Tutorial 3: Solutions to Assignment 1

January 24, 2020

Adam Denny
a.denny15@imperial.ac.uk

Today we will cover...

- Recap of lecture 3
- Solutions to assignment 1

Recap of lecture 3: Evidence on time series predictability

- Fama and French (1989): dividend yield (D/P), default spread (Baa − Aaa yield), and the term spread (long-term minus short-term yields) can forecast excess stock and bond returns over long horizons (2–4 years).
  - Criticism: this result is mechanical due to the fact that these variables are highly persistent (autocorrelated).
- Ang and Bekaert (2007): when correcting for these statistical issues, there is some evidence of predictability up to 5 years ahead. Short-run dividend yield predictability improves significantly when controlling for the level of interest rates.

Recap of lecture 3: Campbell–Shiller decomposition

- Very important equation: (log) price-dividend ratios are a function of expected dividend growth and expected returns:

$$
\begin{aligned}
p_t - d_t &\approx \text{const.} + \mathbb{E}_t \left[\sum_{j=1}^{\infty} \rho^{j-1} \left(\Delta d_{t+j} - r_{t+j} \right) \right] && (1) \\
&\approx \text{const.} + \sum_{j=1}^{\infty} \rho^{j-1} \mathbb{E}_t \left(\Delta d_{t+j} \right) - \sum_{j=1}^{\infty} \rho^{j-1} \mathbb{E}_t \left(r_{t+j} \right) && (2)
\end{aligned}
$$

- If dividend growth and returns are unpredictable then $\mathbb{E}_t(\Delta d_{t+j}) = \mathbb{E}(\Delta d_{t+j})$ & $\mathbb{E}_t(r_{t+j}) = \mathbb{E}(r_{t+j})$ = constant over time, meaning $p_t - d_t$ is constant (which we know is not true)

- Therefore, the price-dividend ratio must forecast high future dividend growth, low future expected returns, or both

- Cochrane (2008): dividend growth is not predictable, therefore expected returns must be predictable (but he doesn't tell us which variables to use)!

Practice MCQ

1. Look at this table. What can we conclude?

Table 1
Forecasting regressions

Regression	b	t	$R^2(\%)$	$\sigma(bx)(\%)$
$R_{t+1} = a + b(D_t/P_t) + \varepsilon_{t+1}$	3.39	2.28	5.8	4.9
$R_{t+1} - R_t^f = a + b(D_t/P_t) + \varepsilon_{t+1}$	3.83	2.61	7.4	5.6
$D_{t+1}/D_t = a + b(D_t/P_t) + \varepsilon_{t+1}$	0.07	0.06	0.0001	0.001

(a) Total returns, excess returns, and dividend growth can be predicted using dividend yields
(b) Total returns and dividend growth can be predicted using dividend yields, excess returns cannot
(c) Total returns and excess returns can be predicted using dividend yields, dividend growth cannot
(d) The dividend yield cannot predict total returns, excess returns, or dividend growth

Practice MCQ

1. Look at this table. What can we conclude?

Table 1
Forecasting regressions

Regression	b	t	R^2(%)	$\sigma(bx)$(%)
$R_{t+1} = a + b(D_t/P_t) + \varepsilon_{t+1}$	3.39	2.28	5.8	4.9
$R_{t+1} - R_t^f = a + b(D_t/P_t) + \varepsilon_{t+1}$	3.83	2.61	7.4	5.6
$D_{t+1}/D_t = a + b(D_t/P_t) + \varepsilon_{t+1}$	0.07	0.06	0.0001	0.001

(c) Total returns and excess returns can be predicted using dividend yields, dividend growth cannot

See the first 3 columns:

- R_{t+1} is the total return. The slope (b) has a t-statistic of 2.28
- $R_{t+1} - R_t^f$ is the excess return. The slope has a t-statistic of 2.61
- D_{t+1}/D_t is the one-period dividend growth. The slope is close to zero with t-statistic of only 0.06

Assignment 1

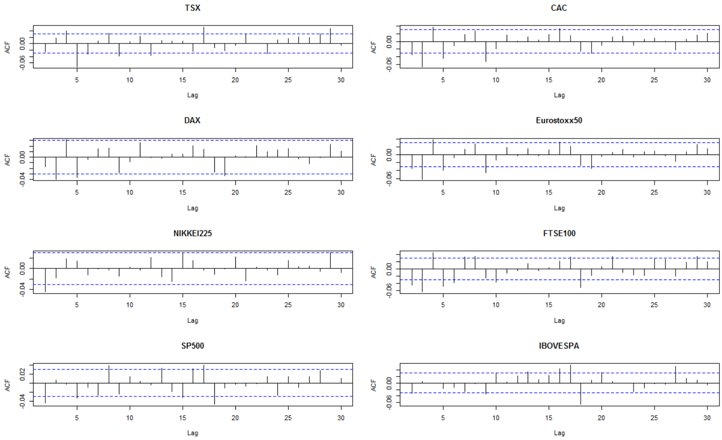
Question 1: Summary statistics

	Mean*	Std. dev.*	Skewness	Kurtosis
TSX	2.56	22.84	−0.72	7.83
CAC	−1.31	26.15	0.02	5.46
DAX	2.75	26.47	−0.01	4.86
Eurostoxx 50	−2.26	26.48	−0.01	4.87
NIKKEI 225	−1.20	24.67	−0.31	5.74
FTSE 100	−1.47	21.74	−0.14	7.07
S&P 500	1.89	19.73	−0.18	8.42
IBOVESPA	0.87	39.39	−0.15	5.30

* In annualised percent.

- Some means are negative. Intuitively, can stock indices have negative expected returns?
- Most index returns are negatively skewed and have high kurtosis relative to a normal distribution (fat tails).

Question 1: Autocorrelations



- Some autocorrelations are significant, but all are small in magnitude (< 0.10). Make sure to check autocorrelations beyond lag 1!

Question 1: Correlation matrix

	TSX	CAC	DAX	Eurostoxx	NIKKEI	FTSE	S&P	IBOV
TSX	1.00	0.61	0.61	0.61	0.13	0.63	0.74	0.60
CAC	0.61	1.00	0.91	0.98	0.21	0.87	0.55	0.48
DAX	0.61	0.91	1.00	0.95	0.19	0.82	0.59	0.50
Eurostoxx	0.61	0.98	0.95	1.00	0.20	0.87	0.56	0.49
NIKKEI	0.13	0.21	0.19	0.20	1.00	0.23	0.00	0.06
FTSE	0.63	0.87	0.82	0.87	0.23	1.00	0.54	0.50
S&P	0.74	0.55	0.59	0.56	0.00	0.54	1.00	0.59
IBOV	0.60	0.48	0.50	0.49	0.06	0.50	0.59	1.00

- Correlations are all positive, and typically high in magnitude.
- Exceptions are NIKKEI and IBOVESPA.

Question 1: Are the index returns iid normal?

- Informal tests:
  - Negative skewness and high kurtosis relative to a normal distribution suggest not.
  - QQ plots, kernel densities, histograms all highlight the deviations from normality in the tails.
- Formal (statistical) tests:
  - Jarque-Bera, Kolmogorov-Smirnov, Lilliefors, ...
  - These tests easily reject the null hypothesis that the returns are normally distributed.
- What would be an alternative distribution to model stock returns?

Question 2: Optimal portfolio weights

Recall the equations to calculate the **unconstrained** tangency and minimum variance portfolios:

$$w_T = \frac{\Sigma^{-1}[E(r) - r_f]}{i'\Sigma^{-1}[E(r) - r_f]} \tag{3}$$

$$w_{MV} = \frac{\Sigma^{-1} i}{i'\Sigma^{-1} i} \tag{4}$$

- Σ is the $N \times N$ covariance matrix
- $E(r) - r_f$ is an $N \times 1$ vector of expected excess returns
- i is an $N \times 1$ vector of ones

Question 2: Optimal portfolio weights

- The short-selling constrained portfolio weights can be computed in two ways (they should both agree):

  1. Solve directly using a numerical optimiser like auglag or solve.QP (from the quadprog package).

  2. Take from the efficient frontier by finding the Sharpe ratio and standard deviation at each point on the efficient frontier.

Question 2: Optimal portfolio weights

	w_T	w_{MV}	w_T (no SS)	w_{MV} (no SS)
TSX	5.225	0.003	0.549	0.000
CAC	38.132	0.011	0.000	0.000
DAX	41.533	−0.038	0.360	0.000
Eurostoxx 50	−76.907	−0.185	0.000	0.000
NIKKEI 225	−1.400	0.329	0.000	0.347
FTSE 100	−3.013	0.380	0.000	0.152
S&P 500	−1.446	0.584	0.090	0.501
IBOVESPA	−1.123	−0.083	0.000	0.000

- Notice the extreme the tangency portfolio weights are when we allow short selling.

Question 2: Calculating the unconstrained efficient frontier

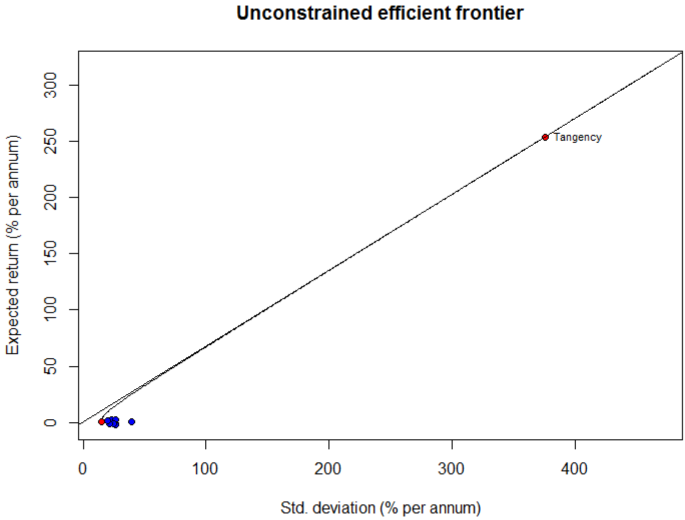
- Most groups successfully constructed the efficient frontier by solving the quadratic programming problem:

$$\begin{aligned} \min_{w} \quad & w'\Sigma w \\ \text{s.t.} \quad & i'w = 1 \\ & w'\mu = \bar{\mu} \end{aligned}$$

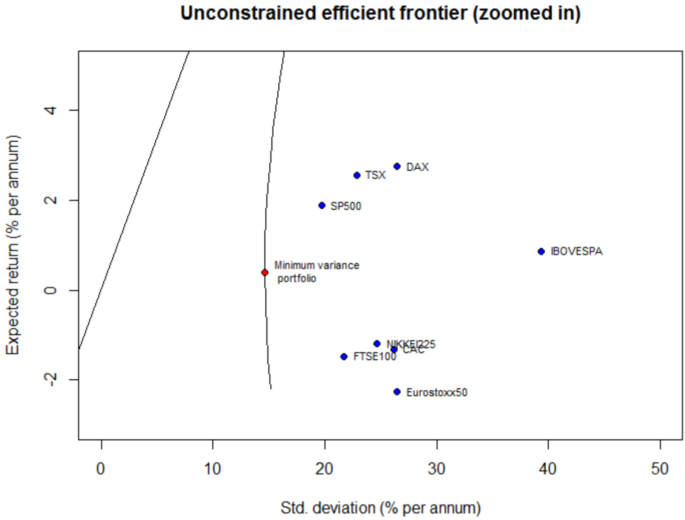
  where w is a $n \times 1$ vector of weights, Σ is an $n \times n$ covariance matrix, i is an $n \times 1$ vector of ones, μ is an $n \times 1$ vector of expected index returns, and $\bar{\mu}$ is a target level of return.

- The idea is to trace out the efficient frontier by finding the portfolio that minimises variance for a range of target expected returns.

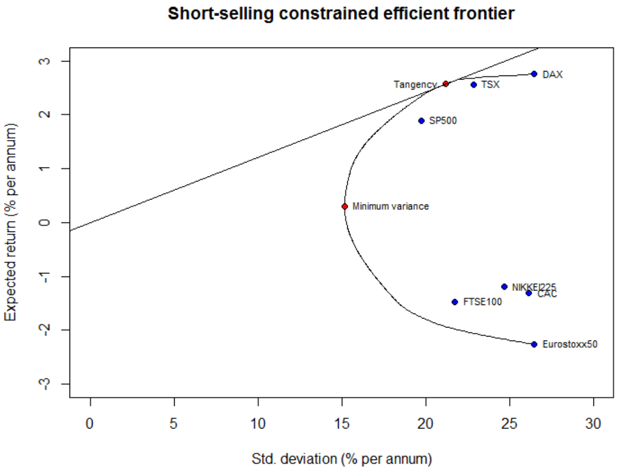
Question 2: Unconstrained efficient frontier



Question 2: Unconstrained efficient frontier - zoomed in

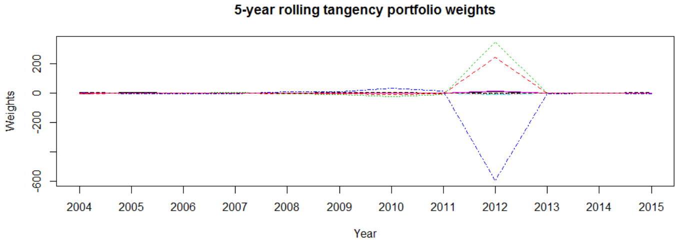


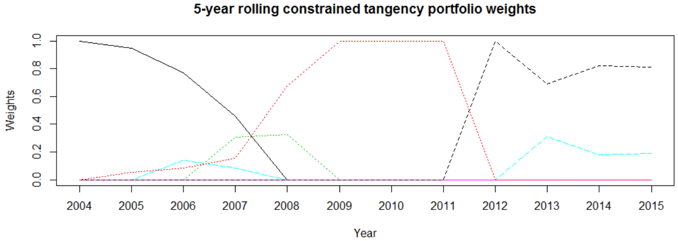
Question 2: Constrained efficient frontier



- Why does the efficient frontier line end at the DAX?

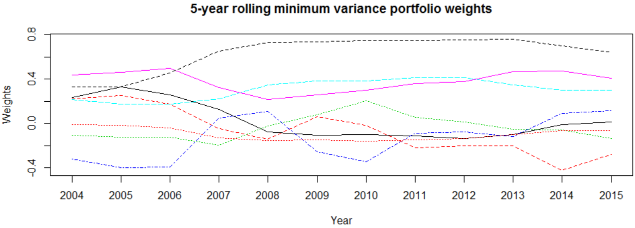
Question 3: Rolling tangency portfolio weights

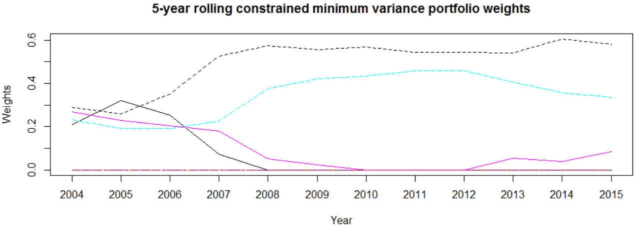




- Weights are very unstable, even if we restrict short selling.

Question 3: Rolling minimum variance portfolio weights





- Weights are much smoother than for the tangency portfolio, particularly after 2008.

Question 3: Mean-variance vs. market cap weights

	w_T	w_T (no SS)	w_{MC}
TSX	5.225	0.549	0.042
CAC	38.132	0.000	0.043
DAX	41.533	0.360	0.036
Eurostoxx 50	−76.907	0.000	0.091
NIKKEI 225	−1.400	0.000	0.093
FTSE 100	−3.013	0.000	0.082
S&P 500	−1.446	0.090	0.602
IBOVESPA	−1.123	0.000	0.011

- The three different methods of portfolio construction differ greatly.

Questions?

If anything comes up later, ask on the Hub or during office hour.