tree.

This status may be returned in relation to the NodeLimit attribute, or some other solver-specific attribute.

source

MathOptInterface.SOLUTION\_LIMIT - Constant.

```
SOLUTION_LIMIT::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

#### About

The algorithm stopped because it found the required number of solutions. This is often used in MIPs to get the solver to return the first feasible solution it encounters.

This status may be returned in relation to the SolutionLimit attribute, or some other solver-specific attribute.

source

MathOptInterface.MEMORY LIMIT - Constant.

```
{\tt MEMORY\_LIMIT::TerminationStatusCode}
```

An instance of the TerminationStatusCode enum.

#### **About**

The algorithm stopped because it ran out of memory.

source

MathOptInterface.OBJECTIVE\_LIMIT - Constant.

```
OBJECTIVE_LIMIT::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

## **About**

The algorithm stopped because it found a solution better than a minimum limit set by the user.

This status may be returned in relation to the <code>ObjectiveLimit</code> attribute, or some other solver-specific attribute.

source

MathOptInterface.NORM\_LIMIT - Constant.

```
{\tt NORM\_LIMIT::TerminationStatusCode}
```

An instance of the TerminationStatusCode enum.

## **About**

The algorithm stopped because the norm of an iterate became too large.

This typically means that the primal problem is unbounded, but that the solver could not prove so.

source

MathOptInterface.OTHER\_LIMIT - Constant.

```
{\tt OTHER\_LIMIT::TerminationStatusCode}
```

An instance of the TerminationStatusCode enum.

#### **About**

The algorithm stopped due to a limit not covered by one of the \_LIMIT\_ statuses above.

source

MathOptInterface.SLOW PROGRESS - Constant.

```
SLOW PROGRESS::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

#### **About**

The algorithm stopped because it was unable to continue making progress towards the solution.

source

 ${\tt MathOptInterface.NUMERICAL\_ERROR-Constant}.$ 

```
NUMERICAL_ERROR::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

## About

The algorithm stopped because it encountered unrecoverable numerical error.

source

MathOptInterface.INVALID\_MODEL - Constant.

```
{\tt INVALID\_MODEL::TerminationStatusCode}
```

An instance of the TerminationStatusCode enum.

## About

The algorithm stopped because the model is invalid.

The reason for this return code is solver-specific, but common causes are that the problem has zero variables or constraints, or that the problem data contains an invalid number such as NaN.

source

MathOptInterface.INVALID\_OPTION - Constant.

```
{\tt INVALID\_OPTION::TerminationStatusCode}
```

An instance of the TerminationStatusCode enum.

#### **About**

The algorithm stopped because it was provided an invalid option.

source

MathOptInterface.INTERRUPTED - Constant.

```
INTERRUPTED::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

#### **About**

The algorithm stopped because of an interrupt signal.

This typically means that the solver was interrupted by the user with CTRL+C.

source

MathOptInterface.OTHER\_ERROR - Constant.

```
OTHER_ERROR::TerminationStatusCode
```

An instance of the TerminationStatusCode enum.

## **About**

The algorithm stopped because of an error not covered by one of the statuses defined above. Check the solver log for further details.

source

MathOptInterface.PrimalStatus - Type.

```
PrimalStatus(result_index::Int = 1)
```

## result\_index

If result\_index is omitted, it defaults to 1.

See ResultCount for information on how the results are ordered.

If result index is larger than the value of ResultCount then NO SOLUTION is returned.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.PrimalStatus)::MOI.ResultStatusCode
```

They should not implement set or supports.

source

MathOptInterface.DualStatus - Type.

```
DualStatus(result_index::Int = 1)
```

An AbstractModelAttribute for the ResultStatusCode of the dual result result\_index.

## result\_index

See ResultCount for information on how the results are ordered.

If result\_index is larger than the value of ResultCount then NO\_SOLUTION is returned.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.PrimalStatus)::MOI.ResultStatusCode
```

They should not implement set or supports.

source

MathOptInterface.RawStatusString - Type.

```
RawStatusString()
```

An AbstractModelAttribute for a solver specific string explaining why the optimizer stopped.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.RawStatusString)::MOI.String
```

They should not implement set or supports.

source

MathOptInterface.ResultCount - Type.

```
ResultCount()
```

An AbstractModelAttribute for the number of results available.

### Order of solutions

A number of attributes contain an index, result\_index, which is used to refer to one of the available results. Thus, result\_index must be an integer between 1 and the number of available results.

As a general rule, the first result (result\_index = 1) is the most important result (for example, an optimal solution or an infeasibility certificate). Other results will typically be alternate solutions that the solver found during the search for the first result.

If a (local) optimal solution is available, that is, TerminationStatus is OPTIMAL or LOCALLY\_SOLVED, the first result must correspond to the (locally) optimal solution. Other results may be alternative optimal solutions, or they may be other suboptimal solutions; use ObjectiveValue to distinguish between them.

If a primal or dual infeasibility certificate is available, that is, TerminationStatus is INFEASIBLE or DUAL\_INFEASIBLE and the corresponding PrimalStatus or DualStatus is INFEASIBILITY\_CERTIFICATE, then the first result must be a certificate. Other results may be alternate certificates, or infeasible points.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.ResultCount)::Int
```

They should not implement set or supports.

source

MathOptInterface.ObjectiveValue - Type.

```
ObjectiveValue(result_index::Int = 1)
```

An AbstractModelAttribute for the objective value of the primal solution result\_index.

## **PrimalStatus**

Before quering this attribute you should first check PrimalStatus to confirm that a primal solution is available.

If the PrimalStatus is NO SOLUTION the result of querying this attribute is undefined.

## result\_index

The optimizer may return multiple primal solutions. See ResultCount for information on how the results are ordered.

If the solver does not have a primal value for the objective because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ObjectiveValue,
)::Union{T, Vector{T}} where {T<:Real}</pre>
```

They should not implement set or supports.

MathOptInterface.DualObjectiveValue - Type.

```
DualObjectiveValue(result_index::Int = 1)
```

An AbstractModelAttribute for the value of the objective function of the dual solution result\_index.

#### **DualStatus**

Before quering this attribute you should first check DualStatus to confirm that a dual solution is avaiable.

If the DualStatus is NO\_SOLUTION the result of querying this attribute is undefined.

## result index

The optimizer may return multiple dual solutions. See ResultCount for information on how the results are ordered.

If the solver does not have a dual value for the objective because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.DualObjectiveValue,
)::Union{T, Vector{T}} where {T<:Real}</pre>
```

They should not implement set or supports.

source

MathOptInterface.ObjectiveBound - Type.

```
ObjectiveBound()
```

An AbstractModelAttribute for the best known bound on the optimal objective value.

source

MathOptInterface.RelativeGap - Type.

```
RelativeGap()
```

An AbstractModelAttribute for the final relative optimality gap.

#### Warning

The definition of this gap is solver-dependent. However, most solvers implementing this attribute define the relative gap as some variation of  $\frac{|b-f|}{|f|}$ , where b is the best bound and f is the best feasible objective value.

MathOptInterface.SolveTimeSec - Type.

```
SolveTimeSec()
```

An AbstractModelAttribute for the total elapsed solution time (in seconds) as reported by the optimizer. source

MathOptInterface.SimplexIterations - Type.

```
SimplexIterations()
```

An AbstractModelAttribute for the cumulative number of simplex iterations while solving a problem.

For a mixed-integer program (MIP), the return value is the total simplex iterations for all nodes.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.SimplexIterations)::Int64
```

They should not implement set or supports.

source

 ${\tt MathOptInterface.BarrierIterations-Type.}$ 

```
BarrierIterations()
```

An AbstractModelAttribute for the cumulative number of barrier iterations while solving a problem.

For a mixed-integer program (MIP), the return value is the total barrier iterations for all nodes.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.BarrierIterations)::Int64
```

They should not implement set or supports.

source

MathOptInterface.NodeCount - Type.

```
NodeCount()
```

An AbstractModelAttribute for the total number of branch-and-bound nodes explored while solving a mixed-integer program (MIP).

#### Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.NodeCount)::Int64
```

They should not implement set or supports.

source

#### ResultStatusCode

MathOptInterface.ResultStatusCode - Type.

```
ResultStatusCode
```

An Enum of possible values for the PrimalStatus and DualStatus attributes.

The values indicate how to interpret the result vector.

#### **Values**

## NO\_SOLUTION

The result vector is empty.

## FEASIBLE\_POINT

The result vector is a feasible point.

```
NEARLY FEASIBLE POINT
```

The result vector is feasible if some constraint tolerances are relaxed.

## **INFEASIBLE POINT**

The result vector is an infeasible point.

## INFEASIBILITY\_CERTIFICATE

The result vector is an infeasibility certificate.

If the PrimalStatus is INFEASIBILITY\_CERTIFICATE, then the primal result vector is a certificate of dual infeasibility.

If the DualStatus is INFEASIBILITY\_CERTIFICATE, then the dual result vector is a proof of primal infeasibility.

## NEARLY\_INFEASIBILITY\_CERTIFICATE

The result satisfies a relaxed criterion for a certificate of infeasibility.

If the PrimalStatus is NEARLY\_INFEASIBILITY\_CERTIFICATE, then the primal result vector is a certificate of dual infeasibility.

If the DualStatus is NEARLY\_INFEASIBILITY\_CERTIFICATE, then the dual result vector is a proof of primal infeasibility.

## REDUCTION\_CERTIFICATE

The result vector is an ill-posed certificate; see this article for details.

If the PrimalStatus is REDUCTION\_CERTIFICATE, then the primal result vector is a proof that the dual problem is ill-posed.

If the DualStatus is REDUCTION\_CERTIFICATE, then the dual result vector is a proof that the primal is ill-posed.

## NEARLY\_REDUCTION\_CERTIFICATE

The result satisfies a relaxed criterion for an ill-posed certificate.

```
UNKNOWN RESULT STATUS
```

The result vector contains a solution with an unknown interpretation. Check the solver log for more details.

```
OTHER RESULT STATUS
```

The result vector contains a solution with an interpretation not covered by one of the statuses defined above. Check the solver log for more details.

source

MathOptInterface.NO SOLUTION - Constant.

```
NO SOLUTION::ResultStatusCode
```

An instance of the ResultStatusCode enum.

## About

The result vector is empty.

source

MathOptInterface.FEASIBLE\_POINT - Constant.

```
FEASIBLE_POINT::ResultStatusCode
```

An instance of the ResultStatusCode enum.

## About

The result vector is a feasible point.

source

MathOptInterface.NEARLY\_FEASIBLE\_POINT - Constant.

```
{\tt NEARLY\_FEASIBLE\_POINT::ResultStatusCode}
```

An instance of the ResultStatusCode enum.

## **About**

The result vector is feasible if some constraint tolerances are relaxed.

source

 ${\tt MathOptInterface.INFEASIBLE\_POINT-Constant}.$ 

```
INFEASIBLE_POINT::ResultStatusCode
```

An instance of the ResultStatusCode enum.

#### About

The result vector is an infeasible point.

source

MathOptInterface.INFEASIBILITY CERTIFICATE - Constant.

```
INFEASIBILITY CERTIFICATE::ResultStatusCode
```

An instance of the ResultStatusCode enum.

#### **About**

The result vector is an infeasibility certificate.

If the PrimalStatus is INFEASIBILITY\_CERTIFICATE, then the primal result vector is a certificate of dual infeasibility.

If the DualStatus is INFEASIBILITY\_CERTIFICATE, then the dual result vector is a proof of primal infeasibility.

source

 ${\tt MathOptInterface.NEARLY\_INFEASIBILITY\_CERTIFICATE-Constant}.$ 

```
NEARLY_INFEASIBILITY_CERTIFICATE::ResultStatusCode
```

An instance of the ResultStatusCode enum.

#### **About**

The result satisfies a relaxed criterion for a certificate of infeasibility.

If the PrimalStatus is NEARLY\_INFEASIBILITY\_CERTIFICATE, then the primal result vector is a certificate of dual infeasibility.

If the DualStatus is NEARLY\_INFEASIBILITY\_CERTIFICATE, then the dual result vector is a proof of primal infeasibility.

source

MathOptInterface.REDUCTION\_CERTIFICATE - Constant.

```
REDUCTION_CERTIFICATE::ResultStatusCode
```

An instance of the ResultStatusCode enum.

#### **About**

The result vector is an ill-posed certificate; see this article for details.

If the PrimalStatus is REDUCTION\_CERTIFICATE, then the primal result vector is a proof that the dual problem is ill-posed.

If the DualStatus is REDUCTION\_CERTIFICATE, then the dual result vector is a proof that the primal is ill-posed.

source

MathOptInterface.NEARLY\_REDUCTION\_CERTIFICATE - Constant.

```
NEARLY_REDUCTION_CERTIFICATE::ResultStatusCode
```

An instance of the ResultStatusCode enum.

#### **About**

The result satisfies a relaxed criterion for an ill-posed certificate.

source

MathOptInterface.UNKNOWN\_RESULT\_STATUS - Constant.

```
UNKNOWN_RESULT_STATUS::ResultStatusCode
```

An instance of the ResultStatusCode enum.

## About

The result vector contains a solution with an unknown interpretation. Check the solver log for more details.

source

MathOptInterface.OTHER\_RESULT\_STATUS - Constant.

```
OTHER_RESULT_STATUS::ResultStatusCode
```

An instance of the ResultStatusCode enum.

#### **About**

The result vector contains a solution with an interpretation not covered by one of the statuses defined above. Check the solver log for more details.

source

## **Conflict Status**

MathOptInterface.compute\_conflict! - Function.

```
compute_conflict!(optimizer::AbstractOptimizer)
```

Computes a minimal subset of constraints such that the model with the other constraint removed is still infeasible.

Some solvers call a set of conflicting constraints an Irreducible Inconsistent Subsystem (IIS).

See also ConflictStatus and ConstraintConflictStatus.

#### Note

If the model is modified after a call to compute\_conflict!, the implementor is not obliged to purge the conflict. Any calls to the above attributes may return values for the original conflict without a warning. Similarly, when modifying the model, the conflict can be discarded.

source

 ${\tt MathOptInterface.ConflictStatus-Type.}$ 

```
ConflictStatus()
```

An AbstractModelAttribute for the ConflictStatusCode explaining why [compute\_conflict!] stopped when computing the conflict.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.ConflictStatus)::MOI.ConflictStatusCode
```

They should not implement set or supports.

source

MathOptInterface.ConflictStatusCode - Type.

```
ConflictStatusCode
```

An Enum of possible values for the ConflictStatus attribute.

This attribute is meant to explain the reason why the conflict finder stopped executing in the most recent call to compute\_conflict!.

## **Values**

```
COMPUTE_CONFLICT_NOT_CALLED
```

The function compute\_conflict! has not yet been called.

```
NO_CONFLICT_EXISTS
```

There is no conflict because the problem is feasible.

```
NO CONFLICT FOUND
```

The solver could not find a conflict.

```
CONFLICT_FOUND
```

The solver found a conflict.

source

 ${\tt MathOptInterface.COMPUTE\_CONFLICT\_NOT\_CALLED-Constant}.$ 

```
COMPUTE_CONFLICT_NOT_CALLED::ConflictStatusCode
```

An instance of the ConflictStatusCode enum.

#### About

The function compute\_conflict! has not yet been called.

source

MathOptInterface.NO\_CONFLICT\_EXISTS - Constant.

```
NO_CONFLICT_EXISTS::ConflictStatusCode
```

An instance of the ConflictStatusCode enum.

## About

There is no conflict because the problem is feasible.

source

MathOptInterface.NO\_CONFLICT\_FOUND - Constant.

```
NO_CONFLICT_FOUND::ConflictStatusCode
```

An instance of the ConflictStatusCode enum.

## About

The solver could not find a conflict.

source

MathOptInterface.CONFLICT\_FOUND - Constant.

```
CONFLICT_FOUND::ConflictStatusCode
```

An instance of the ConflictStatusCode enum.

#### About

The solver found a conflict.

source

MathOptInterface.ConflictCount - Type.

```
ConflictCount()
```

An AbstractModelAttribute for the number of conflicts found by the solver in the most recent call to compute\_conflict!.

## Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.ConflictCount)::Int
```

They should not implement set or supports.

source

MathOptInterface.ConstraintConflictStatus - Type.

```
ConstraintConflictStatus(conflict_index = 1)
```

A constraint attribute to query the ConflictParticipationStatusCode indicating whether the constraint participates in the conflict.

## conflict\_index

The optimizer may return multiple conflicts. See ConflictCount for querying the number of conflicts found.

If the solver does not have a conflict because the conflict\_index is beyond the available solutions (whose number is indicated by the ConflictCount attribute), then MOI.check\_result\_index\_bounds(model, ConstraintConflictStatus(conflict\_index)) will throw a ResultIndexBoundsError.

#### Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ConstraintConflictStatus,
    ::MOI.ConstraintIndex,
)::ConflictParticipationStatusCode
```

They should not implement set or supports.

source

MathOptInterface.ConflictParticipationStatusCode - Type.

```
ConflictParticipationStatusCode
```

An Enum for the value of the ConstraintConflictStatus attribute.

This attribute is meant to indicate whether a given constraint participates or not in the last computed conflict.

## **Values**

```
NOT_IN_CONFLICT
```

The constraint does not participate in the conflict.

```
IN_CONFLICT
```

The constraint participates in the conflict.

```
MAYBE_IN_CONFLICT
```

The solver was not able to prove whether the constraint is required to participate in the conflict.

source

MathOptInterface.NOT\_IN\_CONFLICT - Constant.

```
NOT_IN_CONFLICT::ConflictParticipationStatusCode
```

An instance of the ConflictParticipationStatusCode enum.

### **About**

The constraint does not participate in the conflict.

source

MathOptInterface.IN\_CONFLICT - Constant.

```
IN_CONFLICT::ConflictParticipationStatusCode
```

An instance of the ConflictParticipationStatusCode enum.

## **About**

The constraint participates in the conflict.

source

MathOptInterface.MAYBE\_IN\_CONFLICT - Constant.

```
MAYBE_IN_CONFLICT::ConflictParticipationStatusCode
```

An instance of the ConflictParticipationStatusCode enum.

## About

The solver was not able to prove whether the constraint is required to participate in the conflict.

source

# **Chapter 20**

# **Variables**

## 20.1 Functions

MathOptInterface.add\_variable - Function.

```
add_variable(model::ModelLike)::VariableIndex
```

Add a scalar variable to the model, returning a variable index.

A AddVariableNotAllowed error is thrown if adding variables cannot be done in the current state of the model model.

## **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model)

MOI.VariableIndex(1)
```

source

MathOptInterface.add\_variables - Function.

```
add_variables(model::ModelLike, n::Int)::Vector{VariableIndex}
```

 $\label{eq:Add-number} \mbox{Add n scalar variables to the model, returning a vector of variable indices.}$ 

An AddVariableNotAllowed error is thrown if adding variables cannot be done in the current state of the model model.

## **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> MOI.add_variables(model, 2)
2-element Vector{MathOptInterface.VariableIndex}:
    MOI.VariableIndex(1)
    MOI.VariableIndex(2)
```

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source

MathOptInterface.add\_constrained\_variable - Function.

Add to model a scalar variable constrained to belong to set, returning the index of the variable created and the index of the constraint constraining the variable to belong to set.

By default, this function falls back to creating a free variable with add\_variable and then constraining it to belong to set with add\_constraint.

source

```
add_constrained_variable(
    model::ModelLike,
    set::Tuple{<:GreaterThan,<:LessThan},
)</pre>
```

A special-case method to add a scalar variable with a lower and upper bound.

This method should be implemented by optimizers which have native support for adding a variable with bounds and which cannot performantly modify the variable bounds after creation.

## Example

```
julia> model = MOI.Utilities.Model{Float64}();
julia> set = (MOI.GreaterThan(1.0), MOI.LessThan(2.0));
julia> x, (c_l, c_u) = MOI.add_constrained_variable(model, set);
julia> c_l
MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,
→ MathOptInterface.GreaterThan{Float64}}(1)
julia> c_u
{\tt MathOptInterface.ConstraintIndex\{MathOptInterface.VariableIndex,}
→ MathOptInterface.LessThan{Float64}}(1)
julia> print(model)
Feasibility
Subject to:
VariableIndex-in-GreaterThan{Float64}
v[1] >= 1.0
VariableIndex-in-LessThan{Float64}
v[1] \ll 2.0
```

source

MathOptInterface.add\_constrained\_variables - Function.

```
add_constrained_variables(
    model::ModelLike,
    sets::AbstractVector{<:AbstractScalarSet}
)::Tuple{
    Vector{MOI.VariableIndex},
    Vector{MOI.ConstraintIndex{MOI.VariableIndex,eltype(sets)}},
}</pre>
```

Add to model scalar variables constrained to belong to sets, returning the indices of the variables created and the indices of the constraints constraining the variables to belong to each set in sets. That is, if it returns variables and constraints, constraints[i] is the index of the constraint constraining variable[i] to belong to sets[i].

By default, this function falls back to calling add constrained variable on each set.

source

```
add_constrained_variables(
    model::ModelLike,
    set::AbstractVectorSet,
)::Tuple{
    Vector{MOI.VariableIndex},
    MOI.ConstraintIndex{MOI.VectorOfVariables,typeof(set)},
}
```

Add to model a vector of variables constrained to belong to set, returning the indices of the variables created and the index of the constraint constraining the vector of variables to belong to set.

By default, this function falls back to creating free variables with add\_variables and then constraining it to belong to set with add\_constraint.

source

 ${\tt MathOptInterface.supports\_add\_constrained\_variable-Function}.$ 

```
supports_add_constrained_variable(
   model::ModelLike,
   S::Type{<:AbstractScalarSet}
)::Bool</pre>
```

Return a Bool indicating whether model supports constraining a variable to belong to a set of type S either on creation of the variable with add\_constrained\_variable or after the variable is created with add constraint.

By default, this function falls back to supports\_add\_constrained\_variables (model, Reals) && supports\_constraint (model. VariableIndex, S) which is the correct definition for most models.

## Example

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Suppose that a solver supports only two kind of variables: binary variables and continuous variables with a lower bound. If the solver decides not to support VariableIndex-in-Binary and VariableIndex-in-GreaterThan constraints, it only has to implement add\_constrained\_variable for these two sets which prevents the user to add both a binary constraint and a lower bound on the same variable. Moreover, if the user adds a VariableIndex-in-GreaterThan constraint, implementing this interface (that is, supports\_add\_constrained\_variables) enables the constraint to be transparently bridged into a supported constraint.

source

MathOptInterface.supports\_add\_constrained\_variables - Function.

```
supports_add_constrained_variables(
    model::ModelLike,
    S::Type{<:AbstractVectorSet}
)::Bool</pre>
```

Return a Bool indicating whether model supports constraining a vector of variables to belong to a set of type S either on creation of the vector of variables with add\_constrained\_variables or after the variable is created with add\_constraint.

By default, if S is Reals then this function returns true and otherwise, it falls back to supports\_add\_constrained\_variables (Reals) && supports\_constraint(model, MOI.VectorOfVariables, S) which is the correct definition for most models.

#### **Example**

In the standard conic form (see Duality), the variables are grouped into several cones and the constraints are affine equality constraints. If Reals is not one of the cones supported by the solvers then it needs to implement supports\_add\_constrained\_variables(::0ptimizer, ::Type{Reals}) = false as free variables are not supported. The solvers should then implement supports\_add\_constrained\_variables(::0ptimizer, ::Type{<:SupportedCones}) = true where SupportedCones is the union of all cone types that are supported; it does not have to implement the method supports\_constraint(::Type{VectorOfVariables}, Type{<:SupportedCones}) as it should return false and it's the default. This prevents the user to constrain the same variable in two different cones. When a VectorOfVariables-in-S is added, the variables of the vector have already been created so they already belong to given cones. If bridges are enabled, the constraint will therefore be bridged by adding slack variables in S and equality constraints ensuring that the slack variables are equal to the corresponding variables of the given constraint function.

Note that there may also be sets for which !supports\_add\_constrained\_variables(model, S) and supports\_constraint(model, MOI.VectorOfVariables, S). For instance, suppose a solver supports positive semidefinite variable constraints and two types of variables: binary variables and nonnegative variables. Then the solver should support adding VectorOfVariables-in-PositiveSemidefiniteConeTriangle constraints, but it should not support creating variables constrained to belong to the PositiveSemidefiniteConeTriangle because the variables in PositiveSemidefiniteConeTriangle should first be created as either binary or non-negative.

source

MathOptInterface.is\_valid - Method.

```
is_valid(model::ModelLike, index::Index)::Bool
```

Return a Bool indicating whether this index refers to a valid object in the model model.

source

MathOptInterface.delete - Method.

```
delete(model::ModelLike, index::Index)
```

Delete the referenced object from the model. Throw DeleteNotAllowed if if index cannot be deleted.

The following modifications also take effect if Index is VariableIndex:

- If index used in the objective function, it is removed from the function, that is, it is substituted for zero.
- For each func-in-set constraint of the model:
  - If func isa VariableIndex and func == index then the constraint is deleted.
  - If func isa VectorOfVariables and index in func.variables then
    - \* if length(func.variables) == 1 is one, the constraint is deleted;
    - \* iflength(func.variables) > 1 and supports\_dimension\_update(set) then then the variable is removed from func and set is replaced by update\_dimension(set, MOI.dimension(set)
       1).
    - \* Otherwise, a DeleteNotAllowed error is thrown.
  - Otherwise, the variable is removed from func, that is, it is substituted for zero.

source

MathOptInterface.delete - Method.

```
delete(model::ModelLike, indices::Vector{R<:Index}) where {R}</pre>
```

Delete the referenced objects in the vector indices from the model. It may be assumed that R is a concrete type. The default fallback sequentially deletes the individual items in indices, although specialized implementations may be more efficient.

source

# 20.2 Attributes

MathOptInterface.AbstractVariableAttribute - Type.

```
{\tt AbstractVariableAttribute}
```

Abstract supertype for attribute objects that can be used to set or get attributes (properties) of variables in the model.

source

MathOptInterface.VariableName - Type.

```
VariableName()
```

An AbstractVariableAttribute for a String identifying the variable.

The default name is "" if not set by the user.

### **Duplicate names**

Two variables may have the same name; however, variables with duplicate names cannot be looked up using get.

### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variable(model);
julia> MOI.supports(model, MOI.VariableName(), MOI.VariableIndex)
true
julia> MOI.get(model, MOI.VariableName(), x)

julia> MOI.set(model, MOI.VariableName(), x, "x")
julia> MOI.get(model, MOI.VariableName(), x)

julia> MOI.get(model, MOI.VariableName(), x)
```

# Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.VariableName, ::MOI.VariableIndex)::String
MOI.set(::Optimizer, ::MOI.VariableName, ::MOI.VariableIndex, ::String)::Nothing
MOI.supports(::Optimizer, ::MOI.VariableName, ::Type{MOI.VariableIndex})::Bool
```

source

 ${\tt MathOptInterface.VariablePrimalStart-Type}.$ 

```
VariablePrimalStart()
```

An AbstractVariableAttribute for the initial assignment to the variable's primal value that the optimizer may use to warm-start the solve.

May be a number or nothing (unset).

### **Example**

```
julia> model = MOI.Utilities.UniversalFallback(MOI.Utilities.Model{Float64}());
julia> x = MOI.add_variable(model);
```

```
julia> MOI.supports(model, MOI.VariablePrimalStart(), MOI.VariableIndex)
true
julia> MOI.get(model, MOI.VariablePrimalStart(), x)
julia> MOI.set(model, MOI.VariablePrimalStart(), x, 1.0)
julia> MOI.get(model, MOI.VariablePrimalStart(), x)
1.0
```

### Implementation

Optimizers should implement the following methods:

source

MathOptInterface.VariablePrimal - Type.

```
VariablePrimal(result_index::Int = 1)
```

An AbstractVariableAttribute for the variable's primal value in result result\_index.

## **PrimalStatus**

Before quering this attribute you should first check PrimalStatus to confirm that a primal solution is available.

If the PrimalStatus is NO SOLUTION the result of querying this attribute is undefined.

### result index

The optimizer may return multiple primal solutions. See ResultCount for information on how the results are ordered.

If the solver does not have a primal value for the variable because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError.

### Implementation

Optimizers should implement the following methods:

```
MOI.get(::Optimizer, ::MOI.VariablePrimal, ::MOI.VariableIndex)::T
```

They should not implement set or supports.

 ${\tt MathOptInterface.VariableBasisStatus-Type.}$ 

```
VariableBasisStatus(result_index::Int = 1)
```

An AbstractVariableAttribute for the BasisStatusCode of the variable in result result\_index, with respect to a basic solution.

#### **PrimalStatus**

Before quering this attribute you should first check PrimalStatus to confirm that a primal solution is avaiable.

If the PrimalStatus is NO\_SOLUTION the result of querying this attribute is undefined.

# result\_index

The optimizer may return multiple primal solutions. See ResultCount for information on how the results are ordered.

If the solver does not have a primal value for the variable because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError.

### Implementation

Optimizers should implement the following methods:

```
\texttt{MOI.get(::Optimizer, ::MOI.VariableBasisStatus, ::MOI.VariableIndex)::MOI.BasisStatusCode}
```

They should not implement set or supports.

# **Chapter 21**

# **Constraints**

# **21.1 Types**

 ${\tt MathOptInterface.ConstraintIndex-Type.}$ 

```
ConstraintIndex{F,S}
```

A type-safe wrapper for Int64 for use in referencing F-in-S constraints in a model.

The parameter F is the type of the function in the constraint, and the parameter S is the type of set in the constraint.

To allow for deletion, indices need not be consecutive.

Indices within a constraint type (that is, F-in-S) must be unique, but non-unique indices across different constraint types are allowed.

If F is VariableIndex then the index is equal to the index of the variable. That is for an index::ConstraintIndex{VariableIndex always have

```
index.value == MOI.get(model, MOI.ConstraintFunction(), index).value
```

source

### 21.2 Functions

 ${\tt MathOptInterface.is\_valid-Method}.$ 

```
is_valid(model::ModelLike, index::Index)::Bool
```

Return a Bool indicating whether this index refers to a valid object in the model model.

source

 ${\tt MathOptInterface.add\_constraint-Function}.$ 

```
MOI.add_constraint(map::Map, vi::MOI.VariableIndex, set::MOI.AbstractScalarSet)
```

Record that a constraint vi-in-set is added and throws if a lower or upper bound is set by this constraint and such bound has already been set for vi.

source

```
add_constraint(model::ModelLike, func::F, set::S)::ConstraintIndex{F,S} where {F,S}
```

Add the constraint  $f(x) \in \mathcal{S}$  where f is defined by func, and  $\mathcal{S}$  is defined by set.

Add the constraint  $v \in \mathcal{S}$  where v is the variable (or vector of variables) referenced by v and  $\mathcal{S}$  is defined by set.

- An UnsupportedConstraint error is thrown if model does not support F-in-S constraints,
- a AddConstraintNotAllowed error is thrown if it supports F-in-S constraints but it cannot add the
  constraint in its current state and
- a ScalarFunctionConstantNotZero error may be thrown if func is an AbstractScalarFunction with nonzero constant and set is EqualTo, GreaterThan, LessThan or Interval.
- a LowerBoundAlreadySet error is thrown if F is a VariableIndex and a constraint was already added to this variable that sets a lower bound.
- a UpperBoundAlreadySet error is thrown if F is a VariableIndex and a constraint was already added to this variable that sets an upper bound.

source

MathOptInterface.add\_constraints - Function.

Add the set of constraints specified by each function-set pair in funcs and sets. F and S should be concrete types. This call is equivalent to add\_constraint.(model, funcs, sets) but may be more efficient.

source

MathOptInterface.transform - Function.

```
transform(
  model::ModelLike,
  c::ConstraintIndex{F,S1},
  newset::S2,
)::ConstraintIndex{F,S2}
```

Replace the set in constraint c with newset.

The constraint index c will no longer be valid, and the function returns a new constraint index with the correct type.

Solvers may only support a subset of constraint transforms that they perform efficiently (for example, changing from a LessThan to GreaterThan set). In addition, set modification (where S1 = S2) should be performed via the modify function.

Typically, the user should delete the constraint and add a new one.

# **Example**

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variable(model);
julia> c = MOI.add_constraint(model, 1.0 * x, MOI.LessThan(2.0));
julia> print(model)
Feasibility
Subject to:
ScalarAffineFunction{Float64}-in-LessThan{Float64}
0.0 + 1.0 v[1] \le 2.0
julia> c2 = MOI.transform(model, c, MOI.GreaterThan(0.0))
MathOptInterface.ConstraintIndex{MathOptInterface.ScalarAffineFunction{Float64},
→ MathOptInterface.GreaterThan{Float64}}(1)
julia> print(model)
Feasibility
Subject to:
ScalarAffineFunction{Float64}-in-GreaterThan{Float64}
0.0 + 1.0 \vee [1] >= 0.0
julia> MOI.is_valid(model, c)
false
```

source

MathOptInterface.supports\_constraint - Function.

```
MOI.supports_constraint(
   BT::Type{<:AbstractBridge},
   F::Type{<:MOI.AbstractFunction},
   S::Type{<:MOI.AbstractSet},
)::Bool</pre>
```

Return a Bool indicating whether the bridges of type BT support bridging F-in-S constraints.

# Implementation notes

- This method depends only on the type of the inputs, not the runtime values.
- There is a default fallback, so you need only implement this method for constraint types that the bridge implements.

```
supports_constraint(
   model::ModelLike,
   ::Type{F},
   ::Type{S},
)::Bool where {F<:AbstractFunction,S<:AbstractSet}</pre>
```

Return a Bool indicating whether model supports F-in-S constraints, that is, copy\_to(model, src) does not throw UnsupportedConstraint when src contains F-in-S constraints. If F-in-S constraints are only not supported in specific circumstances, for example, F-in-S constraints cannot be combined with another type of constraint, it should still return true.

source

# 21.3 Attributes

MathOptInterface.AbstractConstraintAttribute - Type.

```
AbstractConstraintAttribute
```

Abstract supertype for attribute objects that can be used to set or get attributes (properties) of constraints in the model.

source

MathOptInterface.ConstraintName - Type.

```
ConstraintName()
```

An AbstractConstraintAttribute for a String identifying the constraint.

The default name is "" if not set by the user.

# **Duplicate names**

Two constraints may have the same name; however, constraints with duplicate names cannot be looked up using get, regardless of whether they have the same F-in-S type.

## VariableIndex connstraints

You should not implement ConstraintName for VariableIndex constraints.

# Example

### Implementation

Optimizers should implement the following methods:

source

 ${\tt MathOptInterface.ConstraintPrimalStart-Type.}$ 

```
ConstraintPrimalStart()
```

An AbstractConstraintAttribute for the initial assignment to the constraint's ConstraintPrimal that the optimizer may use to warm-start the solve.

May be nothing (unset), a number for AbstractScalarFunction, or a vector for AbstractVectorFunction. source

MathOptInterface.ConstraintDualStart - Type.

```
ConstraintDualStart()
```

An AbstractConstraintAttribute for the initial assignment to the constraint's ConstraintDual that the optimizer may use to warm-start the solve.

May be nothing (unset), a number for AbstractScalarFunction, or a vector for AbstractVectorFunction. source

MathOptInterface.ConstraintPrimal - Type.

```
ConstraintPrimal(result_index::Int = 1)
```

An AbstractConstraintAttribute for the constraint's primal value in result result\_index.

### **Definition**

If the constraint is  $f(x) \in S$ , then in most cases the ConstraintPrimal is the value of f, evaluated at the corresponding VariablePrimal solution.

However, some conic solvers reformulate  $b-Ax\in S$  to s=b-Ax and  $s\in S$ . These solvers may return the value of s for ConstraintPrimal, rather than b - Ax. (Although these are constrained by an equality constraint, due to numerical tolerances they may not be identical.)

# **PrimalStatus**

Before quering this attribute you should first check PrimalStatus to confirm that a primal solution is available

If the PrimalStatus is NO SOLUTION the result of querying this attribute is undefined.

### result\_index

The optimizer may return multiple primal solutions. See ResultCount for information on how the results are ordered.

If the solver does not have a primal value for the constraint because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError.

### Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ConstraintPrimal,
    ::MOI.ConstraintIndex{<:MOI.AbstractScalarFunction}
)::T
MOI.get(
    ::Optimizer,
    ::MOI.ConstraintPrimal,
    ::MOI.ConstraintIndex{<:MOI.AbstractVectorFunction}
)::Vector{T}</pre>
```

They should not implement set or supports.

source

MathOptInterface.ConstraintDual - Type.

```
ConstraintDual(result_index::Int = 1)
```

An AbstractConstraintAttribute for the constraint's dual value in result result\_index.

### **DualStatus**

Before quering this attribute you should first check DualStatus to confirm that a dual solution is avaiable.

If the DualStatus is NO\_SOLUTION the result of querying this attribute is undefined.

### result\_index

The optimizer may return multiple dual solutions. See ResultCount for information on how the results are ordered.

If the solver does not have a dual value for the constraint because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must throw a ResultIndexBoundsError.

### Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ConstraintDual,
    ::MOI.ConstraintIndex{<:MOI.AbstractScalarFunction}
)::T

MOI.get(
    ::Optimizer,
    ::MOI.ConstraintDual,
    ::MOI.ConstraintIndex{<:MOI.AbstractVectorFunction}
)::Vector{T}</pre>
```

They should not implement set or supports.

source

MathOptInterface.ConstraintBasisStatus - Type.

```
ConstraintBasisStatus(result_index::Int = 1)
```

An AbstractConstraintAttribute for the BasisStatusCode of the constraint in result result\_index, with respect to a basic solution.

If result\_index is omitted, it is 1 by default.

If the solver does not have a basis status for the constraint because the result\_index is beyond the available solutions (whose number is indicated by the ResultCount attribute), getting this attribute must

throw a ResultIndexBoundsError. Otherwise, if the result is unavailable for another reason (for instance, only a dual solution is available), the result is undefined. Users should first check PrimalStatus before accessing the ConstraintBasisStatus attribute.

See ResultCount for information on how the results are ordered.

#### Notes

For the basis status of a variable, query VariableBasisStatus.

ConstraintBasisStatus does not apply to VariableIndex constraints. You can infer the basis status of a VariableIndex constraint by looking at the result of VariableBasisStatus.

source

MathOptInterface.ConstraintFunction - Type.

```
ConstraintFunction()
```

An AbstractConstraintAttribute for the AbstractFunction object used to define the constraint.

It is guaranteed to be equivalent but not necessarily identical to the function provided by the user.

### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variable(model);
julia> c = MOI.add_constraint(model, 1.0 * x, MOI.GreaterThan(0.0));
julia> MOI.get(model, MOI.ConstraintFunction(), c)
0.0 + 1.0 MOI.VariableIndex(1)
julia> MOI.set(model, MOI.ConstraintFunction(), c, 2.0 * x)
julia> MOI.get(model, MOI.ConstraintFunction(), c)
0.0 + 2.0 MOI.VariableIndex(1)
```

### Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ConstraintFunction,
    ::MOI.ConstraintIndex{F,S},
)::F where {F,S}
```

If the optimizer supports modifying an existing function, it should implement:

```
MOI.set(
    ::Optimizer,
    ::MOI.ConstraintFunction,
```

```
::MOI.ConstraintIndex{F,S},
::F,
)::Nothing where {F,S}
```

It should not implement supports.

source

MathOptInterface.CanonicalConstraintFunction - Type.

```
CanonicalConstraintFunction()
```

An AbstractConstraintAttribute for a canonical representation of the AbstractFunction object used to define the constraint.

Getting this attribute is guaranteed to return a function that is equivalent but not necessarily identical to the function provided by the user.

#### **Fallback**

By default, MOI.get(model, MOI.CanonicalConstraintFunction(), ci) falls back to MOI.Utilities.canonical(MOI.get MOI.ConstraintFunction(), ci)).

However, if model knows that the constraint function is canonical then it can implement a specialized method that directly return the function without calling Utilities.canonical. Therefore, the value returned cannot be assumed to be a copy of the function stored in model.

Moreover, Utilities.Model checks with Utilities.is\_canonical whether the function stored internally is already canonical and if it's the case, then it returns the function stored internally instead of a copy.

### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variable(model);
julia> c = MOI.add_constraint(model, 1.0 * x + 1.0 * x, MOI.GreaterThan(0.0));
julia> MOI.get(model, MOI.CanonicalConstraintFunction(), c)
0.0 + 2.0 MOI.VariableIndex(1)
```

# Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.CanonicalConstraintFunction,
    ::MOI.ConstraintIndex{F,S},
)::F where {F,S}
```

They should not implement set or supports.

MathOptInterface.ConstraintSet - Type.

```
ConstraintSet()
```

An AbstractConstraintAttribute for the AbstractSet object used to define the constraint.

### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model);

julia> c = MOI.add_constraint(model, x, MOI.GreaterThan(0.0));

julia> MOI.get(model, MOI.ConstraintSet(), c)

MathOptInterface.GreaterThan{Float64}(0.0)

julia> MOI.set(model, MOI.ConstraintSet(), c, MOI.GreaterThan(1.0))

julia> MOI.get(model, MOI.ConstraintSet(), c)

MathOptInterface.GreaterThan{Float64}(1.0)
```

### Implementation

Optimizers should implement the following methods:

```
MOI.get(
    ::Optimizer,
    ::MOI.ConstraintSet,
    ::MOI.ConstraintIndex{F,S},
)::S where {F,S}
```

If the optimizer supports modifying an existing set, it should implement:

```
MOI.set(
    ::Optimizer,
    ::MOI.ConstraintSet,
    ::MOI.ConstraintIndex{F,S},
    ::S,
)::Nothing where {F,S}
```

It should not implement supports.

source

 ${\tt MathOptInterface.BasisStatusCode-Type.}$ 

```
BasisStatusCode
```

An Enum for the value of the ConstraintBasisStatus and VariableBasisStatus attributes, explaining the status of a given element with respect to an optimal solution basis.

### Notes

When queried as part of ConstraintBasisStatus, NONBASIC\_AT\_LOWER and NONBASIC\_AT\_UPPER should be returned only for constraints with the Interval set. In this case, they are necessary to distinguish which side of the constraint is active. One-sided constraints (for example, LessThan and GreaterThan) should use NONBASIC instead of the NONBASIC\_AT\_\* values.

This restriction does not apply to VariableBasisStatus, which should return NONBASIC\_AT\_\* regardless of whether the alternative bound exists.

### **Values**

#### BASIC

The element is in the basis.

#### NONBASIC

The element is not in the basis.

### **NONBASIC AT LOWER**

The element is not in the basis and is at its lower bound.

# NONBASIC\_AT\_UPPER

The element is not in the basis and is at its upper bound.

## SUPER BASIC

The element is not in the basis but is also not at one of its bounds.

In a linear program, this status occurs when a variable with no bounds is not in the basis, for example, because it takes the value 0.0.

source

MathOptInterface.BASIC - Constant.

```
BASIC::BasisStatusCode
```

An instance of the BasisStatusCode enum.

### **About**

The element is in the basis.

source

 ${\tt MathOptInterface.NONBASIC-Constant}.$ 

```
NONBASIC::BasisStatusCode
```

An instance of the BasisStatusCode enum.

### **About**

The element is not in the basis.

MathOptInterface.NONBASIC\_AT\_LOWER - Constant.

```
{\tt NONBASIC\_AT\_LOWER::BasisStatusCode}
```

An instance of the BasisStatusCode enum.

### **About**

The element is not in the basis and is at its lower bound.

source

MathOptInterface.NONBASIC\_AT\_UPPER - Constant.

```
NONBASIC_AT_UPPER::BasisStatusCode
```

An instance of the BasisStatusCode enum.

# **About**

The element is not in the basis and is at its upper bound.

source

MathOptInterface.SUPER\_BASIC - Constant.

```
{\tt SUPER\_BASIC::BasisStatusCode}
```

An instance of the BasisStatusCode enum.

# About

The element is not in the basis but is also not at one of its bounds.

In a linear program, this status occurs when a variable with no bounds is not in the basis, for example, because it takes the value 0.0.

# **Chapter 22**

# **Modifications**

MathOptInterface.modify - Function.

```
modify(
    model::ModelLike,
    ci::ConstraintIndex,
    change::AbstractFunctionModification,
)
```

Apply the modification specified by change to the function of constraint ci.

An ModifyConstraintNotAllowed error is thrown if modifying constraints is not supported by the model model.

### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();

julia> x = MOI.add_variable(model);

julia> ci = MOI.add_constraint(model, 1.0 * x, MOI.EqualTo(1.0));

julia> MOI.modify(model, ci, MOI.ScalarConstantChange(10.0))

julia> print(model)
Feasibility

Subject to:

ScalarAffineFunction{Float64}-in-EqualTo{Float64}
10.0 + 1.0 v[1] == 1.0
```

```
modify(
   model::ModelLike,
   cis::AbstractVector{<:ConstraintIndex},
   changes::AbstractVector{<:AbstractFunctionModification},
)</pre>
```

Apply multiple modifications specified by changes to the functions of constraints cis.

A ModifyConstraintNotAllowed error is thrown if modifying constraints is not supported by model.

#### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variables(model, 2);
julia> ci = MOI.add_constraint.(model, 1.0 .* x, MOI.EqualTo(1.0));
julia> MOI.modify(model, ci, MOI.ScalarCoefficientChange.(x, [2.0, 0.5]))
julia> print(model)
Feasibility
Subject to:
ScalarAffineFunction{Float64}-in-EqualTo{Float64}
0.0 + 2.0 v[1] == 1.0
0.0 + 0.5 v[2] == 1.0
```

#### source

```
modify(model::ModelLike, ::ObjectiveFunction, change::AbstractFunctionModification)
```

Apply the modification specified by change to the objective function of model. To change the function completely, call set instead.

An ModifyObjectiveNotAllowed error is thrown if modifying objectives is not supported by the model model.

# **Example**

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variable(model);
julia> MOI.set(model, MOI.ObjectiveSense(), MOI.MIN_SENSE)

julia> f = 1.0 * x;
julia> attr = MOI.ObjectiveFunction{typeof(f)}()
MathOptInterface.ObjectiveFunction{MathOptInterface.ScalarAffineFunction{Float64}}()

julia> MOI.set(model, attr, f)

julia> mOI.modify(model, attr, MOI.ScalarConstantChange(10.0))

julia> print(model)
Minimize ScalarAffineFunction{Float64}:
10.0 + 1.0 v[1]
Subject to:
```

```
modify(
    model::ModelLike,
    attr::ObjectiveFunction,
    changes::AbstractVector{<:AbstractFunctionModification},
)</pre>
```

Apply multiple modifications specified by changes to the functions of constraints cis.

A ModifyObjectiveNotAllowed error is thrown if modifying objective coefficients is not supported by model.

# **Example**

# Example

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variables(model, 2);
julia> MOI.set(model, MOI.ObjectiveSense(), MOI.MIN_SENSE)
julia> f = 1.0 * x[1] + 1.0 * x[2];
julia> attr = MOI.ObjectiveFunction{typeof(f)}()
MathOptInterface.ObjectiveFunction{MathOptInterface.ScalarAffineFunction{Float64}}()
julia> MOI.set(model, attr, f)
julia> MOI.modify(model, attr, MOI.ScalarCoefficientChange.(x, [2.0, 0.5]))
julia> print(model)
Minimize ScalarAffineFunction{Float64}:
0.0 + 2.0 v[1] + 0.5 v[2]
Subject to:
```

source

MathOptInterface.AbstractFunctionModification - Type.

```
AbstractFunctionModification
```

An abstract supertype for structs which specify partial modifications to functions, to be used for making small modifications instead of replacing the functions entirely.

source

MathOptInterface.ScalarConstantChange - Type.

```
ScalarConstantChange{T}(new_constant::T)
```

A struct used to request a change in the constant term of a scalar-valued function.

Applicable to ScalarAffineFunction and ScalarQuadraticFunction.

source

MathOptInterface.VectorConstantChange - Type.

```
VectorConstantChange{T}(new_constant::Vector{T})
```

A struct used to request a change in the constant vector of a vector-valued function.

 ${\bf Applicable\ to\ Vector Affine Function\ and\ Vector Quadratic Function.}$ 

source

MathOptInterface.ScalarCoefficientChange - Type.

```
ScalarCoefficientChange{T}(variable::VariableIndex, new_coefficient::T)
```

A struct used to request a change in the linear coefficient of a single variable in a scalar-valued function.

 ${\bf Applicable\ to\ Scalar Affine Function\ and\ Scalar Quadratic Function.}$ 

source

MathOptInterface.ScalarQuadraticCoefficientChange - Type.

```
ScalarQuadraticCoefficientChange{T}(
   variable_1::VariableIndex,
   variable_2::VariableIndex,
   new_coefficient::T,
)
```

A struct used to request a change in the quadratic coefficient of a ScalarQuadraticFunction.

# **Scaling factors**

A ScalarQuadraticFunction has an implicit 0.5 scaling factor in front of the Q matrix. This modification applies to terms in the Q matrix.

If  $variable_1 == variable_2$ , this modification sets the corresponding diagonal element of the Q matrix to  $new\_coefficient$ .

If variable\_1 != variable\_2, this modification is equivalent to setting both the corresponding upperand lower-triangular elements of the Q matrix to new\_coefficient.

As a consequence:

• to modify the term x^2 to become 2x^2, new\_coefficient must be 4

• to modify the term xy to become 2xy, new\_coefficient must be 2

source

MathOptInterface.MultirowChange - Type.

```
MultirowChange{T}(
    variable::VariableIndex,
    new_coefficients::Vector{Tuple{Int64,T}},
) where {T}
```

A struct used to request a change in the linear coefficients of a single variable in a vector-valued function.

New coefficients are specified by (output\_index, coefficient) tuples.

Applicable to VectorAffineFunction and VectorQuadraticFunction.

# **Chapter 23**

# Nonlinear programming

# **23.1 Types**

 ${\tt MathOptInterface.AbstractNLPEvaluator-Type.}$ 

```
AbstractNLPEvaluator
```

Abstract supertype for the callback object that is used to query function values, derivatives, and expression graphs.

It is used in NLPBlockData.

# Example

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true);

julia> supertype(typeof(evaluator))
MathOptInterface.AbstractNLPEvaluator
```

source

MathOptInterface.NLPBoundsPair - Type.

```
NLPBoundsPair(lower::Float64, upper::Float64)
```

A struct holding a pair of lower and upper bounds.

-Inf and Inf can be used to indicate no lower or upper bound, respectively.

### **Example**

```
julia> bounds = MOI.NLPBoundsPair.([25.0, 40.0], [Inf, 40.0])
2-element Vector{MathOptInterface.NLPBoundsPair}:
MathOptInterface.NLPBoundsPair(25.0, Inf)
MathOptInterface.NLPBoundsPair(40.0, 40.0)
```

MathOptInterface.NLPBlockData - Type.

```
struct NLPBlockData
   constraint_bounds::Vector{NLPBoundsPair}
   evaluator::AbstractNLPEvaluator
   has_objective::Bool
end
```

A struct encoding a set of nonlinear constraints of the form  $lb \leq g(x) \leq ub$  and, if has\_objective == true, a nonlinear objective function f(x).

Nonlinear objectives override any objective set by using the ObjectiveFunction attribute.

The evaluator is a callback object that is used to query function values, derivatives, and expression graphs. If has\_objective == false, then it is an error to query properties of the objective function, and in Hessian-of-the-Lagrangian queries,  $\sigma$  must be set to zero.

#### Note

Throughout the evaluator, all variables are ordered according to ListOfVariableIndices. Hence, MOI copies of nonlinear problems must not re-order variables.

## **Example**

This example uses the Test. HS071 evaluator.

source

### 23.2 Attributes

 ${\tt MathOptInterface.NLPBlock-Type}.$ 

```
NLPBlock()
```

An AbstractModelAttribute that stores an NLPBlockData, representing a set of nonlinear constraints, and optionally a nonlinear objective.

### **Example**

MathOptInterface.NLPBlockDual - Type.

```
NLPBlockDual(result_index::Int = 1)
```

An AbstractModelAttribute for the Lagrange multipliers on the constraints from the NLPBlock in result result\_index.

If result\_index is omitted, it is 1 by default.

# **Example**

```
julia> MOI.NLPBlockDual()
MathOptInterface.NLPBlockDual(1)

julia> MOI.NLPBlockDual(2)
MathOptInterface.NLPBlockDual(2)
```

source

MathOptInterface.NLPBlockDualStart - Type.

```
NLPBlockDualStart()
```

An AbstractModelAttribute for the initial assignment of the Lagrange multipliers on the constraints from the NLPBlock that the solver may use to warm-start the solve.

# **Example**

```
julia> MOI.set(model, MOI.NLPBlock(), block)

julia> MOI.set(model, MOI.NLPBlockDualStart(), [1.0, 2.0])
```

### 23.3 Functions

source

MathOptInterface.initialize - Function.

```
initialize(
    d::AbstractNLPEvaluator,
    requested_features::Vector{Symbol},
)::Nothing
```

Initialize d with the set of features in requested\_features. Check features\_available before calling initialize to see what features are supported by d.

#### Warning

This method must be called before any other methods.

### **Features**

The following features are defined:

- :Grad: enables eval objective gradient
- :Jac: enables eval\_constraint\_jacobian and eval\_constraint\_gradient
- : JacVec: enables eval\_constraint\_jacobian\_product and eval\_constraint\_jacobian\_transpose\_product
- :Hess: enables eval\_hessian\_lagrangian
- :HessVec: enables eval\_hessian\_lagrangian\_product
- :ExprGraph: enables objective\_expr and constraint\_expr.

In all cases, including when requested\_features is empty, eval\_objective and eval\_constraint are supported.

### **Example**

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true);
julia> MOI.initialize(evaluator, [:Grad, :Jac])
```

source

MathOptInterface.features\_available - Function.

```
features_available(d::AbstractNLPEvaluator)::Vector{Symbol}
```

Returns the subset of features available for this problem instance.

See initialize for the list of defined features.

### **Example**

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true, true);

julia> MOI.features_available(evaluator)
6-element Vector{Symbol}:
    :Grad
    :Jac
    :JacVec
    :ExprGraph
    :Hess
    :HessVec
```

source

MathOptInterface.eval\_objective - Function.

```
eval\_objective(d::AbstractNLPEvaluator, \ x:: \textbf{AbstractVector}\{T\}):: T \ where \ \{T\}
```

Evaluate the objective f(x), returning a scalar value.

### Initialize

Before calling this function, you must call initialize, but you do not need to pass a value.

### Example

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true);
julia> MOI.initialize(evaluator, Symbol[])

julia> MOI.eval_objective(evaluator, [1.0, 2.0, 3.0, 4.0])
27.0
```

source

MathOptInterface.eval\_constraint - Function.

```
eval_constraint(
    d::AbstractNLPEvaluator,
    g::AbstractVector{T},
    x::AbstractVector{T},
)::Nothing where {T}
```

Given a set of vector-valued constraints  $l \leq g(x) \leq u$ , evaluate the constraint function g(x), storing the result in the vector g.

#### Initialize

Before calling this function, you must call initialize, but you do not need to pass a value.

# Implementation notes

When implementing this method, you must not assume that g is Vector{Float64}, but you may assume that it supports setindex! and length. For example, it may be the view of a vector.

### **Example**

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true);
julia> MOI.initialize(evaluator, Symbol[])
julia> g = fill(NaN, 2);
julia> MOI.eval_constraint(evaluator, g, [1.0, 2.0, 3.0, 4.0])
julia> g
2-element Vector{Float64}:
24.0
30.0
```

source

MathOptInterface.eval\_objective\_gradient - Function.

```
eval_objective_gradient(
    d::AbstractNLPEvaluator,
    grad::AbstractVector{T},
    x::AbstractVector{T},
)::Nothing where {T}
```

Evaluate the gradient of the objective function  $grad = \nabla f(x)$  as a dense vector, storing the result in the vector grad.

# Initialize

Before calling this function, you must call initialize with :Grad.

### Implementation notes

When implementing this method, you must not assume that grad is Vector{Float64}, but you may assume that it supports setindex! and length. For example, it may be the view of a vector.

# **Example**

```
julia> evaluator = MOI.Test.HS071(true);

julia> MOI.initialize(evaluator, Symbol[:Grad])

julia> grad = fill(NaN, 4);

julia> MOI.eval_objective_gradient(evaluator, grad, [1.0, 2.0, 3.0, 4.0])

julia> grad
4-element Vector{Float64}:
28.0
4.0
5.0
6.0
```

MathOptInterface.jacobian\_structure - Function.

```
jacobian_structure(d::AbstractNLPEvaluator)::Vector{Tuple{Int64,Int64}}
```

Returns a vector of tuples, (row, column), where each indicates the position of a structurally nonzero element in the Jacobian matrix:  $J_g(x) = \begin{bmatrix} \nabla g_1(x) \\ \nabla g_2(x) \\ \vdots \\ \nabla g_m(x) \end{bmatrix}, \text{ where } g_i \text{ is the } i \text{th component of the nonlinear constraints } g(x).$ 

The indices are not required to be sorted and can contain duplicates, in which case the solver should combine the corresponding elements by adding them together.

The sparsity structure is assumed to be independent of the point x.

# Initialize

Before calling this function, you must call initialize with :Jac.

# Example

```
julia> evaluator = MOI.Test.HS071(true);

julia> MOI.initialize(evaluator, Symbol[:Jac])

julia> MOI.jacobian_structure(evaluator)
8-element Vector{Tuple{Int64, Int64}}:
    (1, 1)
    (1, 2)
    (1, 3)
    (1, 4)
    (2, 1)
    (2, 2)
    (2, 3)
    (2, 4)
```

MathOptInterface.eval\_constraint\_gradient - Function.

```
eval_constraint_gradient(
    d::AbstractNLPEvaluator,
    Vg::AbstractVector{T},
    x::AbstractVector{T},
    i::Int,
)::Nothing where {T}
```

Evaluate the gradient of constraint i,  $\nabla g_i(x)$ , and store the non-zero values in  $\nabla g$ , corresponding to the structure returned by constraint\_gradient\_structure.

### Implementation notes

When implementing this method, you must not assume that  $\nabla g$  is Vector{Float64}, but you may assume that it supports setindex! and length. For example, it may be the view of a vector.

#### Initialize

Before calling this function, you must call initialize with :Jac.

### **Example**

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true);

julia> MOI.initialize(evaluator, Symbol[:Jac])

julia> indices = MOI.constraint_gradient_structure(evaluator, 1);

julia> ∇g = zeros(length(indices));

julia> MOI.eval_constraint_gradient(evaluator, ∇g, [1.0, 2.0, 3.0, 4.0], 1)

julia> ∇g
4-element Vector{Float64}:
24.0
12.0
8.0
6.0
```

source

 ${\tt MathOptInterface.constraint\_gradient\_structure - Function}.$ 

```
constraint\_gradient\_structure(d::AbstractNLPEvaluator,\ i:: \textbf{Int}):: \textbf{Vector}\{\textbf{Int64}\}
```

Returns a vector of indices, where each element indicates the position of a structurally nonzero element in the gradient of constraint  $\nabla g_i(x)$ .

The indices are not required to be sorted and can contain duplicates, in which case the solver should combine the corresponding elements by adding them together.

The sparsity structure is assumed to be independent of the point x.

#### Initialize

Before calling this function, you must call initialize with :Jac.

### **Example**

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true);

julia> MOI.initialize(evaluator, Symbol[:Jac])

julia> indices = MOI.constraint_gradient_structure(evaluator, 1)
4-element Vector{Int64}:
    1
    2
    3
    4
```

source

MathOptInterface.eval\_constraint\_jacobian - Function.

```
eval_constraint_jacobian(
    d::AbstractNLPEvaluator,
    J::AbstractVector{T},
    x::AbstractVector{T},
)::Nothing where {T}
```

```
Evaluates the sparse Jacobian matrix J_g(x)=\left[egin{array}{c} \nabla g_1(x) \\ \nabla g_2(x) \\ \vdots \\ \nabla g_m(x) \end{array}\right] .
```

The result is stored in the vector J in the same order as the indices returned by jacobian\_structure.

# Implementation notes

When implementing this method, you must not assume that J is Vector{Float64}, but you may assume that it supports setindex! and length. For example, it may be the view of a vector.

# Initialize

Before calling this function, you must call initialize with :Hess.

### **Example**

```
julia> evaluator = MOI.Test.HS071(true);
julia> MOI.initialize(evaluator, Symbol[:Jac])
```

```
julia> J_indices = MOI.jacobian_structure(evaluator);

julia> J = zeros(length(J_indices));

julia> MOI.eval_constraint_jacobian(evaluator, J, [1.0, 2.0, 3.0, 4.0])

julia> J
8-element Vector{Float64}:
24.0
12.0
8.0
6.0
2.0
4.0
6.0
8.0
```

MathOptInterface.eval\_constraint\_jacobian\_product - Function.

```
eval_constraint_jacobian_product(
    d::AbstractNLPEvaluator,
    y::AbstractVector{T},
    x::AbstractVector{T},
    w::AbstractVector{T},
)::Nothing where {T}
```

Computes the Jacobian-vector product  $y=J_g(x)w$ , storing the result in the vector y.

The vectors have dimensions such that length(w) = length(x), and length(y) is the number of non-linear constraints.

# Implementation notes

When implementing this method, you must not assume that y is Vector{Float64}, but you may assume that it supports setindex! and length. For example, it may be the view of a vector.

### Initialize

Before calling this function, you must call initialize with :JacVec.

### **Example**

```
julia> evaluator = MOI.Test.HS071(true);
julia> MOI.initialize(evaluator, Symbol[:Jac, :JacVec])
julia> y = zeros(2);
julia> x = [1.0, 2.0, 3.0, 4.0];
```

```
julia> w = [1.5, 2.5, 3.5, 4.5];
julia> MOI.eval_constraint_jacobian_product(evaluator, y, x, w)

julia> y
2-element Vector{Float64}:
121.0
70.0
```

MathOptInterface.eval constraint jacobian transpose product - Function.

```
eval_constraint_jacobian_transpose_product(
    d::AbstractNLPEvaluator,
    y::AbstractVector{T},
    x::AbstractVector{T},
    w::AbstractVector{T},
)::Nothing where {T}
```

Computes the Jacobian-transpose-vector product  $y = J_q(x)^T w$ , storing the result in the vector y.

The vectors have dimensions such that length(y) = length(x), and length(w) is the number of non-linear constraints.

### Implementation notes

When implementing this method, you must not assume that y is Vector{Float64}, but you may assume that it supports setindex! and length. For example, it may be the view of a vector.

### Initialize

Before calling this function, you must call initialize with :JacVec.

## **Example**

```
julia> evaluator = MOI.Test.HS071(true);

julia> MOI.initialize(evaluator, Symbol[:Jac, :JacVec])

julia> y = zeros(4);

julia> x = [1.0, 2.0, 3.0, 4.0];

julia> w = [1.5, 2.5];

julia> MOI.eval_constraint_jacobian_transpose_product(evaluator, y, x, w)

julia> y

4-element Vector{Float64}:

41.0
28.0
27.0
29.0
```

MathOptInterface.hessian\_lagrangian\_structure - Function.

```
hessian_lagrangian_structure(
    d::AbstractNLPEvaluator,
)::Vector{Tuple{Int64, Int64}}
```

Returns a vector of tuples, (row, column), where each indicates the position of a structurally nonzero element in the Hessian-of-the-Lagrangian matrix:  $\nabla^2 f(x) + \sum_{i=1}^m \nabla^2 g_i(x)$ .

The indices are not required to be sorted and can contain duplicates, in which case the solver should combine the corresponding elements by adding them together.

Any mix of lower and upper-triangular indices is valid. Elements (i, j) and (j, i), if both present, should be treated as duplicates.

The sparsity structure is assumed to be independent of the point x.

### Initialize

Before calling this function, you must call initialize with :Hess.

### **Example**

This example uses the Test. HS071 evaluator.

source

MathOptInterface.hessian\_objective\_structure - Function.

```
hessian_objective_structure(
    d::AbstractNLPEvaluator,
)::Vector{Tuple{Int64, Int64}}
```

Returns a vector of tuples, (row, column), where each indicates the position of a structurally nonzero element in the Hessian matrix:  $\nabla^2 f(x)$ .

The indices are not required to be sorted and can contain duplicates, in which case the solver should combine the corresponding elements by adding them together.

Any mix of lower and upper-triangular indices is valid. Elements (i, j) and (j, i), if both present, should be treated as duplicates.

The sparsity structure is assumed to be independent of the point x.

### Initialize

Before calling this function, you must call initialize with :Hess.

### **Example**

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true);

julia> MOI.initialize(evaluator, Symbol[:Hess])

julia> MOI.hessian_objective_structure(evaluator)
6-element Vector{Tuple{Int64, Int64}}:
    (1, 1)
    (2, 1)
    (3, 1)
    (4, 1)
    (4, 2)
    (4, 3)
```

source

MathOptInterface.hessian\_constraint\_structure - Function.

```
hessian_constraint_structure(
    d::AbstractNLPEvaluator,
    i::Int64,
)::Vector{Tuple{Int64,Int64}}
```

Returns a vector of tuples, (row, column), where each indicates the position of a structurally nonzero element in the Hessian matrix:  $\nabla^2 g_i(x)$ .

The indices are not required to be sorted and can contain duplicates, in which case the solver should combine the corresponding elements by adding them together.

Any mix of lower and upper-triangular indices is valid. Elements (i, j) and (j, i), if both present, should be treated as duplicates.

The sparsity structure is assumed to be independent of the point x.

### Initialize

Before calling this function, you must call initialize with :Hess.

### **Example**

```
julia> evaluator = MOI.Test.HS071(true);
julia> MOI.initialize(evaluator, Symbol[:Hess])
julia> MOI.hessian_constraint_structure(evaluator, 1)
6-element Vector{Tuple{Int64, Int64}}:
(2, 1)
(3, 1)
(3, 2)
(4, 1)
(4, 2)
(4, 3)
julia> MOI.hessian_constraint_structure(evaluator, 2)
4-element Vector{Tuple{Int64, Int64}}:
(1, 1)
(2, 2)
(3, 3)
(4, 4)
```

MathOptInterface.eval\_hessian\_objective - Function.

```
eval_hessian_objective(
    d::AbstractNLPEvaluator,
    H::AbstractVector{T},
    x::AbstractVector{T},
)::Nothing where {T}
```

This function computes the sparse Hessian matrix:  $\nabla^2 f(x)$ , storing the result in the vector H in the same order as the indices returned by hessian\_objective\_structure.

# Implementation notes

When implementing this method, you must not assume that H is Vector{Float64}, but you may assume that it supports setindex! and length. For example, it may be the view of a vector.

# Initialize

Before calling this function, you must call initialize with :Hess.

### **Example**

```
julia> evaluator = MOI.Test.HS071(true, true);
julia> MOI.initialize(evaluator, Symbol[:Hess])
julia> indices = MOI.hessian_objective_structure(evaluator);
julia> H = zeros(length(indices));
julia> x = [1.0, 2.0, 3.0, 4.0];
```

```
julia> MOI.eval_hessian_objective(evaluator, H, x)

julia> H
6-element Vector{Float64}:
8.0
4.0
4.0
7.0
1.0
1.0
```

 ${\tt MathOptInterface.eval\_hessian\_constraint-Function}.$ 

```
eval_hessian_constraint(
    d::AbstractNLPEvaluator,
    H::AbstractVector{T},
    x::AbstractVector{T},
    i::Int64,
)::Nothing where {T}
```

This function computes the sparse Hessian matrix:  $\nabla^2 g_i(x)$ , storing the result in the vector H in the same order as the indices returned by hessian\_constraint\_structure.

# Implementation notes

When implementing this method, you must not assume that H is Vector{Float64}, but you may assume that it supports setindex! and length. For example, it may be the view of a vector.

### Initialize

Before calling this function, you must call initialize with :Hess.

### **Example**

```
julia> evaluator = MOI.Test.HS071(true, true);

julia> MOI.initialize(evaluator, Symbol[:Hess])

julia> indices = MOI.hessian_constraint_structure(evaluator, 1);

julia> H = zeros(length(indices));

julia> x = [1.0, 2.0, 3.0, 4.0];

julia> MOI.eval_hessian_constraint(evaluator, H, x, 1)

julia> H
6-element Vector{Float64}:
12.0
8.0
```

```
4.0
6.0
3.0
2.0
```

source

 ${\tt MathOptInterface.eval\_hessian\_lagrangian-Function}.$ 

Given scalar weight  $\sigma$  and vector of constraint weights  $\mu$ , this function computes the sparse Hessian-of-the-Lagrangian matrix:  $\sigma \nabla^2 f(x) + \sum_{i=1}^m \mu_i \nabla^2 g_i(x)$ , storing the result in the vector H in the same order as the indices returned by hessian\_lagrangian\_structure.

#### Implementation notes

When implementing this method, you must not assume that H is Vector{Float64}, but you may assume that it supports setindex! and length. For example, it may be the view of a vector.

#### Initialize

Before calling this function, you must call initialize with :Hess.

#### **Example**

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true);
julia> MOI.initialize(evaluator, Symbol[:Hess])
julia> indices = MOI.hessian_lagrangian_structure(evaluator);
julia> H = zeros(length(indices));
julia> x = [1.0, 2.0, 3.0, 4.0];
julia> σ = 1.0;
julia> μ = [1.0, 1.0];
julia> MOI.eval_hessian_lagrangian(evaluator, H, x, σ, μ)
julia> H
10-element Vector{Float64}:
10.0
16.0
```

```
2.0

4.0

2.0

13.0

4.0

3.0

2.0
```

source

MathOptInterface.eval\_hessian\_lagrangian\_product - Function.

```
eval_hessian_lagrangian_product(
    d::AbstractNLPEvaluator,
    h::AbstractVector{T},
    x::AbstractVector{T},
    v::AbstractVector{T},
    p::AbstractVector{T},
    p::Nothing where {T}
```

Given scalar weight  $\sigma$  and vector of constraint weights  $\mu$ , computes the Hessian-of-the-Lagrangian-vector product  $h = \left(\sigma \nabla^2 f(x) + \sum_{i=1}^m \mu_i \nabla^2 g_i(x)\right) v$ , storing the result in the vector  $\mathbf{h}$ .

The vectors have dimensions such that length(h) == length(x) == length(v).

#### Implementation notes

When implementing this method, you must not assume that h is Vector{Float64}, but you may assume that it supports setindex! and length. For example, it may be the view of a vector.

#### Initialize

Before calling this function, you must call initialize with :HessVec.

### Example

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true, true);

julia> MOI.initialize(evaluator, Symbol[:HessVec])

julia> H = fill(NaN, 4);

julia> x = [1.0, 2.0, 3.0, 4.0];

julia> v = [1.5, 2.5, 3.5, 4.5];

julia> σ = 1.0;

julia> μ = [1.0, 1.0];

julia> MOI.eval_hessian_lagrangian_product(evaluator, H, x, v, σ, μ)
```

```
julia> H
4-element Vector{Float64}:
155.5
61.0
48.5
49.0
```

source

MathOptInterface.objective\_expr - Function.

```
objective_expr(d::AbstractNLPEvaluator)::Expr
```

Returns a Julia Expr object representing the expression graph of the objective function.

#### **Format**

The expression has a number of limitations, compared with arbitrary Julia expressions:

- All sums and products are flattened out as simple Expr(:+, ...) and Expr(:\*, ...) objects.
- All decision variables must be of the form Expr(:ref, :x, MOI.VariableIndex(i)), where i is the *i*th variable in ListOfVariableIndices.
- There are currently no restrictions on recognized functions; typically these will be built-in Julia functions like ^, exp, log, cos, tan, sqrt, etc., but modeling interfaces may choose to extend these basic functions, or error if they encounter unsupported functions.

#### Initialize

Before calling this function, you must call initialize with :ExprGraph.

### Example

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true);

julia> MOI.initialize(evaluator, [:ExprGraph])

julia> MOI.objective_expr(evaluator)
:(x[MOI.VariableIndex(1)] * x[MOI.VariableIndex(4)] * (x[MOI.VariableIndex(1)] +

\to x[MOI.VariableIndex(2)] + x[MOI.VariableIndex(3)]) + x[MOI.VariableIndex(3)])
```

source

MathOptInterface.constraint\_expr - Function.

```
constraint_expr(d::AbstractNLPEvaluator, i::Integer)::Expr
```

Returns a Julia Expr object representing the expression graph for the ith nonlinear constraint.

#### **Format**

The format is the same as objective\_expr, with an additional comparison operator indicating the sense of and bounds on the constraint.

For single-sided comparisons, the body of the constraint must be on the left-hand side, and the right-hand side must be a constant.

For double-sided comparisons (that is,  $l \leq g(x) \leq u$ ), the body of the constraint must be in the middle, and the left- and right-hand sides must be constants.

The bounds on the constraints must match the NLPBoundsPairs passed to NLPBlockData.

#### Initialize

Before calling this function, you must call initialize with :ExprGraph.

#### **Example**

This example uses the Test. HS071 evaluator.

```
julia> evaluator = MOI.Test.HS071(true);

julia> MOI.initialize(evaluator, [:ExprGraph])

julia> MOI.constraint_expr(evaluator, 1)
:(x[MOI.VariableIndex(1)] * x[MOI.VariableIndex(2)] * x[MOI.VariableIndex(3)] *

$\to x[MOI.VariableIndex(4)] >= 25.0)

julia> MOI.constraint_expr(evaluator, 2)
:(x[MOI.VariableIndex(1)] ^ 2 + x[MOI.VariableIndex(2)] ^ 2 + x[MOI.VariableIndex(3)] ^ 2 +

$\to x[MOI.VariableIndex(4)] ^ 2 == 40.0)
```

# **Chapter 24**

# **Callbacks**

MathOptInterface.AbstractCallback - Type.

```
abstract type AbstractCallback <: AbstractModelAttribute end</pre>
```

Abstract type for a model attribute representing a callback function. The value set to subtypes of AbstractCallback is a function that may be called during optimize!. As optimize! is in progress, the result attributes (that is, the attributes attr such that is\_set\_by\_optimize(attr)) may not be accessible from the callback, hence trying to get result attributes might throw a OptimizeInProgress error.

At most one callback of each type can be registered. If an optimizer already has a function for a callback type, and the user registers a new function, then the old one is replaced.

The value of the attribute should be a function taking only one argument, commonly called callback\_data, that can be used for instance in LazyConstraintCallback, HeuristicCallback and UserCutCallback.

source

MathOptInterface.AbstractSubmittable - Type.

```
AbstractSubmittable
```

Abstract supertype for objects that can be submitted to the model.

source

 ${\tt MathOptInterface.submit-Function}.$ 

```
submit(
    optimizer::AbstractOptimizer,
    sub::AbstractSubmittable,
    values...,
)::Nothing
```

Submit values to the submittable sub of the optimizer optimizer.

An UnsupportedSubmittable error is thrown if model does not support the attribute attr (see supports) and a SubmitNotAllowed error is thrown if it supports the submittable sub but it cannot be submitted.

#### 24.1 Attributes

MathOptInterface.CallbackNodeStatus - Type.

CallbackNodeStatus(callback\_data)

An optimizer attribute describing the (in)feasibility of the primal solution available from CallbackVariablePrimal during a callback identified by callback\_data.

Returns a CallbackNodeStatusCode Enum.

source

MathOptInterface.CallbackVariablePrimal - Type.

CallbackVariablePrimal(callback data)

An AbstractVariableAttribute for the assignment to the variable's primal value during the callback identified by callback\_data.

source

MathOptInterface.CallbackNodeStatusCode - Type.

CallbackNodeStatusCode

An Enum for the value of the CallbackNodeStatus attribute.

#### **Values**

CALLBACK\_NODE\_STATUS\_INTEGER

The primal solution available from CallbackVariablePrimal is integer feasible.

CALLBACK\_NODE\_STATUS\_FRACTIONAL

The primal solution available from CallbackVariablePrimal is integer infeasible.

CALLBACK\_NODE\_STATUS\_UNKNOWN

The status of the primal solution available from CallbackVariablePrimal is unknown.

source

MathOptInterface.CALLBACK NODE STATUS INTEGER - Constant.

 ${\tt CALLBACK\_NODE\_STATUS\_INTEGER::CallbackNodeStatusCode}$ 

An instance of the CallbackNodeStatusCode enum.

#### **About**

The primal solution available from CallbackVariablePrimal is integer feasible.

 ${\tt MathOptInterface.CALLBACK\_NODE\_STATUS\_FRACTIONAL-Constant}.$ 

```
CALLBACK_NODE_STATUS_FRACTIONAL::CallbackNodeStatusCode
```

An instance of the CallbackNodeStatusCode enum.

#### **About**

The primal solution available from CallbackVariablePrimal is integer infeasible.

source

MathOptInterface.CALLBACK\_NODE\_STATUS\_UNKNOWN - Constant.

```
CALLBACK_NODE_STATUS_UNKNOWN::CallbackNodeStatusCode
```

An instance of the CallbackNodeStatusCode enum.

#### **About**

The status of the primal solution available from CallbackVariablePrimal is unknown.

source

# 24.2 Lazy constraints

 ${\tt MathOptInterface.LazyConstraintCallback-Type.}\\$ 

```
LazyConstraintCallback() <: AbstractCallback
```

The callback can be used to reduce the feasible set given the current primal solution by submitting a LazyConstraint. For instance, it may be called at an incumbent of a mixed-integer problem. Note that there is no guarantee that the callback is called at every feasible primal solution.

The current primal solution is accessed through CallbackVariablePrimal. Trying to access other result attributes will throw OptimizeInProgress as discussed in AbstractCallback.

#### **Example**

```
x = MOI.add_variables(optimizer, 8)
MOI.set(optimizer, MOI.LazyConstraintCallback(), callback_data -> begin
    sol = MOI.get(optimizer, MOI.CallbackVariablePrimal(callback_data), x)
    if # should add a lazy constraint
        func = # computes function
        set = # computes set
        MOI.submit(optimizer, MOI.LazyConstraint(callback_data), func, set)
    end
end)
```

source

MathOptInterface.LazyConstraint - Type.

```
LazyConstraint(callback_data)
```

Lazy constraint func-in-set submitted as func, set. The optimal solution returned by VariablePrimal will satisfy all lazy constraints that have been submitted.

This can be submitted only from the LazyConstraintCallback. The field callback\_data is a solver-specific callback type that is passed as the argument to the feasible solution callback.

#### **Example**

Suppose x and y are VariableIndexs of optimizer. To add a LazyConstraint for  $2x + 3y \le 1$ , write

```
func = 2.0x + 3.0y
set = MOI.LessThan(1.0)
MOI.submit(optimizer, MOI.LazyConstraint(callback_data), func, set)
```

inside a LazyConstraintCallback of data callback\_data.

source

#### 24.3 User cuts

MathOptInterface.UserCutCallback - Type.

```
UserCutCallback() <: AbstractCallback
```

The callback can be used to submit <code>UserCut</code> given the current primal solution. For instance, it may be called at fractional (that is, non-integer) nodes in the branch and bound tree of a mixed-integer problem. Note that there is not guarantee that the callback is called everytime the solver has an infeasible solution.

The infeasible solution is accessed through CallbackVariablePrimal. Trying to access other result attributes will throw OptimizeInProgress as discussed in AbstractCallback.

#### **Example**

```
x = MOI.add_variables(optimizer, 8)
MOI.set(optimizer, MOI.UserCutCallback(), callback_data -> begin
    sol = MOI.get(optimizer, MOI.CallbackVariablePrimal(callback_data), x)
    if # can find a user cut
        func = # computes function
        set = # computes set
        MOI.submit(optimizer, MOI.UserCut(callback_data), func, set)
    end
end
```

source

MathOptInterface.UserCut - Type.

```
UserCut(callback_data)
```

Constraint func-to-set suggested to help the solver detect the solution given by CallbackVariablePrimal as infeasible. The cut is submitted as the tuple (func, set)::Tuple{MOI.AbstractFunction, MOI.AbstractSet}.

Typically CallbackVariablePrimal will violate integrality constraints, and a cut would be of the form ScalarAffineFunction-in-LessThan or ScalarAffineFunction-in-GreaterThan.

Note that, as opposed to LazyConstraint, the provided constraint must not modify the feasible set. The constraint should be redundant, for example, it may be a consequence of affine and integrality constraints.

This can be submitted only from the UserCutCallback. The field callback\_data is a solver-specific callback type that is passed as the argument to the infeasible solution callback.

Note that the solver may silently ignore the provided constraint.

source

#### 24.4 Heuristic solutions

MathOptInterface.HeuristicCallback - Type.

```
HeuristicCallback() <: AbstractCallback</pre>
```

The callback can be used to submit HeuristicSolution given the current primal solution. For example, it may be called at fractional (that is, non-integer) nodes in the branch and bound tree of a mixed-integer problem. Note that there is no guarantee that the callback is called every time the solver has an infeasible colution

The current primal solution is accessed through CallbackVariablePrimal. Trying to access other result attributes will throw OptimizeInProgress as discussed in AbstractCallback.

#### **Example**

source

MathOptInterface.HeuristicSolution - Type.

```
HeuristicSolution(callback_data)
```

Heuristically obtained feasible solution. The solution is submitted as (variables, values)::Tuple{Vector{MOI.VariableI} where values[i] gives the value of variables[i].

The submit call returns a HeuristicSolutionStatus indicating whether the provided solution was accepted or rejected.

This can be submitted only from the <a href="HeuristicCallback">HeuristicCallback</a>. The field callback\_data is a solver-specific callback type that is passed as the argument to the heuristic callback.

Some solvers require a complete solution, others only partial solutions.

source

MathOptInterface.HeuristicSolutionStatus - Type.

HeuristicSolutionStatus

An Enum of possible return values for submit with HeuristicSolution.

This status informs whether the heuristic solution was accepted or rejected.

#### **Values**

**HEURISTIC SOLUTION ACCEPTED** 

The heuristic solution was accepted.

HEURISTIC\_SOLUTION\_REJECTED

The heuristic solution was rejected.

HEURISTIC\_SOLUTION\_UNKNOWN

No information available on the acceptance.

source

MathOptInterface.HEURISTIC\_SOLUTION\_ACCEPTED - Constant.

HEURISTIC\_SOLUTION\_ACCEPTED::HeuristicSolutionStatus

An instance of the HeuristicSolutionStatus enum.

## About

The heuristic solution was accepted.

source

 ${\tt MathOptInterface.HEURISTIC\_SOLUTION\_REJECTED-Constant}.$ 

 ${\tt HEURISTIC\_SOLUTION\_REJECTED::} Heuristic Solution Status$ 

An instance of the HeuristicSolutionStatus enum.

#### **About**

The heuristic solution was rejected.

 ${\tt MathOptInterface.HEURISTIC\_SOLUTION\_UNKNOWN-Constant}.$ 

 $\label{thm:heuristicSolutionStatus} \mbox{HEURISTIC\_SOLUTION\_UNKNOWN::} \mbox{HeuristicSolutionStatus}$ 

An instance of the HeuristicSolutionStatus enum.

### **About**

No information available on the acceptance.

# **Chapter 25**

# **Errors**

When an MOI call fails on a model, precise errors should be thrown when possible instead of simply calling error with a message. The docstrings for the respective methods describe the errors that the implementation should throw in certain situations. This error-reporting system allows code to distinguish between internal errors (that should be shown to the user) and unsupported operations which may have automatic workarounds.

When an invalid index is used in an MOI call, an InvalidIndex is thrown:

MathOptInterface.InvalidIndex - Type.

```
struct InvalidIndex{IndexType<:Index} <: Exception
  index::IndexType
end</pre>
```

An error indicating that the index index is invalid.

source

When an invalid result index is used to retrieve an attribute, a ResultIndexBoundsError is thrown:

 ${\tt MathOptInterface.ResultIndexBoundsError-Type}.$ 

```
struct ResultIndexBoundsError{AttrType} <: Exception
   attr::AttrType
   result_count::Int
end</pre>
```

An error indicating that the requested attribute attr could not be retrieved, because the solver returned too few results compared to what was requested. For instance, the user tries to retrieve VariablePrimal(2) when only one solution is available, or when the model is infeasible and has no solution.

```
See also: check_result_index_bounds.
source
```

MathOptInterface.check\_result\_index\_bounds - Function.

```
check_result_index_bounds(model::ModelLike, attr)
```

This function checks whether enough results are available in the model for the requested attr, using its result\_index field. If the model does not have sufficient results to answer the query, it throws a ResultIndexBoundsError.

source

As discussed in JuMP mapping, for scalar constraint with a nonzero function constant, a Scalar Function Constant Not Zero exception may be thrown:

 ${\tt MathOptInterface.ScalarFunctionConstantNotZero-Type.}$ 

```
struct ScalarFunctionConstantNotZero{T, F, S} <: Exception
    constant::T
end</pre>
```

An error indicating that the constant part of the function in the constraint F-in-S is nonzero.

source

Some VariableIndex constraints cannot be combined on the same variable:

MathOptInterface.LowerBoundAlreadySet - Type.

```
LowerBoundAlreadySet{S1, S2}
```

Error thrown when setting a VariableIndex-in-S2 when a VariableIndex-in-S1 has already been added and the sets S1, S2 both set a lower bound, that is, they are EqualTo, GreaterThan, Interval, Semicontinuous or Semiinteger.

source

MathOptInterface.UpperBoundAlreadySet - Type.

```
UpperBoundAlreadySet{S1, S2}
```

Error thrown when setting a VariableIndex-in-S2 when a VariableIndex-in-S1 has already been added and the sets S1, S2 both set an upper bound, that is, they are EqualTo, LessThan, Interval, Semicontinuous or Semiinteger.

source

As discussed in AbstractCallback, trying to get attributes inside a callback may throw:

MathOptInterface.OptimizeInProgress - Type.

```
struct OptimizeInProgress{AttrType<:AnyAttribute} <: Exception
   attr::AttrType
end</pre>
```

Error thrown from optimizer when MOI.get(optimizer, attr) is called inside an AbstractCallback while it is only defined once optimize! has completed. This can only happen when is\_set\_by\_optimize(attr) is true.

Trying to submit the wrong type of AbstractSubmittable inside an AbstractCallback (for example, a UserCut inside a LazyConstraintCallback) will throw:

MathOptInterface.InvalidCallbackUsage - Type.

```
struct InvalidCallbackUsage{C, S} <: Exception
    callback::C
    submittable::S
end</pre>
```

An error indicating that submittable cannot be submitted inside callback.

For example, UserCut cannot be submitted inside LazyConstraintCallback.

source

The rest of the errors defined in MOI fall in two categories represented by the following two abstract types:

 ${\tt MathOptInterface.UnsupportedError-Type.}$ 

```
UnsupportedError <: Exception
```

Abstract type for error thrown when an element is not supported by the model.

source

MathOptInterface.NotAllowedError - Type.

```
NotAllowedError <: Exception
```

Abstract type for error thrown when an operation is supported but cannot be applied in the current state of the model.

source

The different UnsupportedError and NotAllowedError are the following errors:

MathOptInterface.UnsupportedAttribute - Type.

```
struct UnsupportedAttribute{AttrType} <: UnsupportedError
   attr::AttrType
   message::String
end</pre>
```

An error indicating that the attribute attr is not supported by the model, that is, that supports returns false.

source

MathOptInterface.GetAttributeNotAllowed - Type.

```
struct GetAttributeNotAllowed{AttrType} <: NotAllowedError
  attr::AttrType
  message::String
end</pre>
```

An error indicating that the attribute attr cannot be got for some reason (see the error string).

source

 ${\tt MathOptInterface.SetAttributeNotAllowed-Type}.$ 

```
struct SetAttributeNotAllowed{AttrType} <: NotAllowedError
  attr::AttrType
  message::String
end</pre>
```

An error indicating that the attribute attr is supported (see supports) but cannot be set for some reason (see the error string).

source

MathOptInterface.AddVariableNotAllowed - Type.

```
struct AddVariableNotAllowed <: NotAllowedError
  message::String # Human-friendly explanation why the attribute cannot be set
end</pre>
```

An error indicating that variables cannot be added to the model.

source

MathOptInterface.UnsupportedConstraint - Type.

```
struct UnsupportedConstraint{F<:AbstractFunction,S<:AbstractSet} <: UnsupportedError
   message::String
end</pre>
```

An error indicating that constraints of type F-in-S are not supported by the model, that is, that supports\_constraint returns false.

```
julia> showerror(stdout, MOI.UnsupportedConstraint{MOI.VariableIndex,MOI.ZeroOne}())
UnsupportedConstraint: `MathOptInterface.VariableIndex`-in-`MathOptInterface.ZeroOne`

→ constraints are not supported by the
solver you have chosen, and we could not reformulate your model into a
form that is supported.
To fix this error you must choose a different solver.
```

 ${\tt MathOptInterface.AddConstraintNotAllowed-Type.}$ 

```
struct AddConstraintNotAllowed{F<:AbstractFunction, S<:AbstractSet} <: NotAllowedError
   message::String # Human-friendly explanation why the attribute cannot be set
end</pre>
```

An error indicating that constraints of type F-in-S are supported (see supports\_constraint) but cannot be added.

source

MathOptInterface.ModifyConstraintNotAllowed - Type.

An error indicating that the constraint modification change cannot be applied to the constraint of index ci.

source

MathOptInterface.ModifyObjectiveNotAllowed - Type.

```
struct ModifyObjectiveNotAllowed{C<:AbstractFunctionModification} <: NotAllowedError
    change::C
    message::String
end</pre>
```

An error indicating that the objective modification change cannot be applied to the objective.

source

 ${\tt MathOptInterface.DeleteNotAllowed-Type}.$ 

```
struct DeleteNotAllowed{IndexType <: Index} <: NotAllowedError
  index::IndexType
  message::String
end</pre>
```

An error indicating that the index index cannot be deleted.

source

MathOptInterface.UnsupportedSubmittable - Type.

```
struct UnsupportedSubmitTable{SubmitType} <: UnsupportedError
    sub::SubmitType
    message::String
end</pre>
```

An error indicating that the submittable sub is not supported by the model, that is, that supports returns false.

source

MathOptInterface.SubmitNotAllowed - Type.

```
struct SubmitNotAllowed{SubmitTyp<:AbstractSubmittable} <: NotAllowedError
    sub::SubmitType
    message::String
end</pre>
```

An error indicating that the submittable sub is supported (see supports) but cannot be added for some reason (see the error string).

source

MathOptInterface.UnsupportedNonlinearOperator - Type.

```
UnsupportedNonlinearOperator(head::Symbol[, message::String]) <: UnsupportedError
```

An error thrown by optimizers if they do not support the operator head in a ScalarNonlinearFunction.

#### **Example**

source

Note that setting the ConstraintFunction of a VariableIndex constraint is not allowed:

 ${\tt MathOptInterface.SettingVariableIndexNotAllowed-Type}.$ 

```
SettingVariableIndexNotAllowed()
```

Error type that should be thrown when the user calls set to change the ConstraintFunction of a VariableIndex constraint.

# Part VI

# **Submodules**

# **Chapter 26**

# **Benchmarks**

#### 26.1 Overview

#### The Benchmarks submodule

To aid the development of efficient solver wrappers, MathOptInterface provides benchmarking capability. Benchmarking a wrapper follows a two-step process.

First, prior to making changes, create a baseline for the benchmark results on a given benchmark suite as follows:

```
using SolverPackage # Replace with your choice of solver.
import MathOptInterface as MOI

suite = MOI.Benchmarks.suite() do
    SolverPackage.Optimizer()
end

MOI.Benchmarks.create_baseline(
    suite, "current"; directory = "/tmp", verbose = true
)
```

Use the exclude argument to Benchmarks.suite to exclude benchmarks that the solver doesn't support.

Second, after making changes to the package, re-run the benchmark suite and compare to the prior saved results:

```
using SolverPackage
import MathOptInterface as MOI

suite = MOI.Benchmarks.suite() do
    SolverPackage.Optimizer()
end

MOI.Benchmarks.compare_against_baseline(
    suite, "current"; directory = "/tmp", verbose = true
)
```

This comparison will create a report detailing improvements and regressions.

#### 26.2 API Reference

#### **Benchmarks**

Functions to help benchmark the performance of solver wrappers. See The Benchmarks submodule for more details.

MathOptInterface.Benchmarks.suite - Function.

```
suite(
   new_model::Function;
   exclude::Vector{Regex} = Regex[]
)
```

Create a suite of benchmarks. new\_model should be a function that takes no arguments, and returns a new instance of the optimizer you wish to benchmark.

Use exclude to exclude a subset of benchmarks.

#### **Example**

source

MathOptInterface.Benchmarks.create\_baseline - Function.

```
create_baseline(suite, name::String; directory::String = ""; kwargs...)
```

Run all benchmarks in suite and save to files called name in directory.

 ${\bf Extra\ kwargs\ are\ based\ to\ Benchmark Tools.run.}$ 

#### **Example**

MathOptInterface.Benchmarks.compare\_against\_baseline - Function.

```
compare_against_baseline(
   suite, name::String; directory::String = "",
   report_filename::String = "report.txt"
)
```

Run all benchmarks in suite and compare against files called name in directory that were created by a call to create\_baseline.

A report summarizing the comparison is written to report\_filename in directory.

Extra kwargs are based to BenchmarkTools.run.

### **Example**

# **Chapter 27**

# **Bridges**

#### 27.1 Overview

### The Bridges submodule

The Bridges module simplifies the process of converting models between equivalent formulations.

Tip

Read our paper for more details on how bridges are implemented.

# Why bridges?

A constraint can often be written in a number of equivalent formulations. For example, the constraint  $l \leq a^\top x \leq u$  (ScalarAffineFunction-in-Interval) could be re-formulated as two constraints:  $a^\top x \geq l$  (ScalarAffineFunction-in-GreaterThan) and  $a^\top x \leq u$  (ScalarAffineFunction-in-LessThan). An alternative re-formulation is to add a dummy variable y with the constraints  $l \leq y \leq u$  (VariableIndex-in-Interval) and  $a^\top x - y = 0$  (ScalarAffineFunction-in-EqualTo).

To avoid each solver having to code these transformations manually, MathOptInterface provides bridges.

A bridge is a small transformation from one constraint type to another (potentially collection of) constraint type.

Because these bridges are included in MathOptInterface, they can be re-used by any optimizer. Some bridges also implement constraint modifications and constraint primal and dual translations.

Several bridges can be used in combination to transform a single constraint into a form that the solver may understand. Choosing the bridges to use takes the form of finding a shortest path in the hyper-graph of bridges. The methodology is detailed in the MOI paper.

#### The three types of bridges

There are three types of bridges in MathOptInterface:

- 1. Constraint bridges
- 2. Variable bridges
- 3. Objective bridges

#### **Constraint bridges**

Constraint bridges convert constraints formulated by the user into an equivalent form supported by the solver. Constraint bridges are subtypes of Bridges. Constraint. AbstractBridge.

The equivalent formulation may add constraints (and possibly also variables) in the underlying model.

In particular, constraint bridges can focus on rewriting the function of a constraint, and do not change the set. Function bridges are subtypes of Bridges.Constraint.AbstractFunctionConversionBridge.

Read the list of implemented constraint bridges for more details on the types of transformations that are available. Function bridges are Bridges. Constraint. Scalar Functionize Bridge and Bridges. Constraint. Vector Functionize Bridges.

#### Variable bridges

Variable bridges convert variables added by the user, either free with add\_variable/add\_variables, or constrained with add\_constrained\_variable/add\_constrained\_variables, into an equivalent form supported by the solver. Variable bridges are subtypes of Bridges.Variable.AbstractBridge.

The equivalent formulation may add constraints (and possibly also variables) in the underlying model.

Read the list of implemented variable bridges for more details on the types of transformations that are available.

#### **Objective bridges**

Objective bridges convert the ObjectiveFunction set by the user into an equivalent form supported by the solver. Objective bridges are subtypes of Bridges.Objective.AbstractBridge.

The equivalent formulation may add constraints (and possibly also variables) in the underlying model.

Read the list of implemented objective bridges for more details on the types of transformations that are available.

# ${\bf Bridges.full\_bridge\_optimizer}$

Tip

Unless you have an advanced use-case, this is probably the only function you need to care about.

 $To enable the full power of MathOptInterface's bridges, wrap an optimizer in a {\tt Bridges.full\_bridge\_optimizer}.$ 

```
julia> inner_optimizer = MOI.Utilities.Model{Float64}()
MOIU.Model{Float64}
- ObjectiveSense: FEASIBILITY SENSE
- ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
- NumberOfVariables: 0
L NumberOfConstraints: 0
julia> optimizer = MOI.Bridges.full bridge optimizer(inner optimizer, Float64)
MOIB.LazyBridgeOptimizer{MOIU.Model{Float64}}
- Variable bridges: none
- Constraint bridges: none
- Objective bridges: none
L model: MOIU.Model{Float64}
  - ObjectiveSense: FEASIBILITY SENSE
  - ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
  - NumberOfVariables: 0
  L NumberOfConstraints: 0
```

Now, use optimizer as normal, and bridging will happen lazily behind the scenes. By lazily, we mean that bridging will happen if and only if the constraint is not supported by the inner\_optimizer.

#### Info

Most bridges are added by default in Bridges.full\_bridge\_optimizer. However, for technical reasons, some bridges are not added by default. Three examples include Bridges.Constraint.SOCtoPSDBridge, Bridges.Constraint.SOCtoNonConvexQuadBridge and Bridges.Constraint.RSOCtoNonConvexQuadBridge. See the docs of those bridges for more information.

#### Add a single bridge

If you don't want to use Bridges.full\_bridge\_optimizer, you can wrap an optimizer in a single bridge.

However, this will force the constraint to be bridged, even if the inner\_optimizer supports it.

#### Bridges.LazyBridgeOptimizer

If you don't want to use Bridges.full\_bridge\_optimizer, but you need more than a single bridge (or you want the bridging to happen lazily), you can manually construct a Bridges.LazyBridgeOptimizer.

First, wrap an inner optimizer:

```
| Constraint bridges: none
| Objective bridges: none
| model: MOIU.Model{Float64}
| ObjectiveSense: FEASIBILITY_SENSE
| ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
| NumberOfConstraints: 0
```

Then use Bridges.add bridge to add individual bridges:

```
julia> MOI.Bridges.add_bridge(optimizer, MOI.Bridges.Constraint.SplitIntervalBridge{Float64})

julia> MOI.Bridges.add_bridge(optimizer, MOI.Bridges.Objective.FunctionizeBridge{Float64})
```

Now the constraints will be bridged only if needed:

#### 27.2 Implementation

#### Implementing a bridge

The easiest way to implement a bridge is to follow an existing example. There are three locations of bridges in the source code:

- Constraint bridges are stored in src/Bridges/Constraint/bridges
- Objective bridges are stored in src/Bridges/Objective/bridges
- Variable bridges are stored in src/Bridges/Variable/bridges

The Implementing a constraint bridge tutorial has a more detailed guide on what is required to implement a bridge.

When opening a pull request that adds a new bridge, use the checklist Adding a new bridge.

If you need help or advice, please contact the Developer Chatroom.

#### SetMap bridges

For constraint and variable bridges, a common reformulation is that  $f(x) \in F$  is reformulated to  $g(x) \in G$ . In this case, no additional variables and constraints are added, and the bridge needs only a way to map between the functions f and g and the sets F and G.

To implementation a bridge of this form, subtype the abstract type Bridges.Constraint.SetMapBridge or Bridges.Variable.SetMapBridge and implement the API described in the docstring of each type.

#### final\_touch

Some bridges require information from other parts of the model. One set of examples are the various combinatorial ToMILP bridges, such as Bridges.Constraint.SOS1ToMILPBridge, which require knowledge of the variable bounds.

Bridges requiring information from other parts of the model should implement Bridges.final\_touch and Bridges.needs final touch.

During the bridge's construction, store the function and set and make no changes to the underlying model. Then, in <a href="mailtouch">Bridges.final\_touch</a>, query the additional information and add the reformulated problem to the model.

When implementing, you must consider that:

- Bridges.final\_touch may be called multiple times, so that your reformulation should be applied only if necessary. Sometimes the additional data will be the same, and sometimes it may be different.
- We do not currently support final\_touch bridges that introduce constraints which also require a final\_touch bridge. Therefore, you should implement final\_touch only if necessary, and we recommend that you contact the Developer Chatroom for advice before doing so.

#### **Testing**

Use the Bridges.runtests function to test a bridge. It takes three arguments: the type of the bridge, the input model as a string, and the output model as a string.

Here is an example:

There are a number of other useful keyword arguments.

• eltype can be used to specify the element type of the model (and bridge). It defaults to Float64.

variable\_start and constraint\_start are used as the values to set the VariablePrimalStart and
ConstraintPrimalStart attributes to. They default to 1.2. If you use a different eltype, you must
set appropriate starting values of the same type. The default 1.2 was chosen to minimize the risk that
the starting point is undefined, which could happen for common situations like 0.0 and 1.0. The tests
associated with the starting values do not necessarily check for correctness, only that they can be set
and get to produce the same result.

• print\_inner\_model can be used to print the reformulated output model from the bridge. This is especially helpful during debugging to see what the bridge is doing, and to spot mistakes. It defaults to false.

Here is an example:

```
julia> MOI.Bridges.runtests(
           {\tt MOI.Bridges.Constraint.GreaterToLessBridge,}
           variables: x
          x >= 1
           variables: x
          ::Int: -1 * x <= -1
          ....;
           eltype = Int,
           print_inner_model = true,
           variable_start = 2,
           constraint_start = 2,
      )
Feasibility
Subject to:
ScalarAffineFunction{Int64}-in-LessThan{Int64}
(0) - (1) \times <= (-1)
Test Summary: | Pass Total Time
Bridges.runtests | 29 29 0.0s
```

# 27.3 List of bridges

#### List of bridges

This section describes the Bridges. AbstractBridges that are implemented in MathOptInterface.

# **Constraint bridges**

These bridges are subtypes of Bridges.Constraint.AbstractBridge.

MathOptInterface.Bridges.Constraint.AbstractFunctionConversionBridge - Type.

```
abstract type AbstractFunctionConversionBridge{F,S} <: AbstractBridge end</pre>
```

Abstract type to support writing bridges in which the function changes but the set does not.

By convention, the transformed function is stored in the .constraint field.

source

MathOptInterface.Bridges.Constraint.AbstractToIntervalBridge - Type.

```
AbstractToIntervalBridge{T<: AbstractFloat, S, F}
```

An abstract type that simplifies the creation of other bridges.

#### Warning

T must be a AbstractFloat type because otherwise typemin and typemax would either be not implemented (for example, BigInt), or would not give infinite value (for example, Int). For this reason, this bridge is only added to MOI.Bridges.full\_bridge\_optimizer when T is a subtype of AbstractFloat.

source

MathOptInterface.Bridges.Constraint.AllDifferentToCountDistinctBridge - Type.

```
AllDifferentToCountDistinctBridge{T,F} <: Bridges.Constraint.AbstractBridge
```

AllDifferentToCountDistinctBridge implements the following reformulations:

- $x \in \mathsf{AllDifferent}(d)$  to  $(n,x) \in \mathsf{CountDistinct}(1+d)$  and n=d
- $f(x) \in AllDifferent(d)$  to  $(d, f(x)) \in CountDistinct(1+d)$

#### Source node

AllDifferentToCountDistinctBridge supports:

• Fin MOI.AllDifferent

where F is MOI. VectorOfVariables or MOI. VectorAffineFunction{T}.

#### **Target nodes**

AllDifferentToCountDistinctBridge creates:

- Fin MOI.CountDistinct
- MOI.VariableIndex in MOI.EqualTo{T}

source

 ${\tt MathOptInterface.Bridges.Constraint.BinPackingToMILPBridge-Type.}$ 

```
{\tt BinPackingToMILPBridge\{T,F\}} \; <: \; {\tt Bridges.Constraint.AbstractBridge}
```

BinPackingToMILPBridge implements the following reformulation:

•  $x \in BinPacking(c, w)$  into a mixed-integer linear program.

#### Reformulation

The reformulation is non-trivial, and it depends on the finite domain of each variable  $x_i$ , which we as define  $S_i = \{l_i, \dots, u_i\}$ .

First, we introduce new binary variables  $z_{ij}$ , which are 1 if variable  $x_i$  takes the value j in the optimal solution and 0 otherwise:

$$z_{ij} \in \{0, 1\} \quad \forall i \in 1 \dots d, j \in S_i$$
$$x_i - \sum_{j \in S_i} j \cdot z_{ij} = 0 \quad \forall i \in 1 \dots d$$
$$\sum_{j \in S_i} z_{ij} = 1 \quad \forall i \in 1 \dots d$$

Then, we add the capacity constraint for all possible bins j:

$$\sum_{i} w_i z_{ij} \le c \forall j \in \bigcup_{i=1,\dots,d} S_i$$

#### Source node

BinPackingToMILPBridge supports:

• Fin MOI.BinPacking{T}

### **Target nodes**

BinPackingToMILPBridge creates:

- MOI.VariableIndex in MOI.ZeroOne
- MOI.ScalarAffineFunction{T} in MOI.EqualTo{T}
- MOI.ScalarAffineFunction{T} in MOI.LessThan{T}

source

 ${\tt MathOptInterface.Bridges.Constraint.CircuitToMILPBridge-Type.}$ 

```
CircuitToMILPBridge{T,F} <: Bridges.Constraint.AbstractBridge</pre>
```

CircuitToMILPBridge implements the following reformulation:

x ∈ Circuit(d) to the Miller-Tucker-Zemlin formulation of the Traveling Salesperson Problem.

#### Source node

 ${\tt CircuitToMILPBridge\ supports:}$ 

• Fin MOI.Circuit

where F is MOI. VectorOfVariables or MOI. VectorAffineFunction{T}.

#### **Target nodes**

CircuitToMILPBridge creates:

- MOI.VariableIndex in MOI.ZeroOne
- MOI.VariableIndex in MOI.Integer
- MOI.VariableIndex in MOI.Interval{T}
- MOI.ScalarAffineFunction{T} in MOI.EqualTo{T}
- MOI.ScalarAffineFunction{T} in MOI.LessThan{T}

source

MathOptInterface.Bridges.Constraint.ComplexNormInfinityToSecondOrderConeBridge - Type.

```
ComplexNormInfinityToSecondOrderConeBridge\{T\} <: Bridges.Constraint.AbstractBridge\{T\} <: Bridges.Constraint.AbstractBridge(T) <: Bridges.Constraint.AbstractBridge(T
```

ComplexNormInfinityToSecondOrderConeBridge implements the following reformulation:

•  $(t,x) \in NormInfinity(1+d)$  into  $(t,real(x_i),imag(x_i)) \in SecondOrderCone()$  for all i.

#### Source node

ComplexNormInfinityToSecondOrderConeBridge supports:

• MOI.VectorAffineFunction{Complex{T}} in MOI.NormInfinityCone

# **Target nodes**

 ${\tt ComplexNormInfinityToSecondOrderConeBridge\ creates:}$ 

• MOI. VectorAffineFunction{T} in MOI. SecondOrderCone

source

 ${\tt MathOptInterface.Bridges.Constraint.CountAtLeastToCountBelongsBridge-Type.}$ 

```
CountAtLeastToCountBelongsBridge{T,F} <: Bridges.Constraint.AbstractBridge
```

CountAtLeastToCountBelongsBridge implements the following reformulation:

•  $x \in \text{CountAtLeast}(n, d, \mathcal{S})$  to  $(n_i, x_{d_i}) \in \text{CountBelongs}(1 + d, \mathcal{S})$  and  $n_i \geq n$  for all i.

#### Source node

CountAtLeastToCountBelongsBridge supports:

• Fin MOI.CountAtLeast

where F is MOI. VectorOfVariables or MOI. VectorAffineFunction{T}.

#### **Target nodes**

CountAtLeastToCountBelongsBridge creates:

- Fin MOI.CountBelongs
- MOI.VariableIndex in MOI.GreaterThan{T}

source

 ${\tt MathOptInterface.Bridges.Constraint.CountBelongsToMILPBridge-Type.}$ 

CountBelongsToMILPBridge{T,F} <: Bridges.Constraint.AbstractBridge

CountBelongsToMILPBridge implements the following reformulation:

-  $(n,x) \in \mathsf{CountBelongs}(1+d,\mathcal{S})$  into a mixed-integer linear program.

#### Reformulation

The reformulation is non-trivial, and it depends on the finite domain of each variable  $x_i$ , which we as define  $S_i = \{l_i, \dots, u_i\}$ .

First, we introduce new binary variables  $z_{ij}$ , which are 1 if variable  $x_i$  takes the value j in the optimal solution and 0 otherwise:

$$z_{ij} \in \{0, 1\} \quad \forall i \in 1 \dots d, j \in S_i$$
$$x_i - \sum_{j \in S_i} j \cdot z_{ij} = 0 \quad \forall i \in 1 \dots d$$
$$\sum_{j \in S_i} z_{ij} = 1 \quad \forall i \in 1 \dots d$$

Finally, n is constrained to be the number of  $z_{ij}$  elements that are in  $\mathcal{S}$ :

$$n - \sum_{i \in 1...d, j \in \mathcal{S}} z_{ij} = 0$$

#### Source node

CountBelongsToMILPBridge supports:

• Fin MOI.CountBelongs

where F is MOI. VectorOfVariables or MOI. VectorAffineFunction  $\{T\}$ .

## **Target nodes**

CountBelongsToMILPBridge creates:

• MOI. VariableIndex in MOI. ZeroOne

• MOI.ScalarAffineFunction{T} in MOI.EqualTo{T}

source

MathOptInterface.Bridges.Constraint.CountDistinctToMILPBridge - Type.

CountDistinctToMILPBridge{T,F} <: Bridges.Constraint.AbstractBridge</pre>

CountDistinctToMILPBridge implements the following reformulation:

•  $(n,x) \in CountDistinct(1+d)$  into a mixed-integer linear program.

#### Reformulation

The reformulation is non-trivial, and it depends on the finite domain of each variable  $x_i$ , which we as define  $S_i = \{l_i, \dots, u_i\}$ .

First, we introduce new binary variables  $z_{ij}$ , which are 1 if variable  $x_i$  takes the value j in the optimal solution and 0 otherwise:

$$z_{ij} \in \{0,1\} \quad \forall i \in 1 \dots d, j \in S_i$$
$$x_i - \sum_{j \in S_i} j \cdot z_{ij} = 0 \quad \forall i \in 1 \dots d$$
$$\sum_{j \in S_i} z_{ij} = 1 \quad \forall i \in 1 \dots d$$

Then, we introduce new binary variables  $y_j$ , which are 1 if a variable takes the value j in the optimal solution and 0 otherwise.

$$y_j \in \{0, 1\} \ \forall j \in \bigcup_{i=1, \dots, d} S_i$$
$$y_j \le \sum_{i \in 1 \dots d: j \in S_i} z_{ij} \le M y_j \ \forall j \in \bigcup_{i=1, \dots, d} S_i$$

Finally, n is constrained to be the number of  $y_j$  elements that are non-zero:

$$n - \sum_{j \in \bigcup_{i=1,\dots,d} S_i} y_j = 0$$

#### Formulation (special case)

In the special case that the constraint is [2, x, y] in CountDistinct(3), then the constraint is equivalent to [x, y] in AllDifferent(2), which is equivalent to x = y.

$$(x - y \le -1) \lor (y - x \le -1)$$

which is equivalent to (for suitable M):

$$z \in \{0, 1\}$$
$$x - y - Mz \le -1$$
$$y - x - M(1 - z) \le -1$$

#### Source node

CountDistinctToMILPBridge supports:

• Fin MOI.CountDistinct

where F is MOI. VectorOfVariables or MOI. VectorAffineFunction  $\{T\}$ .

#### **Target nodes**

CountDistinctToMILPBridge creates:

- MOI. VariableIndex in MOI. ZeroOne
- MOI.ScalarAffineFunction{T} in MOI.EqualTo{T}
- MOI.ScalarAffineFunction{T} in MOI.LessThan{T}

source

MathOptInterface.Bridges.Constraint.CountGreaterThanToMILPBridge - Type.

CountGreaterThanToMILPBridge{T,F} <: Bridges.Constraint.AbstractBridge</pre>

CountGreaterThanToMILPBridge implements the following reformulation:

•  $(c, y, x) \in CountGreaterThan()$  into a mixed-integer linear program.

### Source node

CountGreaterThanToMILPBridge supports:

• Fin MOI.CountGreaterThan

# Target nodes

CountGreaterThanToMILPBridge creates:

- MOI.VariableIndex in MOI.ZeroOne
- MOI.ScalarAffineFunction{T} in MOI.EqualTo{T}
- MOI.ScalarAffineFunction{T} in MOI.GreaterThan{T}

source

MathOptInterface.Bridges.Constraint.ExponentialConeToScalarNonlinearFunctionBridge - Type.

```
ExponentialConeToScalarNonlinearFunctionBridge{T,F} <:
    Bridges.Constraint.AbstractBridge</pre>
```

ExponentialConeToScalarNonlinearFunctionBridge implements the following reformulation:

```
• (x, y, z) \in \text{ExponentialCone}() to y \cdot exp(x/y) - z \le 0, y \ge 0.
```

#### Source node

ExponentialConeToScalarNonlinearFunctionBridge supports:

• Fin MOI. ExponentialCone

#### **Target nodes**

 ${\bf Exponential Cone To Scalar Nonlinear Function Bridge\ creates:}$ 

- MOI.ScalarNonlinearFunction in MOI.LessThan{T}
- MOI.ScalarAffineFunction in MOI.GreaterThan{T}

source

MathOptInterface.Bridges.Constraint.FlipSignBridge - Type.

```
FlipSignBridge{T,S1,S2,F,G}
```

An abstract type that simplifies the creation of other bridges.

source

 ${\tt MathOptInterface.Bridges.Constraint.FunctionConversionBridge-Type.}$ 

```
FunctionConversionBridge{T,F,G,S} <: AbstractFunctionConversionBridge{G,S}</pre>
```

 ${\tt FunctionConversionBridge\ implements\ the\ following\ reformulations:}$ 

•  $g(x) \in S$  into  $f(x) \in S$ 

for these pairs of functions:

- MOI.ScalarAffineFunctionto [MOI.ScalarQuadraticFunction'](@ref)
- MOI.ScalarQuadraticFunction to MOI.ScalarNonlinearFunction
- MOI. VectorAffineFunction to MOI. VectorQuadraticFunction

See also SetConversionBridge.

#### Source node

FunctionConversionBridge supports:

• G in S

### **Target nodes**

FunctionConversionBridge creates:

• Fin S

source

 ${\tt MathOptInterface.Bridges.Constraint.GeoMeanBridge-Type.}\\$ 

GeoMeanBridge{T,F,G,H} <: Bridges.Constraint.AbstractBridge</pre>

 $GeoMean Bridge\ implements\ a\ reformulation\ from\ MOI.\ Geometric Mean Cone\ into\ MOI.\ Rotated Second Order Cone.$ 

The reformulation is best described in an example.

Consider the cone of dimension 4:

$$t \le \sqrt[3]{x_1 x_2 x_3}$$

This can be rewritten as  $\exists y \geq 0$  such that:

$$t \le y,$$
  
$$y^4 \le x_1 x_2 x_3 y.$$

Note that we need to create y and not use  $t^4$  directly because t is not allowed to be negative.

This is equivalent to:

$$t \le \frac{y_1}{\sqrt{4}},$$

$$y_1^2 \le 2y_2y_3,$$

$$y_2^2 \le 2x_1x_2,$$

$$y_3^2 \le 2x_3(y_1/\sqrt{4})$$

$$y \ge 0.$$

More generally, you can show how the geometric mean code is recursively expanded into a set of new variables y in MOI.Nonnegatives, a set of MOI.RotatedSecondOrderCone constraints, and a MOI.LessThan constraint between t and  $y_1$ .

### Source node

GeoMeanBridge supports:

• H in MOI.GeometricMeanCone

#### **Target nodes**

GeoMeanBridge creates:

- F in MOI.LessThan{T}
- G in MOI.RotatedSecondOrderCone
- G in MOI.Nonnegatives

source

 ${\tt MathOptInterface.Bridges.Constraint.GeoMeanToPowerBridge-Type.}$ 

```
GeoMeanToPowerBridge{T,F} <: Bridges.Constraint.AbstractBridge
```

 ${\tt GeoMeanToPowerBridge\ implements\ the\ following\ reformulation:}$ 

•  $(y, x...) \in GeometricMeanCone(1+d)$  into  $(x_1, t, y) \in PowerCone(1/d)$  and  $(t, x_2, ..., x_d)inGeometricMeanCone(1+d)$  which is then recursively expanded into more PowerCone constraints.

#### Source node

GeoMeanToPowerBridge supports:

• Fin MOI.GeometricMeanCone

#### **Target nodes**

GeoMeanToPowerBridge creates:

- Fin MOI.PowerCone{T}
- MOI. VectorOfVariables in MOI. Nonnegatives

source

MathOptInterface.Bridges.Constraint.GeoMeantoRelEntrBridge - Type.

```
GeoMeantoRelEntrBridge{T,F,G,H} <: Bridges.Constraint.AbstractBridge</pre>
```

 ${\tt GeoMeantoRelEntrBridge\ implements\ the\ following\ reformulation:}$ 

•  $(u,w) \in GeometricMeanCone$  into  $(0,w,(u+y)\mathbf{1}) \in RelativeEntropyCone$  and  $y \geq 0$ 

#### Source node

GeoMeantoRelEntrBridge supports:

• H in MOI.GeometricMeanCone

### **Target nodes**

GeoMeantoRelEntrBridge creates:

- G in MOI.RelativeEntropyCone
- Fin MOI.Nonnegatives

## **Derivation**

The derivation of the bridge is as follows:

$$(u,w) \in Geometric Mean Cone \iff u \leq \left(\prod_{i=1}^n w_i\right)^{1/n}$$

$$\iff 0 \leq u+y \leq \left(\prod_{i=1}^n w_i\right)^{1/n}, y \geq 0$$

$$\iff 1 \leq \frac{\left(\prod_{i=1}^n w_i\right)^{1/n}}{u+y}, y \geq 0$$

$$\iff 1 \leq \left(\prod_{i=1}^n \frac{w_i}{u+y}\right)^{1/n}, y \geq 0$$

$$\iff 0 \leq \sum_{i=1}^n \log\left(\frac{w_i}{u+y}\right), y \geq 0$$

$$\iff 0 \geq \sum_{i=1}^n \log\left(\frac{u+y}{w_i}\right), y \geq 0$$

$$\iff 0 \geq \sum_{i=1}^n (u+y) \log\left(\frac{u+y}{w_i}\right), y \geq 0$$

$$\iff 0 \geq \sum_{i=1}^n (u+y) \log\left(\frac{u+y}{w_i}\right), y \geq 0$$

$$\iff 0 \leq w \leq (0, w, (u+y)\mathbf{1}) \in Relative Entropy Cone, y \geq 0$$

This derivation assumes that u+y>0, which is enforced by the relative entropy cone.

source

MathOptInterface.Bridges.Constraint.GreaterToIntervalBridge - Type.

GreaterToIntervalBridge{T,F} <: Bridges.Constraint.AbstractBridge</pre>

GreaterToIntervalBridge implements the following reformulations:

• 
$$f(x) \ge l$$
 into  $f(x) \in [l, \infty)$ 

### Source node

GreaterToIntervalBridge supports:

• Fin MOI.GreaterThan{T}

## **Target nodes**

GreaterToIntervalBridge creates:

• Fin MOI.Interval{T}

 ${\tt MathOptInterface.Bridges.Constraint.GreaterToLessBridge-Type.}$ 

```
Greater To Less Bridge \{T,F,G\} <: Bridges.Constraint.Abstract Bridge
```

GreaterToLessBridge implements the following reformulation:

```
• f(x) \ge l into -f(x) \le -l
```

## Source node

GreaterToLessBridge supports:

• G in MOI.GreaterThan{T}

# **Target nodes**

 ${\tt GreaterToLessBridge\ creates:}$ 

• Fin MOI.LessThan{T}

source

MathOptInterface.Bridges.Constraint.HermitianToComplexSymmetricBridge - Type.

```
Hermitian To Complex Symmetric Bridge \{T,F,G\} <: Bridges.Constraint.Abstract Bridge
```

 $\label{thm:lemmatical} Hermitian To Symmetric Bridge\ implements\ the\ following\ reformulation:$ 

• Hermitian positive semidefinite n x n represented as a vector of real entries with real and imaginary parts on different entries to a vector of complex entries.

See also MOI. Bridges. Constraint. HermitianToSymmetricPSDBridge.

### Source node

HermitianToComplexSymmetricBridge supports:

• G in MOI.HermitianPositiveSemidefiniteConeTriangle

## **Target node**

 $\label{lem:hermitianToComplexSymmetricBridge} Hermitian ToComplex Symmetric Bridge\ creates:$ 

 $\bullet \ \ F \ in \ MOI. Positive Semidefinite Cone Triangle$ 

Note that if G is MOI. VectorAffineFunction{T} then F will be MOI. VectorAffineFunction{Complex{T}} source

MathOptInterface.Bridges.Constraint.HermitianToSymmetricPSDBridge - Type.

 $Hermitian To Symmetric PSDB ridge \{T,F,G\} <: Bridges.Constraint.Abstract Bridge$ 

HermitianToSymmetricPSDBridge implements the following reformulation:

Hermitian positive semidefinite n x n complex matrix to a symmetric positive semidefinite 2n x 2n real matrix.

See also MOI.Bridges.Variable.HermitianToSymmetricPSDBridge.

#### Source node

HermitianToSymmetricPSDBridge supports:

• G in MOI.HermitianPositiveSemidefiniteConeTriangle

# Target node

HermitianToSymmetricPSDBridge creates:

• Fin MOI.PositiveSemidefiniteConeTriangle

## Reformulation

The reformulation is best described by example.

The Hermitian matrix:

$$\begin{bmatrix} x_{11} & x_{12} + y_{12}im & x_{13} + y_{13}im \\ x_{12} - y_{12}im & x_{22} & x_{23} + y_{23}im \\ x_{13} - y_{13}im & x_{23} - y_{23}im & x_{33} \end{bmatrix}$$

is positive semidefinite if and only if the symmetric matrix:

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & 0 & y_{12} & y_{13} \\ & x_{22} & x_{23} & -y_{12} & 0 & y_{23} \\ & & x_{33} & -y_{13} & -y_{23} & 0 \\ & & & x_{11} & x_{12} & x_{13} \\ & & & & & x_{22} & x_{23} \\ & & & & & & x_{33} \end{bmatrix}$$

is positive semidefinite.

The bridge achieves this reformulation by constraining the above matrix to belong to the MOI.PositiveSemidefiniteConeTrisource

MathOptInterface.Bridges.Constraint.IndicatorActiveOnFalseBridge - Type.

IndicatorActiveOnFalseBridge{T,F,S} <: Bridges.Constraint.AbstractBridge</pre>

IndicatorActiveOnFalseBridge implements the following reformulation:

• 
$$\neg z \implies f(x) \in S$$
 into  $y \implies f(x) \in S$ ,  $z + y = 1$ , and  $y \in \{0, 1\}$ 

#### Source node

 $Indicator Active On False Bridge\ supports:$ 

• MOI.VectorAffineFunction{T} in MOI.Indicator{MOI.ACTIVATE\_ON\_ZERO,S}

### Target nodes

IndicatorActiveOnFalseBridge creates:

- MOI.VectorAffineFunction{T} in MOI.Indicator{MOI.ACTIVATE\_ON\_ONE,S}
- MOI.ScalarAffineFunction{T} in MOI.EqualTo
- MOI.VariableIndex in MOI.ZeroOne

source

MathOptInterface.Bridges.Constraint.IndicatorGreaterToLessThanBridge - Type.

 $Indicator Greater To Less Than Bridge \{T,A\} \ <: \ Bridges. Constraint. Abstract Bridge$ 

IndicatorGreaterToLessThanBridge implements the following reformulation:

• 
$$z \implies f(x) \ge l \text{ into } z \implies -f(x) \le -l$$

# Source node

 $Indicator {\tt GreaterToLessThanBridge}\ supports:$ 

 $\bullet \ \ \text{MOI.VectorAffineFunction} \{T\} \ \textbf{in} \ \ \text{MOI.Indicator} \{A, \text{MOI.GreaterThan} \{T\}\}$ 

# **Target nodes**

 $Indicator {\tt GreaterToLessThanBridge}\ creates:$ 

• MOI.VectorAffineFunction{T} in MOI.Indicator{A,MOI.LessThan{T}}

source

MathOptInterface.Bridges.Constraint.IndicatorLessToGreaterThanBridge - Type.

 $Indicator Less To Greater Than Bridge \{T,A\} <: Bridges.Constraint.Abstract Bridge$ 

IndicatorLessToGreaterThanBridge implements the following reformulations:

• 
$$z \implies f(x) \le u \text{ into } z \implies -f(x) \ge -u$$

## Source node

IndicatorLessToGreaterThanBridge supports:

• MOI.VectorAffineFunction{T} in MOI.Indicator{A,MOI.LessThan{T}}

## **Target nodes**

IndicatorLessToGreaterThanBridge creates:

• MOI.VectorAffineFunction{T} in MOI.Indicator{A,MOI.GreaterThan{T}}

source

 ${\tt MathOptInterface.Bridges.Constraint.IndicatorSOS1Bridge-Type.}$ 

```
IndicatorSOS1Bridge{T,S} <: Bridges.Constraint.AbstractBridge</pre>
```

IndicatorSOS1Bridge implements the following reformulation:

• 
$$z \implies f(x) \in S \text{ into } f(x) + y \in S, SOS1(y, z)$$

#### Warning

This bridge assumes that the solver supports  $MOI.SOS1\{T\}$  constraints in which one of the variables (y) is continuous.

### Source node

IndicatorSOS1Bridge supports:

• MOI.VectorAffineFunction{T} in MOI.Indicator{MOI.ACTIVATE\_ON\_ONE,S}

# **Target nodes**

IndicatorSOS1Bridge creates:

- MOI.ScalarAffineFunction{T} in S
- MOI. VectorOfVariables in MOI. SOS1{T}

source

MathOptInterface.Bridges.Constraint.IndicatorSetMapBridge - Type.

```
IndicatorSetMapBridge{T,A,S1,S2} <: Bridges.Constraint.AbstractBridge</pre>
```

IndicatorSetMapBridge implements the following reformulations:

• 
$$z \implies f(x) \ge l \text{ into } z \implies -f(x) \le -l$$

• 
$$z \implies f(x) \le u \text{ into } z \implies -f(x) \ge -u$$

# Source node

IndicatorSetMapBridge supports:

• MOI.VectorAffineFunction{T} in MOI.Indicator{A,S1}

## **Target nodes**

IndicatorSetMapBridge creates:

• MOI.VectorAffineFunction{T} in MOI.Indicator{A,S2}

source

MathOptInterface.Bridges.Constraint.IndicatorToMILPBridge - Type.

```
Indicator ToMILPBridge \{T,F,A,S\} <: Bridges.Constraint.AbstractBridge
```

IndicatorToMILPBridge implements the following reformulation:

•  $x \in Indicator(s)$  into a mixed-integer linear program.

#### Source node

IndicatorToMILPBridge supports:

• Fin MOI.Indicator{A,S}

where F is MOI. VectorOfVariables or MOI. VectorAffineFunction{T}.

## **Target nodes**

IndicatorToMILPBridge creates:

- MOI.VariableIndex in MOI.ZeroOne
- MOI.ScalarAffineFunction{T} in S

source

 ${\tt MathOptInterface.Bridges.Constraint.InequalityToComplementsBridge-Type.}$ 

```
Inequality To Complements Bridge \{T,F,S,G\} \ <: \ Bridges.Constraint.Abstract Bridge
```

 $Inequality To {\tt ComplementsBridge}\ implements\ the\ following\ reformulations:$ 

- $f(x) \ge b$  into  $\exists y$  such that  $f(x) b \perp y \ge 0$
- $f(x) \le b$  into  $f(x) b \perp y \le 0$
- f(x) = b into  $f(x) b \perp y$

## Source node

InequalityToComplementsBridge supports:

- F in MOI.GreaterThan{T}
- Fin MOI.LessThan{T}

• Fin MOI.EqualTo

## **Target nodes**

InequalityToComplementsBridge creates:

- MOI.VariableIndex in MOI.LessThan{T}
- MOI.VariableIndex in MOI.GreaterThan{T}
- G in MOI. Complements

source

MathOptInterface.Bridges.Constraint.IntegerToZeroOneBridge - Type.

```
IntegerToZeroOneBridge\{T\} <: Bridges.Constraint.AbstractBridge
```

IntegerToZeroOneBridge implements the following reformulation:

• 
$$x \in \mathbf{Z}$$
 into  $y_i \in \{0, 1\}$ ,  $x == lb + \sum 2^{i-1}y_i$ .

## Source node

IntegerToZeroOneBridge supports:

• VariableIndex in MOI.Integer

# **Target nodes**

IntegerToZeroOneBridge creates:

- MOI.VariableIndex in MOI.ZeroOne
- MOI.ScalarAffineFunction{T} in MOI.EqualTo{T}

# **Developer note**

This bridge is implemented as a constraint bridge instead of a variable bridge because we don't want to substitute the linear combination of y for every instance of x. Doing so would be expensive and greatly reduce the sparsity of the constraints.

source

MathOptInterface.Bridges.Constraint.IntervalToHyperRectangleBridge - Type.

```
IntervalToHyperRectangleBridge{T,F,G} <: Bridges.Constraint.AbstractBridge</pre>
```

 ${\tt IntervalToHyperRectangleBridge\ implements\ the\ following\ reformulations:}$ 

```
• l \leq g(x) \leq u into [g(x)] \in MOI. HyperRectangle([l], [u])
```

where T is the coefficient type of g(x) and the type of l and u.

See also VectorizeBridge for equality and single-sided bound constraints.

#### Source node

IntervalToHyperRectangleBridge supports:

• G in MOI.Interval{T}

## **Target nodes**

IntervalToHyperRectangleBridge creates:

• Fin MOI.HyperRectangle{T}.

source

 ${\tt MathOptInterface.Bridges.Constraint.LessToGreaterBridge-Type.}$ 

 $Less To Greater Bridge \{T,F,G\} <: Bridges.Constraint.Abstract Bridge$ 

LessToGreaterBridge implements the following reformulation:

• 
$$f(x) \le u$$
 into  $-f(x) \ge -u$ 

# Source node

 ${\tt LessToGreaterBridge\ supports:}$ 

• G in MOI.LessThan{T}

## **Target nodes**

LessToGreaterBridge creates:

• F in MOI.GreaterThan{T}

source

 ${\tt MathOptInterface.Bridges.Constraint.LessToIntervalBridge-Type.}$ 

LessToIntervalBridge{T,F} <: Bridges.Constraint.AbstractBridge

LessToIntervalBridge implements the following reformulations:

•  $f(x) \le u$  into  $f(x) \in (-\infty, u]$ 

# Source node

LessToIntervalBridge supports:

• Fin MOI.LessThan{T}

# **Target nodes**

LessToIntervalBridge creates:

• Fin MOI.Interval{T}

source

MathOptInterface.Bridges.Constraint.LogDetBridge - Type.

LogDetBridge{T,F,G,H,I} <: Bridges.Constraint.AbstractBridge</pre>

 $The \ MOI. LogDet ConeTriangle is \ representable \ by \ MOI. Positive Semidefinite ConeTriangle \ and \ MOI. Exponential Cone \ constraints.$ 

Indeed,  $\log \det(X) = \sum\limits_{i=1}^n \log(\delta_i)$  where  $\delta_i$  are the eigenvalues of X.

Adapting the method from [1, p. 149], we see that  $t \leq u \log(\det(X/u))$  for u > 0 if and only if there exists a lower triangular matrix such that

$$\begin{pmatrix} X \\ \top & \text{Diag}() \end{pmatrix} \succeq 0$$
$$t - \sum_{i=1}^{n} u \log \left(\frac{ii}{u}\right) \le 0$$

Which we reformulate further into

$$\begin{pmatrix} X \\ \top & \mathrm{Diag}() \end{pmatrix} \succeq 0$$
 
$$(l_i, u, i_i) \in ExponentialCone \quad \forall i$$
 
$$t - \sum_{i=1}^n l_i \leq 0$$

# Source node

LogDetBridge supports:

• I in MOI.LogDetConeTriangle

# **Target nodes**

 ${\bf LogDetBridge\ creates:}$ 

- Fin MOI.PositiveSemidefiniteConeTriangle
- G in MOI. Exponential Cone
- H in MOI.LessThan{T}

[1] Ben-Tal, Aharon, and Arkadi Nemirovski. Lectures on modern convex optimization: analysis, algorithms, and engineering applications. Society for Industrial and Applied Mathematics, 2001.

 ${\tt MathOptInterface.Bridges.Constraint.MultiSetMapBridge-Type.}$ 

```
abstract type MultiSetMapBridge{T,S1,G} <: AbstractBridge end</pre>
```

Same as SetMapBridge but the output constraint type does not only depend on the input constraint type.

When subtyping MultiSetMapBridge, added\_constraint\_types and supports should additionally be implemented by the bridge.

For example, if a bridge BridgeType may create either a constraint of type F2-in-S2 or F3-in-S3, these methods should be implemented as follows:

source

 ${\tt MathOptInterface.Bridges.Constraint.NonnegToNonposBridge-Type.}\\$ 

```
NonnegToNonposBridge \{T,F,G\} \ <: \ Bridges.Constraint.AbstractBridge
```

NonnegToNonposBridge implements the following reformulation:

```
• f(x) \in \mathbb{R}_+ into -f(x) \in \mathbb{R}_-
```

# Source node

 ${\tt NonnegToNonposBridge\ supports:}$ 

• G in MOI.Nonnegatives

## **Target nodes**

NonnegToNonposBridge creates:

• Fin MOI.Nonpositives

source

MathOptInterface.Bridges.Constraint.NonposToNonnegBridge - Type.

 $NonposToNonnegBridge\{T,F,G\} <: Bridges.Constraint.AbstractBridge$ 

 ${\tt NonposToNonnegBridge\ implements\ the\ following\ reformulation:}$ 

•  $f(x) \in \mathbb{R}_-$  into  $-f(x) \in \mathbb{R}_+$ 

## Source node

NonposToNonnegBridge supports:

• G in MOI.Nonpositives

## **Target nodes**

NonposToNonnegBridge creates:

• Fin MOI.Nonnegatives

source

MathOptInterface.Bridges.Constraint.NormInfinityBridge - Type.

 $NormInfinityBridge \{T,F,G\} <: Bridges.Constraint.AbstractBridge$ 

NormInfinityBridge implements the following reformulation:

•  $|x|_{\infty} \leq t$  into  $[t - x_i, t + x_i] \in \mathbb{R}_+$ .

## Source node

NormInfinityBridge supports:

• G in MOI.NormInfinityCone{T}

# **Target nodes**

 ${\tt NormInfinityBridge\ creates:}$ 

• Fin MOI.Nonnegatives

source

 ${\tt MathOptInterface.Bridges.Constraint.NormInfinityConeToNormConeBridge-Type.}$ 

NormInfinityConeToNormConeBridge{T,F} <: Bridges.Constraint.AbstractBridge

NormInfinityConeToNormConeBridge implements the following reformulations:

• (t,x)inNormInfinityCone(d) into (t,x)inNormCone(Inf,d)

# Source node

NormInfinityConeToNormConeBridge supports:

• Fin MOI.NormInfinityCone

# **Target nodes**

NormInfinityConeToNormConeBridge creates:

• Fin MOI.NormCone

source

MathOptInterface.Bridges.Constraint.NormNuclearBridge - Type.

NormNuclearBridge{T,F,G,H} <: Bridges.Constraint.AbstractBridge

NormNuclearBridge implements the following reformulation:

• 
$$t \geq \sum_i \sigma_i(X)$$
 into  $\left[ egin{array}{cc} U & X^\top \\ X & V \end{array} 
ight] \succeq 0$  and  $2t \geq tr(U) + tr(V).$ 

## Source node

NormNuclearBridge supports:

• Hin MOI.NormNuclearCone

# **Target nodes**

NormNuclearBridge creates:

- F in MOI.GreaterThan{T}
- G in MOI.PositiveSemidefiniteConeTriangle

source

MathOptInterface.Bridges.Constraint.NormOneBridge - Type.

 $NormOneBridge \{T,F,G\} <: Bridges.Constraint.AbstractBridge$ 

NormOneBridge implements the following reformulation:

• 
$$\sum |x_i| \le t$$
 into  $[t - \sum y_i, y_i - x_i, y_i + x_i] \in \mathbb{R}_+$ .

## Source node

NormOneBridge supports:

• G in MOI.NormOneCone{T}

# **Target nodes**

NormOneBridge creates:

• Fin MOI.Nonnegatives

source

MathOptInterface.Bridges.Constraint.NormOneConeToNormConeBridge - Type.

 $NormOneConeToNormConeBridge \{T,F\} <: Bridges.Constraint.AbstractBridge$ 

 ${\tt NormOneConeToNormConeBridge\ implements\ the\ following\ reformulations:}$ 

• (t,x)inNormOneCone(d) into (t,x)inNormCone(1,d)

## Source node

NormOneConeToNormConeBridge supports:

• Fin MOI.NormOneCone

# **Target nodes**

NormOneConeToNormConeBridge creates:

• Fin MOI.NormCone

source

MathOptInterface.Bridges.Constraint.NormSpectralBridge - Type.

 $NormSpectralBridge\{T,F,G\} <: Bridges.Constraint.AbstractBridge$ 

NormSpectralBridge implements the following reformulation:

• 
$$t \geq \sigma_1(X)$$
 into  $\left[ egin{array}{cc} t {f I} & X^\top \ X & t {f I} \end{array} \right] \succeq 0$ 

## Source node

NormSpectralBridge supports:

• G in MOI.NormSpectralCone

# **Target nodes**

NormSpectralBridge creates:

• Fin MOI.PositiveSemidefiniteConeTriangle

source

MathOptInterface.Bridges.Constraint.NormToPowerBridge - Type.

NormToPowerBridge{T,F} <: Bridges.Constraint.AbstractBridge

 ${\tt NormToPowerBridge\ implements\ the\ following\ reformulation:}$ 

•  $(t,x) \in NormCone(p,1+d)$  into  $(r_i,t,x_i) \in PowerCone(1/p)$  for all i, and  $\sum_i r_i == t$ .

For details, see Alizadeh, F., and Goldfarb, D. (2001). "Second-order cone programming." Mathematical Programming, Series B, 95:3-51.

## Source node

NormToPowerBridge supports:

• Fin MOI.NormCone

# **Target nodes**

NormToPowerBridge creates:

- F in MOI.PowerCone{T}
- MOI.ScalarAffineFunction in MOI.EqualTo

source

MathOptInterface.Bridges.Constraint.NumberConversionBridge - Type.

NumberConversionBridge{T,F1,S1,F2,S2} <: Bridges.Constraint.AbstractBridge</pre>

 ${\tt NumberConversionBridge\ implements\ the\ following\ reformulation:}$ 

•  $f1(x) \in S1$  to  $f2(x) \in S2$ 

where f and S are the same functional form, but differ in their coefficient type.

### Source node

NumberConversionBridge supports:

• F1 in S1

# **Target node**

 ${\tt NumberConversionBridge\ creates:}$ 

• F2 in S2

source

 ${\tt MathOptInterface.Bridges.Constraint.QuadtoSOCBridge-Type.}$ 

QuadtoSOCBridge{T} <: Bridges.Constraint.AbstractBridge</pre>

QuadtoSOCBridge converts quadratic inequalities

$$\frac{1}{2}x^TQx + a^Tx \le ub$$

 ${\tt into\ MOI.Rotated Second Order Cone\ constraints}, \ {\tt but\ it\ only\ applies\ when\ } Q \ {\tt is\ positive\ definite}.$ 

This is because, if Q is positive definite, there exists U such that  $Q=U^TU$ , and so the inequality can then be rewritten as;

$$||Ux||_2^2 \le 2(-a^Tx + ub)$$

Therefore, QuadtoSOCBridge implements the following reformulations:

- $\frac{1}{2}x^TQx + a^Tx \le ub$  into  $(1, -a^Tx + ub, Ux) \in RotatedSecondOrderCone$  where  $Q = U^TU$
- $\bullet \ \ \tfrac{1}{2}x^TQx + a^Tx \geq lb \ \text{into} \ (1, a^Tx lb, Ux) \in RotatedSecondOrderCone \ \text{where} \ -Q = U^TU$

## Source node

QuadtoSOCBridge supports:

- MOI.ScalarAffineFunction{T} in MOI.LessThan{T}
- MOI.ScalarAffineFunction{T} in MOI.GreaterThan{T}

## **Target nodes**

RelativeEntropyBridge creates:

• MOI.VectorAffineFunction{T} in MOI.RotatedSecondOrderCone

## **Errors**

This bridge errors if Q is not positive definite.

source

MathOptInterface.Bridges.Constraint.RSOCtoNonConvexQuadBridge - Type.

 $RSOC to NonConvex Quad Bridge \{T\} <: Bridges.Constraint.Abstract Bridge$ 

RSOCtoNonConvexQuadBridge implements the following reformulations:

•  $||x||_2^2 \le 2tu$  into  $\sum x^2 - 2tu \le 0$ ,  $1t + 0 \ge 0$ , and  $1u + 0 \ge 0$ .

The MOI.ScalarAffineFunctions 1t+0 and 1u+0 are used in case the variables have other bound constraints.

#### Warning

This transformation starts from a convex constraint and creates a non-convex constraint. Unless the solver has explicit support for detecting rotated second-order cones in quadratic form, this may (wrongly) be interpreted by the solver as being non-convex. Therefore, this bridge is not added automatically by MOI.Bridges.full\_bridge\_optimizer. Care is recommended when adding this bridge to a optimizer.

## Source node

RSOCtoNonConvexQuadBridge supports:

• MOI.VectorOfVariables in MOI.RotatedSecondOrderCone

## **Target nodes**

RSOCtoNonConvexQuadBridge creates:

- MOI.ScalarQuadraticFunction{T} in MOI.LessThan{T}
- MOI.ScalarAffineFunction{T} in MOI.GreaterThan{T}

source

MathOptInterface.Bridges.Constraint.RSOCtoPSDBridge - Type.

 $RSOCtoPSDBridge\{T,F,G\} <: Bridges.Constraint.AbstractBridge$ 

 ${\tt RSOCtoPSDBridge\ implements\ the\ following\ reformulation:}$ 

$$\bullet \ ||x||_2^2 \leq 2t \cdot u \text{ into } \left[ \begin{array}{cc} t & x^\top \\ x & 2tu\mathbf{I} \end{array} \right] \succeq 0$$

## Source node

RSOCtoPSDBridge supports:

• G in MOI.RotatedSecondOrderCone

## **Target nodes**

RSOCtoPSDBridge creates:

• F in MOI.PositiveSemidefiniteConeTriangle

source

MathOptInterface.Bridges.Constraint.RSOCtoSOCBridge - Type.

RSOCtoSOCBridge{T,F,G} <: Bridges.Constraint.AbstractBridge

RSOCtoSOCBridge implements the following reformulation:

•  $||x||_2^2 \le 2tu$  into  $||\frac{t-u}{\sqrt{2}}, x||_2 \le \frac{t+u}{\sqrt{2}}$ 

## Source node

RSOCtoSOCBridge supports:

• G in MOI.RotatedSecondOrderCone

## Target node

RSOCtoSOCBridge creates:

• Fin MOI.SecondOrderCone

source

MathOptInterface.Bridges.Constraint.ReifiedAllDifferentToCountDistinctBridge - Type.

```
ReifiedAllDifferentToCountDistinctBridge{T,F} <:
Bridges.Constraint.AbstractBridge
```

 $Reified \verb|AllDifferentToCountDistinctBridge| implements the following reformulations:$ 

- $r \iff x \in \mathsf{AllDifferent}(d) \text{ to } r \iff (n,x) \in \mathsf{CountDistinct}(1+d) \text{ and } n=d$
- $r \iff f(x) \in \mathsf{AllDifferent}(d) \text{ to } r \iff (d, f(x)) \in \mathsf{CountDistinct}(1+d)$

# Source node

ReifiedAllDifferentToCountDistinctBridge supports:

• Fin MOI.Reified{MOI.AllDifferent}

where F is MOI. VectorOfVariables or MOI. VectorAffineFunction{T}.

### **Target nodes**

ReifiedAllDifferentToCountDistinctBridge creates:

- Fin MOI.Reified{MOI.CountDistinct}
- MOI.VariableIndex in MOI.EqualTo{T}

source

MathOptInterface.Bridges.Constraint.ReifiedCountDistinctToMILPBridge - Type.

 $Reified Count Distinct To MILP Bridge \{T,F\} <: Bridges.Constraint.Abstract Bridge$ 

ReifiedCountDistinctToMILPBridge implements the following reformulation:

•  $r \iff (n,x) \in \mathsf{CountDistinct}(1+d)$  into a mixed-integer linear program.

#### Reformulation

The reformulation is non-trivial, and it depends on the finite domain of each variable  $x_i$ , which we as define  $S_i = \{l_i, \dots, u_i\}$ .

First, we introduce new binary variables  $z_{ij}$ , which are 1 if variable  $x_i$  takes the value j in the optimal solution and 0 otherwise:

$$z_{ij} \in \{0,1\} \quad \forall i \in 1 \dots d, j \in S_i$$
$$x_i - \sum_{j \in S_i} j \cdot z_{ij} = 0 \quad \forall i \in 1 \dots d$$
$$\sum_{j \in S_i} z_{ij} = 1 \quad \forall i \in 1 \dots d$$

Then, we introduce new binary variables  $y_j$ , which are 1 if a variable takes the value j in the optimal solution and 0 otherwise.

$$y_j \in \{0,1\} \ \forall j \in \bigcup_{i=1,\dots,d} S_i$$
$$y_j \le \sum_{i \in 1\dots d: j \in S_i} z_{ij} \le My_j \ \forall j \in \bigcup_{i=1,\dots,d} S_i$$

Finally, n is constrained to be the number of  $y_j$  elements that are non-zero, with some slack:

$$n - \sum_{j \in \bigcup_{i=1,\dots,d} S_i} y_j = \delta^+ - \delta^-$$

And then the slack is constrained to respect the reif variable r:

$$d_{1} \leq \delta^{+} \leq Md_{1}$$

$$d_{2} \leq \delta^{-} \leq Md_{s}$$

$$d_{1} + d_{2} + r = 1$$

$$d_{1}, d_{2} \in \{0, 1\}$$

# Source node

ReifiedCountDistinctToMILPBridge supports:

• Fin MOI.Reified{MOI.CountDistinct}

where F is MOI. VectorOfVariables or MOI. VectorAffineFunction{T}.

# **Target nodes**

ReifiedCountDistinctToMILPBridge creates:

- MOI.VariableIndex in MOI.ZeroOne
- MOI.ScalarAffineFunction{T} in MOI.EqualTo{T}
- MOI.ScalarAffineFunction{T} in MOI.LessThan{T}

source

MathOptInterface.Bridges.Constraint.RelativeEntropyBridge - Type.

```
Relative Entropy Bridge \{T,F,G,H\} <: Bridges.Constraint.AbstractBridge
```

RelativeEntropyBridge implements the following reformulation that converts a MOI.RelativeEntropyCone into an MOI.ExponentialCone:

• 
$$u \geq \sum_{i=1}^n w_i \log\left(\frac{w_i}{v_i}\right)$$
 into  $y_i \geq 0$ ,  $u \geq \sum_{i=1}^n y_i$ , and  $(-y_i, w_i, v_i) \in ExponentialCone$ .

#### Source node

RelativeEntropyBridge supports:

• H in MOI.RelativeEntropyCone

# **Target nodes**

RelativeEntropyBridge creates:

- F in MOI.GreaterThan{T}
- G in MOI. ExponentialCone

source

 ${\tt MathOptInterface.Bridges.Constraint.RootDetBridge-Type.}\\$ 

```
RootDetBridge{T,F,G,H} <: Bridges.Constraint.AbstractBridge</pre>
```

The MOI.RootDetConeTriangle is representable by MOI.PositiveSemidefiniteConeTriangle and MOI.GeometricMeanCone constraints, see [1, p. 149].

Indeed,  $t \leq \det(X)^{1/n}$  if and only if there exists a lower triangular matrix such that:

$$\begin{pmatrix} X \\ \top & \text{Diag}() \end{pmatrix} \succeq 0$$
$$(t, \text{Diag}()) \in GeometricMeanCone$$

# Source node

RootDetBridge supports:

• I in MOI.RootDetConeTriangle

# **Target nodes**

RootDetBridge creates:

- Fin MOI.PositiveSemidefiniteConeTriangle
- G in MOI.GeometricMeanCone

[1] Ben-Tal, Aharon, and Arkadi Nemirovski. Lectures on modern convex optimization: analysis, algorithms, and engineering applications. Society for Industrial and Applied Mathematics, 2001.

source

MathOptInterface.Bridges.Constraint.SOCtoNonConvexQuadBridge - Type.

```
SOCtoNonConvexQuadBridge{T} <: Bridges.Constraint.AbstractBridge
```

SOCtoNonConvexQuadBridge implements the following reformulations:

•  $||x||_2 \le t$  into  $\sum x^2 - t^2 \le 0$  and  $1t + 0 \ge 0$ 

The MOI. Scalar Affine Function 1t+0 is used in case the variable has other bound constraints.

### Warning

This transformation starts from a convex constraint and creates a non-convex constraint. Unless the solver has explicit support for detecting second-order cones in quadratic form, this may (wrongly) be interpreted by the solver as being non-convex. Therefore, this bridge is not added automatically by MOI.Bridges.full\_bridge\_optimizer. Care is recommended when adding this bridge to a optimizer.

### Source node

 ${\tt SOCtoNonConvexQuadBridge\ supports:}$ 

• MOI. VectorOfVariables in MOI. SecondOrderCone

# **Target nodes**

SOCtoNonConvexQuadBridge creates:

- MOI.ScalarQuadraticFunction{T} in MOI.LessThan{T}
- MOI.ScalarAffineFunction{T} in MOI.GreaterThan{T}

source

MathOptInterface.Bridges.Constraint.SOCtoPSDBridge - Type.

```
SOCtoPSDBridge{T,F,G} <: Bridges.Constraint.AbstractBridge
```

SOCtoPSDBridge implements the following reformulation:

• 
$$||x||_2 \le t$$
 into  $\begin{bmatrix} t & x^\top \\ x & t\mathbf{I} \end{bmatrix} \succeq 0$ 

#### Warning

This bridge is not added by default by MOI.Bridges.full\_bridge\_optimizer because bridging second order cone constraints to semidefinite constraints can be achieved by the SOCtoRSOCBridge followed by the RSOCtoPSDBridge, while creating a smaller semidefinite constraint.

## Source node

SOCtoPSDBridge supports:

• G in MOI.SecondOrderCone

# **Target nodes**

SOCtoPSDBridge creates:

• Fin MOI.PositiveSemidefiniteConeTriangle

source

MathOptInterface.Bridges.Constraint.SOCtoRSOCBridge - Type.

 ${\tt SOCtoRSOCBridge\{T,F,G\}} \; <: \; Bridges.Constraint.AbstractBridge$ 

SOCtoRSOCBridge implements the following reformulation:

•  $||x||_2 \le t$  into  $(t+x_1)(t-x_1) \ge ||(x_2,\ldots,x_N)||_2^2$ 

## **Assumptions**

• SOCtoRSOCBridge assumes that the length of x is at least one.

# Source node

SOCtoRSOCBridge supports:

• G in MOI. SecondOrderCone

# Target node

SOCtoRSOCBridge creates:

• Fin MOI.RotatedSecondOrderCone

source

 ${\tt MathOptInterface.Bridges.Constraint.SOS1ToMILPBridge-Type.}$ 

```
{\tt SOS1ToMILPBridge\{T,F\}} \; <: \; Bridges.Constraint.AbstractBridge
```

SOS1ToMILPBridge implements the following reformulation:

•  $x \in SOS1(d)$  into a mixed-integer linear program.

## Source node

SOS1ToMILPBridge supports:

• Fin MOI.SOS1

where F is MOI. VectorOfVariables or MOI. VectorAffineFunction{T}.

## **Target nodes**

SOS1ToMILPBridge creates:

- MOI.VariableIndex in MOI.ZeroOne
- MOI.ScalarAffineFunction{T} in MOI.EqualTo{T}
- MOI.ScalarAffineFunction{T} in MOI.LessThan{T}

source

MathOptInterface.Bridges.Constraint.SOS2ToMILPBridge - Type.

```
{\tt SOS2ToMILPBridge\{T,F\}} \ <: \ {\tt Bridges.Constraint.AbstractBridge}
```

SOS2ToMILPBridge implements the following reformulation:

•  $x \in SOS2(d)$  into a mixed-integer linear program.

## Source node

SOS2ToMILPBridge supports:

• Fin MOI.SOS2

where F is MOI. VectorOfVariables or MOI. VectorAffineFunction{T}.

## **Target nodes**

SOS2ToMILPBridge creates:

- MOI.VariableIndex in MOI.ZeroOne
- MOI.ScalarAffineFunction{T} in MOI.EqualTo{T}
- MOI.ScalarAffineFunction{T} in MOI.LessThan{T}

source

 ${\tt MathOptInterface.Bridges.Constraint.ScalarFunctionizeBridge-Type.}$ 

ScalarFunctionizeBridge implements the following reformulations:

```
• x \in S into 1x + 0 \in S
```

# Source node

ScalarFunctionizeBridge supports:

• MOI.VariableIndex in S

### **Target nodes**

ScalarFunctionizeBridge creates:

• MOI.ScalarAffineFunction{T} in S

source

 ${\tt MathOptInterface.Bridges.Constraint.ScalarSlackBridge-Type.}$ 

```
ScalarSlackBridge{T,F,S} <: Bridges.Constraint.AbstractBridge</pre>
```

ScalarSlackBridge implements the following reformulation:

```
• f(x) \in S into f(x) - y == 0 and y \in S
```

# Source node

ScalarSlackBridge supports:

• G in S, where G is not MOI. VariableIndex and S is not MOI. EqualTo

# **Target nodes**

 ${\tt ScalarSlackBridge\ creates:}$ 

- F in MOI.EqualTo{T}
- MOI. VariableIndex in S

source

 ${\tt MathOptInterface.Bridges.Constraint.ScalarizeBridge-Type.}$ 

```
{\sf ScalarizeBridge}\{{\sf T,F,S}\}
```

ScalarizeBridge implements the following reformulations:

```
• f(x) - a \in \mathbb{R}_+ into f_i(x) \ge a_i for all i
```

- $f(x) a \in \mathbb{R}_-$  into  $f_i(x) \le a_i$  for all i
- $f(x) a \in \{0\}$  into  $f_i(x) == a_i$  for all i

#### Source node

ScalarizeBridge supports:

- G in MOI.Nonnegatives{T}
- G in MOI.Nonpositives{T}
- G in MOI.Zeros{T}

# **Target nodes**

ScalarizeBridge creates:

• F in S, where S is one of MOI.GreaterThan{T}, MOI.LessThan{T}, and MOI.EqualTo{T}, depending on the type of the input set.

source

 ${\tt MathOptInterface.Bridges.Constraint.SecondOrderConeToNormConeBridge-Type.}$ 

```
SecondOrderConeToNormConeBridge{T,F} <: Bridges.Constraint.AbstractBridge
```

 ${\tt SecondOrderConeToNormConeBridge\ implements\ the\ following\ reformulations:}$ 

• (t,x)inSecondOrderCone(d) into (t,x)inNormCone(2,d)

# Source node

SecondOrderConeToNormConeBridge supports:

• Fin MOI.SecondOrderCone

## **Target nodes**

SecondOrderConeToNormConeBridge creates:

• Fin MOI.NormCone

source

 ${\tt MathOptInterface.Bridges.Constraint.SemiToBinaryBridge-Type.}$ 

```
SemiToBinaryBridge \{T,S\} <: Bridges.Constraint.AbstractBridge
```

SemiToBinaryBridge implements the following reformulations:

•  $x \in \{0\} \cup [l, u]$  into

$$x \le zu$$
$$x \ge zl$$
$$z \in \{0, 1\}$$

•  $x \in \{0\} \cup \{l, ..., u\}$  into

$$x \le zu$$
$$x \ge zl$$
$$z \in \{0, 1\}$$
$$x \in \mathbb{Z}$$

#### Source node

SemiToBinaryBridge supports:

- MOI.VariableIndex in MOI.Semicontinuous{T}
- MOI.VariableIndex in MOI.Semiinteger{T}

## **Target nodes**

SemiToBinaryBridge creates:

- MOI.VariableIndex in MOI.ZeroOne
- MOI.ScalarAffineFunction{T} in MOI.LessThan{T}
- MOI.ScalarAffineFunction{T} in MOI.GreaterThan{T}
- MOI.VariableIndex{T} in MOI.Integer (if S is MOI.Semiinteger{T}

source

MathOptInterface.Bridges.Constraint.SetConversionBridge - Type.

```
SetConversionBridge{T,S2,S1,F} <:
    MOI.Bridges.Constraint.SetMapBridge{T,S2,S1,F,F}</pre>
```

SetConversionBridge implements the following reformulations:

```
• f(x) \in S1 into f(x) \in S2
```

In order to add this bridge, you need to create a bridge specific for a given type T and set S2:

```
MOI.Bridges.add_bridge(model, MOI.Bridges.Constraint.SetConversionBridge{T,S2})
```

In order to define a bridge with S2 specified but T unspecified, for example for JuMP.add\_bridge, you can use

```
\label{eq:constraint} \textbf{Constraint.SetConversionBridge} \{\texttt{T,S1,F}\} \ = \ \texttt{MOI.Bridges.Constraint.SetConversionBridge} \{\texttt{T,S2,S1,F}\}
```

See also FunctionConversionBridge.

#### Source node

SetConversionBridge supports:

• F in S1

## **Target nodes**

SetConversionBridge creates:

• F in S2

source

MathOptInterface.Bridges.Constraint.SetDotInverseScalingBridge - Type.

```
SetDotInverseScalingBridge{T,S,F,G} <: Bridges.Constraint.AbstractBridge
```

 $SetDotInverseScalingBridge\ implements\ the\ reformulation\ from\ constraints\ in\ the\ MOI.Scaled \{S\}\ to\ constraints\ in\ the\ S.$ 

## Source node

SetDotInverseScalingBridge supports:

• G in MOI.Scaled{S}

# Target node

SetDotInverseScalingBridge creates:

• Fin S

source

MathOptInterface.Bridges.Constraint.SetDotScalingBridge - Type.

```
SetDotScalingBridge{T,S,F,G} <: Bridges.Constraint.AbstractBridge
```

SetDotScalingBridge implements the reformulation from constraints in S to constraints in MOI. Scaled {S}.

## Source node

SetDotScalingBridge supports:

• G in S

# **Target node**

SetDotScalingBridge creates:

• F in MOI.Scaled{S}

source

MathOptInterface.Bridges.Constraint.SetMapBridge - Type.

```
abstract type SetMapBridge{T,S2,S1,F,G} <: MultiSetMapBridge{T,S1,G} end</pre>
```

Consider two type of sets, S1 and S2, and a linear mapping A such that the image of a set of type S1 under A is a set of type S2.

A SetMapBridge{T,S2,S1,F,G} is a bridge that maps G-in-S1 constraints into F-in-S2 by mapping the function through A.

The linear map A is described by;

- MOI.Bridges.map set
- MOI.Bridges.map\_function.

Implementing a method for these two functions is sufficient to bridge constraints. However, in order for the getters and setters of attributes such as dual solutions and starting values to work as well, a method for the following functions must be implemented:

- MOI.Bridges.inverse\_map\_set
- MOI.Bridges.inverse\_map\_function
- MOI.Bridges.adjoint\_map\_function
- MOI.Bridges.inverse\_adjoint\_map\_function

See the docstrings of each function to see which feature would be missing if it was not implemented for a given bridge.

source

MathOptInterface.Bridges.Constraint.SplitComplexEqualToBridge - Type.

```
SplitComplexEqualToBridge{T,F,G} <: Bridges.Constraint.AbstractBridge</pre>
```

SplitComplexEqualToBridge implements the following reformulation:

```
• f(x) + g(x) * im = a + b * im into f(x) = a and g(x) = b
```

# Source node

SplitComplexEqualToBridge supports:

• G in MOI.EqualTo{Complex{T}}}

where G is a function with Complex coefficients.

# **Target nodes**

SplitComplexEqualToBridge creates:

• F in MOI.EqualTo{T}

where F is the type of the real/imaginary part of G.

source

MathOptInterface.Bridges.Constraint.SplitComplexZerosBridge - Type.

```
SplitComplexZerosBridge{T,F,G} <: Bridges.Constraint.AbstractBridge</pre>
```

SplitComplexZerosBridge implements the following reformulation:

• 
$$f(x) \in \{0\}^n$$
 into  $\text{Re}(f(x)) \in \{0\}^n$  and  $\text{Im}(f(x)) \in \{0\}^n$ 

# Source node

SplitComplexZerosBridge supports:

• G in MOI.Zeros

where G is a function with Complex coefficients.

# **Target nodes**

SplitComplexZerosBridge creates:

• Fin MOI.Zeros

where  ${\sf F}$  is the type of the real/imaginary part of  ${\sf G}.$ 

source

MathOptInterface.Bridges.Constraint.SplitHyperRectangleBridge - Type.

```
SplitHyperRectangleBridge\{T,G,F\} <: Bridges.Constraint.AbstractBridge
```

 ${\tt SplitHyperRectangleBridge\ implements\ the\ following\ reformulation:}$ 

```
• f(x) \in \mathsf{HyperRectangle}(l,u) to [f(x)-l;u-f(x)] \in \mathbb{R}_+.
```

## Source node

SplitHyperRectangleBridge supports:

• Fin MOI. HyperRectangle

# **Target nodes**

SplitHyperRectangleBridge creates:

• G in MOI.Nonnegatives

source

 ${\tt MathOptInterface.Bridges.Constraint.SplitIntervalBridge-Type.}$ 

```
SplitIntervalBridge{T,F,S,LS,US} <: Bridges.Constraint.AbstractBridge</pre>
```

SplitIntervalBridge implements the following reformulations:

```
• l \le f(x) \le u into f(x) \ge l and f(x) \le u
```

- f(x) = b into  $f(x) \ge b$  and  $f(x) \le b$
- $f(x) \in \{0\}$  into  $f(x) \in \mathbb{R}_+$  and  $f(x) \in \mathbb{R}_-$

## Source node

SplitIntervalBridge supports:

- Fin MOI.Interval{T}
- Fin MOI.EqualTo{T}
- Fin MOI.Zeros

## **Target nodes**

SplitIntervalBridge creates:

- Fin MOI.LessThan{T}
- Fin MOI.GreaterThan{T}

or

- Fin MOI.Nonnegatives
- Fin MOI.Nonpositives

### Note

If T<:AbstractFloat and S is MOI.Interval{T} then no lower (resp. upper) bound constraint is created if the lower (resp. upper) bound is typemin(T) (resp. typemax(T)). Similarly, when MOI.ConstraintSet is set, a lower or upper bound constraint may be deleted or created accordingly.

source

MathOptInterface.Bridges.Constraint.SquareBridge - Type.

```
Square Bridge \{T,F,G,TT,ST\} \ <: \ Bridges.Constraint.AbstractBridge
```

SquareBridge implements the following reformulations:

- $\bullet \ \ (t,u,X) \in LogDetConeSquare \ {\tt into} \ (t,u,Y) in LogDetConeTriangle$
- $(t, X) \in RootDetConeSquare into (t, Y)inRootDetConeTriangle$
- $\bullet \ \ X \in AbstractSymmetricMatrixSetSquare \ {\tt into} \ YinAbstractSymmetricMatrixSetTriangle$

where Y is the upper triangluar component of X.

In addition, constraints are added as necessary to constrain the matrix X to be symmetric. For example, the constraint for the matrix:

$$\begin{pmatrix} 1 & 1+x & 2-3x \\ 1+x & 2+x & 3-x \\ 2-3x & 2+x & 2x \end{pmatrix}$$

can be broken down to the constraint of the symmetric matrix

$$\begin{pmatrix} 1 & 1+x & 2-3x \\ \cdot & 2+x & 3-x \\ \cdot & \cdot & 2x \end{pmatrix}$$

and the equality constraint between the off-diagonal entries (2, 3) and (3, 2) 3-x==2+x. Note that no symmetrization constraint needs to be added between the off-diagonal entries (1, 2) and (2, 1) or between (1, 3) and (3, 1) because the expressions are the same.

## Source node

SquareBridge supports:

• F in ST

# **Target nodes**

SquareBridge creates:

• G in TT

source

 ${\tt MathOptInterface.Bridges.Constraint.TableToMILPBridge-Type.}$ 

TableToMILPBridge{T,F} <: Bridges.Constraint.AbstractBridge</pre>

TableToMILPBridge implements the following reformulation:

•  $x \in Table(t)$  into

$$z_{j} \in \{0, 1\} \quad \forall i, j$$

$$\sum_{j=1}^{n} z_{j} = 1$$

$$\sum_{j=1}^{n} t_{ij} z_{j} = x_{i} \quad \forall i$$

# Source node

TableToMILPBridge supports:

• Fin MOI.Table{T}

# **Target nodes**

TableToMILPBridge creates:

- MOI.VariableIndex in MOI.ZeroOne
- MOI.ScalarAffineFunction{T} in MOI.EqualTo{T}

source

MathOptInterface.Bridges.Constraint.ToScalarNonlinearBridge - Type.

 $To Scalar Nonlinear Bridge \{T,G,S\} <: AbstractFunction Conversion Bridge \{G,S\}$ 

ToScalarNonlinearBridge implements the following reformulation:

• 
$$g(x) \in S$$
 into  $f(x) \in S$ 

where g is an abstract scalar function and f is a  ${\tt MOI.ScalarNonlinearFunction}$ .

### Source node

ToScalarNonlinearBridge supports:

• G<:AbstractScalarFunction in S

# **Target nodes**

ToScalarNonlinearBridge creates:

• MOI.ScalarNonlinearFunction in S

source

 ${\tt MathOptInterface.Bridges.Constraint.ToScalarQuadraticBridge-Type.}\\$ 

```
ToScalarQuadraticBridge{T,G,S} <: AbstractFunctionConversionBridge{G,S}
```

ToScalarQuadraticBridge implements the following reformulation:

• 
$$g(x) \in S$$
 into  $f(x) \in S$ 

where g is an abstract scalar function and f is a  ${\tt MOI.ScalarQuadraticFunction}$ .

# Source node

ToScalarQuadraticBridge supports:

• G<:AbstractScalarFunction in S

## **Target nodes**

ToScalarQuadraticBridge creates:

• MOI.ScalarQuadraticFunction in S

source

MathOptInterface.Bridges.Constraint.ToVectorQuadraticBridge - Type.

```
To Vector Quadratic Bridge \{T,G,S\} <: AbstractFunction Conversion Bridge \{G,S\}
```

ToVectorQuadraticBridge implements the following reformulation:

• 
$$g(x) \in S$$
 into  $f(x) \in S$ 

where g is an abstract vector function and f is a MOI. VectorQuadraticFunction.

#### Source node

ToVectorQuadraticBridge supports:

• G<:AbstractVectorFunction in S

# **Target nodes**

ToVectorQuadraticBridge creates:

• MOI.VectorQuadraticFunction in S

source

 ${\tt MathOptInterface.Bridges.Constraint.VectorFunctionizeBridge-Type.}$ 

```
\label{lem:vectorFunction} VectorFunctionize Bridge \{T,S\} \ = \ Function Conversion Bridge \{T,MOI.Vector Affine Function \{T\},S\}
```

 $\label{lem:vectorFunction} \mbox{VectorFunctionizeBridge implements the following reformulations:} \\$ 

•  $x \in S$  into  $Ix + 0 \in S$ 

## Source node

VectorFunctionizeBridge supports:

• MOI. VectorOfVariables in S

# **Target nodes**

 $\label{lem:vectorFunctionizeBridge} VectorFunctionizeBridge\ creates:$ 

• MOI.VectorAffineFunction{T} in S

source

 ${\tt MathOptInterface.Bridges.Constraint.VectorSlackBridge-Type.}$ 

```
VectorSlackBridge{T,F,S} <: Bridges.Constraint.AbstractBridge</pre>
```

VectorSlackBridge implements the following reformulation:

• 
$$f(x) \in S$$
 into  $f(x) - y \in \{0\}$  and  $y \in S$ 

## Source node

VectorSlackBridge supports:

• G in S, where G is not MOI. VectorOfVariables and S is not MOI. Zeros

# **Target nodes**

VectorSlackBridge creates:

- Fin MOI.Zeros
- MOI. VectorOfVariables in S

source

MathOptInterface.Bridges.Constraint.VectorizeBridge - Type.

```
VectorizeBridge{T,F,S,G} <: Bridges.Constraint.AbstractBridge</pre>
```

VectorizeBridge implements the following reformulations:

- $g(x) \ge a$  into  $[g(x) a] \in \mathbb{R}_+$
- $g(x) \le a$  into  $[g(x) a] \in \mathbb{R}_-$
- g(x) == a into  $[g(x) a] \in \{0\}$

where T is the coefficient type of g(x) - a.

See also IntervalToHyperRectangleBridge for double-sided bound constraints.

## Source node

VectorizeBridge supports:

- G in MOI.GreaterThan{T}
- G in MOI.LessThan{T}
- G in MOI.EqualTo{T}

# **Target nodes**

VectorizeBridge creates:

• F in S, where S is one of MOI.Nonnegatives, MOI.Nonpositives, MOI.Zeros depending on the type of the input set.

source

 ${\tt MathOptInterface.Bridges.Constraint.ZeroOneBridge-Type.}\\$ 

```
ZeroOneBridge{T} <: Bridges.Constraint.AbstractBridge</pre>
```

ZeroOneBridge implements the following reformulation:

•  $x \in \{0, 1\}$  into  $x \in \mathbb{Z}$ ,  $1x \in [0, 1]$ .

# Note

ZeroOneBridge adds a linear constraint instead of adding variable bounds to avoid conflicting with bounds set by the user.

## Source node

ZeroOneBridge supports:

• MOI.VariableIndex in MOI.ZeroOne

## **Target nodes**

ZeroOneBridge creates:

- MOI.VariableIndex in MOI.Integer
- MOI.ScalarAffineFunction{T} in MOI.Interval{T}

source

## **Objective bridges**

These bridges are subtypes of Bridges.Objective.AbstractBridge.

 ${\tt MathOptInterface.Bridges.Objective.FunctionConversionBridge-Type.}\\$ 

```
Function Conversion Bridge \{T,F,G\} <: AbstractBridge
```

FunctionConversionBridge implements the following reformulations:

- $\min\{g(x)\}\$ into  $\min\{f(x)\}$
- $\max\{g(x)\}\$ into  $\max\{f(x)\}\$

for these pairs of functions:

• MOI.ScalarAffineFunction to MOI.ScalarQuadraticFunction

- MOI.ScalarQuadraticFunction to MOI.ScalarNonlinearFunction
- MOI. VectorAffineFunction to MOI. VectorQuadraticFunction

## Source node

FunctionConversionBridge supports:

• MOI.ObjectiveFunction{G}

# **Target nodes**

FunctionConversionBridge creates:

• One objective node: MOI.ObjectiveFunction{F}

source

MathOptInterface.Bridges.Objective.FunctionizeBridge - Type.

```
Functionize Bridge \{T,G\} <: Function Conversion Bridge \{T,MOI.Scalar Affine Function \{T\},G\} \\
```

FunctionizeBridge implements the following reformulations:

- $\min\{x\}$  into  $\min\{1x+0\}$
- $\max\{x\}$  into  $\max\{1x+0\}$

where T is the coefficient type of 1 and 0.

### Source node

FunctionizeBridge supports:

• MOI.ObjectiveFunction{G}

# **Target nodes**

 ${\tt FunctionizeBridge\ creates:}$ 

• One objective node: MOI.ObjectiveFunction{MOI.ScalarAffineFunction{T}}

source

MathOptInterface.Bridges.Objective.QuadratizeBridge - Type.

```
QuadratizeBridge{T,G} <: FunctionConversionBridge{T,MOI.ScalarQuadraticFunction{T},G}</pre>
```

QuadratizeBridge implements the following reformulations:

- $\min\{a^{\top}x+b\}$  into  $\min\{x^{\top}\mathbf{0}x+a^{\top}x+b\}$
- $\max\{a^{\top}x+b\}$  into  $\max\{x^{\top}\mathbf{0}x+a^{\top}x+b\}$

where T is the coefficient type of  $\theta$ .

## Source node

QuadratizeBridge supports:

• MOI.ObjectiveFunction{G}

# **Target nodes**

QuadratizeBridge creates:

• One objective node: MOI.ObjectiveFunction{MOI.ScalarQuadraticFunction{T}}

source

MathOptInterface.Bridges.Objective.SlackBridge - Type.

```
SlackBridge{T,F,G}
```

SlackBridge implements the following reformulations:

- $\min\{f(x)\}$  into  $\min\{y \mid f(x) y \le 0\}$
- $\max\{f(x)\}$  into  $\max\{y \mid f(x) y \ge 0\}$

where F is the type of f(x) - y, G is the type of f(x), and T is the coefficient type of f(x).

# Source node

SlackBridge supports:

• MOI.ObjectiveFunction{G}

# **Target nodes**

SlackBridge creates:

- One variable node: MOI.VariableIndex in MOI.Reals
- One objective node: MOI.ObjectiveFunction{MOI.VariableIndex}
- One constraint node, that depends on the MOI.ObjectiveSense:
  - F-in-MOI.LessThan if MIN\_SENSE
  - F-in-MOI.GreaterThan if MAX\_SENSE

### Warning

When using this bridge, changing the optimization sense is not supported. Set the sense to MOI.FEASIBILITY\_SENSE first to delete the bridge, then set MOI.ObjectiveSense and re-add the objective.

source

MathOptInterface.Bridges.Objective.ToScalarNonlinearBridge - Type.

 $To Scalar Nonlinear Bridge \{T,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Nonlinear Function,G\} \ <: \ Function Conversion Bridge \{T,MOI.Scalar Non$ 

ToScalarNonlinearBridge implements the following reformulations:

- $\min\{x^{\top}x + a^{\top}x + b\}$  into  $\min\{f(x)\}$
- $\max\{x^{\top}x + a^{\top}x + b\}$  into  $\max\{f(x)\}$

where f(x) is a MOI. Scalar Nonlinear Function.

### Source node

ToScalarNonlinearBridge supports:

• MOI.ObjectiveFunction{G}

### **Target nodes**

ToScalarNonlinearBridge creates:

• One objective node: MOI.ObjectiveFunction{MOI.ScalarNonlinearFunction}

source

 ${\tt MathOptInterface.Bridges.Objective.VectorFunctionizeBridge-Type.}$ 

 $\label{thm:conversionBridge} VectorFunctionize Bridge \{T,G\} <: FunctionConversionBridge \{T,MOI.VectorAffineFunction \{T\},G\} = \{T,T\} + \{T,T\} +$ 

VectorFunctionizeBridge implements the following reformulations:

- $\min\{x\}$  into  $\min\{1x+0\}$
- $\max\{x\}$  into  $\max\{1x+0\}$

where T is the coefficient type of 1 and 0.

# Source node

VectorFunctionizeBridge supports:

• MOI.ObjectiveFunction{G}

# Target nodes

 $\label{lem:vectorFunctionizeBridge} VectorFunctionizeBridge\ creates:$ 

• One objective node: MOI.ObjectiveFunction{MOI.VectorAffineFunction{T}}

source

 ${\tt MathOptInterface.Bridges.Objective.VectorSlackBridge-Type.}$ 

```
VectorSlackBridge{T,F,G}
```

VectorSlackBridge implements the following reformulations:

```
• \min\{f(x)\}\ into \min\{y\mid y-f(x)\in\mathbb{R}_+\}
```

```
• \max\{f(x)\} into \max\{y \mid f(x) - y \in \mathbb{R}_+\}
```

where F is the type of f(x) - y, G is the type of f(x), and T is the coefficient type of f(x).

#### Source node

VectorSlackBridge supports:

• MOI.ObjectiveFunction{G}

#### **Target nodes**

VectorSlackBridge creates:

- One variable node: MOI.VectorOfVariables in MOI.Reals
- One objective node: MOI.ObjectiveFunction{MOI.VectorOfVariables}
- One constraint node: F-in-MOI.Nonnegatives

#### Warning

When using this bridge, changing the optimization sense is not supported. Set the sense to MOI.FEASIBILITY\_SENSE first to delete the bridge, then set MOI.ObjectiveSense and re-add the objective.

source

# Variable bridges

These bridges are subtypes of Bridges.Variable.AbstractBridge.

 ${\tt MathOptInterface.Bridges.Variable.FlipSignBridge-Type}.$ 

```
abstract type FlipSignBridge{T,S1,S2} <: SetMapBridge{T,S2,S1} end</pre>
```

An abstract type that simplifies the creation of other bridges.

source

MathOptInterface.Bridges.Variable.FreeBridge - Type.

```
FreeBridge{T} <: Bridges.Variable.AbstractBridge</pre>
```

FreeBridge implements the following reformulation:

•  $x \in \mathbb{R}$  into  $y, z \ge 0$  with the substitution rule x = y - z,

where T is the coefficient type of y - z.

### Source node

FreeBridge supports:

• MOI. VectorOfVariables in MOI. Reals

### **Target nodes**

FreeBridge creates:

• One variable node: MOI.VectorOfVariables in MOI.Nonnegatives

source

MathOptInterface.Bridges.Variable.HermitianToSymmetricPSDBridge - Type.

```
Hermitian To Symmetric PSDB ridge \{T\} <: Bridges. Variable. Abstract Bridge
```

HermitianToSymmetricPSDBridge implements the following reformulation:

• Hermitian positive semidefinite n x n complex matrix to a symmetric positive semidefinite 2n x 2n real matrix satisfying equality constraints described below.

# Source node

HermitianToSymmetricPSDBridge supports:

• MOI. VectorOfVariables in MOI. HermitianPositiveSemidefiniteConeTriangle

# **Target node**

HermitianToSymmetricPSDBridge creates:

- MOI.VectorOfVariables in MOI.PositiveSemidefiniteConeTriangle
- MOI.ScalarAffineFunction{T} in MOI.EqualTo{T}

# Reformulation

The reformulation is best described by example.

The Hermitian matrix:

$$\begin{bmatrix} x_{11} & x_{12} + y_{12}im & x_{13} + y_{13}im \\ x_{12} - y_{12}im & x_{22} & x_{23} + y_{23}im \\ x_{13} - y_{13}im & x_{23} - y_{23}im & x_{33} \end{bmatrix}$$

is positive semidefinite if and only if the symmetric matrix:

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & 0 & y_{12} & y_{13} \\ & x_{22} & x_{23} & -y_{12} & 0 & y_{23} \\ & & x_{33} & -y_{13} & -y_{23} & 0 \\ & & & x_{11} & x_{12} & x_{13} \\ & & & & x_{22} & x_{23} \\ & & & & & x_{33} \end{bmatrix}$$

is positive semidefinite.

The bridge achieves this reformulation by adding a new set of variables in MOI. PositiveSemidefiniteConeTriangle(6), and then adding three groups of equality constraints to:

- · constrain the two x blocks to be equal
- force the diagonal of the y blocks to be 0
- force the lower triangular of the y block to be the negative of the upper triangle.

source

MathOptInterface.Bridges.Variable.NonposToNonnegBridge - Type.

```
NonposToNonnegBridge{T} <: Bridges.Variable.AbstractBridge
```

 ${\tt NonposToNonnegBridge\ implements\ the\ following\ reformulation:}$ 

•  $x \in \mathbb{R}_-$  into  $y \in \mathbb{R}_+$  with the substitution rule x = -y,

where T is the coefficient type of -y.

# Source node

NonposToNonnegBridge supports:

• MOI.VectorOfVariables in MOI.Nonpositives

# **Target nodes**

NonposToNonnegBridge creates:

• One variable node: MOI.VectorOfVariables in MOI.Nonnegatives,

source

 ${\tt MathOptInterface.Bridges.Variable.ParameterToEqualToBridge-Type.}$ 

```
ParameterToEqualToBridge{T} <: Bridges.Variable.AbstractBridge
```

ParameterToEqualToBridge implements the following reformulation:

•  $x \in Parameter(v)$  into x == v

### Source node

ParameterToEqualToBridge supports:

• MOI. VariableIndex in MOI. Parameter

### **Target nodes**

ParameterToEqualToBridge creates:

• One variable node: MOI.VariableIndex in MOI.EqualTo{T}

source

MathOptInterface.Bridges.Variable.RSOCtoPSDBridge - Type.

```
RSOCtoPSDBridge{T} <: Bridges.Variable.AbstractBridge
```

RSOCtoPSDBridge implements the following reformulation:

•  $||x||_2^2 \leq 2tu$  where  $t,u \geq 0$  into  $Y \succeq 0$ , with the substitution rule:  $Y = \left[ \begin{array}{cc} t & x^\top \\ x & 2u\mathbf{I} \end{array} \right]$ .

Additional bounds are added to ensure the off-diagonals of the 2uI submatrix are 0, and linear constraints are added to ensure the diagonal of 2uI takes the same values.

As a special case, if |x||=0, then RSOCtoPSDBridge reformulates into  $(t,u)\in\mathbb{R}_+$ .

# Source node

RSOCtoPSDBridge supports:

• MOI.VectorOfVariables in MOI.RotatedSecondOrderCone

#### **Target nodes**

RSOCtoPSDBridge creates:

- One variable node that depends on the input dimension:
  - ${\tt MOI.VectorOfVariables}$  in  ${\tt MOI.Nonnegatives}$  if dimension is 1 or 2
  - MOI. VectorOfVariables in

MOI.PositiveSemidefiniteConeTriangle otherwise

- The constraint node MOI. VariableIndex in MOI. EqualTo
- The constant node MOI.ScalarAffineFunction in MOI.EqualTo

source

MathOptInterface.Bridges.Variable.RSOCtoSOCBridge - Type.

```
RSOC to SOCBridge \{T\} <: \ Bridges. Variable. Abstract Bridge
```

RSOCtoSOCBridge implements the following reformulation:

•  $||x||_2^2 \le 2tu$  into  $||v||_2 \le w$ , with the substitution rules  $t = \frac{w}{\sqrt{2}} + \frac{v_1}{\sqrt{2}}$ ,  $u = \frac{w}{\sqrt{2}} - \frac{v_1}{\sqrt{2}}$ , and  $x = (v_2, \dots, v_N)$ .

### Source node

RSOCtoSOCBridge supports:

• MOI. VectorOfVariables in MOI. RotatedSecondOrderCone

### Target node

RSOCtoSOCBridge creates:

• MOI. VectorOfVariables in MOI. SecondOrderCone

source

MathOptInterface.Bridges.Variable.SOCtoRSOCBridge - Type.

```
SOCtoRSOCBridge{T} <: Bridges.Variable.AbstractBridge</pre>
```

SOCtoRSOCBridge implements the following reformulation:

•  $||x||_2 \le t$  into  $2uv \ge ||w||_2^2$ , with the substitution rules  $t = \frac{u}{\sqrt{2}} + \frac{v}{\sqrt{2}}$ ,  $x = (\frac{u}{\sqrt{2}} - \frac{v}{\sqrt{2}}, w)$ .

#### **Assumptions**

• SOCtoRSOCBridge assumes that  $|x| \ge 1$ .

### Source node

SOCtoRSOCBridge supports:

• MOI. VectorOfVariables in MOI. SecondOrderCone

# Target node

SOCtoRSOCBridge creates:

• MOI. VectorOfVariables in MOI. RotatedSecondOrderCone

source

MathOptInterface.Bridges.Variable.SetMapBridge - Type.

```
abstract type SetMapBridge{T,S1,S2} <: AbstractBridge end</pre>
```

Consider two type of sets, S1 and S2, and a linear mapping A such that the image of a set of type S1 under A is a set of type S2.

A SetMapBridge{T,S1,S2} is a bridge that substitutes constrained variables in S2 into the image through A of constrained variables in S1.

The linear map A is described by:

```
• MOI.Bridges.map_set
```

```
• MOI.Bridges.map function
```

Implementing a method for these two functions is sufficient to bridge constrained variables. However, in order for the getters and setters of attributes such as dual solutions and starting values to work as well, a method for the following functions must be implemented:

```
MOI.Bridges.inverse_map_setMOI.Bridges.inverse_map_functionMOI.Bridges.adjoint_map_function
```

• MOI.Bridges.inverse\_adjoint\_map\_function.

See the docstrings of each function to see which feature would be missing if it was not implemented for a given bridge.

### **Fieldnames**

If S1 and S2 are [MOI.AbstractScalarSet], the struct must have fields:

```
• variable::MOI.VariableIndex
```

• constraint::MOI.ConstraintIndex{MOI.VariableIndex,S1}

If S1 and S2 are [MOI.AbstractVectorSet], the struct must have fields:

```
variable::Vector{MOI.VariableIndex}
```

• constraint::MOI.ConstraintIndex{MOI.VectorOfVariables,S1}

source

MathOptInterface.Bridges.Variable.VectorizeBridge - Type.

```
VectorizeBridge{T,S} <: Bridges.Variable.AbstractBridge</pre>
```

 $\label{lem:vectorizeBridge} \textbf{VectorizeBridge} \ implements \ the \ following \ reformulations:$ 

```
• x \ge a into [y] \in \mathbb{R}_+ with the substitution rule x = a + y
```

```
• x \leq a into [y] \in \mathbb{R}_- with the substitution rule x = a + y
```

• x == a into  $[y] \in \{0\}$  with the substitution rule x = a + y

where T is the coefficient type of a + y.

### Source node

VectorizeBridge supports:

```
• MOI.VariableIndex in MOI.GreaterThan{T}
```

- MOI.VariableIndex in MOI.LessThan{T}
- MOI. VariableIndex in MOI. EqualTo{T}

# **Target nodes**

VectorizeBridge creates:

• One variable node: MOI. VectorOfVariables in S, where S is one of MOI. Nonnegatives, MOI. Nonpositives, MOI. Zeros depending on the type of S.

source

MathOptInterface.Bridges.Variable.ZerosBridge - Type.

```
ZerosBridge{T} <: Bridges.Variable.AbstractBridge</pre>
```

ZerosBridge implements the following reformulation:

•  $x \in \{0\}$  into the substitution rule x = 0,

where T is the coefficient type of 0.

### Source node

ZerosBridge supports:

• MOI. VectorOfVariables in MOI. Zeros

### **Target nodes**

ZerosBridge does not create target nodes. It replaces all instances of x with 0 via substitution. This means that no variables are created in the underlying model.

### **Caveats**

The bridged variables are similar to parameters with zero values. Parameters with non-zero values can be created with constrained variables in MOI. EqualTo by combining a VectorizeBridge and this bridge.

However, functions modified by ZerosBridge cannot be unbridged. That is, for a given function, we cannot determine if the bridged variables were used.

A related implication is that this bridge does not support MOI. ConstraintDual. However, if a MOI. Utilities. CachingOptimi is used, the dual can be determined by the bridged optimizer using MOI. Utilities.get\_fallback because the caching optimizer records the unbridged function.

source

### 27.4 API Reference

# **Bridges**

### AbstractBridge API

 ${\tt MathOptInterface.Bridges.AbstractBridge-Type.}$ 

abstract type AbstractBridge end

An abstract type representing a bridged constraint or variable in a MOI.Bridges.AbstractBridgeOptimizer.

All bridges must implement:

```
    added_constrained_variable_types
    added_constraint_types
    MOI.get(::AbstractBridge, ::MOI.NumberOfVariables)
    MOI.get(::AbstractBridge, ::MOI.ListOfVariableIndices)
    MOI.get(::AbstractBridge, ::MOI.NumberOfConstraints)
    MOI.get(::AbstractBridge, ::MOI.ListOfConstraintIndices)
```

Subtypes of AbstractBridge may have additional requirements. Consult their docstrings for details.

In addition, all subtypes may optionally implement the following constraint attributes with the bridge in place of the constraint index:

- MOI.ConstraintDual
- MOI.ConstraintPrimal

source

MathOptInterface.Bridges.added\_constrained\_variable\_types - Function.

```
added_constrained_variable_types(
   BT::Type{<:AbstractBridge},
)::Vector{Tuple{Type}}</pre>
```

Return a list of the types of constrained variables that bridges of concrete type BT add.

# Implementation notes

- This method depends only on the type of the bridge, not the runtime value. If the bridge may add a constrained variable, the type must be included in the return vector.
- If the bridge adds a free variable via MOI.add\_variable or MOI.add\_variables, the return vector must include (MOI.Reals,).

# **Example**

source

MathOptInterface.Bridges.added\_constraint\_types - Function.

```
added_constraint_types(
   BT::Type{<:AbstractBridge},
)::Vector{Tuple{Type,Type}}</pre>
```

Return a list of the types of constraints that bridges of concrete type BT add.

# Implementation notes

• This method depends only on the type of the bridge, not the runtime value. If the bridge may add a constraint, the type must be included in the return vector.

### **Example**

source

MathOptInterface.get - Method.

```
MOI.get(b::AbstractBridge, ::MOI.NumberOfVariables)::Int64
```

Return the number of variables created by the bridge b in the model.

See also MOI.NumberOfConstraints.

# Implementation notes

• There is a default fallback, so you need only implement this if the bridge adds new variables.

source

MathOptInterface.get - Method.

```
MOI.get(b::AbstractBridge, ::MOI.ListOfVariableIndices)
```

Return the list of variables created by the bridge b.

See also MOI.ListOfVariableIndices.

# Implementation notes

• There is a default fallback, so you need only implement this if the bridge adds new variables.

source

MathOptInterface.get - Method.

```
\label{eq:MOI.get} MOI.get(b::AbstractBridge, ::MOI.NumberOfConstraints\{F,S\}):: \textbf{Int64} \ \ \text{where} \ \ \{F,S\}
```

Return the number of constraints of the type F-in-S created by the bridge b.

See also MOI.NumberOfConstraints.

# Implementation notes

• There is a default fallback, so you need only implement this for the constraint types returned by added\_constraint\_types.

source

MathOptInterface.get - Method.

```
\label{eq:MOI.get} MOI.get (b::AbstractBridge, ::MOI.ListOfConstraintIndices \{F,S\}) \ \ where \ \ \{F,S\}
```

 $Return\ a\ Vector\{ConstraintIndex\{F,S\}\}\ with\ indices\ of\ all\ constraints\ of\ type\ F-in-S\ created\ by\ the\ bride\ b$ 

See also MOI.ListOfConstraintIndices.

### Implementation notes

 There is a default fallback, so you need only implement this for the constraint types returned by added\_constraint\_types.

source

MathOptInterface.Bridges.needs\_final\_touch - Function.

```
needs_final_touch(bridge::AbstractBridge)::Bool
```

Return whether final\_touch is implemented by bridge.

source

MathOptInterface.Bridges.final\_touch - Function.

```
final_touch(bridge::AbstractBridge, model::MOI.ModelLike)::Nothing
```

A function that is called immediately prior to MOI.optimize! to allow bridges to modify their reformulations with respect to other variables and constraints in model.

For example, if the correctness of bridge depends on the bounds of a variable or the fact that variables are integer, then the bridge can implement final\_touch to check assumptions immediately before a call to MOI.optimize!.

If you implement this method, you must also implement needs final touch.

source

MathOptInterface.Bridges.bridging\_cost - Function.

```
bridging_cost(b::AbstractBridgeOptimizer, S::Type{<:MOI.AbstractSet}})</pre>
```

Return the cost of bridging variables constrained in S on creation, is\_bridged(b, S) is assumed to be true.

```
bridging_cost(
    b::AbstractBridgeOptimizer,
    F::Type{<:MOI.AbstractFunction},
    S::Type{<:MOI.AbstractSet},
)</pre>
```

Return the cost of bridging F-in-S constraints.

is\_bridged(b, S) is assumed to be true.

source

MathOptInterface.Bridges.runtests - Function.

```
runtests(
    Bridge::Type{<:AbstractBridge},
    input_fn::Function,
    output_fn::Function;
    variable_start = 1.2,
    constraint_start = 1.2,
    eltype = Float64,
    cannot_unbridge::Bool = false,
)</pre>
```

Run a series of tests that check the correctness of Bridge.

input\_fn and output\_fn are functions such that input\_fn(model) and output\_fn(model) load the corresponding model into model.

Set cannot\_unbridge to true if the bridge transformation is not invertible. If Bridge is a variable bridge this allows Variable.unbridged\_map to returns nothing so that the tests allow errors that can be raised due to this. If Bridge is a constraint bridge this allows the getter of MOI.ConstraintFunction and MOI.ConstraintPrimalStart to throw MOI.GetAttributeNotAllowed.

#### **Example**

source

```
runtests(
    Bridge::Type{<:AbstractBridge},
    input::String,
    output::String;
    variable_start = 1.2,
    constraint_start = 1.2,
    eltype = Float64,
)</pre>
```

Run a series of tests that check the correctness of Bridge.

input and output are models in the style of MOI. Utilities.loadfromstring!.

### **Example**

source

# **Constraint bridge API**

MathOptInterface.Bridges.Constraint.AbstractBridge - Type.

```
abstract type AbstractBridge <: MOI.Bridges.AbstractType</pre>
```

 $Subtype\ of\ {\tt MOI.Bridges.AbstractBridge}\ for\ constraint\ bridges.$ 

In addition to the required implementation described in MOI. Bridges. AbstractBridge, subtypes of AbstractBridge must additionally implement:

- MOI.supports\_constraint(::Type{<:AbstractBridge}, ::Type{<:MOI.AbstractFunction}, ::Type{<:MOI.AbstractFunction}
- concrete\_bridge\_type
- bridge\_constraint

source

 ${\tt MathOptInterface.Bridges.Constraint.SingleBridgeOptimizer-Type.}$ 

```
SingleBridgeOptimizer{BT<:AbstractBridge}(model::MOI.ModelLike)
```

Return AbstractBridgeOptimizer that always bridges any objective function supported by the bridge BT.

This is in contrast with the MOI.Bridges.LazyBridgeOptimizer, which only bridges the objective function if it is supported by the bridge BT and unsupported by model.

### **Example**

#### Implementation notes

All bridges should simplify the creation of SingleBridgeOptimizers by defining a constant that wraps the bridge in a SingleBridgeOptimizer.

This enables users to create bridged models as follows:

```
julia> MyNewBridgeModel{Float64}(MOI.Utilities.Model{Float64}());
```

MathOptInterface.supports\_constraint - Method.

```
MOI.supports_constraint(
   BT::Type{<:AbstractBridge},
   F::Type{<:MOI.AbstractFunction},
   S::Type{<:MOI.AbstractSet},
)::Bool</pre>
```

Return a Bool indicating whether the bridges of type BT support bridging F-in-S constraints.

# Implementation notes

source

- This method depends only on the type of the inputs, not the runtime values.
- There is a default fallback, so you need only implement this method for constraint types that the bridge implements.

source

MathOptInterface.Bridges.Constraint.concrete\_bridge\_type - Function.

```
concrete_bridge_type(
   BT::Type{<:AbstractBridge},
   F::Type{<:MOI.AbstractFunction},
   S::Type{<:MOI.AbstractSet}
)::Type</pre>
```

Return the concrete type of the bridge supporting F-in-S constraints.

This function can only be called if MOI.supports\_constraint(BT, F, S) is true.

#### **Example**

The SplitIntervalBridge bridges a MOI. VariableIndex-in-MOI. Interval constraint into a MOI. VariableIndex-in-MOI. GreaterThan and a MOI. VariableIndex-in-MOI. LessThan constraint.

source

MathOptInterface.Bridges.Constraint.bridge\_constraint - Function.

```
bridge_constraint(
   BT::Type{<:AbstractBridge},
   model::MOI.ModelLike,
   func::AbstractFunction,
   set::MOI.AbstractSet,
)::BT</pre>
```

Bridge the constraint func-in-set using bridge BT to model and returns a bridge object of type BT.

# Implementation notes

• The bridge type BT should be a concrete type, that is, all the type parameters of the bridge must be set.

source

MathOptInterface.Bridges.Constraint.add all bridges - Function.

```
add_all_bridges(model, ::Type{T}) where {T}
```

Add all bridges defined in the Bridges. Constraint submodule to model.

The coefficient type used is T.

source

MathOptInterface.Bridges.Constraint.conversion\_cost - Function.

```
conversion_cost(
    F::Type{<:MOI.AbstractFunction},
    G::Type{<:MOI.AbstractFunction},
)::Float64</pre>
```

Return a Float64 returning the cost of converting any function of type G to a function of type F with convert.

This cost is used to compute MOI.Bridges.bridging cost.

The default cost is Inf, which means that MOI.Bridges.Constraint.FunctionConversionBridge should not attempt the conversion.

source

# Objective bridge API

MathOptInterface.Bridges.Objective.AbstractBridge - Type.

```
abstract type AbstractBridge <: MOI.Bridges.AbstractBridge end</pre>
```

Subtype of MOI.Bridges.AbstractBridge for objective bridges.

In addition to the required implementation described in MOI. Bridges. AbstractBridge, subtypes of AbstractBridge must additionally implement:

- supports\_objective\_function
- concrete\_bridge\_type
- bridge\_objective
- MOI.Bridges.set\_objective\_function\_type

source

MathOptInterface.Bridges.Objective.SingleBridgeOptimizer - Type.

```
SingleBridgeOptimizer{BT<:AbstractBridge}(model::MOI.ModelLike)
```

Return AbstractBridgeOptimizer that always bridges any objective function supported by the bridge BT.

This is in contrast with the MOI.Bridges.LazyBridgeOptimizer, which only bridges the objective function if it is supported by the bridge BT and unsupported by model.

#### **Example**

#### Implementation notes

All bridges should simplify the creation of SingleBridgeOptimizers by defining a constant that wraps the bridge in a SingleBridgeOptimizer.

This enables users to create bridged models as follows:

```
julia> MyNewBridgeModel{Float64}(MOI.Utilities.Model{Float64}());
```

source

MathOptInterface.Bridges.Objective.supports\_objective\_function - Function.

```
supports_objective_function(
   BT::Type{<:MOI.Bridges.Objective.AbstractBridge},
   F::Type{<:MOI.AbstractFunction},
)::Bool</pre>
```

Return a Bool indicating whether the bridges of type BT support bridging objective functions of type F.

# Implementation notes

- This method depends only on the type of the inputs, not the runtime values.
- There is a default fallback, so you need only implement this method For objective functions that the bridge implements.

source

MathOptInterface.Bridges.set\_objective\_function\_type - Function.

```
set_objective_function_type(
   BT::Type{<:Objective.AbstractBridge},
)::Type{<:MOI.AbstractFunction}</pre>
```

Return the type of objective function that bridges of concrete type BT set.

# Implementation notes

• This method depends only on the type of the bridge, not the runtime value.

# **Example**

source

MathOptInterface.Bridges.Objective.concrete bridge type - Function.

```
concrete_bridge_type(
   BT::Type{<:MOI.Bridges.Objective.AbstractBridge},
   F::Type{<:MOI.AbstractFunction},
)::Type</pre>
```

Return the concrete type of the bridge supporting objective functions of type F.

This function can only be called if  $MOI.supports\_objective\_function(BT, F)$  is true.

source

MathOptInterface.Bridges.Objective.bridge\_objective - Function.

```
bridge_objective(
   BT::Type{<:MOI.Bridges.Objective.AbstractBridge},
   model::MOI.ModelLike,
   func::MOI.AbstractFunction,
)::BT</pre>
```

Bridge the objective function func using bridge BT to model and returns a bridge object of type BT.

### Implementation notes

 The bridge type BT must be a concrete type, that is, all the type parameters of the bridge must be set.

source

MathOptInterface.Bridges.Objective.add all bridges - Function.

```
add_all_bridges(model, ::Type{T}) where {T}
```

Add all bridges defined in the Bridges.Objective submodule to model.

The coefficient type used is T.

source

### Variable bridge API

MathOptInterface.Bridges.Variable.AbstractBridge - Type.

```
abstract type AbstractBridge <: MOI.Bridges.AbstractBridge end</pre>
```

Subtype of MOI.Bridges.AbstractBridge for variable bridges.

In addition to the required implementation described in MOI. Bridges. AbstractBridge, subtypes of AbstractBridge must additionally implement:

- supports constrained variable
- concrete\_bridge\_type
- bridge\_constrained\_variable

source

MathOptInterface.Bridges.Variable.SingleBridgeOptimizer - Type.

```
SingleBridgeOptimizer{BT<:AbstractBridge}(model::MOI.ModelLike)
```

Return MOI.Bridges.AbstractBridgeOptimizer that always bridges any variables constrained on creation supported by the bridge BT.

This is in contrast with the MOI.Bridges.LazyBridgeOptimizer, which only bridges the variables constrained on creation if they are supported by the bridge BT and unsupported by model.

#### Warning

Two SingleBridgeOptimizers cannot be used together as both of them assume that the underlying model only returns variable indices with nonnegative values. Use MOI.Bridges.LazyBridgeOptimizer instead.

#### **Example**

# Implementation notes

All bridges should simplify the creation of SingleBridgeOptimizers by defining a constant that wraps the bridge in a SingleBridgeOptimizer.

This enables users to create bridged models as follows:

```
julia> MyNewBridgeModel{Float64}(MOI.Utilities.Model{Float64}());
```

source

MathOptInterface.Bridges.Variable.supports\_constrained\_variable - Function.

```
supports_constrained_variable(
   BT::Type{<:AbstractBridge},
   S::Type{<:MOI.AbstractSet},
)::Bool</pre>
```

Return a Bool indicating whether the bridges of type BT support bridging constrained variables in S. That is, it returns true if the bridge of type BT converts constrained variables of type S into a form supported by the solver.

# Implementation notes

- This method depends only on the type of the bridge and set, not the runtime values.
- There is a default fallback, so you need only implement this method for sets that the bridge implements.

### **Example**

source

MathOptInterface.Bridges.Variable.concrete bridge type - Function.

```
concrete_bridge_type(
   BT::Type{<:AbstractBridge},
   S::Type{<:MOI.AbstractSet},
)::Type</pre>
```

Return the concrete type of the bridge supporting variables in S constraints.

This function can only be called if MOI.supports\_constrained\_variable(BT, S) is true.

# **Example**

As a variable in MOI. GreaterThan is bridged into variables in MOI. Nonnegatives by the VectorizeBridge:

source

MathOptInterface.Bridges.Variable.bridge\_constrained\_variable - Function.

```
bridge_constrained_variable(
   BT::Type{<:AbstractBridge},
   model::MOI.ModelLike,
   set::MOI.AbstractSet,
)::BT</pre>
```

Bridge the constrained variable in set using bridge BT to model and returns a bridge object of type BT.

# Implementation notes

 The bridge type BT must be a concrete type, that is, all the type parameters of the bridge must be set.

source

 ${\tt MathOptInterface.Bridges.Variable.add\_all\_bridges-Function}.$ 

```
add_all_bridges(model, ::Type{T}) where {T}
```

 ${\bf Add\ all\ bridges\ defined\ in\ the\ Bridges. Variable\ submodule\ to\ model}.$ 

The coefficient type used is T.

source

MathOptInterface.Bridges.Variable.unbridged\_map - Function.

```
unbridged_map(
  bridge::MOI.Bridges.Variable.AbstractBridge,
  vi::MOI.VariableIndex,
)
```

For a bridged variable in a scalar set, return a tuple of pairs mapping the variables created by the bridge to an affine expression in terms of the bridged variable vi.

```
unbridged_map(
    bridge::MOI.Bridges.Variable.AbstractBridge,
    vis::Vector{MOI.VariableIndex},
)
```

For a bridged variable in a vector set, return a tuple of pairs mapping the variables created by the bridge to an affine expression in terms of the bridged variable vis. If this method is not implemented, it falls back to calling the following method for every variable of vis.

```
unbridged_map(
    bridge::MOI.Bridges.Variable.AbstractBridge,
    vi::MOI.VariableIndex,
    i::MOI.Bridges.IndexInVector,
)
```

For a bridged variable in a vector set, return a tuple of pairs mapping the variables created by the bridge to an affine expression in terms of the bridged variable vi corresponding to the ith variable of the vector.

If there is no way to recover the expression in terms of the bridged variable(s) vi(s), return nothing. See ZerosBridge for an example of bridge returning nothing.

source

# AbstractBridgeOptimizer API

MathOptInterface.Bridges.AbstractBridgeOptimizer - Type.

```
abstract type AbstractBridgeOptimizer <: MOI.AbstractOptimizer end</pre>
```

An abstract type that implements generic functions for bridges.

#### Implementation notes

By convention, the inner optimizer should be stored in a model field. If not, the optimizer must implement MOI.optimize!.

source

MathOptInterface.Bridges.bridged variable function - Function.

```
bridged_variable_function(
    b::AbstractBridgeOptimizer,
    vi::MOI.VariableIndex,
)
```

Return a MOI. AbstractScalarFunction of variables of b.model that equals vi. That is, if the variable vi is bridged, it returns its expression in terms of the variables of b.model. Otherwise, it returns vi.

source

MathOptInterface.Bridges.unbridged\_variable\_function - Function.

```
unbridged_variable_function(
    b::AbstractBridgeOptimizer,
    vi::MOI.VariableIndex,
)
```

Return a MOI.AbstractScalarFunction of variables of b that equals vi. That is, if the variable vi is an internal variable of b.model created by a bridge but not visible to the user, it returns its expression in terms of the variables of bridged variables. Otherwise, it returns vi.

source

MathOptInterface.Bridges.bridged\_function - Function.

```
bridged_function(b::AbstractBridgeOptimizer, value)::typeof(value)
```

Substitute any bridged MOI. VariableIndex in value by an equivalent expression in terms of variables of b.model.

source

MathOptInterface.Bridges.supports\_constraint\_bridges - Function.

```
supports_constraint_bridges(b::AbstractBridgeOptimizer)::Bool
```

Return a Bool indicating if b supports MOI.Bridges.Constraint.AbstractBridge.

source

MathOptInterface.Bridges.recursive\_model - Function.

```
recursive_model(b::AbstractBridgeOptimizer)
```

If a variable, constraint, or objective is bridged, return the context of the inner variables. For most optimizers, this should be b.model.

source

MathOptInterface.Bridges.FirstBridge - Type.

```
struct FirstBridge <: MOI.AbstractConstraintAttribute end
```

Returns the first bridge used to bridge the constraint.

#### Warning

The indices of the bridge correspond to internal indices and may not correspond to indices of the model this attribute is got from.

# LazyBridgeOptimizer API

MathOptInterface.Bridges.LazyBridgeOptimizer - Type.

```
LazyBridgeOptimizer(model::MOI.ModelLike)
```

The LazyBridgeOptimizer is a bridge optimizer that supports multiple bridges, and only bridges things which are not supported by the internal model.

Internally, the LazyBridgeOptimizer solves a shortest hyper-path problem to determine which bridges to use.

In general, you should use full\_bridge\_optimizer instead of this constructor because full\_bridge\_optimizer automatically adds a large number of supported bridges.

See also: add\_bridge, remove\_bridge, has\_bridge and full\_bridge\_optimizer.

### **Example**

```
julia> model = MOI.Bridges.LazyBridgeOptimizer(MOI.Utilities.Model{Float64}());

julia> MOI.Bridges.add_bridge(model, MOI.Bridges.Variable.FreeBridge{Float64})

julia> MOI.Bridges.has_bridge(model, MOI.Bridges.Variable.FreeBridge{Float64})

true
```

source

MathOptInterface.Bridges.full\_bridge\_optimizer - Function.

```
full_bridge_optimizer(model::MOI.ModelLike, ::Type{T}) where {T}
```

Returns a LazyBridgeOptimizer bridging model for every bridge defined in this package (see below for the few exceptions) and for the coefficient type T, as well as the bridges in the list returned by the ListOfNonstandardBridges attribute.

#### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();
julia> bridged_model = MOI.Bridges.full_bridge_optimizer(model, Float64);
```

### **Exceptions**

The following bridges are not added by full\_bridge\_optimizer, except if they are in the list returned by the ListOfNonstandardBridges attribute:

- Constraint.SOCtoNonConvexQuadBridge
- Constraint.RSOCtoNonConvexQuadBridge](@ref)
- Constraint.SOCtoPSDBridge

• If T is not a subtype of AbstractFloat, subtypes of Constraint.AbstractToIntervalBridge

```
- Constraint.GreaterToIntervalBridge
```

```
- Constraint.LessToIntervalBridge)
```

See the docstring of the each bridge for the reason they are not added.

source

MathOptInterface.Bridges.ListOfNonstandardBridges - Type.

```
ListOfNonstandardBridges{T}() <: MOI.AbstractOptimizerAttribute
```

Any optimizer can be wrapped in a LazyBridgeOptimizer using full\_bridge\_optimizer. However, by default LazyBridgeOptimizer uses a limited set of bridges that are:

- implemented in MOI.Bridges
- 2. generally applicable for all optimizers.

For some optimizers however, it is useful to add additional bridges, such as those that are implemented in external packages (for example, within the solver package itself) or only apply in certain circumstances (for example, Constraint.SOCtoNonConvexQuadBridge).

Such optimizers should implement the ListOfNonstandardBridges attribute to return a vector of bridge types that are added by full\_bridge\_optimizer in addition to the list of default bridges.

Note that optimizers implementing ListOfNonstandardBridges may require package-specific functions or sets to be used if the non-standard bridges are not added. Therefore, you are recommended to use model = MOI.instantiate(Package.Optimizer; with\_bridge\_type = T) instead of model = MOI.instantiate(Package.Optimizer) See MOI.instantiate.

#### **Example**

# An optimizer using a non-default bridge in MOI.Bridges

Solvers supporting MOI.ScalarQuadraticFunction can support MOI.SecondOrderCone and MOI.RotatedSecondOrderCone by defining:

```
function MOI.get(::MyQuadraticOptimizer, ::ListOfNonstandardBridges{Float64})
    return Type[
         MOI.Bridges.Constraint.SOCtoNonConvexQuadBridge{Float64},
         MOI.Bridges.Constraint.RSOCtoNonConvexQuadBridge{Float64},
    ]
end
```

# An optimizer defining an internal bridge

Suppose an optimizer can exploit specific structure of a constraint, for example, it can exploit the structure of the matrix A in the linear system of equations A \* x = b.

The optimizer can define the function:

```
struct MatrixAffineFunction{T} <: MOI.AbstractVectorFunction
    A::SomeStructuredMatrixType{T}
    b::Vector{T}
end</pre>
```

and then a bridge

```
struct MatrixAffineFunctionBridge{T} <: MOI.Constraint.AbstractBridge
    # ...
end
# ...</pre>
```

from  $VectorAffineFunction\{T\}$  to the MatrixAffineFunction. Finally, it defines:

```
function MOI.get(::Optimizer{T}, ::ListOfNonstandardBridges{T}) where {T}
    return Type[MatrixAffineFunctionBridge{T}]
end
```

source

 ${\tt MathOptInterface.Bridges.add\_bridge-Function}.$ 

```
add_bridge(b::LazyBridgeOptimizer, BT::Type{<:AbstractBridge})</pre>
```

Enable the use of the bridges of type BT by b.

source

MathOptInterface.Bridges.remove\_bridge - Function.

```
remove_bridge(b::LazyBridgeOptimizer, BT::Type{<:AbstractBridge})</pre>
```

Disable the use of the bridges of type BT by b.

source

 ${\tt MathOptInterface.Bridges.has\_bridge-Function}.$ 

```
has_bridge(b::LazyBridgeOptimizer, BT::Type{<:AbstractBridge})</pre>
```

Return a Bool indicating whether the bridges of type BT are used by b.

source

 ${\tt MathOptInterface.Bridges.print\_active\_bridges-Function}.$ 

```
print_active_bridges([io::I0=stdout,] b::MOI.Bridges.LazyBridgeOptimizer)
```

Print the set of bridges that are active in the model b.

source

```
print_active_bridges(
    [io::I0=stdout,]
    b::MOI.Bridges.LazyBridgeOptimizer,
    F::Type{<:MOI.AbstractFunction}
)</pre>
```

Print the set of bridges required for an objective function of type F.

source

```
print_active_bridges(
    [io::I0=stdout,]
    b::MOI.Bridges.LazyBridgeOptimizer,
    F::Type{<:MOI.AbstractFunction},
    S::Type{<:MOI.AbstractSet},
)</pre>
```

Print the set of bridges required for a constraint of type F-in-S.

source

```
print_active_bridges(
    [io::I0=stdout,]
    b::MOI.Bridges.LazyBridgeOptimizer,
    S::Type{<:MOI.AbstractSet}
)</pre>
```

Print the set of bridges required for a variable constrained to set S.

source

MathOptInterface.Bridges.print\_graph - Function.

```
print_graph([io::I0 = stdout,] b::LazyBridgeOptimizer)
```

Print the hyper-graph containing all variable, constraint, and objective types that could be obtained by bridging the variables, constraints, and objectives that are present in the model by all the bridges added to b.

Each node in the hyper-graph corresponds to a variable, constraint, or objective type.

- Variable nodes are indicated by [ ]
- Constraint nodes are indicated by ( )
- Objective nodes are indicated by | |

The number inside each pair of brackets is an index of the node in the hyper-graph.

Note that this hyper-graph is the full list of possible transformations. When the bridged model is created, we select the shortest hyper-path from this graph, so many nodes may be un-used.

To see which nodes are used, call print\_active\_bridges.

For more information, see Legat, B., Dowson, O., Garcia, J., and Lubin, M. (2020). "MathOptInterface: a data structure for mathematical optimization problems." URL

source

MathOptInterface.Bridges.debug supports constraint - Function.

```
debug_supports_constraint(
    b::LazyBridgeOptimizer,
    F::Type{<:MOI.AbstractFunction},
    S::Type{<:MOI.AbstractSet};
    io::IO = Base.stdout,
)</pre>
```

Prints to io explanations for the value of MOI.supports\_constraint with the same arguments.

source

MathOptInterface.Bridges.debug\_supports - Function.

```
debug_supports(
    b::LazyBridgeOptimizer,
    ::MOI.ObjectiveFunction{F};
    io::I0 = Base.stdout,
) where F
```

Prints to io explanations for the value of MOI. supports with the same arguments.

source

# SetMap API

 ${\tt MathOptInterface.Bridges.MapNotInvertible-Type}.\\$ 

```
struct MapNotInvertible <: Exception
  message::String
end</pre>
```

An error thrown by inverse\_map\_function or inverse\_adjoint\_map\_function indicating that the linear map A defined in Variable.SetMapBridge and Constraint.SetMapBridge is not invertible.

source

MathOptInterface.Bridges.map\_set - Function.

```
map_set(bridge::MOI.Bridges.AbstractBridge, set)
map_set(::Type{BT}, set) where {BT}
```

 $Return \ the \ image \ of \ set\ through \ the \ linear \ map\ A\ defined \ in\ Variable\ . Set\ MapBridge\ and\ Constraint\ . Set\ MapBridge\ .$ 

This function is used for bridging the constraint and setting the MOI.ConstraintSet.

source

MathOptInterface.Bridges.inverse\_map\_set - Function.

```
inverse_map_set(bridge::MOI.Bridges.AbstractBridge, set)
inverse_map_set(::Type{BT}, set) where {BT}
```

Return the preimage of set through the linear map A defined in Variable. SetMapBridge and Constraint. SetMapBridge.

This function is used for getting the MOI. ConstraintSet.

The method can alternatively be defined on the bridge type. This legacy interface is kept for backward compatibility.

source

MathOptInterface.Bridges.map function - Function.

```
map_function(bridge::MOI.Bridges.AbstractBridge, func)
map_function(::Type{BT}, func) where {BT}
```

Return the image of func through the linear map A defined in Variable. SetMapBridge and Constraint. SetMapBridge.

This function is used for getting the MOI. ConstraintPrimal of variable bridges. For constraint bridges, this is used for bridging the constraint, setting the MOI. ConstraintFunction and MOI. ConstraintPrimalStart and modifying the function with MOI. modify.

The default implementation of Constraint.bridge\_constraint uses map\_function with the bridge type so if this function is defined on the bridge type, Constraint.bridge\_constraint does not need to be implemented.

source

```
map_function(::Type{BT}, func, i::IndexInVector) where {BT}
```

Return the scalar function at the ith index of the vector function that would be returned by map\_function(BT, func) except that it may compute the ith element. This is used by bridged\_function and for getting the MOI.VariablePrimal and MOI.VariablePrimalStart of variable bridges.

source

MathOptInterface.Bridges.inverse\_map\_function - Function.

```
inverse_map_function(bridge::MOI.Bridges.AbstractBridge, func)
inverse_map_function(::Type{BT}, func) where {BT}
```

Return the image of func through the inverse of the linear map A defined in Variable.SetMapBridge and Constraint.SetMapBridge.

This function is used by Variable.unbridged\_map and for setting the MOI.VariablePrimalStart of variable bridges and for getting the MOI.ConstraintFunction, the MOI.ConstraintPrimal and the MOI.ConstraintPrimalStart of constraint bridges.

If the linear map A is not invertible, the error MapNotInvertible is thrown.

The method can alternatively be defined on the bridge type. This legacy interface is kept for backward compatibility.

source

MathOptInterface.Bridges.adjoint\_map\_function - Function.

```
adjoint_map_function(bridge::MOI.Bridges.AbstractBridge, func)
adjoint_map_function(::Type{BT}, func) where {BT}
```

Return the image of func through the adjoint of the linear map A defined in Variable.SetMapBridge and Constraint.SetMapBridge.

This function is used for getting the MOI.ConstraintDual and MOI.ConstraintDualStart of constraint bridges.

The method can alternatively be defined on the bridge type. This legacy interface is kept for backward compatibility.

source

MathOptInterface.Bridges.inverse\_adjoint\_map\_function - Function.

```
inverse_adjoint_map_function(bridge::MOI.Bridges.AbstractBridge, func)
inverse_adjoint_map_function(::Type{BT}, func) where {BT}
```

Return the image of func through the inverse of the adjoint of the linear map A defined in Variable. SetMapBridge and Constraint. SetMapBridge.

This function is used for getting the MOI.ConstraintDual of variable bridges and setting the MOI.ConstraintDualStart of constraint bridges.

If the linear map A is not invertible, the error MapNotInvertible is thrown.

The method can alternatively be defined on the bridge type. This legacy interface is kept for backward compatibility.

source

### **Bridging graph API**

MathOptInterface.Bridges.Graph - Type.

```
Graph()
```

A type-stable datastructure for computing the shortest hyperpath problem.

# Nodes

There are three types of nodes in the graph:

- VariableNode
- ConstraintNode

• ObjectiveNode

Add nodes to the graph using add\_node.

#### **Edges**

There are two types of edges in the graph:

- Edge
- ObjectiveEdge

Add edges to the graph using add\_edge.

For the ability to add a variable constrained on creation as a free variable followed by a constraint, use set\_variable\_constraint\_node.

### Optimal hyper-edges

Use bridge index to compute the minimum-cost bridge leaving a node.

Note that <code>bridge\_index</code> lazy runs a Bellman-Ford algorithm to compute the set of minimum cost edges. Thus, the first call to <code>bridge\_index</code> after adding new nodes or edges will take longer than subsequent calls.

source

MathOptInterface.Bridges.VariableNode - Type.

```
VariableNode(index::Int)
```

A node in Graph representing a variable constrained on creation.

source

 ${\tt MathOptInterface.Bridges.ConstraintNode-Type}.\\$ 

```
ConstraintNode(index::Int)
```

A node in Graph representing a constraint.

source

MathOptInterface.Bridges.ObjectiveNode - Type.

```
ObjectiveNode(index::Int)
```

A node in Graph representing an objective function.

source

MathOptInterface.Bridges.Edge - Type.

```
Edge(
    bridge_index::Int,
    added_variables::Vector{VariableNode},
    added_constraints::Vector{ConstraintNode},
    cost::Float64 = 1.0,
)
```

Return a new datastructure representing an edge in Graph that starts at a VariableNode or a ConstraintNode.

source

MathOptInterface.Bridges.ObjectiveEdge - Type.

```
ObjectiveEdge(
    bridge_index::Int,
    added_variables::Vector{VariableNode},
    added_constraints::Vector{ConstraintNode},
)
```

Return a new datastructure representing an edge in Graph that starts at an ObjectiveNode.

source

MathOptInterface.Bridges.add\_node - Function.

```
add_node(graph::Graph, ::Type{VariableNode})::VariableNode
add_node(graph::Graph, ::Type{ConstraintNode})::ConstraintNode
add_node(graph::Graph, ::Type{ObjectiveNode})::ObjectiveNode
```

Add a new node to graph.

source

MathOptInterface.Bridges.add\_edge - Function.

```
add_edge(graph::Graph, node::VariableNode, edge::Edge)::Nothing
add_edge(graph::Graph, node::ConstraintNode, edge::Edge)::Nothing
add_edge(graph::Graph, node::ObjectiveNode, edge::ObjectiveEdge)::Nothing
```

Add edge to graph, where edge starts at node and connects to the nodes defined in edge.

source

MathOptInterface.Bridges.set\_variable\_constraint\_node - Function.

```
set_variable_constraint_node(
    graph::Graph,
    variable_node::VariableNode,
    constraint_node::ConstraintNode,
    cost::Int,
)
```

As an alternative to variable\_node, add a virtual edge to graph that represents adding a free variable, followed by a constraint of type constraint\_node, with bridging cost cost.

# Why is this needed?

Variables can either be added as a variable constrained on creation, or as a free variable which then has a constraint added to it.

source

MathOptInterface.Bridges.bridge\_index - Function.

```
bridge_index(graph::Graph, node::VariableNode)::Int
bridge_index(graph::Graph, node::ConstraintNode)::Int
bridge_index(graph::Graph, node::ObjectiveNode)::Int
```

Return the optimal index of the bridge to chose from node.

source

 ${\tt MathOptInterface.Bridges.is\_variable\_edge\_best-Function}.$ 

```
is_variable_edge_best(graph::Graph, node::VariableNode)::Bool
```

Return a Bool indicating whether node should be added as a variable constrained on creation, or as a free variable followed by a constraint.

source

# **Chapter 28**

# **FileFormats**

# 28.1 Overview

### The FileFormats submodule

The FileFormats module provides functions for reading and writing MOI models using write\_to\_file and read\_from\_file.

### Supported file types

You must read and write files to a FileFormats. Model object. Specific the file-type by passing a FileFormats. FileFormat enum. For example:

### The Conic Benchmark Format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_CBF)
MOI.FileFormats.CBF.Model
| ObjectiveSense: FEASIBILITY_SENSE
| ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
L NumberOfConstraints: 0
```

# The LP file format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_LP)
MOI.FileFormats.LP.Model
| ObjectiveSense: FEASIBILITY_SENSE
| ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
L NumberOfConstraints: 0
```

# The MathOptFormat file format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MOF)
MOI.FileFormats.MOF.Model
| ObjectiveSense: FEASIBILITY_SENSE
| ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
```

```
| NumberOfVariables: 0
| NumberOfConstraints: 0
```

### The MPS file format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MPS)
MOI.FileFormats.MPS.Model
| ObjectiveSense: FEASIBILITY_SENSE
| ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
L NumberOfConstraints: 0
```

### The NL file format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_NL)
MOI.FileFormats.NL.Model
| ObjectiveSense: unknown
| ObjectiveFunctionType: unknown
| NumberOfVariables: unknown
L NumberOfConstraints: unknown
```

#### The REW file format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_REW)
MOI.FileFormats.MPS.Model
| ObjectiveSense: FEASIBILITY_SENSE
| ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
| NumberOfConstraints: 0
```

Note that the REW format is identical to the MPS file format, except that all names are replaced with generic identifiers.

### The SDPA file format

```
julia> model = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_SDPA)
MOI.FileFormats.SDPA.Model
| ObjectiveSense: FEASIBILITY_SENSE
| ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
L NumberOfConstraints: 0
```

# Write to file

To write a model src to a MathOptFormat file, use:

```
julia> src = MOI.Utilities.Model{Float64}();
julia> MOI.add_variable(src);
julia> dest = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MOF);
julia> MOI.copy_to(dest, src)
MathOptInterface.Utilities.IndexMap with 1 entry:
    MOI.VariableIndex(1) => MOI.VariableIndex(1)
julia> MOI.write_to_file(dest, "file.mof.json")
julia> print(read("file.mof.json", String))
{"name":"MathOptFormat
    → Model", "version":{"major":1, "minor":7}, "variables":[{"name":"x1"}], "objective":{"sense":"feasibility"}, "constraint"
```

#### Read from file

To read a MathOptFormat file, use:

```
julia> dest = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MOF);

julia> MOI.read_from_file(dest, "file.mof.json")

julia> MOI.get(dest, MOI.ListOfVariableIndices())
1-element Vector{MathOptInterface.VariableIndex}:
    MOI.VariableIndex(1)

julia> rm("file.mof.json") # Clean up after ourselves.
```

# **Detecting the file-type automatically**

Instead of the format keyword, you can also use the filename keyword argument to FileFormats. Model. This will attempt to automatically guess the format from the file extension. For example:

```
julia> src = MOI.Utilities.Model{Float64}();
julia> dest = MOI.FileFormats.Model(filename = "file.cbf.gz");
julia> MOI.copy_to(dest, src)
MathOptInterface.Utilities.IndexMap()

julia> MOI.write_to_file(dest, "file.cbf.gz")
julia> src_2 = MOI.FileFormats.Model(filename = "file.cbf.gz");
julia> mOI.read_from_file(src_2, "file.cbf.gz")
julia> rm("file.cbf.gz") # Clean up after ourselves.
```

Note how the compression format (GZip) is also automatically detected from the filename.

### **Unsupported constraints**

In some cases src may contain constraints that are not supported by the file format (for example, the CBF format supports integer variables but not binary). If so, copy src to a bridged model using Bridges.full\_bridge\_optimizer:

```
julia> src = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variable(src);
julia> MOI.add_constraint(src, x, MOI.ZeroOne());
julia> dest = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_CBF);
julia> bridged = MOI.Bridges.full_bridge_optimizer(dest, Float64);
julia> MOI.copy_to(bridged, src);
julia> MOI.write_to_file(dest, "my_model.cbf")
julia> rm("my_model.cbf") # Clean up after ourselves.
```

#### Note

Even after bridging, it may still not be possible to write the model to file because of unsupported constraints (for example, PSD variables in the LP file format).

### Read and write to io

In addition to write\_to\_file and read\_from\_file, you can read and write directly from IO streams using Base.write and Base.read!:

```
julia> src = MOI.Utilities.Model{Float64}();

julia> dest = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MPS);

julia> MOI.copy_to(dest, src)

MathOptInterface.Utilities.IndexMap()

julia> io = IOBuffer();

julia> write(io, dest)

julia> seekstart(io);

julia> src_2 = MOI.FileFormats.Model(format = MOI.FileFormats.FORMAT_MPS);

julia> read!(io, src_2);
```

# ScalarNonlinearFunction

By default, reading a .nl or .mof.json that contains nonlinear expressions will create an NLPBlock.

To instead read nonlinear expressions as ScalarNonlinearFunction, pass the use\_nlp\_block = false keyword argument to the Model constructor:

#### Validating MOF files

MathOptFormat files are governed by a schema. Use JSONSchema.jl to check if a .mof.json file satisfies the schema.

First, construct the schema object as follows:

```
julia> import JSON, JSONSchema

julia> schema = JSONSchema.Schema(JSON.parsefile(MOI.FileFormats.MOF.SCHEMA_PATH))
A JSONSchema
```

Then, check if a model file is valid using isvalid:

If we construct an invalid file, for example by mis-typing name as NaMe, the validation fails:

```
julia> isvalid(schema, bad_model)
false
```

Use JSONSchema.validate to obtain more insight into why the validation failed:

```
julia> JSONSchema.validate(schema, bad_model)
Validation failed:
path: [variables][1]
instance: Dict{String, Any}("NaMe" => "x")
schema key: required
schema value: Any["name"]
```

### 28.2 API Reference

#### **File Formats**

Functions to help read and write MOI models to/from various file formats. See The FileFormats submodule for more details.

MathOptInterface.FileFormats.Model - Function.

```
Model(;
   format::FileFormat = FORMAT_AUTOMATIC,
   filename::Union{Nothing, String} = nothing,
   kwargs...
)
```

Return model corresponding to the FileFormats.FileFormat format, or, if format == FORMAT\_AUTOMATIC, guess the format from filename.

The filename argument is only needed if format == FORMAT\_AUTOMATIC.

kwargs are passed to the underlying model constructor.

source

MathOptInterface.FileFormats.FileFormat - Type.

```
FileFormat
```

List of accepted export formats.

### **Values**

FORMAT\_AUTOMATIC

Try to detect the file format based on the file name.

FORMAT\_CBF

The Conic Benchmark format.

See FileFormats.CBF.Model for more details. FORMAT\_LP The LP file format. See FileFormats.LP.Model for more details. FORMAT\_MOF The MathOptFormat file format. See FileFormats.MOF.Model for more details. FORMAT\_MPS The MPS file format. See FileFormats.MPS.Model for more details. FORMAT\_NL The AMPL .nl file format. See FileFormats.NL.Model for more details. FORMAT\_REW The .rew file format, which is equivalent to the MPS format (FileFormats.FORMAT\_MPS) with the generic\_names = true keyword argument set by default. See FileFormats.MPS.Model for more details. FORMAT\_SDPA The SemiDefinite Programming Algorithm format. See FileFormats.SDPA.Model for more details. source

MathOptInterface.FileFormats.FORMAT\_AUTOMATIC - Constant.

```
FORMAT_AUTOMATIC::FileFormat
```

An instance of the FileFormat enum.

### **About**

Try to detect the file format based on the file name.

source

MathOptInterface.FileFormats.FORMAT\_CBF - Constant.

```
FORMAT_CBF::FileFormat
```

An instance of the FileFormat enum.

#### **About**

The Conic Benchmark format.

See FileFormats.CBF.Model for more details.

source

MathOptInterface.FileFormats.CBF.Model - Type.

```
Model()
```

Create an empty instance of FileFormats.CBF.Model.

source

MathOptInterface.FileFormats.FORMAT\_LP - Constant.

```
FORMAT_LP::FileFormat
```

An instance of the FileFormat enum.

#### **About**

The LP file format.

See FileFormats.LP.Model for more details.

source

MathOptInterface.FileFormats.LP.Model - Type.

```
Model(;
    maximum_length::Int = 255,
    warn::Bool = false,
    coefficient_type::Type{T} = Float64,
) where {T}
```

Create an empty instance of FileFormats.LP.Model.

Keyword arguments are:

- maximum\_length::Int=255: the maximum length for the name of a variable. lp\_solve 5.0 allows only 16 characters, while CPLEX 12.5+ allow 255.
- warn::Bool=false: print a warning when variables or constraints are renamed.
- coefficient\_type::Type{T} = Float64: the supported type to use when reading and writing files.

source

MathOptInterface.FileFormats.FORMAT\_MOF - Constant.

```
FORMAT_MOF::FileFormat
```

An instance of the FileFormat enum.

#### **About**

The MathOptFormat file format.

See FileFormats.MOF.Model for more details.

source

MathOptInterface.FileFormats.MOF.Model - Type.

```
Model(;
    print_compact::Bool = false,
    warn::Bool = false,
    differentiation_backend::MOI.Nonlinear.AbstractAutomaticDifferentiation =
        MOI.Nonlinear.SparseReverseMode(),
    use_nlp_block::Union{Bool,Nothing} = nothing,
    coefficient_type::Type{T} = Float64,
) where {T}
```

Create an empty instance of FileFormats.MOF.Model.

Keyword arguments are:

- print compact::Bool=false: print the JSON file in a compact format without spaces or newlines.
- · warn::Bool=false: print a warning when variables or constraints are renamed
- differentiation\_backend::MOI.Nonlinear.AbstractAutomaticDifferentiation = MOI.Nonlinear.SparseRever automatic differentiation backend to use when reading models with nonlinear constraints and objectives.

use\_nlp\_block::Bool=true: if true parse "ScalarNonlinearFunction" into an MOI.NLPBlock. If false, "ScalarNonlinearFunction" are parsed as MOI.ScalarNonlinearFunction functions.

• coefficient\_type::Type{T} = Float64: the supported type to use when reading and writing files.

source

MathOptInterface.FileFormats.FORMAT MPS - Constant.

```
FORMAT_MPS::FileFormat
```

An instance of the FileFormat enum.

## **About**

The MPS file format.

See  $\label{lem:file_formats.MPS.Model} For more details.$ 

source

MathOptInterface.FileFormats.MPS.Model - Type.

```
Model(;
  warn::Bool = false,
  print_objsense::Bool = false,
  generic_names::Bool = false,
  quadratic_format::QuadraticFormat = kQuadraticFormatGurobi,
  coefficient_type::Type{T} = Float64,
) where {T}
```

Create an empty instance of FileFormats.MPS.Model.

Keyword arguments are:

- warn::Bool=false: print a warning when variables or constraints are renamed.
- print\_objsense::Bool=false: print the OBJSENSE section when writing
- generic\_names::Bool=false: strip all names in the model and replace them with the generic names C\$i and R\$i for the i'th column and row respectively.
- quadratic\_format::QuadraticFormat = kQuadraticFormatGurobi: specify the solver-specific extension used when writing the quadratic components of the model. Options are kQuadraticFormatGurobi, kQuadraticFormatCPLEX, and kQuadraticFormatMosek.
- coefficient type::Type{T}=Float64: the supported type to use when reading and writing files.

source

MathOptInterface.FileFormats.FORMAT\_NL - Constant.

```
FORMAT_NL::FileFormat
```

An instance of the FileFormat enum.

#### **About**

The AMPL .nl file format.

See FileFormats.NL.Model for more details.

source

MathOptInterface.FileFormats.NL.Model - Type.

```
Model(; use_nlp_block::Bool = true)
```

Create a new Optimizer object.

source

MathOptInterface.FileFormats.FORMAT REW - Constant.

```
FORMAT_REW::FileFormat
```

An instance of the FileFormat enum.

#### **About**

The .rew file format, which is equivalent to the MPS format (FileFormats.FORMAT\_MPS) with the generic\_names = true keyword argument set by default.

See FileFormats.MPS.Model for more details.

source

MathOptInterface.FileFormats.FORMAT\_SDPA - Constant.

```
FORMAT SDPA::FileFormat
```

An instance of the FileFormat enum.

#### **About**

The SemiDefinite Programming Algorithm format.

See FileFormats.SDPA.Model for more details.

source

MathOptInterface.FileFormats.SDPA.Model - Type.

```
Model(; coefficient_type::Type{T} = Float64) where {T}
```

Create an empty instance of FileFormats.SDPA.Model{T}.

It is important to be aware that the SDPA file format is interpreted in geometric form and not standard conic form. The standard conic form and geometric conic form are two dual standard forms for semidefinite programs (SDPs). The geometric conic form of an SDP is as follows:

$$\min_{y \in \mathbb{D}^m} \qquad b^T y \tag{28.1}$$

s.t. 
$$\sum_{i=1}^m A_i y_i - C \in \mathbb{K}$$
 (28.2)

where  $\mathcal{K}$  is a cartesian product of nonnegative orthant and positive semidefinite matrices that align with a block diagonal structure shared with the matrices  $A_i$  and C.

In other words, the geometric conic form contains free variables and affine constraints in either the nonnegative orthant or the positive semidefinite cone. That is, in the MathOptInterface's terminology, MOI. VectorAffineFunction-in-MOI.Nonnegatives and MOI.VectorAffineFunction-in-MOI.PositiveSemidefiniteConeTriangle constraints

The corresponding standard conic form of the dual SDP is as follows:

$$\max_{X\in\mathbb{K}} \qquad \operatorname{tr}(CX) \tag{28.3}$$

s.t. 
$$\operatorname{tr}(A_i X) = b_i$$
  $i = 1, \dots, m$ . (28.4)

In other words, the standard conic form contains nonnegative and positive semidefinite variables with equality constraints. That is, in the MathOptInterface's terminology, MOI.VectorOfVariables-in-MOI.Nonnegatives, MOI.VectorOfVariables-in-MOI.PositiveSemidefiniteConeTriangle and MOI.ScalarAffineFunction-in-MOI.EqualTo constraints.

If a model is in standard conic form, use Dualization.jl to transform it into the geometric conic form before writting it. Otherwise, the nonnegative (resp. positive semidefinite) variables will be bridged into free variables with affine constraints constraining them to belong to the nonnegative orthant (resp. positive semidefinite cone) by the MOI.Bridges.Constraint.VectorFunctionizeBridge. Moreover, equality constraints will be bridged into pairs of affine constraints in the nonnegative orthant by the MOI.Bridges.Constraint.SplitInte and then the MOI.Bridges.Constraint.VectorizeBridge.

If a solver is in standard conic form, use Dualization.jl to transform the model read into standard conic form before copying it to the solver. Otherwise, the free variables will be bridged into pairs of variables in the nonnegative orthant by the MOI.Bridges.Variable.FreeBridge and affine constraints will be bridged into equality constraints by creating a slack variable by the MOI.Bridges.Constraint.VectorSlackBridge.

source

#### Other helpers

MathOptInterface.FileFormats.NL.SolFileResults - Type.

```
SolFileResults(
   filename::String,
   model::Model;
   suffix_lower_bound_duals::Vector{String} =
        ["ipopt_zL_out", "lower_bound_duals"],
   suffix_uuper_bound_duals::Vector{String} =
        ["ipopt_zU_out", "upper_bound_duals"],
)
```

Parse the .sol file filename created by solving model and return a SolFileResults struct.

The returned struct supports the MOI.get API for querying result attributes such as MOI.TerminationStatus, MOI.VariablePrimal, and MOI.ConstraintDual.

source

```
SolFileResults(
    raw_status::String,
    termination_status::MOI.TerminationStatusCode,
)
```

Return a SolFileResults struct with MOI.RawStatusString set to raw\_status, MOI.TerminationStatus set to termination\_status, and MOI.PrimalStatus and MOI.DualStatus set to NO\_SOLUTION.

All other attributes are un-set.

source

# 28.3 The LP file format

### The LP file format

The purpose of this page is to document the LP file format, and the various differences that occur between solvers.

#### Resources

There are a bunch of different descriptions of the LP file format on the internet.

- CPLEX
- FICO
- Gurobi
- Ipsolve
- Mosek
- QSopt

#### Grammar

This section gives the grammar of an LP file as we implement it. This grammar may be different to that of particular solvers.

The syntax rules for the grammar are:

- $\bullet\,\,$  <name>: the name of a symbol in the grammar
- A :== B: A is equivalent to B
- A | B: either A or B
- i"": case insensitive string
- [A]: A is optional
- (A)\*: there are 0 or more repeats of A
- (A)+: there is at least one or more repeats of A

In addition to the grammar, there are the following rules:

- Comments begin with \, and run until the next \n character
- Whitespace is ignored
- · Newlines are ignored, except where explicitly described

```
<lp-file> :==
    <keyword-objective>\n
    [<section-objective>\n]
    <keyword-constraints>\n
    (<constraint>)*
    [<keyword-bounds>\n (<bound-expression>)+]
    [<keyword-general>\n (<identifier>)+]
    [<keyword-binary>\n (<identifier>)+]
    [<keyword-sos>\n (<constraint-sos>)+]
    [<keyword-end>]
<keyword-objective> :==
    i"min" | i"minimum" | i"minimize" | i"minimise"
  | i"max" | i"maximum" | i"maximize" | i"maximise"
<keyword-constraints> :==
    (i"subject to" | i"st" | i"st." | i"s.t." | i"such that")[":"]
<keyword-bounds> :== i"bound" | i"bounds"
<keyword-general> :== i"gen" | i"general" | i"generals"
<keyword-binary> :== i"bin" | i"binary" | i"binaries"
<keyword-sos> :== i"sos"
<keyword-end> :== i"end"
<digit> :== "0" | "1" | "2" | "3" | "4" | "5" | "6" | "7" | "8" | "9"
<char> :== a-z | A-Z | !"#$%&()/,;?@_'`|~
<identifier> :== <char> (<char> | <digit> | ".")*
<number> :==
   "+" <number>
  | "-" <number>
 | <digit>+[.(<digit>)*][("e" | "E")("+" | "-")(<digit>)+]
 | i"inf"
  | i"infinity"
<quadratic-term> :==
   "+" <quadratic-term>
  | "-" <quadratic-term>
  | \ [ < number > \ ["*"] ] < identifier > "^" \ "2"
  | \ [< number > \ ["*"]] < identifier > "*" < identifier >
<quadratic-expression> :==
   "[" <quadratic-term> (("+" | "-") <quadratic-term>)* "]" ["/" "2"]
<term> :==
   "+" <term>
  | "-" <term>
  | <identifier>
  | <number>
```

```
| <number> <identifier>
 | <number> "*" <identifier>
 | <quadratic-expression>
<expression> :== <term> (("+" | "-") <term>)*
<name> :== [<identifier>":"]
<objective> :== <name> [<expression>]
<inequality> :== "<" | "<=" | "=<" | ">" | ">=" | "=>" | "=="
<set-suffix> := <inequality> <number>
<set-prefix> := <number> <inequality>
<constraint-indicator> :== <identifier> "=" (0 \mid 1) "->" <expression> <set-suffix>
<constraint-sos> :==
   "S1::" (<identifier>":"<number>)+\n
 | "S2::" (<identifier>":"<number>)+\n
<constraint> :==
   <name> <expression> <set-suffix>
 | <name> <constraint-indicator>
 | <name> <sos-constraint>
<bound-expression> :==
   <identifier> i"free"
 | <identifier> <set-suffix>
 | <set-prefix> <identifier>
 | <set-prefix> <identifier> <set-suffix>
```

# **Differences**

There are many differences in how solvers parse an LP file.

#### The "integer" section

Consider the section:

```
integers x
```

Gurobi will interpret this as x in MOI.Integer(). FICO Xpress will interpret this as x in MOI.ZeroOne().

FICO document this behavior, but they're an outlier.

We choose to interpret integers as MOI.Integer().

# Whether variables can have the same name as a keyword

Consider the file

```
min
st
st
st
end
```

or even the horrific

```
min st st st >= 0 end
```

Gurobi will complain up front that the keyword st appears twice, whereas Xpress will read the file as equivalent to :

```
minimize
x
subject to
x >= 0
end
```

We choose to allow variables to be named as keywords, and we use context to disambiguate.

### Whitespace

Consider the file

```
minimize
2x
end
```

Gurobi will interpret this as a single variable with the name 2x, where as Xpress will interpret this as the expression 2 \* x.

Gurobi document this behavior, saying that they require whitespace around all tokens, but they're an outlier.

We choose to allow juxtaposted tokens without whitespace.

#### **Identifiers**

In general, an identifier may contain the letters a-z, A-Z, the digits 0-9, and the characters  $! "#\$%\&()/, .; ?@_'`|\sim$ . Additional solvers put additional restrictions:

- In (all?) solvers except Gurobi, the identifier must not start with a digit or a . (in Gurobi, identifiers must be separated by whitespace)
- Identifiers in Mosek and Ipsolve may not start with the letter e or E
- · Keywords must not be used as names in Gurobi or Mosek, but they may in Xpress
- Many solvers actually support reading any UTF-8 string as the identifier, but they will normalize to the legal letters on write

We choose to allow any valid UTF-8 names.

# **Chapter 29**

# **Nonlinear**

### 29.1 Overview

#### **Nonlinear**

The Nonlinear submodule contains data structures and functions for working with a nonlinear optimization problem in the form of an expression graph. This page explains the API and describes the rationale behind its design.

#### Standard form

Nonlinear programs (NLPs) are a class of optimization problems in which some of the constraints or the objective function are nonlinear:

$$\min_{x \in \mathbb{R}^n} f_0(x) \tag{29.1}$$

$$s.t.l_j \le f_j(x) \le u_j \qquad \qquad j = 1 \dots m \tag{29.2}$$

There may be additional constraints, as well as things like variable bounds and integrality restrictions, but we do not consider them here because they are best dealt with by other components of MathOptInterface.

#### **API** overview

The core element of the Nonlinear submodule is Nonlinear. Model:

```
julia> const Nonlinear = MOI.Nonlinear;

julia> model = Nonlinear.Model()
A Nonlinear.Model with:
0 objectives
0 parameters
0 expressions
0 constraints
```

Nonlinear. Model is a mutable struct that stores all of the nonlinear information added to the model.

#### **Decision variables**

Decision variables are represented by VariableIndexes. The user is responsible for creating these using MOI.VariableIndex(i), where i is the column associated with the variable.

#### **Expressions**

The input data structure is a Julia Expr. The input expressions can incorporate VariableIndexes, but these must be interpolated into the expression with \$:

```
julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)

julia> input = :(1 + sin($x)^2)
:(1 + sin(MathOptInterface.VariableIndex(1)) ^ 2)
```

There are a number of restrictions on the input Expr:

- · It cannot contain macros
- · It cannot contain broadcasting
- It cannot contain splatting (except in limited situations)
- It cannot contain linear algebra, such as matrix-vector products
- It cannot contain generator expressions, including sum(i for i in S)

Given an input expression, add an expression using Nonlinear.add\_expression:

```
julia> expr = Nonlinear.add_expression(model, input)
MathOptInterface.Nonlinear.ExpressionIndex(1)
```

The return value, expr, is a Nonlinear. ExpressionIndex that can then be interpolated into other input expressions.

Looking again at model, we see:

```
julia> model
A Nonlinear.Model with:
0 objectives
0 parameters
1 expression
0 constraints
```

### **Parameters**

In addition to constant literals like 1 or 1.23, you can create parameters. Parameters are placeholders whose values can change before passing the expression to the solver. Create a parameter using Nonlinear.add\_parameter, which accepts a default value:

```
julia> p = Nonlinear.add_parameter(model, 1.23)
MathOptInterface.Nonlinear.ParameterIndex(1)
```

The return value, p, is a Nonlinear. Parameter Index that can then be interpolated into other input expressions.

Looking again at model, we see:

```
julia> model
A Nonlinear.Model with:
0 objectives
1 parameter
1 expression
0 constraints
```

Update a parameter as follows:

```
julia> model[p]
1.23

julia> model[p] = 4.56
4.56

julia> model[p]
4.56
```

# **Objectives**

Set a nonlinear objective using Nonlinear.set\_objective:

```
julia> Nonlinear.set_objective(model, :($p + $expr + $x))

julia> model
A Nonlinear.Model with:
1 objective
1 parameter
1 expression
0 constraints
```

Clear a nonlinear objective by passing nothing:

```
julia> Nonlinear.set_objective(model, nothing)

julia> model
A Nonlinear.Model with:
0 objectives
1 parameter
1 expression
0 constraints
```

But we'll re-add the objective for later:

```
julia> Nonlinear.set_objective(model, :($p + $expr + $x));
```

#### **Constraints**

Add a constraint using Nonlinear.add\_constraint:

```
julia> c = Nonlinear.add_constraint(model, :(1 + sqrt($x)), MoI.LessThan(2.0))
MathOptInterface.Nonlinear.ConstraintIndex(1)

julia> model
A Nonlinear.Model with:
1 objective
1 parameter
1 expression
1 constraint
```

The return value, c, is a Nonlinear.ConstraintIndex that is a unique identifier for the constraint. Interval constraints are also supported:

```
julia> c2 = Nonlinear.add_constraint(model, :(1 + sqrt($x)), MoI.Interval(-1.0, 2.0))
MathOptInterface.Nonlinear.ConstraintIndex(2)

julia> model
A Nonlinear.Model with:
1 objective
1 parameter
1 expression
2 constraints
```

Delete a constraint using Nonlinear.delete:

```
julia> Nonlinear.delete(model, c2)

julia> model
A Nonlinear.Model with:
1 objective
1 parameter
1 expression
1 constraint
```

# **User-defined operators**

By default, Nonlinear supports a wide range of univariate and multivariate operators. However, you can also define your own operators by registering them.

### **Univariate operators**

Register a univariate user-defined operator using Nonlinear.register\_operator:

```
julia> f(x) = 1 + sin(x)^2
f (generic function with 1 method)

julia> Nonlinear.register_operator(model, :my_f, 1, f)
```

Now, you can use :my\_f in expressions:

```
julia> new_expr = Nonlinear.add_expression(model, :(my_f($x + 1)))
MathOptInterface.Nonlinear.ExpressionIndex(2)
```

By default, Nonlinear will compute first- and second-derivatives of the registered operator using ForwardDiff.jl. Override this by passing functions which compute the respective derivative:

```
julia> f'(x) = 2 * sin(x) * cos(x)
f' (generic function with 1 method)

julia> Nonlinear.register_operator(model, :my_f2, 1, f, f')
```

or

```
julia> f''(x) = 2 * (cos(x)^2 - sin(x)^2)
f'' (generic function with 1 method)

julia> Nonlinear.register_operator(model, :my_f3, 1, f, f', f'')
```

#### **Multivariate operators**

Register a multivariate user-defined operator using Nonlinear.register\_operator:

```
julia> g(x...) = x[1]^2 + x[1] * x[2] + x[2]^2
g (generic function with 1 method)

julia> Nonlinear.register_operator(model, :my_g, 2, g)
```

Now, you can use :my\_g in expressions:

```
julia> new_expr = Nonlinear.add_expression(model, :(my_g($x + 1, $x)))
MathOptInterface.Nonlinear.ExpressionIndex(3)
```

By default, Nonlinear will compute the gradient of the registered operator using ForwardDiff.jl. (Hessian information is not supported.) Override this by passing a function to compute the gradient:

### MathOptInterface

MathOptInterface communicates the nonlinear portion of an optimization problem to solvers using concrete subtypes of AbstractNLPEvaluator, which implement the Nonlinear programming API.

Create an AbstractNLPEvaluator from Nonlinear. Model using Nonlinear. Evaluator.

Nonlinear. Evaluator requires an Nonlinear. AbstractAutomaticDifferentiation backend and an ordered list of the variables that are included in the model.

There following backends are available to choose from within MOI, although other packages may add more options by sub-typing Nonlinear. AbstractAutomaticDifferentiation:

- Nonlinear.ExprGraphOnly
- Nonlinear.SparseReverseMode.

```
julia> evaluator = Nonlinear.Evaluator(model, Nonlinear.ExprGraphOnly(), [x])
Nonlinear.Evaluator with available features:
  * :ExprGraph
```

The functions of the Nonlinear programming API implemented by Nonlinear. Evaluator depends upon the chosen Nonlinear. AbstractAutomaticDifferentiation backend.

The :ExprGraph feature means we can call objective\_expr and constraint\_expr to retrieve the expression graph of the problem. However, we cannot call gradient terms such as eval\_objective\_gradient because Nonlinear.ExprGraphOnly does not have the capability to differentiate a nonlinear expression.

If, instead, we pass Nonlinear. SparseReverseMode, then we get access to : Grad, the gradient of the objective function, : Jac, the Jacobian matrix of the constraints, : JacVec, the ability to compute Jacobian-vector products, and :ExprGraph.

However, before using the evaluator, we need to call initialize:

```
julia> MOI.initialize(evaluator, [:Grad, :JacVec, :ExprGraph])
```

Now we can call methods like eval\_objective:

```
julia> x = [1.0]
1-element Vector{Float64}:
1.0
```

```
julia> MOI.eval_objective(evaluator, x)
7.268073418273571
```

and eval\_objective\_gradient:

```
julia> grad = [0.0]
1-element Vector{Float64}:
    0.0

julia> MOI.eval_objective_gradient(evaluator, grad, x)

julia> grad
1-element Vector{Float64}:
    1.909297426825682
```

Instead of passing Nonlinear. Evaluator directly to solvers, solvers query the NLPBlock attribute, which returns an NLPBlockData. This object wraps an Nonlinear. Evaluator and includes other information such as constraint bounds and whether the evaluator has a nonlinear objective. Create and set NLPBlockData as follows:

```
julia> block = MOI.NLPBlockData(evaluator);
julia> model = MOI.Utilities.UniversalFallback(MOI.Utilities.Model{Float64}());
julia> MOI.set(model, MOI.NLPBlock(), block);
```

#### Warning

Only call NLPBlockData once you have finished modifying the problem in model.

Putting everything together, you can create a nonlinear optimization problem in MathOptInterface as follows:

```
import MathOptInterface as MoI

function build_model(
    model::MOI.ModelLike;
    backend::MOI.Nonlinear.AbstractAutomaticDifferentiation,
)
    x = MOI.add_variable(model)
    y = MOI.add_variable(model)
    MOI.set(model, MOI.ObjectiveSense(), MOI.MIN_SENSE)
    nl_model = MOI.Nonlinear.Model()
    MOI.Nonlinear.set_objective(nl_model, :($x^2 + $y^2))
    evaluator = MOI.Nonlinear.Evaluator(nl_model, backend, [x, y])
    MOI.set(model, MOI.NLPBlock(), MOI.NLPBlockData(evaluator))
    return
end

# Replace `model` and `backend` with your optimizer and backend of choice.
```

```
model = MOI.Utilities.UniversalFallback(MOI.Utilities.Model{Float64}())
build_model(model; backend = MOI.Nonlinear.SparseReverseMode())
```

#### **Expression-graph representation**

Nonlinear. Model stores nonlinear expressions in Nonlinear. Expressions. This section explains the design of the expression graph data structure in Nonlinear. Expression.

Given a nonlinear function like  $f(x) = \sin(x)^2 + x$ , a conceptual aid for thinking about the graph representation of the expression is to convert it into Polish prefix notation:

```
f(x, y) = (+ (^ (\sin x) 2) x)
```

This format identifies each operator (function), as well as a list of arguments. Operators can be univariate, like sin, or multivariate, like +.

A common way of representing Polish prefix notation in code is as follows:

This data structure follows our Polish prefix notation very closely, and we can easily identify the arguments to an operator. However, it has a significant draw-back: each node in the graph requires a Vector, which is heap-allocated and tracked by Julia's garbage collector (GC). For large models, we can expect to have millions of nodes in the expression graph, so this overhead quickly becomes prohibitive for computation.

An alternative is to record the expression as a linear tape:

```
julia> expr = Any[:+, 2, :^, 2, :sin, 1, x, 2.0, x]
9-element Vector{Any}:
    :+
2
    :^
2
    :sin
1
    MOI.VariableIndex(1)
2.0
    MOI.VariableIndex(1)
```

The Int after each operator Symbol specifies the number of arguments.

This data-structure is a single vector, which resolves our problem with the GC, but each element is the abstract type, Any, and so any operations on it will lead to slower dynamic dispatch. It's also hard to identify the children of each operation without reading the entire tape.

To summarize, representing expression graphs in Julia has the following challenges:

- Nodes in the expression graph should not contain a heap-allocated object
- All data-structures should be concretely typed
- It should be easy to identify the children of a node

### Sketch of the design in Nonlinear

Nonlinear overcomes these problems by decomposing the data structure into a number of different concretetyped vectors.

First, we create vectors of the supported uni- and multivariate operators.

```
julia> const UNIVARIATE_OPERATORS = [:sin];
julia> const MULTIVARIATE_OPERATORS = [:+, :^];
```

In practice, there are many more supported operations than the ones listed here.

Second, we create an enum to represent the different types of nodes present in the expression graph:

In practice, there are node types other than the ones listed here.

Third, we create two concretely typed structs as follows:

For each node node in the .nodes field, if node.type is:

- NODE\_CALL\_MULTIVARIATE, we look up MULTIVARIATE\_OPERATORS[node.index] to retrieve the operator
- NODE\_CALL\_UNIVARIATE, we look up UNIVARIATE\_OPERATORS[node.index] to retrieve the operator
- NODE\_VARIABLE, we create MOI.VariableIndex(node.index)
- NODE\_VALUE, we look up values[node.index]

The .parent field of each node is the integer index of the parent node in .nodes. For the first node, the parent is -1 by convention.

Therefore, we can represent our function as:

The ordering of the nodes in the tape must satisfy two rules:

- The children of a node must appear after the parent. This means that the tape is ordered topologically, so that a reverse pass of the nodes evaluates all children nodes before their parent
- The arguments for a CALL node are ordered in the tape based on the order in which they appear in the function call.

### **Design goals**

This is less readable than the other options, but does this data structure meet our design goals?

Instead of a heap-allocated object for each node, we only have two Vectors for each expression, nodes and values, as well as two constant vectors for the OPERATORS. In addition, all fields are concretely typed, and there are no Union or Any types.

For our third goal, it is not easy to identify the children of a node, but it is easy to identify the parent of any node. Therefore, we can use Nonlinear.adjacency\_matrix to compute a sparse matrix that maps parents to their children.

#### The design in practice

In practice, Node and Expression are exactly Nonlinear. Node and Nonlinear. Expression. However, Nonlinear. NodeType has more fields to account for comparison operators such as :>= and :<=, logic operators such as :&& and :||, nonlinear parameters, and nested subexpressions.

Moreover, instead of storing the operators as global constants, they are stored in Nonlinear.OperatorRegistry, and it also stores a vector of logic operators and a vector of comparison operators. In addition to Nonlinear.DEFAULT\_UNIVARIATE and Nonlinear.DEFAULT\_MULTIVARIATE\_OPERATORS, you can register user-defined functions using Nonlinear.register\_operat

Nonlinear. Model is a struct that stores the Nonlinear. OperatorRegistry, as well as a list of parameters and subexpressions in the model.

#### ReverseAD

Nonlinear.ReverseAD is a submodule for computing derivatives of a nonlinear optimization problem using sparse reverse-mode automatic differentiation (AD).

This section does not attempt to explain how sparse reverse-mode AD works, but instead explains why MOI contains its own implementation, and highlights notable differences from similar packages.

#### Warning

Don't use the API in ReverseAD to compute derivatives. Instead, create a Nonlinear. Evaluator object with Nonlinear. SparseReverseMode as the backend, and then query the MOI API methods.

#### **Design goals**

The JuliaDiff organization maintains a list of packages for doing AD in Julia. At last count, there were at least ten packages——not including ReverseAD——for reverse-mode AD in Julia. ReverseAD exists because it has a different set of design goals.

- Goal: handle scale and sparsity. The types of nonlinear optimization problems that MOI represents can be large scale (10^5 or more functions across 10^5 or more variables) with very sparse derivatives. The ability to compute a sparse Hessian matrix is essential. To the best of our knowledge, ReverseAD is the only reverse-mode AD system in Julia that handles sparsity by default.
- Goal: limit the scope to improve robustness. Most other AD packages accept arbitrary Julia functions as input and then trace an expression graph using operator overloading. This means they must deal (or detect and ignore) with control flow, I/O, and other vagaries of Julia. In contrast, ReverseAD only accepts functions in the form of Nonlinear. Expression, which greatly limits the range of syntax that it must deal with. By reducing the scope of what we accept as input to functions relevant for mathematical optimization, we can provide a simpler implementation with various performance optimizations.
- Goal: provide outputs which match what solvers expect. Other AD packages focus on differentiating individual Julia functions. In contrast, ReverseAD has a very specific use-case: to generate outputs needed by the MOI nonlinear API. This means it needs to efficiently compute sparse Hessians, and it needs subexpression handling to avoid recomputing subexpressions that are shared between functions.

#### History

ReverseAD started life as ReverseDiffSparse.jl, development of which began in early 2014(!). This was well before the other AD packages started development. Because we had a well-tested, working AD in JuMP, there was less motivation to contribute to and explore other AD packages. The lack of historical interaction also meant that other packages were not optimized for the types of problems that JuMP is built for (that is, large-scale sparse problems). When we first created MathOptInterface, we kept the AD in JuMP to simplify the transition, and post-poned the development of a first-class nonlinear interface in MathOptInterface.

Prior to the introduction of Nonlinear, JuMP's nonlinear implementation was a confusing mix of functions and types spread across the code base and in the private \_Derivatives submodule. This made it hard to swap the AD system for another. The main motivation for refactoring JuMP to create the Nonlinear submodule in MathOptInterface was to abstract the interface between JuMP and the AD system, allowing us to swap-in and test new AD systems in the future.

# 29.2 SymbolicAD

# **SymbolicAD**

The Nonlinear. SymbolicAD submodule contains data structures and functions for working with the symbolic derivatives of a nonlinear optimization problem. This page explains the API and describes the rationale behind its design.

### **Background**

The code in SymbolicAD is inspired by Hassan Hijazi's work on coin-or/gravity, a high-performance algebraic modeling language in C++.

Hassan made the following observations:

- For large scale models, symbolic differentiation is slower than other automatic differentiation techniques, such as the reverse mode algorithm implemented in MOI.Nonlinear.ReverseAD.
- However, most large-scale nonlinear programs have a lot of structure.
- Gravity asks the user to provide structure in the form of template constraints, where the user gives the symbolic form of the constraint as well as a set of data to convert from a symbolic form to the numerical form.
- Instead of differentiating each constraint in its numerical form, we can compute one symbolic derivative
  of the constraint in symbolic form, and then plug in the data in to get the numerical derivative of each
  function.
- As a final step, if users don't provide the structure, we can still infer it -perhaps with less accuracy-by comparing the expression tree of each constraint.

The symbolic differentiation approach of Gravity works well when the problem is large with few unique constraints. For example, a model like:

```
model = Model()
@variable(model, 0 <= x[1:10_000] <= 1)
@constraint(model, [i=1:10_000], sin(x[i]) <= 1)
@objective(model, Max, sum(x))</pre>
```

is ideal, because although the Jacobian matrix has 10,000 rows, we can compute the derivative of sin(x[i]) as cos(x[i]), and then fill in the Jacobian by evaluating the derivative function instead of having to differentiation 10,000 expressions.

The symbolic differentiation approach of Gravity works poorly if there are a large number of unique constraints in the model (which would require a lot of expressions to be symbolically differentiated), or if the nonlinear functions contain a large number of nonlinear terms (which would make the symbolic derivative expensive to compute).

SymbolicAD started life as MathOptSymbolicAD.jl, development of which began in early 2022. This initial version of SymbolicAD used the Symbolics.jl package to compute the symbolic derivatives. In 2025, we rewrote MathOptSymbolicAD.jl to remove the dependence on Symbolics.jl, and, since the rewrite depended only on MathOptInterface, we added it to MOI.Nonlinear as a new submodule.

For more details on MathOptSymbolicAD.jl, see Oscar's JuMP-dev 2022 talk, although note that the syntax has changed since the original recording.

#### Use SymbolicAD with JuMP

To use SymbolicAD with JuMP, set the AutomaticDifferentiationBackend attribute to Nonlinear.SymbolicMode:

```
using JuMP, Ipopt
model = Model(Ipopt.Optimizer)
set_attribute(
    model,
    MOI.AutomaticDifferentiationBackend(),
    MOI.Nonlinear.SymbolicMode(),
)
@variable(model, x[1:2])
@objective(model, Min, (1 - x[1])^2 + 100 * (x[2] - x[1]^2)^2)
optimize!(model)
```

To revert back to the default sparse reverse mode algorithm, set the AutomaticDifferentiationBackend attribute to Nonlinear.SparseReverseMode.

### simplify

Use Nonlinear. SymbolicAD. simplify to simplify nonlinear expressions. The simplification algorithm performs simple rewrites such as lifting nested summations:

and trivial identities such as  $x^1 = x$ :

```
julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)

julia> f = MOI.ScalarNonlinearFunction(:^, Any[x, 1])
^(MOI.VariableIndex(1), (1))

julia> MOI.Nonlinear.SymbolicAD.simplify(f)
MOI.VariableIndex(1)
```

The list of rewrites that will be made is intentionally limited to keep the codebase simple. Nonlinear. SymbolicAD is not a substitute for a Computer Algebraic System (CAS). For example, we do not detect the relationship  $sin(x)^2 + cos(x)^2 = 1$ :

```
julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)

julia> sin_x = MOI.ScalarNonlinearFunction(:sin, Any[x])
sin(MOI.VariableIndex(1))
```

In addition to Nonlinear. SymbolicAD. simplify, there is an in-place version Nonlinear. SymbolicAD. simplify! that may make changes to the existing function.

#### variables

Use Nonlinear. SymbolicAD. variables to return a sorted list of the variables that appear in the function:

```
julia> x = MOI.VariableIndex.(1:3)
3-element Vector{MathOptInterface.VariableIndex}:
MOI.VariableIndex(1)
MOI.VariableIndex(2)
MOI.VariableIndex(3)

julia> f = MOI.ScalarNonlinearFunction(:atan, Any[x[3], 2.0 * x[1]])
atan(MOI.VariableIndex(3), 0.0 + 2.0 MOI.VariableIndex(1))

julia> MOI.Nonlinear.SymbolicAD.variables(f)
2-element Vector{MathOptInterface.VariableIndex}:
MOI.VariableIndex(1)
MOI.VariableIndex(3)
```

#### derivative

Use Nonlinear. SymbolicAD. derivative to compute the symbolic derivative of a function with respect to a decision variable:

```
julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)

julia> f = MOI.ScalarNonlinearFunction(:sin, Any[x])
sin(MOI.VariableIndex(1))

julia> MOI.Nonlinear.SymbolicAD.derivative(f, x)
cos(MOI.VariableIndex(1))
```

Note that the resultant expression can often be simplified. Thus, in most cases you should call Nonlinear. SymbolicAD.simplify on the expression before using it in other places:

```
julia> x = MOI.VariableIndex(1)

julia> f = MOI.ScalarNonlinearFunction(:sin, Any[x + 1.0])
sin(1.0 + 1.0 MOI.VariableIndex(1))

julia> df_dx = MOI.Nonlinear.SymbolicAD.derivative(f, x)
*(cos(1.0 + 1.0 MOI.VariableIndex(1)), 1.0)

julia> MOI.Nonlinear.SymbolicAD.simplify!(df_dx)
cos(1.0 + 1.0 MOI.VariableIndex(1))
```

### gradient\_and\_hessian

In some cases, you may want to compute the gradient and (sparse) Hessian matrix of a function. One way to achieve this is by recursively calling Nonlinear. SymbolicAD.derivative on the result of Nonlinear. SymbolicAD.derivative for each variable in the list of Nonlinear. SymbolicAD.variables. But, to simplify the process, you should use Nonlinear. SymbolicAD.gradient and hessian:

```
julia> x = MOI.VariableIndex.(1:2);
julia> f = MOI.ScalarNonlinearFunction(:sin, Any[1.0 * x[1] + 2.0 * x[2]])
sin(0.0 + 1.0 MOI.VariableIndex(1) + 2.0 MOI.VariableIndex(2))
julia> y, \nabla f, H, \nabla^2 f = MOI.Nonlinear.SymbolicAD.gradient_and_hessian(f);
julia> y
2-element Vector{MathOptInterface.VariableIndex}:
MOI.VariableIndex(1)
MOI.VariableIndex(2)
julia> ∇f
2-element Vector{Any}:
cos(0.0 + 1.0 MOI.VariableIndex(1) + 2.0 MOI.VariableIndex(2))
*(\cos(0.0 + 1.0 \text{ MOI.VariableIndex}(1) + 2.0 \text{ MOI.VariableIndex}(2)), 2.0)
julia> H
3-element Vector{Tuple{Int64, Int64}}:
(1, 1)
(1, 2)
(2, 2)
julia> ∇²f
3-element Vector{Any}:
-(sin(0.0 + 1.0 MOI.VariableIndex(1) + 2.0 MOI.VariableIndex(2)))
*(-(\sin(0.0 + 1.0 \text{ MOI.VariableIndex}(1) + 2.0 \text{ MOI.VariableIndex}(2))), 2.0)
*(-(\sin(0.0 + 1.0 \text{ MOI.VariableIndex}(1) + 2.0 \text{ MOI.VariableIndex}(2))), 4.0)
```

### where:

- · y is the list of variables that appear in f
- Vf is the first partial derivatives of f with respect to each variable in y

H and ∇²f form a sparse Hessian matrix, were H is the (row, column) index of each element, and the
corresponding element in ∇²f is the second partial derivative of f with respect to y[row] and y[column].

Unlike Nonlinear. SymbolicAD. derivative, the gradient and Hessian expressions have already been simplified; you do not need to call Nonlinear. SymbolicAD. simplify.

#### 29.3 API Reference

# **Nonlinear Modeling**

More information can be found in the Nonlinear section of the manual.

MathOptInterface.Nonlinear.Model - Type.

```
Model()
```

The core datastructure for representing a nonlinear optimization problem.

It has the following fields:

- objective::Union{Nothing,Expression} : holds the nonlinear objective function, if one exists, otherwise nothing.
- expressions::Vector{Expression}: a vector of expressions in the model.
- constraints::OrderedDict{ConstraintIndex,Constraint}: a map from ConstraintIndex to the
  corresponding Constraint. An OrderedDict is used instead of a Vector to support constraint deletion.
- parameters::Vector{Float64}: holds the current values of the parameters.
- $\bullet \ \ \text{operators::0peratorRegistry: stores the operators used in the model}.$

source

# **Expressions**

 ${\tt MathOptInterface.Nonlinear.ExpressionIndex-Type.}\\$ 

```
ExpressionIndex
```

An index to a nonlinear expression that is returned by add expression.

Given data::Model and ex::ExpressionIndex, use data[ex] to retrieve the corresponding Expression. source

MathOptInterface.Nonlinear.add expression - Function.

```
add_expression(model::Model, expr)::ExpressionIndex
```

Parse expr into a Expression and add to model. Returns an ExpressionIndex that can be interpolated into other input expressions.

expr must be a type that is supported by parse\_expression.

#### **Example**

```
julia> model = MOI.Nonlinear.Model();
julia> x = MOI.VariableIndex(1);
julia> ex = MOI.Nonlinear.add_expression(model, :($x^2 + 1))
MathOptInterface.Nonlinear.ExpressionIndex(1)
julia> MOI.Nonlinear.set_objective(model, :(sqrt($ex)))
```

source

### **Parameters**

MathOptInterface.Nonlinear.ParameterIndex - Type.

```
ParameterIndex
```

An index to a nonlinear parameter that is returned by add\_parameter. Given data::Model and p::ParameterIndex, use data[p] to retrieve the current value of the parameter and data[p] = value to set a new value.

source

 ${\tt MathOptInterface.Nonlinear.add\_parameter-Function}.$ 

```
add_parameter(model::Model, value::Float64)::ParameterIndex
```

Add a new parameter to model with the default value value. Returns a <u>ParameterIndex</u> that can be interpolated into other input expressions and used to modify the value of the parameter.

# **Example**

```
julia> model = MOI.Nonlinear.Model()
A Nonlinear.Model with:
0 objectives
0 parameters
0 expressions
0 constraints

julia> x = MOI.VariableIndex(1)

MOI.VariableIndex(1)

julia> p = MOI.Nonlinear.add_parameter(model, 1.2)

MathOptInterface.Nonlinear.ParameterIndex(1)

julia> c = MOI.Nonlinear.add_constraint(model, :($x^2 - $p), MOI.LessThan(0.0))

MathOptInterface.Nonlinear.ConstraintIndex(1)
```

source

### **Objectives**

 ${\tt MathOptInterface.Nonlinear.set\_objective-Function}.$ 

```
set_objective(model::Model, obj)::Nothing
```

Parse obj into a Expression and set as the objective function of model.

obj must be a type that is supported by parse\_expression.

To remove the objective, pass nothing.

### **Example**

```
julia> model = MOI.Nonlinear.Model()
A Nonlinear.Model with:
0 objectives
0 parameters
0 expressions
0 constraints

julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)

julia> MOI.Nonlinear.set_objective(model, :($x^2 + 1))

julia> MOI.Nonlinear.set_objective(model, x)

julia> MOI.Nonlinear.set_objective(model, nothing)
```

source

#### **Constraints**

 ${\tt MathOptInterface.Nonlinear.ConstraintIndex-Type.}\\$ 

```
ConstraintIndex
```

An index to a nonlinear constraint that is returned by add\_constraint.

 $\label{lem:constraint} \textbf{Given data::} \textbf{Model and } \textbf{c::} \textbf{ConstraintIndex}, \textbf{use data[c] to retrieve the corresponding } \textbf{Constraint.}$ 

source

MathOptInterface.Nonlinear.add\_constraint - Function.

```
add_constraint(
   model::Model,
   func,
   set::Union{
```

```
MOI.GreaterThan{Float64},
    MOI.LessThan{Float64},
    MOI.Interval{Float64},
    MOI.EqualTo{Float64},
},
```

Parse func and set into a Constraint and add to model. Returns a ConstraintIndex that can be used to delete the constraint or query solution information.

### **Example**

```
julia> model = MOI.Nonlinear.Model();
julia> x = MOI.VariableIndex(1);
julia> c = MOI.Nonlinear.add_constraint(model, :($x^2), MOI.LessThan(1.0))
MathOptInterface.Nonlinear.ConstraintIndex(1)
```

source

MathOptInterface.Nonlinear.delete - Function.

```
delete(model::Model, c::ConstraintIndex)::Nothing
```

Delete the constraint index c from model.

#### **Example**

```
julia> model = MOI.Nonlinear.Model()
A Nonlinear. Model with:
 0 objectives
 0 parameters
 0 expressions
 0 constraints
julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)
\textbf{julia>} \ c \ = \ \texttt{MOI.Nonlinear.add\_constraint(model, :($x^2), \ \texttt{MOI.LessThan}(1.0))}
MathOptInterface.Nonlinear.ConstraintIndex(1)
julia> model
A Nonlinear. Model with:
0 objectives
 0 parameters
 0 expressions
 1 constraint
julia> MOI.Nonlinear.delete(model, c)
julia> model
```

```
A Nonlinear.Model with:
0 objectives
0 parameters
0 expressions
0 constraints
```

source

# **User-defined operators**

MathOptInterface.Nonlinear.OperatorRegistry - Type.

```
OperatorRegistry()
```

Create a new OperatorRegistry to store and evaluate univariate and multivariate operators.

source

 ${\tt MathOptInterface.Nonlinear.DEFAULT\_UNIVARIATE\_OPERATORS-Constant}.$ 

```
DEFAULT_UNIVARIATE_OPERATORS
```

The list of univariate operators that are supported by default.

# **Example**

```
julia> MOI.Nonlinear.DEFAULT_UNIVARIATE_OPERATORS
73-element Vector{Symbol}:
 :+
 :abs
 :sign
 :sqrt
 :cbrt
 :abs2
 :inv
 :log
 :log10
 :airybi
 :airyaiprime
 :airybiprime
 :besselj0
 :besselj1
 :bessely0
 :bessely1
 :erfcx
 :dawson
```

 ${\tt MathOptInterface.Nonlinear.DEFAULT\_MULTIVARIATe\_OPERATORS-Constant.}$ 

```
DEFAULT_MULTIVARIATE_OPERATORS
```

The list of multivariate operators that are supported by default.

# **Example**

```
julia> MOI.Nonlinear.DEFAULT_MULTIVARIATE_OPERATORS
9-element Vector{Symbol}:
    :+
    :-
    :*
    :^
    :/
    :ifelse
    :atan
    :min
:max
```

source

MathOptInterface.Nonlinear.register\_operator - Function.

```
register_operator(
   model::Model,
   op::Symbol,
   nargs::Int,
   f::Function,
   [∇f::Function],
   [∇²f::Function],
)
```

Register the user-defined operator op with nargs input arguments in model.

### **Univariate functions**

- f(x::T)::T must be a function that takes a single input argument x and returns the function evaluated at x. If  $\nabla f$  and  $\nabla^2 f$  are not provided, f must support any Real input type T.
- ∇f(x::T)::T is a function that takes a single input argument x and returns the first derivative of f with respect to x. If ∇²f is not provided, ∇f must support any Real input type T.
- \(\nabla^2 f(x::T)::T\) is a function that takes a single input argument x and returns the second derivative
  of f with respect to x.

# **Multivariate functions**

- f(x::T...)::T must be a function that takes a nargs input arguments x and returns the function evaluated at x. If  $\nabla f$  and  $\nabla^2 f$  are not provided, f must support any Real input type T.
- ∇f(g::AbstractVector{T}, x::T...)::T is a function that takes a cache vector g of length length(x), and fills each element g[i] with the partial derivative of f with respect to x[i].

∇²f(H::AbstractMatrix, x::T...)::T is a function that takes a matrix H and fills the lower-triangular components H[i, j] with the Hessian of f with respect to x[i] and x[j] for i >= j.

#### Notes for multivariate Hessians

- H has size(H) == (length(x), length(x)), but you must not access elements H[i, j] for i > j.
- H is dense, but you do not need to fill structural zeros.

source

 ${\tt MathOptInterface.Nonlinear.register\_operator\_if\_needed-Function}.$ 

```
register_operator_if_needed(
    registry::OperatorRegistry,
    op::Symbol,
    nargs::Int,
    f::Function;
)
```

Similar to register\_operator, but this function warns if the function is not registered, and skips silently if it already is.

source

MathOptInterface.Nonlinear.assert registered - Function.

```
assert_registered(registry::OperatorRegistry, op::Symbol, nargs::Int)
```

Throw an error if op is not registered in registry with nargs arguments.

source

MathOptInterface.Nonlinear.check\_return\_type - Function.

```
check_return_type(::Type{T}, ret::S) where {T,S}
```

Overload this method for new types S to throw an informative error if a user-defined function returns the type S instead of T.

source

MathOptInterface.Nonlinear.eval\_univariate\_function - Function.

```
eval_univariate_function(
    registry::OperatorRegistry,
    op::Union{Symbol,Integer},
    x::T,
) where {T}
```

Evaluate the operator op(x)::T, where op is a univariate function in registry.

If op isa Integer, then op is the index in registry.univariate\_operators[op].

#### **Example**

```
julia> r = MOI.Nonlinear.OperatorRegistry();

julia> MOI.Nonlinear.eval_univariate_function(r, :abs, -1.2)
1.2

julia> r.univariate_operators[3]
:abs

julia> MOI.Nonlinear.eval_univariate_function(r, 3, -1.2)
1.2
```

source

MathOptInterface.Nonlinear.eval\_univariate\_gradient - Function.

```
eval_univariate_gradient(
    registry::OperatorRegistry,
    op::Union{Symbol,Integer},
    x::T,
) where {T}
```

Evaluate the first-derivative of the operator op(x)::T, where op is a univariate function in registry. If op isa Integer, then op is the index in registry.univariate\_operators[op].

# **Example**

```
julia> r = MOI.Nonlinear.OperatorRegistry();

julia> MOI.Nonlinear.eval_univariate_gradient(r, :abs, -1.2)
-1.0

julia> r.univariate_operators[3]
:abs

julia> MOI.Nonlinear.eval_univariate_gradient(r, 3, -1.2)
-1.0
```

source

 ${\tt MathOptInterface.Nonlinear.eval\_univariate\_function\_and\_gradient-Function}.$ 

```
eval_univariate_function_and_gradient(
    registry::OperatorRegistry,
    op::Union{Symbol,Integer},
    x::T,
)::Tuple{T,T} where {T}
```

Evaluate the function and first-derivative of the operator op(x)::T, where op is a univariate function in registry.

If op isa Integer, then op is the index in registry.univariate operators[op].

#### **Example**

```
julia> r = MOI.Nonlinear.OperatorRegistry();

julia> MOI.Nonlinear.eval_univariate_function_and_gradient(r, :abs, -1.2)
(1.2, -1.0)

julia> r.univariate_operators[3]
:abs

julia> MOI.Nonlinear.eval_univariate_function_and_gradient(r, 3, -1.2)
(1.2, -1.0)
```

source

MathOptInterface.Nonlinear.eval\_univariate\_hessian - Function.

```
eval_univariate_hessian(
    registry::OperatorRegistry,
    op::Union{Symbol,Integer},
    x::T,
) where {T}
```

Evaluate the second-derivative of the operator op(x)::T, where op is a univariate function in registry. If op isa Integer, then op is the index in registry.univariate\_operators[op].

# Example

```
julia> r = MOI.Nonlinear.OperatorRegistry();

julia> MOI.Nonlinear.eval_univariate_hessian(r, :sin, 1.0)
-0.8414709848078965

julia> r.univariate_operators[16]
:sin

julia> MOI.Nonlinear.eval_univariate_hessian(r, 16, 1.0)
-0.8414709848078965

julia> -sin(1.0)
-0.8414709848078965
```

source

MathOptInterface.Nonlinear.eval\_multivariate\_function - Function.

```
eval_multivariate_function(
    registry::OperatorRegistry,
    op::Symbol,
    x::AbstractVector{T},
) where {T}
```

Evaluate the operator op(x)::T, where op is a multivariate function in registry.

source

MathOptInterface.Nonlinear.eval\_multivariate\_gradient - Function.

```
eval_multivariate_gradient(
    registry::OperatorRegistry,
    op::Symbol,
    g::AbstractVector{T},
    x::AbstractVector{T},
) where {T}
```

Evaluate the gradient of operator  $g := \nabla op(x)$ , where op is a multivariate function in registry.

source

MathOptInterface.Nonlinear.eval multivariate hessian - Function.

```
eval_multivariate_hessian(
    registry::OperatorRegistry,
    op::Symbol,
    H::AbstractMatrix,
    x::AbstractVector{T},
)::Bool where {T}
```

Evaluate the Hessian of operator  $\nabla^2 op(x)$ , where op is a multivariate function in registry.

The Hessian is stored in the lower-triangular part of the matrix H.

Returns a Bool indicating whether non-zeros were stored in the matrix.

#### Note

Implementations of the Hessian operators will not fill structural zeros. Therefore, before calling this function you should pre-populate the matrix H with  $\theta$ .

source

MathOptInterface.Nonlinear.eval\_logic\_function - Function.

```
eval_logic_function(
    registry::OperatorRegistry,
    op::Symbol,
```

```
lhs::T,
  rhs::T,
)::Bool where {T}
```

Evaluate (lhs op rhs)::Bool, where op is a logic operator in registry.

source

MathOptInterface.Nonlinear.eval\_comparison\_function - Function.

```
eval_comparison_function(
    registry::OperatorRegistry,
    op::Symbol,
    lhs::T,
    rhs::T,
)::Bool where {T}
```

Evaluate (lhs op rhs)::Bool, where op is a comparison operator in registry.

source

#### **Automatic-differentiation backends**

MathOptInterface.Nonlinear.Evaluator - Type.

```
Evaluator(
   model::Model,
   backend::AbstractAutomaticDifferentiation,
   ordered_variables::Vector{MOI.VariableIndex},
)
```

Create Evaluator, a subtype of MOI. AbstractNLPEvaluator, from Model.

source

 ${\tt MathOptInterface.Nonlinear.AbstractAutomaticDifferentiation-Type.}$ 

```
AbstractAutomaticDifferentiation
```

An abstract type for extending Evaluator.

source

MathOptInterface.Nonlinear.ExprGraphOnly - Type.

```
ExprGraphOnly() <: AbstractAutomaticDifferentiation</pre>
```

 $The \ default implementation \ of \ Abstract Automatic Differentiation. \ The \ only \ supported \ feature \ is \ : Expr Graph.$ 

source

MathOptInterface.Nonlinear.SparseReverseMode - Type.

```
SparseReverseMode() <: AbstractAutomaticDifferentiation
```

An implementation of AbstractAutomaticDifferentiation that uses sparse reverse-mode automatic differentiation to compute derivatives. Supports all features in the MOI nonlinear interface.

source

MathOptInterface.Nonlinear.SymbolicMode - Type.

```
SymbolicMode() <: AbstractAutomaticDifferentiation</pre>
```

A type for setting as the value of the MOI.AutomaticDifferentiationBackend() attribute to enable symbolic automatic differentiation.

source

#### **Data-structure**

MathOptInterface.Nonlinear.Node - Type.

```
struct Node
   type::NodeType
   index::Int
   parent::Int
end
```

A single node in a nonlinear expression tree. Used by Expression.

See the MathOptInterface documentation for information on how the nodes and values form an expression tree

source

MathOptInterface.Nonlinear.NodeType - Type.

```
NodeType
```

An enum describing the possible node types. Each Node has a .index field, which should be interpreted as follows:

- NODE\_CALL\_MULTIVARIATE: the index into operators.multivariate\_operators
- NODE\_CALL\_UNIVARIATE: the index into operators.univariate\_operators
- NODE\_LOGIC: the index into operators.logic\_operators
- NODE\_COMPARISON: the index into operators.comparison\_operators
- NODE\_MOI\_VARIABLE: the value of MOI.VariableIndex(index) in the user's space of the model.

- NODE\_VARIABLE: the 1-based index of the internal vector
- NODE\_VALUE: the index into the .values field of Expression
- NODE\_PARAMETER: the index into data.parameters
- NODE\_SUBEXPRESSION: the index into data.expressions

source

MathOptInterface.Nonlinear.Expression - Type.

```
struct Expression
  nodes::Vector{Node}
  values::Vector{Float64}
end
```

The core type that represents a nonlinear expression. See the MathOptInterface documentation for information on how the nodes and values form an expression tree.

source

MathOptInterface.Nonlinear.Constraint - Type.

```
struct Constraint
  expression::Expression
  set::Union{
     MOI.LessThan{Float64},
     MOI.GreaterThan{Float64},
     MOI.EqualTo{Float64},
     MOI.Interval{Float64},
}
```

A type to hold information relating to the nonlinear constraint f(x) in S, where f(x) is defined by .expression, and S is .set.

source

MathOptInterface.Nonlinear.adjacency\_matrix - Function.

```
adjacency_matrix(nodes::Vector{Node})
```

Compute the sparse adjacency matrix describing the parent-child relationships in nodes.

The element (i, j) is true if there is an edge from node[j] to node[i]. Since we get a column-oriented matrix, this gives us a fast way to look up the edges leaving any node (that is, the children).

source

MathOptInterface.Nonlinear.parse\_expression - Function.

```
parse_expression(data::Model, input)::Expression
```

Parse input into a Expression.

source

```
parse_expression(
   data::Model,
   expr::Expression,
   input::Any,
   parent_index::Int,
)::Expression
```

Parse input into a Expression, and add it to expr as a child of expr.nodes[parent\_index]. Existing subexpressions and parameters are stored in data.

You can extend parsing support to new types of objects by overloading this method with a different type on input::Any.

source

 ${\tt MathOptInterface.Nonlinear.convert\_to\_expr-Function}.$ 

```
convert_to_expr(data::Model, expr::Expression)
```

Convert the Expression expr into a Julia Expr.

- subexpressions are represented by a ExpressionIndex object.
- parameters are represented by a ParameterIndex object.
- variables are represented by an MOI. VariableIndex object.

source

```
convert_to_expr(
    evaluator::Evaluator,
    expr::Expression;
    moi_output_format::Bool,
)
```

Convert the Expression expr into a Julia Expr.

If moi\_output\_format = true:

- subexpressions will be converted to Julia Expr and substituted into the output expression.
- the current value of each parameter will be interpolated into the expression
- variables will be represented in the form x[MOI.VariableIndex(i)]

```
If moi_output_format = false:
```

- subexpressions will be represented by a ExpressionIndex object.
- parameters will be represented by a ParameterIndex object.
- variables will be represented by an MOI. VariableIndex object.

#### Warning

To use moi\_output\_format = true, you must have first called MOI.initialize with :ExprGraph as a requested feature.

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source

MathOptInterface.Nonlinear.ordinal\_index - Function.

```
ordinal_index(evaluator::Evaluator, c::ConstraintIndex)::Int
```

Return the 1-indexed value of the constraint index c in evaluator.

#### **Example**

```
julia> model = MOI.Nonlinear.Model()
A Nonlinear.Model with:
 0 objectives
 0 parameters
 0 expressions
 0 constraints
julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)
julia> c1 = MOI.Nonlinear.add_constraint(model, :($x^2), MOI.LessThan(1.0))
MathOptInterface.Nonlinear.ConstraintIndex(1)
julia > c2 = MOI.Nonlinear.add_constraint(model, :($x^2), MOI.LessThan(1.0))
MathOptInterface.Nonlinear.ConstraintIndex(2)
julia> evaluator = MOI.Nonlinear.Evaluator(model)
Nonlinear. Evaluator with available features:
  * :ExprGraph
julia> MOI.initialize(evaluator, Symbol[])
julia> MOI.Nonlinear.ordinal_index(evaluator, c2) # Returns 2
julia> MOI.Nonlinear.delete(model, c1)
julia> evaluator = MOI.Nonlinear.Evaluator(model)
Nonlinear. Evaluator with available features:
  * :ExprGraph
julia> MOI.initialize(evaluator, Symbol[])
```

```
julia> MOI.Nonlinear.ordinal_index(evaluator, c2) # Returns 1
```

source

# **SymbolicAD**

MathOptInterface.Nonlinear.SymbolicAD.simplify - Function.

```
simplify(f)
```

Return a simplified copy of the function f.

#### Warning

This function is not type stable by design.

#### **Example**

```
julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)

julia> f = MOI.ScalarNonlinearFunction(:^, Any[x, 1])
^(MOI.VariableIndex(1), (1))

julia> MOI.Nonlinear.SymbolicAD.simplify(f)
MOI.VariableIndex(1)
```

source

MathOptInterface.Nonlinear.SymbolicAD.simplify! - Function.

```
simplify!(f)
```

Simplify the function f in-place and return either the function f or a new object if f can be represented in a simpler type.

#### Warning

This function is not type stable by design.

#### **Example**

```
julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)

julia> f = MOI.ScalarNonlinearFunction(
```

source

```
simplify!(::Val{head}, f::MOI.ScalarNonlinearFunction)
```

Simplify the function f in-place and return either the function f or a new object if f can be represented in a simpler type.

#### Val

The head in Val{head} is taken from f.head. This function should be called as:

```
f = simplify!(Val(f.head), f)
```

Implementing a method that dispatches on head enables custom simplification rules for different operators without needing a giant switch statement.

#### Note

It is important that this function does not recursively call simplify!. Deal only with the immediate operator. The children arguments will already be simplified.

source

MathOptInterface.Nonlinear.SymbolicAD.variables - Function.

```
variables(f::Union{Real, MOI.AbstractScalarFunction})
```

Return a sorted list of the MOI. VariableIndex present in the function f.

#### **Example**

```
julia> x = MOI.VariableIndex.(1:3)
3-element Vector{MathOptInterface.VariableIndex}:
MOI.VariableIndex(1)
MOI.VariableIndex(2)
MOI.VariableIndex(3)

julia> f = MOI.ScalarNonlinearFunction(:atan, Any[x[3], 2.0 * x[1]])
atan(MOI.VariableIndex(3), 0.0 + 2.0 MOI.VariableIndex(1))

julia> MOI.Nonlinear.SymbolicAD.variables(f)
```

```
2-element Vector{MathOptInterface.VariableIndex}:
MOI.VariableIndex(1)
MOI.VariableIndex(3)
```

source

MathOptInterface.Nonlinear.SymbolicAD.derivative - Function.

```
\label{eq:derivative} derivative (\texttt{f}:: \textbf{Union} \{ \textbf{Real}_{.} \texttt{MOI}. AbstractScalarFunction} \}, \ \ \textbf{x}:: \texttt{MOI}. VariableIndex})
```

Return an expression representing the partial derivative of f with respect to x.

#### **Expression swelling**

With few exceptions, the algorithm used to compute the derivative does not perform simplications. As a result, the returned expression may contain terms like \*(false, g) that can be trivially simplified to false.

In most cases, you should call simplify!(derivative(f, x)) to return a simplified expression of the derivative.

## **Example**

```
julia> x = MOI.VariableIndex(1)
MOI.VariableIndex(1)

julia> f = MOI.ScalarNonlinearFunction(:sin, Any[x])
sin(MOI.VariableIndex(1))

julia> df_dx = MOI.Nonlinear.SymbolicAD.derivative(f, x)
cos(MOI.VariableIndex(1))
```

source

MathOptInterface.Nonlinear.SymbolicAD.gradient\_and\_hessian - Function.

```
gradient_and_hessian(
    [filter_fn::Function = x -> true,]
    f::MOI.AbstractScalarFunction,
)
```

Compute the symbolic gradient and Hessian of f, and return the result as a tuple of four elements:

- 1. x::Vector{MOI.VariableIndex}: the list of variables that appear in f
- 2.  $\nabla f:: Vector{Any}: a vector for the first partial derivative of f with respect to each element in x$
- 3. H::Vector{Tuple{Int,Int}}: a vector of (row, column) tuples that list the non-zero entries in the Hessian of f
- 4. ∇²f::Vector{Any}: a vector of expressions, in the same order as H, for the non-zero entries in the Hessian of f

# filter\_fn

This argument is a function,  $filter_fn(::MOI.VariableIndex)::Bool that returns true if the gradient and Hessian of f should be computed with respect to it.$ 

Use this argument to filter out constant parameters from decision variables.

source

# **Chapter 30**

# **Utilities**

# 30.1 Overview

#### The Utilities submodule

The Utilities submodule provides a variety of functions and datastructures for managing MOI.ModelLike objects.

#### **Utilities.Model**

Utilities.Model provides an implementation of a ModelLike that efficiently supports all functions and sets defined within MOI. However, given the extensibility of MOI, this might not cover all use cases.

Create a model as follows:

```
julia> model = MOI.Utilities.Model{Float64}()
MOIU.Model{Float64}
| ObjectiveSense: FEASIBILITY_SENSE
| ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
L NumberOfConstraints: 0
```

## **Utilities.UniversalFallback**

Utilities.UniversalFallback is a layer that sits on top of any ModelLike and provides non-specialized (slower) fallbacks for constraints and attributes that the underlying ModelLike does not support.

For example, Utilities.Model doesn't support some variable attributes like VariablePrimalStart, so JuMP uses a combination of Universal fallback and Utilities.Model as a generic problem cache:

```
julia> model = MOI.Utilities.UniversalFallback(MOI.Utilities.Model{Float64}())
MOIU.UniversalFallback{MOIU.Model{Float64}}
| ObjectiveSense: FEASIBILITY_SENSE
| ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
L NumberOfConstraints: 0
```

#### Warning

Adding a UniversalFallback means that your model will now support all constraints, even if the innermodel does not. This can lead to unexpected behavior.

#### Utilities.@model

For advanced use cases that need efficient support for functions and sets defined outside of MOI (but still known at compile time), we provide the Utilities.@model macro.

The @model macro takes a name (for a new type, which must not exist yet), eight tuples specifying the types of constraints that are supported, and then a Bool indicating the type is a subtype of MOI.AbstractOptimizer (if true) or MOI.ModelLike (if false).

The eight tuples are in the following order:

- 1. Un-typed scalar sets, for example, Integer
- 2. Typed scalar sets, for example, LessThan
- 3. Un-typed vector sets, for example, Nonnegatives
- 4. Typed vector sets, for example, PowerCone
- 5. Un-typed scalar functions, for example, VariableIndex
- 6. Typed scalar functions, for example, ScalarAffineFunction
- 7. Un-typed vector functions, for example, VectorOfVariables
- 8. Typed vector functions, for example, VectorAffineFunction

The tuples can contain more than one element. Typed-sets must be specified without their type parameter, for example, MOI.LessThan, not MOI.LessThan{Float64}.

Here is an example:

```
julia> MOI.Utilities.@model(
            MyNewModel,
            (MOI.Integer,),
                                                # Un-typed scalar sets
            (MOI.GreaterThan,),
                                                # Typed scalar sets
            (MOI.Nonnegatives,),
                                               # Un-typed vector sets
            (MOI.PowerCone,),
                                                # Typed vector sets
            (MOI.VariableIndex,),
                                               # Un-typed scalar functions
            (MOI.ScalarAffineFunction,), # Typed scalar functions
            (MOI.VectorOfVariables,),
                                               # Un-typed vector functions
            (MOI.VectorAffineFunction,), # Typed vector functions
            true,
                                                # <:MOI.AbstractOptimizer?</pre>
MathOptInterface.Utilities.GenericOptimizer{T, MathOptInterface.Utilities.ObjectiveContainer{T},
\quad \hookrightarrow \quad \mathsf{MathOptInterface}. \\ \mathsf{Utilities}. \\ \mathsf{VariablesContainer} \\ \mathsf{T} \}, \ \mathsf{MyNewModelFunctionConstraints} \\ \mathsf{T} \} \} \ \mathsf{where} \ \mathsf{T} 
julia> model = MyNewModel{Float64}()
MOIU.GenericOptimizer{Float64, MOIU.ObjectiveContainer{Float64}, MOIU.VariablesContainer{Float64},

→ MyNewModelFunctionConstraints{Float64}}
- ObjectiveSense: FEASIBILITY_SENSE
```

```
| ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
| NumberOfConstraints: 0
```

#### Warning

MyNewModel supports every VariableIndex-in-Set constraint, as well as VariableIndex, ScalarAffineFunction, and ScalarQuadraticFunction objective functions. Implement MOI.supports as needed to forbid constraint and objective function combinations.

As another example, PATHSolver, which only supports VectorAffineFunction-in-Complements defines its optimizer as:

However, PathOptimizer does not support some VariableIndex-in-Set constraints, so we must explicitly define:

Finally, PATH doesn't support an objective function, so we need to add:

```
julia> MOI.supports(::PathOptimizer, ::MOI.ObjectiveFunction) = false
```

#### Warning

This macro creates a new type, so it must be called from the top-level of a module, for example, it cannot be called from inside a function.

## **Utilities.CachingOptimizer**

A [Utilities.CachingOptimizer] is an MOI layer that abstracts the difference between solvers that support incremental modification (for example, they support adding variables one-by-one), and solvers that require the entire problem in a single API call (for example, they only accept the A, b and c matrices of a linear program).

It has two parts:

- 1. A cache, where the model can be built and modified incrementally
- 2. An optimizer, which is used to solve the problem

```
julia> model = MOI.Utilities.CachingOptimizer(
          MOI.Utilities.Model{Float64}(),
          PathOptimizer{Float64}(),
      )
MOIU.CachingOptimizer
- state: EMPTY OPTIMIZER
- mode: AUTOMATIC
- model cache: MOIU.Model{Float64}
| - ObjectiveSense: FEASIBILITY_SENSE
| - ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
- NumberOfVariables: 0
 L NumberOfConstraints: 0
optimizer: MOIU.GenericOptimizer{Float64, MOIU.ObjectiveContainer{Float64},
→ MOIU.VariablesContainer{Float64}, MOIU.VectorOfConstraints{MOI.VectorAffineFunction{Float64},
→ MOI.Complements}}
  - ObjectiveSense: FEASIBILITY_SENSE
  - ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
  - NumberOfVariables: 0
  L NumberOfConstraints: 0
```

A Utilities.CachingOptimizer may be in one of three possible states:

- NO\_OPTIMIZER: The CachingOptimizer does not have any optimizer.
- EMPTY\_OPTIMIZER: The CachingOptimizer has an empty optimizer, and it is not synchronized with the cached model. Modifications are forwarded to the cache, but not to the optimizer.
- ATTACHED\_OPTIMIZER: The CachingOptimizer has an optimizer, and it is synchronized with the cached model. Modifications are forwarded to the optimizer. If the optimizer does not support modifications, and error will be thrown.

Use Utilities.attach\_optimizer to go from EMPTY\_OPTIMIZER to ATTACHED\_OPTIMIZER:

```
julia> MOI.Utilities.attach_optimizer(model)

julia> MOI.Utilities.state(model)

ATTACHED_OPTIMIZER::CachingOptimizerState = 2
```

#### Info

You must be in ATTACHED OPTIMIZER to use optimize!.

Use Utilities.reset optimizer to go from ATTACHED OPTIMIZER to EMPTY OPTIMIZER:

```
julia> MOI.Utilities.reset_optimizer(model)

julia> MOI.Utilities.state(model)

EMPTY_OPTIMIZER::CachingOptimizerState = 1
```

#### Info

Calling MOI.empty! (model) also resets the state to EMPTY\_OPTIMIZER. So after emptying a model, the modification will only be applied to the cache.

Use Utilities.drop\_optimizer to go from any state to NO\_OPTIMIZER:

 $Pass\ an\ empty\ optimizer\ to\ Utilities.reset\_optimizer\ to\ go\ from\ NO\_OPTIMIZER\ to\ EMPTY\_OPTIMIZER:$ 

```
- ObjectiveSense: FEASIBILITY_SENSE
- ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
- NumberOfVariables: 0
- NumberOfConstraints: 0
```

Deciding when to attach and reset the optimizer is tedious, and you will often write code like this:

```
try
    # modification
catch
    MOI.Utilities.reset_optimizer(model)
    # Re-try modification
end
```

To make this easier, Utilities.CachingOptimizer has two modes of operation:

- AUTOMATIC: The CachingOptimizer changes its state when necessary. Attempting to add a constraint or perform a modification not supported by the optimizer results in a drop to EMPTY\_OPTIMIZER mode.
- MANUAL: The user must change the state of the CachingOptimizer. Attempting to perform an operation in the incorrect state results in an error.

By default, AUTOMATIC mode is chosen. However, you can create a CachingOptimizer in MANUAL mode as follows:

```
julia> model = MOI.Utilities.CachingOptimizer(
          MOI.Utilities.Model{Float64}(),
          MOI.Utilities.MANUAL,
      )
MOIU.CachingOptimizer
- state: NO OPTIMIZER
- mode: MANUAL
- model_cache: MOIU.Model{Float64}
| - ObjectiveSense: FEASIBILITY_SENSE
| - ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
 L NumberOfConstraints: 0
L optimizer: nothing
julia> MOI.Utilities.reset_optimizer(model, PathOptimizer{Float64}())
julia> model
MOIU.CachingOptimizer
- state: EMPTY OPTIMIZER
- mode: MANUAL
- model_cache: MOIU.Model{Float64}
| - ObjectiveSense: FEASIBILITY_SENSE
| - ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
L NumberOfConstraints: 0
optimizer: MOIU.GenericOptimizer{Float64, MOIU.ObjectiveContainer{Float64},
→ MOIU.VariablesContainer{Float64}, MOIU.VectorOfConstraints{MOI.VectorAffineFunction{Float64},
→ MOI.Complements}}
```

```
| ObjectiveSense: FEASIBILITY_SENSE
| ObjectiveFunctionType: MOI.ScalarAffineFunction{Float64}
| NumberOfVariables: 0
| NumberOfConstraints: 0
```

#### **Printing**

Use print to print the formulation of the model.

Use Utilities.latex\_formulation to display the model in LaTeX form:

```
julia> MOI.Utilities.latex_formulation(model)

$$ \begin{aligned}
\max\quad & x\_var \\
\text{Subject to}\\
 & \text{VariableIndex-in-ZeroOne} \\
 & x\_var \in \{0, 1\} \\
\end{aligned} $$
```

# Tip

In IJulia, calling print or ending a cell with Utilities.latex\_formulation will render the model in LaTeX.

# **Utilities.PenaltyRelaxation**

Pass Utilities.PenaltyRelaxation to modify to relax the problem by adding penalized slack variables to the constraints. This is helpful when debugging sources of infeasible models.

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variable(model);
julia> MOI.set(model, MOI.VariableName(), x, "x")
julia> c = MOI.add_constraint(model, 1.0 * x, MOI.LessThan(2.0));
julia> map = MOI.modify(model, MOI.Utilities.PenaltyRelaxation(Dict(c => 2.0)));
julia> print(model)
Minimize ScalarAffineFunction{Float64}:
0.0 + 2.0 v[2]
Subject to:
ScalarAffineFunction{Float64}-in-LessThan{Float64}
0.0 + 1.0 x - 1.0 v[2] <= 2.0
VariableIndex-in-GreaterThan{Float64}
v[2] >= 0.0
julia> map[c]
0.0 + 1.0 MOI.VariableIndex(2)
```

You can also modify a single constraint using Utilities.ScalarPenaltyRelaxation:

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variable(model);
julia> MOI.set(model, MOI.VariableName(), x, "x")

julia> c = MOI.add_constraint(model, 1.0 * x, MOI.LessThan(2.0));
julia> f = MOI.modify(model, c, MOI.Utilities.ScalarPenaltyRelaxation(2.0));
julia> print(model)
Minimize ScalarAffineFunction{Float64}:
0.0 + 2.0 v[2]

Subject to:

ScalarAffineFunction{Float64}-in-LessThan{Float64}
0.0 + 1.0 x - 1.0 v[2] <= 2.0

VariableIndex-in-GreaterThan{Float64}
v[2] >= 0.0

julia> f
0.0 + 1.0 MOI.VariableIndex(2)
```

## **Utilities.MatrixOfConstraints**

The constraints of Utilities. Model are stored as a vector of tuples of function and set in a Utilities. VectorOfConstraints.

 $Other \, representations \, can \, be \, used \, by \, parameterizing \, the \, type \, {\tt Utilities.GenericModel} \, (resp. \, {\tt Utilities.GenericOptimizer}).$ 

For example, if all non-VariableIndex constraints are affine, the coefficients of all the constraints can be stored in a single sparse matrix using Utilities.MatrixOfConstraints.

The constraints storage can even be customized up to a point where it exactly matches the storage of the solver of interest, in which case copy\_to can be implemented for the solver by calling copy\_to to this custom model.

For example, Clp.jl defines the following model:

```
julia> MOI.Utilities.@product of sets(
           SupportedSets,
           MOI.EqualTo{T},
           MOI.LessThan{T},
          MOI.GreaterThan{T},
          MOI.Interval{T},
      );
julia> const OptimizerCache = MOI.Utilities.GenericModel{
           Float64.
           MOI.Utilities.ObjectiveContainer{Float64},
           MOI.Utilities.VariablesContainer{Float64},
           MOI.Utilities.MatrixOfConstraints{
               Float64,
               MOI.Utilities.MutableSparseMatrixCSC{
                   # The data type of the coefficients
                  Float64,
                  # The data type of the variable indices
                   # Can also be MOI.Utilities.OneBasedIndexing
                  MOI.Utilities.ZeroBasedIndexing,
              },
               MOI.Utilities.Hyperrectangle{Float64},
               SupportedSets{Float64},
           },
      };
```

Given the input model:

```
z == 5.0
""",
)
```

We can construct a new cached model and copy src to it:

```
julia> dest = OptimizerCache();
julia> index_map = MOI.copy_to(dest, src);
```

From dest, we can access the A matrix in sparse matrix form:

```
julia> A = dest.constraints.coefficients;
julia> A.n
julia> A.m
julia> A.colptr
4-element Vector{Int32}:
2
5
julia> A.rowval
5-element Vector{Int32}:
0
1
1
2
0
julia> A.nzval
5-element Vector{Float64}:
-4.0
1.0
1.0
 2.0
1.0
```

The lower and upper row bounds:

```
julia> row_bounds = dest.constraints.constants;

julia> row_bounds.lower
3-element Vector{Float64}:
    5.0
-Inf
    3.0
```

```
julia> row_bounds.upper
3-element Vector{Float64}:
    5.0
    1.0
Inf
```

The lower and upper variable bounds:

```
julia> dest.variables.lower
3-element Vector{Float64}:
    0.0
-Inf
    5.0

julia> dest.variables.upper
3-element Vector{Float64}:
    1.0
    10.0
    5.0
```

Because of larger variations between solvers, the objective can be queried using the standard MOI methods:

```
julia> MOI.get(dest, MOI.ObjectiveSense())
MAX_SENSE::OptimizationSense = 1

julia> F = MOI.get(dest, MOI.ObjectiveFunctionType())
MathOptInterface.ScalarAffineFunction{Float64}

julia> F = MOI.get(dest, MOI.ObjectiveFunction{F}())
0.0 + 1.0 MOI.VariableIndex(1) + 2.0 MOI.VariableIndex(2) - 3.1 MOI.VariableIndex(3)
```

Thus, Clp.jl implements copy\_to methods similar to the following:

```
# This method copies from the cache to the `Clp.Optimizer` object.
function MOI.copy_to(dest::Optimizer, src::OptimizerCache)
   @assert MOI.is_empty(dest)
   A = {\tt src.constraints.coefficients}
    row_bounds = src.constraints.constants
    Clp_loadProblem(
        dest,
        A.n,
        A.m,
        A.colptr,
        A.rowval,
        A.nzval,
        src.lower_bound,
        src.upper_bound,
        # (...) objective vector (omitted),
        row bounds.lower,
        row_bounds.upper,
```

```
return MOI.Utilities.identity_index_map(src)
end
# This method copies from an arbitrary model to the optimizer, by the
# intermediate `OptimizerCache` representation.
function MOI.copy_to(dest::Optimizer, src::MOI.ModelLike)
    cache = OptimizerCache()
   index_map = MOI.copy_to(cache, src)
   MOI.copy_to(dest, cache)
    return index_map
end
# This is a special method that gets called in some cases when `OptimizerCache`
# is used as the backing data structure in a `MOI.Utilities.CachingOptimizer`.
# It is needed for performance, but not correctness.
function MOI.copy_to(
    dest::Optimizer,
    src::MOI.Utilities.UniversalFallback{OptimizerCache},
   MOI.Utilities.throw_unsupported(src)
    return MOI.copy_to(dest, src.model)
end
```

#### Tip

For other examples of Utilities.MatrixOfConstraints, see:

- Cbc.jl
- ECOS.jl
- SCS.jl

## ModelFilter

Utilities provides Utilities.ModelFilter as a useful tool to copy a subset of a model. For example, given an infeasible model, we can copy the irreducible infeasible subsystem (for models implementing ConstraintConflictStatus) as follows:

```
my_filter(::Any) = true
function my_filter(ci::MOI.ConstraintIndex)
    status = MOI.get(dest, MOI.ConstraintConflictStatus(), ci)
    return status != MOI.NOT_IN_CONFLICT
end
filtered_src = MOI.Utilities.ModelFilter(my_filter, src)
index_map = MOI.copy_to(dest, filtered_src)
```

# Fallbacks

The value of some attributes can be inferred from the value of other attributes.

For example, the value of ObjectiveValue can be computed using ObjectiveFunction and VariablePrimal.

When a solver gives direct access to an attribute, it is better to return this value. However, if this is not the case, Utilities.get\_fallback can be used instead. For example:

```
function MOI.get(model::Optimizer, attr::MOI.ObjectiveFunction)
    return MOI.Utilities.get_fallback(model, attr)
end
```

#### **DoubleDicts**

When writing MOI interfaces, we often need to handle situations in which we map ConstraintIndexs to different values. For example, to a string for ConstraintName.

One option is to use a dictionary like Dict{MOI.ConstraintIndex,String}. However, this incurs a performance cost because the key is not a concrete type.

The DoubleDicts submodule helps this situation by providing two types main types Utilities.DoubleDicts.DoubleDict and Utilities.DoubleDicts.IndexDoubleDict. These types act like normal dictionaries, but internally they use more efficient dictionaries specialized to the type of the function-set pair.

The most common usage of a DoubleDict is in the index\_map returned by copy\_to. Performance can be improved, by using a function barrier. That is, instead of code like:

```
index_map = MOI.copy_to(dest, src)
for (F, S) in MOI.get(src, MOI.ListOfConstraintTypesPresent())
    for ci in MOI.get(src, MOI.ListOfConstraintIndices{F,S}())
        dest_ci = index_map[ci]
        # ...
    end
end
```

use instead:

```
function function_barrier(
    dest,
    src,
    index_map::MOI.Utilities.DoubleDicts.IndexDoubleDictInner{F,S},
) where {F,S}
    for ci in MOI.get(src, MOI.ListOfConstraintIndices{F,S}())
        dest_ci = index_map[ci]
        # ...
    end
    return
end

index_map = MOI.copy_to(dest, src)
for (F, S) in MOI.get(src, MOI.ListOfConstraintTypesPresent())
        function_barrier(dest, src, index_map[F, S])
end
```

## 30.2 API Reference

#### **Utilities.Model**

MathOptInterface.Utilities.Model - Type.

```
MOI.Utilities.Model{T}() where {T}
```

An implementation of ModelLike that supports all functions and sets defined in MOI. It is parameterized by the coefficient type.

## **Example**

source

#### Utilities.UniversalFallback

MathOptInterface.Utilities.UniversalFallback - Type.

```
UniversalFallback
```

The UniversalFallback can be applied on a MOI.ModelLike model to create the model UniversalFallback (model) supporting any constraint and attribute. This allows to have a specialized implementation in model for performance critical constraints and attributes while still supporting other attributes with a small performance penalty. Note that model is unaware of constraints and attributes stored by UniversalFallback so this is not appropriate if model is an optimizer (for this reason, MOI.optimize! has not been implemented). In that case, optimizer bridges should be used instead.

source

# Utilities.@model

MathOptInterface.Utilities.@model - Macro.

```
macro model(
    model_name,
    scalar_sets,
    typed_scalar_sets,
    vector_sets,
    typed_vector_sets,
    scalar_functions,
    typed_scalar_functions,
    vector_functions,
    typed_vector_functions,
    is_optimizer = false
)
```

Creates a type model\_name implementing the MOI model interface and supporting all combinations of the provided functions and sets.

Each typed\_ scalar/vector sets/functions argument is a tuple of types. A type is "typed" if it has a coefficient {T} as the first type parameter.

#### **Tuple syntax**

To give no set/function, write (). To give one set or function X, write (X,).

## is optimizer

If is\_optimizer = true, the resulting struct is a of GenericOptimizer, which is a subtype of MOI. AbstractOptimizer, otherwise, it is a GenericModel, which is a subtype of MOI. ModelLike.

#### VariableIndex

- The function MOI. VariableIndex must not be given in scalar\_functions.
- The model supports MOI. VariableIndex-in-S constraints where S is MOI. EqualTo, MOI. GreaterThan, MOI.LessThan, MOI.Interval, MOI.Integer, MOI.ZeroOne, MOI.Semicontinuous or MOI.Semiinteger.
- The sets supported with MOI. VariableIndex cannot be controlled from the macro; use UniversalFallback to support more sets.

#### **Example**

The model describing a linear program would be:

```
@model(
                                                     # model name
   LPModel,
                                                     # untyped scalar sets
   (),
    (MOI.EqualTo, MOI.GreaterThan, MOI.LessThan, MOI.Interval), # typed scalar sets
    (MOI.Zeros, MOI.Nonnegatives, MOI.Nonpositives), # untyped vector sets
   (),
                                                     # typed vector sets
   (),
                                                     # untyped scalar functions
    (MOI.ScalarAffineFunction,),
                                                     # typed scalar functions
    (MOI.VectorOfVariables.).
                                                     # untyped vector functions
    (MOI. VectorAffineFunction,),
                                                     # typed vector functions
   false.
                                                     # is_optimizer
```

source

MathOptInterface.Utilities.GenericModel - Type.

```
mutable struct GenericModel{T,0,V,C} <: AbstractModelLike{T}</pre>
```

Implements a model supporting coefficients of type T and:

- An objective function stored in .objective::0
- Variables and VariableIndex constraints stored in .variable\_bounds::V
- F-in-S constraints (excluding VariableIndex constraints) stored in .constraints::C

All interactions take place via the MOI interface, so the types 0, V, and C must implement the API as needed for their functionality.

source

MathOptInterface.Utilities.GenericOptimizer - Type.

```
mutable struct GenericOptimizer{T,0,V,C} <: AbstractOptimizer{T}</pre>
```

Implements a model supporting coefficients of type T and:

- An objective function stored in .objective::0
- Variables and VariableIndex constraints stored in .variable\_bounds::V
- F-in-S constraints (excluding VariableIndex constraints) stored in .constraints::C

All interactions take place via the MOI interface, so the types 0, V, and C must implement the API as needed for their functionality.

source

# .objective

MathOptInterface.Utilities.ObjectiveContainer - Type.

```
ObjectiveContainer{T}
```

A helper struct to simplify the handling of objective functions in Utilities. Model.

source

## .variables

MathOptInterface.Utilities.VariablesContainer - Type.

```
struct VariablesContainer{T} <: AbstractVectorBounds
   set_mask::Vector{UInt16}
   lower::Vector{T}
   upper::Vector{T}
end</pre>
```

A struct for storing variables and VariableIndex-related constraints. Used in MOI.Utilities.Model by default.

source

 ${\tt MathOptInterface.Utilities.FreeVariables-Type}.$ 

```
mutable struct FreeVariables <: MOI.ModelLike
   n::Int64
   FreeVariables() = new(θ)
end</pre>
```

A struct for storing free variables that can be used as the variables field of GenericModel or GenericModel. It represents a model that does not support any constraint nor objective function.

#### **Example**

false

julia> MOI.Utilities.@product\_of\_sets(

julia> MOI.Bridges.is\_bridged(model, bridge.vector\_constraint)

The following model type represents a conic model in geometric form. As opposed to VariablesContainer, FreeVariables does not support constraint bounds so they are bridged into an affine constraint in the MOI.Nonnegatives cone as expected for the geometric conic form.

```
Cones.
    MOI.Zeros,
    MOI.Nonnegatives,
    MOI.SecondOrderCone,
    MOI.PositiveSemidefiniteConeTriangle,
);
julia> const ConicModel{T} = MOI.Utilities.GenericOptimizer{
    MOI.Utilities.ObjectiveContainer{T},
    MOI.Utilities.FreeVariables,
    MOI.Utilities.MatrixOfConstraints{
        MOI.Utilities.MutableSparseMatrixCSC{
            Τ,
            Int.
            MOI.Utilities.OneBasedIndexing,
        Vector{T},
        Cones{T},
    },
};
julia> model = MOI.instantiate(ConicModel{Float64}, with_bridge_type=Float64);
julia> x = MOI.add_variable(model)
MathOptInterface.VariableIndex(1)
julia> c = MOI.add_constraint(model, x, MOI.GreaterThan(1.0))
MathOptInterface.ConstraintIndex{MathOptInterface.VariableIndex,
→ MathOptInterface.GreaterThan{Float64}}(1)
julia> MOI.Bridges.is_bridged(model, c)
true
julia> bridge = MOI.Bridges.bridge(model, c)
MathOptInterface.Bridges.Constraint.VectorizeBridge{Float64,
\quad \hookrightarrow \quad \mathsf{MathOptInterface.VectorAffineFunction\{Float64\}}, \ \mathsf{MathOptInterface.Nonnegatives}, \\
→ MathOptInterface.VariableIndex}(MathOptInterface.ConstraintIndex{MathOptInterface.VectorAffineFunction{Float64
\hookrightarrow MathOptInterface.Nonnegatives}(1), 1.0)
julia> bridge.vector_constraint
MathOptInterface.ConstraintIndex{MathOptInterface.VectorAffineFunction{Float64},
→ MathOptInterface.Nonnegatives}(1)
```

source

## .constraints

MathOptInterface.Utilities.VectorOfConstraints - Type.

```
mutable struct VectorOfConstraints{
    F<:MOI.AbstractFunction,
    S<:MOI.AbstractSet,
} <: MOI.ModelLike
    constraints::CleverDicts.CleverDict{
        MOI.ConstraintIndex{F,S},
        Tuple{F,S},
        typeof(CleverDicts.key_to_index),
        typeof(CleverDicts.index_to_key),
    }
end</pre>
```

A struct storing F-in-S constraints as a mapping between the constraint indices to the corresponding tuple of function and set.

source

MathOptInterface.Utilities.StructOfConstraints - Type.

```
abstract type StructOfConstraints <: MOI.ModelLike end</pre>
```

A struct storing a subfields other structs storing constraints of different types.

See Utilities.@struct\_of\_constraints\_by\_function\_types and Utilities.@struct\_of\_constraints\_by\_set\_types. source

 ${\tt MathOptInterface.Utilities.@struct\_of\_constraints\_by\_function\_types-Macro.}$ 

```
Utilities.@struct_of_constraints_by_function_types(name, func_types...)
```

Given a vector of n function types (F1, F2,..., Fn) in func\_types, defines a subtype of StructOfConstraints of name name and which type parameters  $\{T, C1, C2, ..., Cn\}$ . It contains n field where the ith field has type Ci and stores the constraints of function type Fi.

The expression Fi can also be a union in which case any constraint for which the function type is in the union is stored in the field with type Ci.

source

MathOptInterface.Utilities.@struct\_of\_constraints\_by\_set\_types - Macro.

```
Utilities.@struct_of_constraints_by_set_types(name, func_types...)
```

Given a vector of n set types (S1, S2,..., Sn) in func\_types, defines a subtype of StructOfConstraints of name name and which type parameters {T, C1, C2, ..., Cn}. It contains n field where the ith field has type Ci and stores the constraints of set type Si. The expression Si can also be a union in which case any constraint for which the set type is in the union is stored in the field with type Ci. This can be useful if Ci is a MatrixOfConstraints in order to concatenate the coefficients of constraints of several different set types in the same matrix.

source

MathOptInterface.Utilities.struct of constraint code - Function.

```
struct_of_constraint_code(struct_name, types, field_types = nothing)
```

Given a vector of n Union{SymbolFun,\_UnionSymbolFS{SymbolFun}} or Union{SymbolSet,\_UnionSymbolFS{SymbolSet}} in types, defines a subtype of StructOfConstraints of name name and which type parameters {T, F1, F2, ..., Fn} if field\_types is nothing and a {T} otherwise. It contains n field where the ith field has type Ci if field\_types is nothing and type field\_types[i] otherwise. If types is vector of Union{SymbolFun,\_UnionSymbolFS{SymbolFun}} (resp. Union{SymbolSet,\_UnionSymbolFS{SymbolSet}}) then the constraints of that function (resp. set) type are stored in the corresponding field.

This function is used by the macros @model, @struct\_of\_constraints\_by\_function\_types and @struct\_of\_constraints\_by\_source

#### Caching optimizer

MathOptInterface.Utilities.CachingOptimizer - Type.

```
CachingOptimizer
```

CachingOptimizer is an intermediate layer that stores a cache of the model and links it with an optimizer. It supports incremental model construction and modification even when the optimizer doesn't.

#### Mode

A Utilities.CachingOptimizer has two modes of operation: Utilities.AUTOMATIC and Utilities.MANUAL. See their docstrings for details.

Use Utilities.mode to query the mode of a Utilities.CachingOptimizer.

#### State

A Utilities.CachingOptimizer may be in one of three possible states: NO\_OPTIMIZER, Utilities.EMPTY\_OPTIMIZER, and Utilities.ATTACHED OPTIMIZER. See their docstrings for details.

Use Utilities.state to query the state of a Utilities.CachingOptimizer.

#### Constructor with optimizer

CachingOptimizer(cache::MOI.ModelLike, optimizer::AbstractOptimizer)

Creates a CachingOptimizer in AUTOMATIC mode, with the optimizer optimizer.

The type of the optimizer returned is CachingOptimizer(typeof(optimizer), typeof(cache)) so it does not support the function Utilities.reset\_optimizer(::CachingOptimizer, new\_optimizer) if the type of new\_optimizer is different from the type of optimizer.

# Constructor without optimizer

CachingOptimizer(cache::MOI.ModelLike, mode::CachingOptimizerMode)

 $Creates\ a\ Caching 0 ptimizer\ in\ the\ NO\_OPTIMIZER\ Utilities\ .\ Caching 0 ptimizer\ State\ and\ the\ Utilities\ .\ Caching 0 ptimizer\ Mode.$ 

The type of the optimizer returned is CachingOptimizer{MOI.AbstractOptimizer, typeof(cache)} so it does support the function reset\_optimizer(::CachingOptimizer, new\_optimizer) if the type of new\_optimizer is different from the type of optimizer.

source

MathOptInterface.Utilities.CachingOptimizerState - Type.

CachingOptimizerState

A Utilities.CachingOptimizer may be in one of three possible states.

# Values

NO OPTIMIZER

The CachingOptimizer does not have any optimizer.

**EMPTY OPTIMIZER** 

The CachingOptimizer has an optimizer. The optimizer is empty and it is not synchronized with the cached model.

ATTACHED\_OPTIMIZER

The CachingOptimizer has an optimizer, and it is synchronized with the cached model.

source

MathOptInterface.Utilities.NO\_OPTIMIZER - Constant.

 ${\tt NO\_OPTIMIZER::CachingOptimizerState}$ 

An instance of the CachingOptimizerState enum.

## **About**

The CachingOptimizer does not have any optimizer.

source

MathOptInterface.Utilities.EMPTY\_OPTIMIZER - Constant.

```
{\tt EMPTY\_OPTIMIZER::CachingOptimizerState}
```

An instance of the CachingOptimizerState enum.

#### **About**

The CachingOptimizer has an optimizer. The optimizer is empty and it is not synchronized with the cached model.

source

MathOptInterface.Utilities.ATTACHED\_OPTIMIZER - Constant.

```
ATTACHED_OPTIMIZER::CachingOptimizerState
```

An instance of the CachingOptimizerState enum.

## **About**

The CachingOptimizer has an optimizer, and it is synchronized with the cached model.

source

MathOptInterface.Utilities.state - Function.

```
state(m::CachingOptimizer)::CachingOptimizerState
```

Returns the state of the CachingOptimizer m.

See Utilities.CachingOptimizer.

source

 ${\tt MathOptInterface.Utilities.CachingOptimizerMode-Type.}\\$ 

```
CachingOptimizerMode
```

A Utilities.CachingOptimizer has two modes of operation.

# **Values**

#### MANUAL

The only methods that change the state of the CachingOptimizer are Utilities.reset\_optimizer, Utilities.drop\_optimizer, and Utilities.attach\_optimizer.

Attempting to perform an operation in the incorrect state results in an error.

**AUTOMATIC** 

The Utilities.CachingOptimizer changes its state when necessary. For example, MOI.optimize! will automatically call Utilities.attach\_optimizer (an optimizer must have been previously set). Attempting to add a constraint or perform a modification not supported by the optimizer results in a drop to the EMPTY\_OPTIMIZER state.

source

MathOptInterface.Utilities.AUTOMATIC - Constant.

```
AUTOMATIC::CachingOptimizerMode
```

An instance of the CachingOptimizerMode enum.

#### **About**

The Utilities.CachingOptimizer changes its state when necessary. For example, MOI.optimize! will automatically call Utilities.attach\_optimizer (an optimizer must have been previously set). Attempting to add a constraint or perform a modification not supported by the optimizer results in a drop to the EMPTY\_OPTIMIZER state.

source

MathOptInterface.Utilities.MANUAL - Constant.

```
MANUAL::CachingOptimizerMode
```

An instance of the CachingOptimizerMode enum.

# About

The only methods that change the state of the CachingOptimizer are Utilities.reset\_optimizer, Utilities.drop\_optimizer, and Utilities.attach\_optimizer.

Attempting to perform an operation in the incorrect state results in an error.

source

MathOptInterface.Utilities.mode - Function.

```
mode(m::CachingOptimizer)::CachingOptimizerMode
```

Returns the operating mode of the CachingOptimizer m.

See Utilities.CachingOptimizer.

source

 ${\tt MathOptInterface.Utilities.attach\_optimizer-Function}.$ 

```
attach_optimizer(model::CachingOptimizer)
```

Attaches the optimizer to model, copying all model data into it. Can be called only from the EMPTY\_OPTIMIZER state. If the copy succeeds, the CachingOptimizer will be in state ATTACHED\_OPTIMIZER after the call, otherwise an error is thrown; see MOI.copy\_to for more details on which errors can be thrown.

source

MathOptInterface.Utilities.reset\_optimizer - Function.

```
reset_optimizer(m::CachingOptimizer, optimizer::MOI.AbstractOptimizer)
```

Sets or resets m to have the given empty optimizer optimizer.

Can be called from any state. An assertion error will be thrown if optimizer is not empty.

The CachingOptimizer m will be in state EMPTY\_OPTIMIZER after the call.

source

```
reset_optimizer(m::CachingOptimizer)
```

Detaches and empties the current optimizer. Can be called from ATTACHED\_OPTIMIZER or EMPTY\_OPTIMIZER state. The CachingOptimizer will be in state EMPTY\_OPTIMIZER after the call.

source

 ${\tt MathOptInterface.Utilities.drop\_optimizer-Function}.$ 

```
drop_optimizer(m::CachingOptimizer)
```

Drops the optimizer, if one is present. Can be called from any state. The CachingOptimizer will be in state NO\_OPTIMIZER after the call.

source

## Mock optimizer

MathOptInterface.Utilities.MockOptimizer - Type.

```
MockOptimizer
```

MockOptimizer is a fake optimizer especially useful for testing. Its main feature is that it can store the values that should be returned for each attribute.

source

# **Printing**

 ${\tt MathOptInterface.Utilities.latex\_formulation-Function}.$ 

```
latex_formulation(model::MOI.ModelLike; kwargs...)
```

Wrap model in a type so that it can be pretty-printed as text/latex in a notebook like IJulia, or in Documenter.

To render the model, end the cell with latex\_formulation(model), or call display(latex\_formulation(model)) in to force the display of the model from inside a function.

Possible keyword arguments are:

- simplify\_coefficients : Simplify coefficients if possible by omitting them or removing trailing zeros.
- default\_name : The name given to variables with an empty name.
- print\_types : Print the MOI type of each function and set for clarity.

source

# Copy utilities

MathOptInterface.Utilities.default\_copy\_to - Function.

```
default_copy_to(dest::MOI.ModelLike, src::MOI.ModelLike)
```

A default implementation of MOI.copy\_to(dest, src) for models that implement the incremental interface, that is, MOI.supports\_incremental\_interface returns true.

source

MathOptInterface.Utilities.IndexMap - Type.

```
IndexMap()
```

The dictionary-like object returned by  ${\tt MOI.copy\_to}$ .

source

 ${\tt MathOptInterface.Utilities.identity\_index\_map-Function}.$ 

```
identity_index_map(model::MOI.ModelLike)
```

Return an IndexMap that maps all variable and constraint indices of model to themselves.

source

MathOptInterface.Utilities.ModelFilter - Type.

```
ModelFilter(filter::Function, model::MOI.ModelLike)
```

A layer to filter out various components of model.

The filter function takes a single argument, which is each element from the list returned by the attributes below. It returns true if the element should be visible in the filtered model and false otherwise.

The components that are filtered are:

- Entire constraint types via:
  - MOI.ListOfConstraintTypesPresent
- · Individual constraints via:
  - MOI.ListOfConstraintIndices{F,S}
- Specific attributes via:
  - MOI.ListOfModelAttributesSet
  - MOI.ListOfConstraintAttributesSet
  - MOI.ListOfVariableAttributesSet

#### Warning

The list of attributes filtered may change in a future release. You should write functions that are generic and not limited to the five types listed above. Thus, you should probably define a fallback filter(::Any) = true.

See below for examples of how this works.

## Note

This layer has a limited scope. It is intended by be used in conjunction with MOI.copy\_to.

# Example: copy model excluding integer constraints

Use the do syntax to provide a single function.

```
filtered_src = MOI.Utilities.ModelFilter(src) do item
    return item != (MOI.VariableIndex, MOI.Integer)
end
MOI.copy_to(dest, filtered_src)
```

# Example: copy model excluding names

Use type dispatch to simplify the implementation:

```
my_filter(::Any) = true # Note the generic fallback
my_filter(::MOI.VariableName) = false
my_filter(::MOI.ConstraintName) = false
filtered_src = MOI.Utilities.ModelFilter(my_filter, src)
MOI.copy_to(dest, filtered_src)
```

## Example: copy irreducible infeasible subsystem

```
my_filter(::Any) = true # Note the generic fallback
function my_filter(ci::MOI.ConstraintIndex)
    status = MOI.get(dest, MOI.ConstraintConflictStatus(), ci)
    return status != MOI.NOT_IN_CONFLICT
end
filtered_src = MOI.Utilities.ModelFilter(my_filter, src)
MOI.copy_to(dest, filtered_src)
```

source

MathOptInterface.Utilities.loadfromstring! - Function.

```
loadfromstring!(model, s)
```

A utility function to aid writing tests.

#### Warning

This function is not intended for widespread use. It is mainly used as a tool to simplify writing tests in MathOptInterface. Do not use it as an exchange format for storing or transmitting problem instances. Use the FileFormats submodule instead.

## **Example**

#### **Notes**

Special labels are:

- variables
- minobjective
- maxobjectives

Everything else denotes a constraint with a name.

Append :: T to use an element type of T when parsing the function.

Do not name VariableIndex constraints.

## **Exceptions**

```
• x - y does NOT currently parse. Instead, write x + -1.0 * y.
```

• x^2 does NOT currently parse. Instead, write x \* x.

source

#### Penalty relaxation

MathOptInterface.Utilities.PenaltyRelaxation - Type.

```
PenaltyRelaxation(
    penalties = Dict{MOI.ConstraintIndex,Float64}();
    default::Union{Nothing,T} = 1.0,
    warn::Bool = true,
)
```

A problem modifier that, when passed to MOI.modify, destructively modifies the model in-place to create a penalized relaxation of the constraints.

#### Warning

This is a destructive routine that modifies the model in-place. If you don't want to modify the original model, use JuMP.copy\_model to create a copy before calling MOI.modify.

#### Reformulation

See Utilities.ScalarPenaltyRelaxation for details of the reformulation.

For each constraint ci, the penalty passed to Utilities.ScalarPenaltyRelaxation is get(penalties, ci, default). If the value is nothing, because ci does not exist in penalties and default = nothing, then the constraint is skipped.

#### Return value

MOI.modify(model, PenaltyRelaxation()) returns a Dict{MOI.ConstraintIndex,MOI.ScalarAffineFunction} that maps each constraint index to the corresponding y + z as a MOI.ScalarAffineFunction. In an optimal solution, query the value of these functions to compute the violation of each constraint.

## Relax a subset of constraints

To relax a subset of constraints, pass a penalties dictionary and set default = nothing.

## Supported constraint types

The penalty relaxation is currently limited to modifying MOI.ScalarAffineFunction and MOI.ScalarQuadraticFunction constraints in the linear sets MOI.LessThan, MOI.GreaterThan, MOI.EqualTo and MOI.Interval.

It does not include variable bound or integrality constraints, because these cannot be modified in-place.

To modify variable bounds, rewrite them as linear constraints.

If a constraint cannot be modified, a warning is logged and the constraint is skipped. The warning can be disabled by setting warn = false.

#### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variable(model);
julia> c = MOI.add_constraint(model, 1.0 * x, MOI.LessThan(2.0));
julia> map = MOI.modify(model, MOI.Utilities.PenaltyRelaxation(default = 2.0));
julia> print(model)
Minimize ScalarAffineFunction{Float64}:
0.0 + 2.0 v[2]
Subject to:
ScalarAffineFunction{Float64}-in-LessThan{Float64}
0.0 + 1.0 v[1] - 1.0 v[2] <= 2.0
VariableIndex-in-GreaterThan{Float64}
v[2] >= 0.0
julia> map[c] isa MOI.ScalarAffineFunction{Float64}
true
```

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variable(model);
julia> c = MOI.add_constraint(model, 1.0 * x, MOI.LessThan(2.0));
julia> map = MOI.modify(model, MOI.Utilities.PenaltyRelaxation(Dict(c => 3.0)));
julia> print(model)
Minimize ScalarAffineFunction{Float64}:
    0.0 + 3.0 v[2]
Subject to:
ScalarAffineFunction{Float64}-in-LessThan{Float64}
    0.0 + 1.0 v[1] - 1.0 v[2] <= 2.0
VariableIndex-in-GreaterThan{Float64}
    v[2] >= 0.0
julia> map[c] isa MOI.ScalarAffineFunction{Float64}
true
```

source

MathOptInterface.Utilities.ScalarPenaltyRelaxation - Type.

```
ScalarPenaltyRelaxation(penalty::T) where {T}
```

A problem modifier that, when passed to MOI.modify, destructively modifies the constraint in-place to create a penalized relaxation of the constraint.

#### Warning

This is a destructive routine that modifies the constraint in-place. If you don't want to modify the original model, use JuMP.copy\_model to create a copy before calling MOI.modify.

#### Reformulation

The penalty relaxation modifies constraints of the form  $f(x) \in S$  into  $f(x) + y - z \in S$ , where  $y, z \ge 0$ , and then it introduces a penalty term into the objective of  $a \times (y+z)$  (if minimizing, else -a), where a is penalty

When S is MOI.LessThan or MOI.GreaterThan, we omit y or z respectively as a performance optimization.

#### **Return value**

MOI.modify(model, ci, ScalarPenaltyRelaxation(penalty)) returns y + z as a MOI.ScalarAffineFunction. In an optimal solution, query the value of this function to compute the violation of the constraint.

#### **Example**

```
julia> model = MOI.Utilities.Model{Float64}();
julia> x = MOI.add_variable(model);
julia> c = MOI.add_constraint(model, 1.0 * x, MOI.LessThan(2.0));
julia> f = MOI.modify(model, c, MOI.Utilities.ScalarPenaltyRelaxation(2.0));
julia> print(model)
Minimize ScalarAffineFunction{Float64}:
0.0 + 2.0 v[2]
Subject to:
ScalarAffineFunction{Float64}-in-LessThan{Float64}
0.0 + 1.0 v[1] - 1.0 v[2] <= 2.0
VariableIndex-in-GreaterThan{Float64}
v[2] >= 0.0
julia> f isa MOI.ScalarAffineFunction{Float64}
true
```

source

## MatrixOfConstraints

MathOptInterface.Utilities.MatrixOfConstraints - Type.

```
mutable struct MatrixOfConstraints{T,AT,BT,ST} <: MOI.ModelLike
   coefficients::AT
   constants::BT</pre>
```

```
sets::ST
caches::Vector{Any}
are_indices_mapped::Vector{BitSet}
final_touch::Bool
end
```

Represent ScalarAffineFunction and VectorAffinefunction constraints in a matrix form where the linear coefficients of the functions are stored in the coefficients field, the constants of the functions or sets are stored in the constants field. Additional information about the sets are stored in the sets field.

This model can only be used as the constraints field of a MOI.Utilities.AbstractModel.

When the constraints are added, they are stored in the caches field. They are only loaded in the coefficients and constants fields once MOI.Utilities.final\_touch is called. For this reason, MatrixOfConstraints should not be used by an incremental interface. Use MOI.copy to instead.

The constraints can be added in two different ways:

- 1. With add\_constraint, in which case a canonicalized copy of the function is stored in caches.
- 2. With pass\_nonvariable\_constraints, in which case the functions and sets are stored themselves in caches without mapping the variable indices. The corresponding index in caches is added in are\_indices\_mapped. This avoids doing a copy of the function in case the getter of CanonicalConstraintFunction does not make a copy for the source model, for example, this is the case of VectorOfConstraints.

We illustrate this with an example. Suppose a model is copied from a src::MOI.Utilities.Model to a bridged model with a MatrixOfConstraints. For all the types that are not bridged, the constraints will be copied with pass\_nonvariable\_constraints. Hence the functions stored in caches are exactly the same as the ones stored in src. This is ok since this is only during the copy\_to operation during which src cannot be modified. On the other hand, for the types that are bridged, the functions added may contain duplicates even if the functions did not contain duplicates in src so duplicates are removed with MOI.Utilities.canonical.

## Interface

The .coefficients::AT type must implement:

```
AT()
MOI.empty(::AT)!
MOI.Utilities.add_column
MOI.Utilities.set_number_of_rows
MOI.Utilities.allocate_terms
MOI.Utilities.load_terms
MOI.Utilities.final_touch
```

The .constants::BT type must implement:

```
BT()Base.empty!(::BT)Base.resize(::BT)MOI.Utilities.load_constants
```

```
MOI.Utilities.set_from_constants
The .sets::ST type must implement:

ST()
MOI.is_empty(::ST)
MOI.empty(::ST)
MOI.dimension(::ST)
MOI.is_valid(::ST, ::MOI.ConstraintIndex)
MOI.get(::ST, ::MOI.ListOfConstraintTypesPresent)
MOI.get(::ST, ::MOI.NumberOfConstraints)
MOI.get(::ST, ::MOI.ListOfConstraintIndices)
MOI.Utilities.set_types
MOI.Utilities.set_index
MOI.Utilities.add_set
MOI.Utilities.rows
MOI.Utilities.final_touch
```

• MOI.Utilities.function\_constants

# .coefficients

source

MathOptInterface.Utilities.add\_column - Function.

```
add_column(coefficients)::Nothing
```

Tell coefficients to pre-allocate datastructures as needed to store one column.

source

MathOptInterface.Utilities.allocate\_terms - Function.

```
allocate_terms(coefficients, index_map, func)::Nothing
```

Tell coefficients that the terms of the function func where the variable indices are mapped with index\_map will be loaded with load\_terms.

The function func must be canonicalized before calling allocate\_terms. See is\_canonical.

source

MathOptInterface.Utilities.set\_number\_of\_rows - Function.

```
set_number_of_rows(coefficients, n)::Nothing
```

Tell coefficients to pre-allocate datastructures as needed to store n rows.

source

MathOptInterface.Utilities.load\_terms - Function.

```
load_terms(coefficients, index_map, func, offset)::Nothing
```

Loads the terms of func to coefficients, mapping the variable indices with index\_map.

The ith dimension of func is loaded at the (offset +i)th row of coefficients.

The function must be allocated first with allocate\_terms.

The function func must be canonicalized, see is\_canonical.

source

MathOptInterface.Utilities.final\_touch - Function.

```
final_touch(coefficients)::Nothing
```

Informs the coefficients that all functions have been added with load\_terms. No more modification is allowed unless MOI.empty! is called.

```
final_touch(sets)::Nothing
```

Informs the sets that all functions have been added with add\_set. No more modification is allowed unless MOI.empty! is called.

source

 ${\tt MathOptInterface.Utilities.extract\_function-Function}.$ 

```
extract_function(coefficients, row::Integer, constant::T) where {T}
```

Return the MOI.ScalarAffineFunction{T} function corresponding to row row in coefficients.

```
extract_function(
   coefficients,
   rows::UnitRange,
   constants::Vector{T},
) where{T}
```

Return the MOI. VectorAffineFunction{T} function corresponding to rows rows in coefficients.

source

MathOptInterface.Utilities.MutableSparseMatrixCSC - Type.

```
mutable struct MutableSparseMatrixCSC{Tv,Ti<:Integer,I<:AbstractIndexing}
    indexing::I
    m::Int
    n::Int
    colptr::Vector{Ti}
    rowval::Vector{Ti}
    nzval::Vector{Ti}
end</pre>
```

Matrix type loading sparse matrices in the Compressed Sparse Column format. The indexing used is indexing, see AbstractIndexing. The other fields have the same meaning than for SparseArrays.SparseMatrixCSC except that the indexing is different unless indexing is OneBasedIndexing. In addition, nz\_added is used to cache the number of non-zero terms that have been added to each column due to the incremental nature of load terms.

The matrix is loaded in 5 steps:

- MOI.empty! is called.
- 2. MOI.Utilities.add\_column and MOI.Utilities.allocate\_terms are called in any order.
- 3. MOI.Utilities.set number of rows is called.
- 4. MOI.Utilities.load\_terms is called for each affine function.
- MOI.Utilities.final\_touch is called.

source

MathOptInterface.Utilities.AbstractIndexing - Type.

```
abstract type AbstractIndexing end
```

Indexing to be used for storing the row and column indices of MutableSparseMatrixCSC. See ZeroBasedIndexing and OneBasedIndexing.

source

 ${\tt MathOptInterface.Utilities.ZeroBasedIndexing-Type.}$ 

```
struct ZeroBasedIndexing <: AbstractIndexing end
```

Zero-based indexing: the ith row or column has index i - 1. This is useful when the vectors of row and column indices need to be communicated to a library using zero-based indexing such as C libraries.

source

MathOptInterface.Utilities.OneBasedIndexing - Type.

```
struct ZeroBasedIndexing <: AbstractIndexing end
```

One-based indexing: the ith row or column has index i. This enables an allocation-free conversion of MutableSparseMatrixCSC to SparseArrays.SparseMatrixCSC.

source

#### .constants

MathOptInterface.Utilities.load\_constants - Function.

```
load_constants(constants, offset, func_or_set)::Nothing
```

This function loads the constants of func\_or\_set in constants at an offset of offset. Where offset is the sum of the dimensions of the constraints already loaded. The storage should be preallocated with resize! before calling this function.

This function should be implemented to be usable as storage of constants for MatrixOfConstraints.

The constants are loaded in three steps:

- Base.empty! is called.
- 2. Base.resize! is called with the sum of the dimensions of all constraints.
- MOI.Utilities.load\_constants is called for each function for vector constraint or set for scalar constraint.

source

MathOptInterface.Utilities.function\_constants - Function.

```
function_constants(constants, rows)
```

This function returns the function constants that were loaded with load\_constants at the rows rows.

This function should be implemented to be usable as storage of constants for MatrixOfConstraints.

source

MathOptInterface.Utilities.set\_from\_constants - Function.

```
set_from_constants(constants, S::Type, rows)::S
```

This function returns an instance of the set S for which the constants where loaded with load\_constants at the rows rows.

This function should be implemented to be usable as storage of constants for MatrixOfConstraints.

source

MathOptInterface.Utilities.modify\_constants - Function.

```
modify_constants(constants, row::Integer, new_constant::T) where {T}
modify_constants(
    constants,
    rows::AbstractVector{<:Integer},
    new_constants::AbstractVector{T},
) where {T}</pre>
```

Modify constants in-place to store new\_constant in the row row, or rows rows.

 $This function \ must be implemented to enable \ MOI. Scalar Constant Change \ and \ MOI. Vector Constant Change for \ Matrix Of Constraints.$ 

source

MathOptInterface.Utilities.Hyperrectangle - Type.

```
struct Hyperrectangle{T} <: AbstractVectorBounds
    lower::Vector{T}
    upper::Vector{T}
end</pre>
```

A struct for the .constants field in MatrixOfConstraints.

source

#### .sets

MathOptInterface.Utilities.set\_index - Function.

```
set_index(sets, ::Type{S})::Union{Int,Nothing} where {S<:MOI.AbstractSet}</pre>
```

Return an integer corresponding to the index of the set type in the list given by set\_types.

If S is not part of the list, return nothing.

source

MathOptInterface.Utilities.set\_types - Function.

```
set_types(sets)::Vector{Type}
```

Return the list of the types of the sets allowed in sets.

source

MathOptInterface.Utilities.add\_set - Function.

```
add_set(sets, i)::Int64
```

Add a scalar set of type index i.

```
add_set(sets, i, dim)::Int64
```

Add a vector set of type index i and dimension dim.

Both methods return a unique Int64 of the set that can be used to reference this set.

source

MathOptInterface.Utilities.rows - Function.

```
rows(sets, ci::MOI.ConstraintIndex)::Union{Int,UnitRange{Int}}
```

Return the rows in 1:MOI.dimension(sets) corresponding to the set of id ci.value.

For scalar sets, this returns an Int. For vector sets, this returns an UnitRange{Int}.

source

MathOptInterface.Utilities.num rows - Function.

```
num_rows(sets::OrderedProductOfSets, ::Type{S}) where {S}
```

Return the number of rows corresponding to a set of type S. That is, it is the sum of the dimensions of the sets of type S.

source

MathOptInterface.Utilities.set\_with\_dimension - Function.

```
set_with_dimension(::Type{S}, dim) where {S<:MOI.AbstractVectorSet}</pre>
```

Returns the instance of S of MOI.dimension dim. This needs to be implemented for sets of type S to be useable with MatrixOfConstraints.

source

MathOptInterface.Utilities.ProductOfSets - Type.

```
abstract type ProductOfSets{T} end
```

Represents a cartesian product of sets of given types.

source

MathOptInterface.Utilities.MixOfScalarSets - Type.

```
abstract type MixOfScalarSets{T} <: ProductOfSets{T} end</pre>
```

Product of scalar sets in the order the constraints are added, mixing the constraints of different types.

Use @mix\_of\_scalar\_sets to generate a new subtype.

source

MathOptInterface.Utilities.@mix\_of\_scalar\_sets - Macro.

```
@mix_of_scalar_sets(name, set_types...)
```

Generate a new MixOfScalarSets subtype.

#### **Example**

source

MathOptInterface.Utilities.OrderedProductOfSets - Type.

```
abstract type OrderedProductOfSets{T} <: ProductOfSets{T} end</pre>
```

Product of sets in the order the constraints are added, grouping the constraints of the same types contiguously.

Use @product\_of\_sets to generate new subtypes.

source

MathOptInterface.Utilities.@product\_of\_sets - Macro.

```
@product_of_sets(name, set_types...)
```

Generate a new OrderedProductOfSets subtype.

## **Example**

source

## Fallbacks

MathOptInterface.Utilities.get\_fallback - Function.

```
get_fallback(model::MOI.ModelLike, ::MOI.ObjectiveValue)
```

Compute the objective function value using the VariablePrimal results and the ObjectiveFunction value.

source

```
get_fallback(
   model::MOI.ModelLike,
   ::MOI.DualObjectiveValue,
   ::Type{T},
)::T where {T}
```

Compute the dual objective value of type T using the ConstraintDual results and the ConstraintFunction and ConstraintSet values.

Note that the nonlinear part of the model is ignored.

source

```
get_fallback(
    model::MOI.ModelLike,
    ::MOI.ConstraintPrimal,
    constraint_index::MOI.ConstraintIndex,
)
```

Compute the value of the function of the constraint of index constraint\_index using the VariablePrimal results and the ConstraintFunction values.

source

```
get_fallback(
    model::MOI.ModelLike,
    attr::MOI.ConstraintDual,
    ci::MOI.ConstraintIndex{Union{MOI.VariableIndex,MOI.VectorOfVariables}},
    ::Type{T} = Float64,
) where {T}
```

Compute the dual of the constraint of index ci using the ConstraintDual of other constraints and the ConstraintFunction values.

Throws an error if some constraints are quadratic or if there is one another MOI.VariableIndex-in-S or MOI.VectorOfVariables-in-S constraint with one of the variables in the function of the constraint ci.

source

## **Function utilities**

The following utilities are available for functions:

MathOptInterface.Utilities.eval\_variables - Function.

```
eval_variables(value_fn::Function, f::MOI.AbstractFunction)
```

Returns the value of function f if each variable index vi is evaluated as value fn(vi).

Note that value\_fn must return a Number. See substitute\_variables for a similar function where value\_fn returns an MOI.AbstractScalarFunction.

#### Warning

The two-argument version of eval\_variables is deprecated and may be removed in MOI v2.0.0. Use the three-argument method eval\_variables(::Function, ::MOI.ModelLike, ::MOI.AbstractFunction) instead.

source

MathOptInterface.Utilities.map\_indices - Function.

```
map_indices(index_map::Function, attr::MOI.AnyAttribute, x::X)::X where {X}
```

Substitute any MOI. VariableIndex (resp. MOI. ConstraintIndex) in x by the MOI. VariableIndex (resp. MOI. ConstraintIndex) of the same type given by index\_map(x).

#### When to implement this method for new types X

This function is used by implementations of MOI.copy\_to on constraint functions, attribute values and submittable values. If you define a new attribute whose values x::X contain variable or constraint indices, you must also implement this function.

source

```
map_indices(
    variable_map::AbstractDict{T,T},
    x::X,
)::X where {T<:MOI.Index,X}</pre>
```

Shortcut for map\_indices(vi -> variable\_map[vi], x).

source

 ${\tt MathOptInterface.Utilities.substitute\_variables-Function}.$ 

```
substitute_variables(variable_map::Function, x)
```

Substitute any MOI.VariableIndex in x by variable\_map(x). The variable\_map function returns either MOI.VariableIndex or MOI.ScalarAffineFunction, see eval\_variables for a similar function where variable\_map returns a number.

This function is used by bridge optimizers on constraint functions, attribute values and submittable values when at least one variable bridge is used hence it needs to be implemented for custom types that are meant to be used as attribute or submittable value.

#### Note

When implementing a new method, don't use substitute\_variables(::Function, because Julia will not specialize on it. Use instead substitute\_variables(::F, ...) where {F<:Function}.

source

MathOptInterface.Utilities.filter\_variables - Function.

```
filter_variables(keep::Function, f::AbstractFunction)
```

Return a new function f with the variable vi such that !keep(vi) removed.

WARNING: Don't define filter\_variables(::Function, ...) because Julia will not specialize on this. Define instead filter\_variables(::F, ...) where {F<:Function}.

source

MathOptInterface.Utilities.remove variable - Function.

```
remove_variable(f::AbstractFunction, vi::VariableIndex)
```

Return a new function f with the variable vi removed.

source

```
remove_variable(
    f::MOI.AbstractFunction,
    s::MOI.AbstractSet,
    vi::MOI.VariableIndex,
)
```

Return a tuple (g, t) representing the constraint f-in-s with the variable vi removed. That is, the terms containing the variable vi in the function f are removed and the dimension of the set s is updated if needed (for example, when f is a VectorOfVariables with vi being one of the variables).

source

MathOptInterface.Utilities.all\_coefficients - Function.

```
all_coefficients(p::Function, f::MOI.AbstractFunction)
```

Determine whether predicate p returns true for all coefficients of f, returning false as soon as the first coefficient of f for which p returns false is encountered (short-circuiting). Similar to all.

source

 ${\tt MathOptInterface.Utilities.unsafe\_add-Function}.$ 

```
unsafe_add(t1::MOI.ScalarAffineTerm, t2::MOI.ScalarAffineTerm)
```

Sums the coefficients of t1 and t2 and returns an output MOI.ScalarAffineTerm. It is unsafe because it uses the variable of t1 as the variable of the output without checking that it is equal to that of t2.

source

```
unsafe_add(t1::MOI.ScalarQuadraticTerm, t2::MOI.ScalarQuadraticTerm)
```

Sums the coefficients of t1 and t2 and returns an output MOI. ScalarQuadraticTerm. It is unsafe because it uses the variable's of t1 as the variable's of the output without checking that they are the same (up to permutation) to those of t2.

source

```
unsafe_add(t1::M0I.VectorAffineTerm, t2::M0I.VectorAffineTerm)
```

Sums the coefficients of t1 and t2 and returns an output MOI.VectorAffineTerm. It is unsafe because it uses the output\_index and variable of t1 as the output\_index and variable of the output term without checking that they are equal to those of t2.

source

MathOptInterface.Utilities.isapprox\_zero - Function.

```
isapprox_zero(f::MOI.AbstractFunction, tol)
```

Return a Bool indicating whether the function f is approximately zero using tol as a tolerance.

## Important note

This function assumes that f does not contain any duplicate terms, you might want to first call canonical if that is not guaranteed.

### **Example**

```
julia> x = MOI.VariableIndex(1)

julia> f = MOI.ScalarAffineFunction(MOI.ScalarAffineTerm.([1, -1], [x, x]), 0)
(0) + (1) MOI.VariableIndex(1) - (1) MOI.VariableIndex(1)

julia> MOI.Utilities.isapprox_zero(f, 1e-8)
false

julia> MOI.Utilities.isapprox_zero(MOI.Utilities.canonical(f), 1e-8)
true
```

source

MathOptInterface.Utilities.modify\_function - Function.

```
modify_function(f::AbstractFunction, change::AbstractFunctionModification)
```

Return a copy of the function f, modified according to change.

source

MathOptInterface.Utilities.zero\_with\_output\_dimension - Function.

```
zero_with_output_dimension(::Type{T}, output_dimension::Integer) where {T}
```

Create an instance of type T with the output dimension output\_dimension.

This is mostly useful in Bridges, when code needs to be agnostic to the type of vector-valued function that is passed in.

source

The following functions can be used to canonicalize a function:

MathOptInterface.Utilities.is\_canonical - Function.

```
is_canonical(f::Union{ScalarAffineFunction, VectorAffineFunction})
```

Returns a Bool indicating whether the function is in canonical form. See canonical.

source

```
is_canonical(f::Union{ScalarQuadraticFunction, VectorQuadraticFunction})
```

Returns a Bool indicating whether the function is in canonical form. See canonical.

source

MathOptInterface.Utilities.canonical - Function.

```
canonical(f::MOI.AbstractFunction)
```

Returns the function in a canonical form, that is,

- A term appear only once.
- The coefficients are nonzero.
- The terms appear in increasing order of variable where there the order of the variables is the order of their value.
- For a AbstractVectorFunction, the terms are sorted in ascending order of output index.

The output of canonical can be assumed to be a copy of f, even for VectorOfVariables.

## Example

source

MathOptInterface.Utilities.canonicalize! - Function.

```
canonicalize!(f::Union{ScalarAffineFunction, VectorAffineFunction})
```

Convert a function to canonical form in-place, without allocating a copy to hold the result. See canonical.

source

```
canonicalize!(f::Union{ScalarQuadraticFunction, VectorQuadraticFunction})
```

Convert a function to canonical form in-place, without allocating a copy to hold the result. See canonical. source

The following functions can be used to manipulate functions with basic algebra:

MathOptInterface.Utilities.scalar type - Function.

```
scalar_type(F::Type{<:MOI.AbstractVectorFunction})</pre>
```

 $\label{thm:continuous} \mbox{Type of functions obtained by indexing objects obtained by calling each scalar on functions of type F.$ 

source

MathOptInterface.Utilities.scalarize - Function.

```
scalarize(func::MOI.VectorOfVariables, ignore_constants::Bool = false)
```

Returns a vector of scalar functions making up the vector function in the form of a Vector{MOI.SingleVariable}. See also each scalar.

source

```
scalarize(func::MOI.VectorAffineFunction{T}, ignore_constants::Bool = false)
```

Returns a vector of scalar functions making up the vector function in the form of a  $Vector\{MOI.ScalarAffineFunction\{T\}\}$ .

See also each scalar.

source

```
scalarize(func::MOI.VectorQuadraticFunction{T}, ignore_constants::Bool = false)
```

Returns a vector of scalar functions making up the vector function in the form of a  $Vector\{MOI.ScalarQuadraticFunction\{T\}\}$  See also each scalar.

source

MathOptInterface.Utilities.eachscalar - Function.

```
eachscalar(f::M0I.AbstractVectorFunction)
```

Returns an iterator for the scalar components of the vector function.

See also scalarize.

source

```
eachscalar(f::MOI.AbstractVector)
```

Returns an iterator for the scalar components of the vector.

source

 ${\tt MathOptInterface.Utilities.promote\_operation-Function}.$ 

```
promote_operation(
    op::Function,
    ::Type{T},
    ArgsTypes::Type{<:Union{T,AbstractVector{T},MOI.AbstractFunction}}...,
) where {T<:Number}</pre>
```

Compute the return type of the call operate(op, T, args...), where the types of the arguments args are ArgsTypes.

One assumption is that the element type T is invariant under each operation. That is, op(::T, ::T)::T where op is a +, -, \*, and /.

There are six methods for which we implement Utilities.promote operation:

```
    + a. promote_operation(::typeof(+), ::Type{T}, ::Type{F1}, ::Type{F2})
    - a. promote_operation(::typeof(-), ::Type{T}, ::Type{F}) b. promote_operation(::typeof(-), ::Type{T}, ::Type{F1}, ::Type{F2})
```

6. imag a. promote\_operation(::typeof(imag), ::Type{T}, ::Type{F}) where F is VariableIndex or VectorOfVariables

In each case, F (or F1 and F2) is one of the ten supported types, with a restriction that the mathematical operation makes sense, for example, we don't define promote\_operation(-, T, F1, F2) where F1 is a scalar-valued function and F2 is a vector-valued function. The ten supported types are:

- 1. ::T
- 2. ::VariableIndex
- ::ScalarAffineFunction{T}
- 4. ::ScalarQuadraticFunction{T}
- 5. ::ScalarNonlinearFunction
- 6. ::AbstractVector{T}
- 7. ::VectorOfVariables
- 8. ::VectorAffineFunction{T}
- 9. ::VectorQuadraticFunction{T}
- 10. ::VectorNonlinearFunction

source

MathOptInterface.Utilities.operate - Function.

```
operate(
    op::Function,
    ::Type{T},
    args::Union{T,MOI.AbstractFunction}...,
)::MOI.AbstractFunction where {T<:Number}</pre>
```

Returns an MOI.AbstractFunction representing the function resulting from the operation op(args...) on functions of coefficient type T.

No argument can be modified.

## Methods

```
    +a. operate(::typeof(+), ::Type{T}, ::F1) b. operate(::typeof(+), ::Type{T}, ::F1, ::F2) c. operate(::typeof(+), ::Type{T}, ::F1...)
    -a. operate(::typeof(-), ::Type{T}, ::F) b. operate(::typeof(-), ::Type{T}, ::F1, ::F2)
    *a. operate(::typeof(*), ::Type{T}, ::T, ::F) b. operate(::typeof(*), ::Type{T}, ::F, ::T) c. operate(::typeof(*), ::Type{T}, ::F1, ::F2) where F1 and F2 are VariableIndex or ScalarAffineFunction d. operate(::typeof(*), ::Type{T}, ::Diagonal{T}, ::F)
```

```
4. / a. operate(::typeof(/), ::Type{T}, ::F, ::T)
5. vcat a. operate(::typeof(vcat), ::Type{T}, ::F...)
6. imag a. operate(::typeof(imag), ::Type{T}, ::F) where F is VariableIndex or VectorOfVariables
```

One assumption is that the element type T is invariant under each operation. That is, op(::T, ::T)::T where op is a +, -, \*, and /.

In each case, F (or F1 and F2) is one of the ten supported types, with a restriction that the mathematical operation makes sense, for example, we don't define promote\_operation(-, T, F1, F2) where F1 is a scalar-valued function and F2 is a vector-valued function. The ten supported types are:

- 1. ::T
- 2. ::VariableIndex
- 3. ::ScalarAffineFunction{T}
- 4. ::ScalarQuadraticFunction{T}
- 5. ::ScalarNonlinearFunction
- 6. ::AbstractVector{T}
- 7. ::VectorOfVariables
- 8. ::VectorAffineFunction{T}
- 9. ::VectorQuadraticFunction{T}
- 10. ::VectorNonlinearFunction

source

MathOptInterface.Utilities.operate! - Function.

```
operate!(
    op::Function,
    ::Type{T},
    args::Union{T,MOI.AbstractFunction}...,
)::MOI.AbstractFunction where {T<:Number}</pre>
```

Returns an MOI.AbstractFunction representing the function resulting from the operation op(args...) on functions of coefficient type T.

The first argument may be modified, in which case the return value is identical to the first argument. For operations which cannot be implemented in-place, this function returns a new object.

source

MathOptInterface.Utilities.operate\_output\_index! - Function.

```
operate_output_index!(
    op::Union{typeof(+),typeof(-)},
    ::Type{T},
    output_index::Integer,
    f::Union{AbstractVector{T},MOI.AbstractVectorFunction}
    g::Union{T,MOI.AbstractScalarFunction}...
) where {T<:Number}</pre>
```

Return an MOI.AbstractVectorFunction in which the scalar function in row output\_index is the result of op(f[output\_index], g).

The functions at output index different to output\_index are the same as the functions at the same output index in func. The first argument may be modified.

#### Methods

```
    + a. operate_output_index!(+, ::Type{T}, ::Int, ::VectorF, ::ScalarF)
    - a. operate_output_index!(-, ::Type{T}, ::Int, ::VectorF, ::ScalarF)

source
```

MathOptInterface.Utilities.vectorize - Function.

```
vectorize(x::AbstractVector{<:Number})

Returns x.
source

vectorize(x::AbstractVector{MOI.VariableIndex})</pre>
```

Returns the vector of scalar affine functions in the form of a MOI. VectorAffineFunction{T}.

source

```
vectorize(funcs:: \textbf{AbstractVector} \{ \texttt{MOI.ScalarAffineFunction} \{ T \} \}) \ \ where \ \ T
```

Returns the vector of scalar affine functions in the form of a MOI. Vector Affine Function  $\{T\}$ .

source

```
vectorize(funcs:: \textbf{AbstractVector} \{ \texttt{MOI.ScalarQuadraticFunction} \{ \texttt{T} \} \}) \ \ where \ \ \texttt{T}
```

 $Returns\ the\ vector\ of\ scalar\ quadratic\ functions\ in\ the\ form\ of\ a\ MOI. Vector Quadratic Function \{T\}.$ 

source

## **Constraint utilities**

The following utilities are available for moving the function constant to the set for scalar constraints: MathOptInterface.Utilities.shift\_constant - Function.

```
shift_constant(set::MOI.AbstractScalarSet, offset)
```

Returns a new scalar set new\_set such that func-in-set is equivalent to func + offset-in-new\_set.

Use  ${\it supports\_shift\_constant}$  to check if the set supports shifting:

```
if MOI.Utilities.supports_shift_constant(typeof(set))
    new_set = MOI.Utilities.shift_constant(set, -func.constant)
    func.constant = 0
    MOI.add_constraint(model, func, new_set)
else
    MOI.add_constraint(model, func, set)
end
```

## Note for developers

Only define this function if it makes sense and you have implemented supports\_shift\_constant to return true.

#### **Example**

```
julia> set = MOI.Interval(-2.0, 3.0)
MathOptInterface.Interval{Float64}(-2.0, 3.0)

julia> MOI.Utilities.supports_shift_constant(typeof(set))
true

julia> MOI.Utilities.shift_constant(set, 1.0)
MathOptInterface.Interval{Float64}(-1.0, 4.0)
```

source

MathOptInterface.Utilities.supports\_shift\_constant - Function.

```
supports_shift_constant(::Type{S}) where {S<:MOI.AbstractSet}</pre>
```

Return true if shift\_constant is defined for set S.

See also shift constant.

## **Example**

```
julia> MOI.Utilities.supports_shift_constant(MOI.Interval{Float64})
true

julia> MOI.Utilities.supports_shift_constant(MOI.ZeroOne)
false
```

source

MathOptInterface.Utilities.normalize\_and\_add\_constraint - Function.

```
normalize_and_add_constraint(
   model::MOI.ModelLike,
   func::MOI.AbstractScalarFunction,
   set::MOI.AbstractScalarSet;
   allow_modify_function::Bool = false,
)
```

Adds the scalar constraint obtained by moving the constant term in func to the set in model. If allow\_modify\_function is true then the function func can be modified.

source

MathOptInterface.Utilities.normalize\_constant - Function.

```
normalize_constant(
   func::MOI.AbstractScalarFunction,
   set::MOI.AbstractScalarSet;
   allow_modify_function::Bool = false,
)
```

Return the func-in-set constraint in normalized form. That is, if func is MOI.ScalarQuadraticFunction or MOI.ScalarAffineFunction, the constant is moved to the set. If allow\_modify\_function is true then the function func can be modified.

source

The following utility identifies those constraints imposing bounds on a given variable, and returns those bound values:

MathOptInterface.Utilities.get\_bounds - Function.

```
get_bounds(model::MOI.ModelLike, ::Type{T}, x::MOI.VariableIndex)
```

Return a tuple (lb, ub) of type  $Tuple\{T, T\}$ , where lb and ub are lower and upper bounds, respectively, imposed on x in model.

source

```
get_bounds(
    model::MOI.ModelLike,
    bounds_cache::Dict{MOI.VariableIndex,NTuple{2,T}},
    f::MOI.ScalarAffineFunction{T},
) where {T} ---> Union{Nothing,NTuple{2,T}}
```

Return the lower and upper bound of f as a tuple. If the domain is not bounded, return nothing.

source

```
get_bounds(
    model::MOI.ModelLike,
    bounds_cache::Dict{MOI.VariableIndex,NTuple{2,T}},
    x::MOI.VariableIndex,
) where {T} --> Union{Nothing,NTuple{2,T}}
```

Return the lower and upper bound of x as a tuple. If the domain is not bounded, return nothing.

Similar to get\_bounds(::MOI.ModelLike, ::Type{T}, ::MOI.VariableIndex), except that the second argument is a cache which maps variables to their bounds and avoids repeated lookups.

source

The following utilities are useful when working with symmetric matrix cones.

MathOptInterface.Utilities.is\_diagonal\_vectorized\_index - Function.

```
is_diagonal_vectorized_index(index::Base.Integer)
```

Return whether index is the index of a diagonal element in a MOI. AbstractSymmetricMatrixSetTriangle set.

source

MathOptInterface.Utilities.side\_dimension\_for\_vectorized\_dimension - Function.

```
side_dimension_for_vectorized_dimension(n::Integer)
```

Return the dimension d such that MOI.dimension(MOI.PositiveSemidefiniteConeTriangle(d)) is n. source

## Set utilities

The following utilities are available for sets:

MathOptInterface.Utilities.AbstractDistance - Type.

```
abstract type AbstractDistance end
```

An abstract type used to enable dispatch of Utilities.distance\_to\_set.

source

 ${\tt MathOptInterface.Utilities.ProjectionUpperBoundDistance-Type.}\\$ 

```
ProjectionUpperBoundDistance() <: AbstractDistance</pre>
```

An upper bound on the minimum distance between point and the closest feasible point in set.

## **Definition of distance**

The minimum distance is computed as:

$$d(x, \mathcal{K}) = \min_{y \in \mathcal{K}} ||x - y||$$

where x is point and  $\mathcal K$  is set. The norm is computed as:

$$||x|| = \sqrt{f(x, x, \mathcal{K})}$$

where f is Utilities.set\_dot.

In the default case, where the set does not have a specialized method for Utilities.set\_dot, the norm is equivalent to the Euclidean norm  $||x|| = \sqrt{\sum x_i^2}$ .

#### Why an upper bound?

In most cases, distance\_to\_set should return the smallest upper bound, but it may return a larger value if the smallest upper bound is expensive to compute.

For example, given an epigraph from of a conic set,  $\{(t,x)|f(x) \leq t\}$ , it may be simpler to return  $\delta$  such that  $f(x) \leq t + \delta$ , rather than computing the nearest projection onto the set.

If the distance is not the smallest upper bound, the docstring of the appropriate distance\_to\_set method must describe the way that the distance is computed.

source

MathOptInterface.Utilities.distance\_to\_set - Function.

```
distance_to_set(
   [d::AbstractDistance = ProjectionUpperBoundDistance()],]
   point::T,
   set::MOI.AbstractScalarSet,
) where {T}

distance_to_set(
   [d::AbstractDistance = ProjectionUpperBoundDistance(),]
   point::AbstractVector{T},
   set::MOI.AbstractVectorSet,
) where {T}
```

Compute the distance between point and set using the distance metric d. If point is in the set set, this function must return zero(T).

If d is omitted, the default distance is Utilities.ProjectionUpperBoundDistance.

source

```
\label{linear_distance_to_set} distance\_to\_set(::ProjectionUpperBoundDistance, \ x, \ ::M0I.RotatedSecondOrderCone)
```

Let (t, u, y...) = x. Return the 2-norm of the vector d such that in x + d, u is projected to 1 if  $u \le 0$ , and t is increased such that x + d belongs to the set.

source

```
distance_to_set(::ProjectionUpperBoundDistance, x, ::MOI.ExponentialCone)
```

Let (u, v, w) = x. If v > 0, return the epigraph distance d such that (u, v, w + d) belongs to the set.

If  $v \le 0$  return the 2-norm of the vector d such that x + d = (u, 1, z) where z satisfies the constraints.

source

```
distance_to_set(::ProjectionUpperBoundDistance, x, ::MOI.DualExponentialCone)
```

Let (u, v, w) = x. If u < 0, return the epigraph distance d such that (u, v, w + d) belongs to the set.

If u >= 0 return the 2-norm of the vector d such that x + d = (u, -1, z) where z satisfies the constraints.

source

```
\label{linear_distance_to_set} \textbf{distance\_to\_set}(::ProjectionUpperBoundDistance, \ x, \ ::MOI.GeometricMeanCone)
```

Let (t, y...) = x. If all y are non-negative, return the epigraph distance d such that (t + d, y...) belongs to the set.

If any y are strictly negative, return the 2-norm of the vector d that projects negative y elements to 0 and t to  $\mathbb{R}_{-}$ .

source

```
distance_to_set(::ProjectionUpperBoundDistance, x, ::MOI.PowerCone)
```

Let (a, b, c) = x. If a and b are non-negative, return the epigraph distance required to increase c such that the constraint is satisfied.

If a or b is strictly negative, return the 2-norm of the vector d such that in the vector x + d: c, and any negative a and b are projected to  $\theta$ .

source

```
distance_to_set(::ProjectionUpperBoundDistance, x, ::MOI.DualPowerCone)
```

Let (a, b, c) = x. If a and b are non-negative, return the epigraph distance required to increase c such that the constraint is satisfied.

If a or b is strictly negative, return the 2-norm of the vector d such that in the vector x + d: c, and any negative a and b are projected to  $\theta$ .

source

```
\label{linear_distance_to_set} \\ \mbox{distance_to\_set(::ProjectionUpperBoundDistance, x, ::MOI.NormOneCone)} \\
```

Let (t, y...) = x. Return the epigraph distance d such that (t + d, y...) belongs to the set.

source

```
distance to set(::ProjectionUpperBoundDistance, x, ::MOI.NormInfinityCone)
```

Let (t, y...) = x. Return the epigraph distance d such that (t + d, y...) belongs to the set.

source

```
distance_to_set(::ProjectionUpperBoundDistance, x, ::MOI.RelativeEntropyCone)
```

Let (u, v..., w...) = x. If v and w are strictly positive, return the epigraph distance required to increase u such that the constraint is satisfied.

If any elements in v or w are non-positive, return the 2-norm of the vector d such that in the vector d + d: any non-positive elements in d and d are projected to 1, and d is projected such that the epigraph constraint holds.

source

```
distance_to_set(::ProjectionUpperBoundDistance, x, set::MOI.NormCone)
```

Let (t, y...) = x. Return the epigraph distance d such that (t + d, y...) belongs to the set.

source

```
distance_to_set(
    ::ProjectionUpperBoundDistance,
    x::AbstractVector,
    set::Union{
        MOI.PositiveSemidefiniteConeSquare,
            MOI.PositiveSemidefiniteConeTriangle,
     },
}
```

Let X be x reshaped into the appropriate matrix. The returned distance is  $||X-Y||_2^2$  where Y is the eigen decomposition of X with negative eigen values removed.

source

 ${\tt MathOptInterface.Utilities.set\_dot-Function}.$ 

```
set_dot(x::AbstractVector, y::AbstractVector, set::AbstractVectorSet)
```

Return the scalar product between a vector x of the set set and a vector y of the dual of the set s.

source

```
set_dot(x, y, set::AbstractScalarSet)
```

Return the scalar product between a number x of the set set and a number y of the dual of the set s.

source

#### **DoubleDicts**

MathOptInterface.Utilities.DoubleDicts.DoubleDict - Type.

```
DoubleDict{V}
```

An optimized dictionary to map MOI.ConstraintIndex to values of type V.

Works as a AbstractDict{MOI.ConstraintIndex,V} with minimal differences.

If V is also a MOI.ConstraintIndex, use IndexDoubleDict.

Note that MOI. ConstraintIndex is not a concrete type, opposed to MOI. ConstraintIndex {MOI. VariableIndex, MOI. Integers}, which is a concrete type.

When looping through multiple keys of the same Function-in-Set type, use

```
inner = dict[F, S]
```

to return a type-stable DoubleDictInner.

source

MathOptInterface.Utilities.DoubleDicts.DoubleDictInner - Type.

```
DoubleDictInner{F,S,V}
```

A type stable inner dictionary of DoubleDict.

source

MathOptInterface.Utilities.DoubleDicts.IndexDoubleDict - Type.

```
IndexDoubleDict
```

A specialized version of [DoubleDict] in which the values are of type MOI.ConstraintIndex

When looping through multiple keys of the same Function-in-Set type, use

```
inner = dict[F, S]
```

to return a type-stable IndexDoubleDictInner.

source

MathOptInterface.Utilities.DoubleDicts.IndexDoubleDictInner - Type.

```
IndexDoubleDictInner{F,S}
```

A type stable inner dictionary of IndexDoubleDict.

source

MathOptInterface.Utilities.DoubleDicts.outer\_keys - Function.

```
outer_keys(d::AbstractDoubleDict)
```

Return an iterator over the outer keys of the AbstractDoubleDict d. Each outer key is a Tuple{Type, Type} so that a double loop can be easily used:

```
for (F, S) in DoubleDicts.outer_keys(dict)
  for (k, v) in dict[F, S]
  # ...
  end
end
```

For performance, it is recommended that the inner loop lies in a separate function to guarantee type-stability. Some outer keys (F, S) might lead to an empty dict[F, S]. If you want only nonempty dict[F, S], use nonempty\_outer\_keys.

source

 ${\tt MathOptInterface.Utilities.DoubleDicts.nonempty\_outer\_keys-Function}.$ 

```
nonempty_outer_keys(d::AbstractDoubleDict)
```

Return a vector of outer keys of the AbstractDoubleDict d.

Only outer keys that have a nonempty set of inner keys will be returned.

Each outer key is a Tuple{Type, Type} so that a double loop can be easily used

```
for (F, S) in DoubleDicts.nonempty_outer_keys(dict)
    for (k, v) in dict[F, S]
        # ...
    end
end
For performance, it is recommended that the inner loop lies in a separate
function to guarantee type-stability.

If you want an iterator of all current outer keys, use [`outer_keys`](@ref).
```

source

# **Chapter 31**

## **Test**

## 31.1 Overview

#### The Test submodule

The Test submodule provides tools to help solvers implement unit tests in order to ensure they implement the MathOptInterface API correctly, and to check for solver-correctness.

We use a centralized repository of tests, so that if we find a bug in one solver, instead of adding a test to that particular repository, we add it here so that all solvers can benefit.

## How to test a solver

The skeleton below can be used for the wrapper test file of a solver named FooBar.

```
module TestFooBar
import FooBar
using Test
import MathOptInterface as MOI
const OPTIMIZER = MOI.instantiate(
   MOI.OptimizerWithAttributes(FooBar.Optimizer, MOI.Silent() => true),
const BRIDGED = MOI.instantiate(
   MOI.OptimizerWithAttributes(FooBar.Optimizer, MOI.Silent() => true),
   with_bridge_type = Float64,
# See the docstring of MOI.Test.Config for other arguments.
const CONFIG = MOI.Test.Config(
   # Modify tolerances as necessary.
   atol = 1e-6,
   rtol = 1e-6,
   # Use MOI.LOCALLY SOLVED for local solvers.
   optimal status = MOI.OPTIMAL,
   # Pass attributes or MOI functions to `exclude` to skip tests that
   # rely on this functionality.
```

```
exclude = Any[MOI.VariableName, MOI.delete],
)
    runtests()
This function runs all functions in the this Module starting with `test_`.
function runtests()
    for name in names(@__MODULE__; all = true)
        if startswith("$(name)", "test_")
            @testset "$(name)" begin
                getfield(@__MODULE__, name)()
            end
        end
    end
end
    test_runtests()
This function runs all the tests in MathOptInterface.Test.
Pass arguments to `exclude` to skip tests for functionality that is not
implemented or that your solver doesn't support.
function test_runtests()
    MOI.Test.runtests(
        BRIDGED,
        CONFIG,
        exclude = [
            "test attribute NumberOfThreads",
            "test_quadratic_",
        ],
        # This argument is useful to prevent tests from failing on future
        # releases of MOI that add new tests. Don't let this number get too far
        # behind the current MOI release though. You should periodically check
        # for new tests to fix bugs and implement new features.
        exclude_tests_after = v"0.10.5",
    )
    return
end
    test SolverName()
You can also write new tests for solver-specific functionality. Write each new
test as a function with a name beginning with `test_`.
function test_SolverName()
    @test MOI.get(FooBar.Optimizer(), MOI.SolverName()) == "FooBar"
    return
end
end # module TestFooBar
```

```
# This line at the end of the file runs all the tests!
TestFooBar.runtests()
```

Then modify your runtests.jl file to include the MOI\_wrapper.jl file:

#### Info

The optimizer BRIDGED constructed with instantiate automatically bridges constraints that are not supported by OPTIMIZER using the bridges listed in Bridges. It is recommended for an implementation of MOI to only support constraints that are natively supported by the solver and let bridges transform the constraint to the appropriate form. For this reason it is expected that tests may not pass if OPTIMIZER is used instead of BRIDGED.

## How to debug a failing test

When writing a solver, it's likely that you will initially fail many tests. Some failures will be bugs, but other failures you may choose to exclude.

There are two ways to exclude tests:

• Exclude tests whose names contain a string using:

```
MOI.Test.runtests(
    model,
    config;
    exclude = String["test_to_exclude", "test_conic_"],
)
```

This will exclude tests whose name contains either of the two strings provided.

• Exclude tests which rely on specific functionality using:

```
MOI.Test.Config(exclude = Any[MOI.VariableName, MOI.optimize!])
```

This will exclude tests which use the MOI.VariableName attribute, or which call MOI.optimize!.

Each test that fails can be independently called as:

```
model = FooBar.Optimizer()
config = MOI.Test.Config()
MOI.empty!(model)
MOI.Test.test_category_name_that_failed(model, config)
```

You can look-up the source code of the test that failed by searching for it in the src/Test/test\_category.jl file.

```
Tip

Each test function also has a docstring that explains what the test is for. Use ?

MOI.Test.test_category_name_that_failed from the REPL to read it.
```

Periodically, you should re-run excluded tests to see if they now pass. The easiest way to do this is to swap the exclude keyword argument of runtests to include. For example:

```
MOI.Test.runtests(
    model,
    config;
    exclude = String["test_to_exclude", "test_conic_"],
)
```

becomes

```
MOI.Test.runtests(
    model,
    config;
    include = String["test_to_exclude", "test_conic_"],
)
```

#### How to add a test

To detect bugs in solvers, we add new tests to MOI.Test.

As an example, ECOS errored calling optimize! twice in a row. (See ECOS.jl PR #72.) We could add a test to ECOS.jl, but that would only stop us from re-introducing the bug to ECOS.jl in the future, but it would not catch other solvers in the ecosystem with the same bug. Instead, if we add a test to MOI.Test, then all solvers will also check that they handle a double optimize call.

For this test, we care about correctness, rather than performance. therefore, we don't expect solvers to efficiently decide that they have already solved the problem, only that calling optimize! twice doesn't throw an error or give the wrong answer.

## Step 1

Install the MathOptInterface julia package in dev mode:

```
julia> ]
(@v1.6) pkg> dev MathOptInterface
```

From here on, proceed with making the following changes in the ~/.julia/dev/MathOptInterface folder (or equivalent dev path on your machine).

#### Step 3

Since the double-optimize error involves solving an optimization problem, add a new test to src/Test/test\_solve.jl:

```
test_unit_optimize!_twice(model::MOI.ModelLike, config::Config)
Test that calling `MOI.optimize!` twice does not error.
This problem was first detected in ECOS.jl PR#72:
https://github.com/jump-dev/ECOS.jl/pull/72
function test_unit_optimize!_twice(
   model::MOI.ModelLike,
   config::Config{T},
) where {T}
   # Use the `@requires` macro to check conditions that the test function
   # requires to run. Models failing this `@requires` check will silently skip
   # the test.
   @requires MOI.supports_constraint(
       model,
       MOI.VariableIndex,
       MOI.GreaterThan{Float64},
   @requires _supports(config, MOI.optimize!)
   # If needed, you can test that the model is empty at the start of the test.
   # You can assume that this will be the case for tests run via `runtests`.
   # User's calling tests individually need to call `MOI.empty!` themselves.
   @test MOI.is empty(model)
   # Create a simple model. Try to make this as simple as possible so that the
   # majority of solvers can run the test.
   x = MOI.add_variable(model)
   {\tt MOI.add\_constraint(model,\ x,\ MOI.GreaterThan(one(T)))}
   MOI.set(model, MOI.ObjectiveSense(), MOI.MIN_SENSE)
   MOI.set(
       MOI.ObjectiveFunction{MOI.VariableIndex}(),
   )
   # The main component of the test: does calling `optimize!` twice error?
   MOI.optimize!(model)
   MOI.optimize!(model)
   # Check we have a solution.
   @test MOI.get(model, MOI.TerminationStatus()) == MOI.OPTIMAL
   # There is a three-argument version of `Base.isapprox` for checking
   # approximate equality based on the tolerances defined in `config`:
   @test isapprox(MOI.get(model, MOI.VariablePrimal(), x), one(T), config)
   # For code-style, these tests should always `return` `nothing`.
    return
end
```

#### Info

Make sure the function is agnostic to the number type T; don't assume it is a Float64 capable solver.

We also need to write a test for the test. Place this function immediately below the test you just wrote in the same file:

```
function setup_test(
    ::typeof(test_unit_optimize!_twice),
    model::MOI.Utilities.MockOptimizer,
    ::Config,
)

MOI.Utilities.set_mock_optimize!(
    model,
    (mock::MOI.Utilities.MockOptimizer) -> MOIU.mock_optimize!(
    mock,
    MOI.OPTIMAL,
    (MOI.FEASIBLE_POINT, [1.0]),
    ),
    )
    return
end
```

Finally, you also need to implement Test.version\_added. If we added this test when the latest released version of MOI was v0.10.5, define:

```
version_added(::typeof(test_unit_optimize!_twice)) = v"0.10.6"
```

## Step 6

Commit the changes to git from ~/.julia/dev/MathOptInterface and submit the PR for review.

## Tip

If you need help writing a test, open an issue on GitHub, or ask the Developer Chatroom.

#### 31.2 API Reference

## The Test submodule

Functions to help test implementations of MOI. See The Test submodule for more details.

MathOptInterface.Test.Config - Type.

```
Config(
    ::Type{T} = Float64;
    atol::Real = Base.rtoldefault(T),
    rtol::Real = Base.rtoldefault(T),
    optimal_status::MOI.TerminationStatusCode = MOI.OPTIMAL,
    infeasible_status::MOI.TerminationStatusCode = MOI.INFEASIBLE,
    exclude::Vector{Any} = Any[],
) where {T}
```

Return an object that is used to configure various tests.

## **Configuration arguments**

- atol::Real = Base.rtoldefault(T): Control the absolute tolerance used when comparing solutions.
- rtol::Real = Base.rtoldefault(T): Control the relative tolerance used when comparing solutions.
- optimal\_status = MOI.OPTIMAL: Set to MOI.LOCALLY\_SOLVED if the solver cannot prove global optimality.
- infeasible\_status = MOI.INFEASIBLE: Set to MOI.LOCALLY\_INFEASIBLE if the solver cannot prove global infeasibility.
- exclude = Vector{Any}: Pass attributes or functions to exclude to skip parts of tests that require certain functionality. Common arguments include:
  - MOI.delete to skip deletion-related tests
  - MOI.optimize! to skip optimize-related tests
  - MOI.ConstraintDual to skip dual-related tests
  - MOI. VariableName to skip setting variable names
  - MOI.ConstraintName to skip setting constraint names

#### **Example**

For a nonlinear solver that finds local optima and does not support finding dual variables or constraint names:

source

MathOptInterface.Test.runtests - Function.

```
runtests(
   model::MOI.ModelLike,
   config::Config;
   include::Vector{Union{String,Regex}} = String[],
   exclude::Vector{Union{String,Regex}} = String[],
   warn_unsupported::Bool = false,
   exclude_tests_after::VersionNumber = v"999.0.0",
   verbose::Bool = false,
   test_module = MathOptInterface.Test,
)
```

Run all tests in test\_module, which defaults to MathOptInterface.Test, on model.

## **Configuration arguments**

- config is a Test.Config object that can be used to modify the behavior of tests.
- If include is not empty, only run tests if an element from include occursin the name of the test.
- · If exclude is not empty, skip tests if an element from exclude occursin the name of the test.
- exclude takes priority over include.
- If warn\_unsupported is false, runtests will silently skip tests that fail with a MOI.NotAllowedError,
  MOI.UnsupportedError, or RequirementUnmet error. (The latter is thrown when an @requires statement returns false.) When warn\_unsupported is true, a warning will be printed. For most cases
  the default behavior, false, is what you want, since these tests likely test functionality that is not
  supported by model. However, it can be useful to run warn\_unsupported = true to check you are
  not skipping tests due to a missing supports\_constraint method or equivalent.
- exclude\_tests\_after is a version number that excludes any tests to MOI added after that version number. This is useful for solvers who can declare a fixed set of tests, and not cause their tests to break if a new patch of MOI is released with a new test.
- verbose is a Bool that controls whether the name of the test is printed before executing it. This can be helpful when debugging.
- test module is a Module where all the functions starting with test are considered as tests.

See also: setup\_test.

## **Example**

```
config = MathOptInterface.Test.Config()
MathOptInterface.Test.runtests(
    model,
    config;
    include = ["test_linear_", r"^test_model_Name$"],
    exclude = ["VariablePrimalStart"],
    warn_unsupported = true,
    verbose = true,
    exclude_tests_after = v"0.10.5",
)
```

source

MathOptInterface.Test.setup\_test - Function.

```
setup_test(::typeof(f), model::MOI.ModelLike, config::Config)
```

Overload this method to modify model before running the test function f on model with config. You can also modify the fields in config (for example, to loosen the default tolerances).

This function should either return nothing, or return a function which, when called with zero arguments, undoes the setup to return the model to its previous state. You do not need to undo any modifications to config.

This function is most useful when writing new tests of the tests for MOI, but it can also be used to set test-specific tolerances, etc.

See also: runtests

## **Example**

```
function MOI.Test.setup_test(
    ::typeof(MOI.Test.test_linear_VariablePrimalStart_partial),
    mock::MOIU.MockOptimizer,
    ::MOI.Test.Config,
)

MOIU.set_mock_optimize!(
    mock,
    (mock::MOIU.MockOptimizer) -> MOIU.mock_optimize!(mock, [1.0, 0.0]),
)
mock.eval_variable_constraint_dual = false

function reset_function()
    mock.eval_variable_constraint_dual = true
    return
end
return reset_function
end
```

source

MathOptInterface.Test.version\_added - Function.

```
version_added(::typeof(function_name))
```

Returns the version of MOI in which the test function\_name was added.

This method should be implemented for all new tests.

See the exclude\_tests\_after keyword of runtests for more details.

source

MathOptInterface.Test.@requires - Macro.

```
@requires(x)
```

Check that the condition x is true. Otherwise, throw an RequirementUnmet error to indicate that the model does not support something required by the test function.

## **Example**

```
@requires MOI.supports(model, MOI.Silent())
@test MOI.get(model, MOI.Silent())
```

source

MathOptInterface.Test.RequirementUnmet - Type.

```
RequirementUnmet(msg::String) <: Exception
```

An error for throwing in tests to indicate that the model does not support some requirement expected by the test function.

source

MathOptInterface.Test.HS071 - Type.

```
HS071(
    enable_hessian::Bool,
    enable_hessian_vector_product::Bool = false,
)
```

An MOI. AbstractNLPEvaluator for the problem:

$$\begin{aligned} \min x_1 * x_4 * (x_1 + x_2 + x_3) + x_3 \\ \text{subject to } x_1 * x_2 * x_3 * x_4 &\geq 25 \\ x_1^2 + x_2^2 + x_3^2 + x_4^2 &= 40 \\ 1 &\leq x_1, x_2, x_3, x_4 \leq 5 \end{aligned}$$

The optimal solution is [1.000, 4.743, 3.821, 1.379].

source

# Part VII

# **Developer Docs**

## **Chapter 32**

# **Checklists**

The purpose of this page is to collate a series of checklists for commonly performed changes to the source code of MathOptInterface.

In each case, copy the checklist into the description of the pull request.

## 32.1 Making a release

Use this checklist when making a release of the MathOptInterface repository.

## 32.2 Adding a new set

Use this checklist when adding a new set to the MathOptInterface repository.

```
## Basic

- [ ] Add a new `AbstractScalarSet` or `AbstractVectorSet` to `src/sets.jl`
- [ ] If `isbitstype(S) == false`, implement `Base.copy(set::S)`
- [ ] If `isbitstype(S) == false`, implement `Base.:(==)(x::S, y::S)`
- [ ] If an `AbstractVectorSet`, implement `dimension(set::S)`, unless the
```

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```
dimension is given by `set.dimension`.
## Utilities
 - [ ] If an `AbstractVectorSet`, implement `Utilities.set_dot`,
       unless the dot product between two vectors in the set is equivalent to
       `LinearAlgebra.dot`
 - [ ] If an `AbstractVectorSet`, implement `Utilities.set_with_dimension` in
       `src/Utilities/matrix_of_constraints.jl`
 - [ ] Add the set to the `@model` macro at the bottom of `src/Utilities.model.jl`
## Documentation
 - [ ] Add a docstring, which gives the mathematical definition of the set,
       along with an `## Example` block containing a `jldoctest`
 - [ ] Add the docstring to `docs/src/reference/standard_form.md`
 - [ ] Add the set to the relevant table in `docs/src/manual/standard form.md`
## Tests
- [ ] Define a new `_set(::Type{S})` method in `src/Test/test_basic_constraint.jl`
       and add the name of the set to the list at the bottom of that files
 - [ ] If the set has any checks in its constructor, add tests to `test/sets.jl`
## MathOptFormat
 - [ ] Open an issue at `https://github.com/jump-dev/MathOptFormat` to add
       support for the new set {{ replace with link to the issue }}
## Optional
- [ ] Implement `dual_set(::S)` and `dual_set_type(::Type{S})`
- [ ] Add new tests to the `Test` submodule exercising your new set
- [ ] Add new bridges to convert your set into more commonly used sets
```

## 32.3 Adding a new bridge

Use this checklist when adding a new bridge to the MathOptInterface repository.

The steps are mostly the same, but locations depend on whether the bridge is a Constraint, Objective, or Variable bridge. In each case below, replace XXX with the appropriate type of bridge.

```
## Basic
[ ] Create a new file in `src/Bridges/XXX/bridges` named after the type of the bridge
[ ] Define the bridge, following existing examples. The name of the bridge struct must end in `Bridge`
[ ] Check if your bridge can be a subtype of [`MOI.Bridges.Constraint.SetMapBridge`](@ref)
[ ] Define a new `const` that is a `SingleBridgeOptimizer` wrapping the new bridge. The name of the const must be the name of the bridge, less the `Bridge` suffix
[ ] If the bridge should be enabled by default, add the bridge to `add_all_bridges` at the bottom of `src/Bridges/XXX/XXX.jl`
```

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## 32.4 Updating MathOptFormat

Use this checklist when updating the version of MathOptFormat.