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Problem Set 9: Differential Equation

Calculus for Computer Science

These are selected problem for practice. Please hand-in the bold problems.

Chapter 9.1

1-5

6, 7, 8, **12**

Chapter 9.2

3-6

9, 10

25 (all)

26

Chapter 9.3

1, **2**, 6, 7, 10

13, 14, **15**, 19, 20

21,

22,

9.1 Exercises

1–5 Write a differential equation that models the given situation. In each case the stated rate of change is with respect to time t.

1. The rate of change of the radius r of a tree trunk is inversely proportional to the radius.
2. The rate of change of the velocity v of a falling body is constant.
3. For a car with maximum velocity M, the rate of change of the velocity v of the car is proportional to the difference between M and v.
4. When an infectious disease is introduced into a city of fixed population N, the rate of change of the number y of infected individuals is proportional to the product of the number of infected individuals and the number of noninfected individuals.
5. When an advertising campaign for a new product is introduced into a city of fixed population N, the rate of change of the number y of individuals who have heard about the product at time t is proportional to the number of individuals in the population who have not yet heard about the product.

1) $\frac{dr}{dt} = cr^{-1} = \frac{c}{r}$ เมื่อ C เป็นค่าคงที่

2) $\frac{dr}{dt} = c$

3) $\frac{dv}{dt} = c\,(M-v)$

4) $\frac{dv}{dt} = c\,(y)\,(N-y)$

5) $\frac{dy}{dt} = c\,(N-y)$

12. $y = \ln x; \quad xy'' - y' = 0$

$y' = \frac{1}{x}$, $y'' = -\frac{1}{x^2}$

$\therefore x\left(-\frac{1}{x^2}\right) - \left(\frac{1}{x}\right) = 0$

$-\frac{1}{x} - \frac{1}{x} = 0$

$-\frac{2}{x} = 0 \rightarrow$ เป็นเท็จ

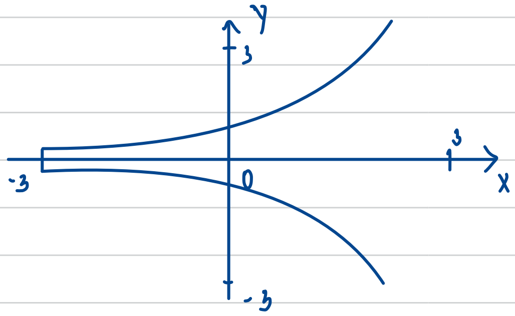
$y = \ln x$ ไม่ใช่คำตอบของ $xy'' - y' = 0$ #

9. $y' = \frac{1}{2}y$

x	y	$y' = \frac{1}{2}y$
0	0	0
0	1	0.5
0	2	1
0	-3	-1.5
0	-2	-1

$y = 0$, $y' = 0$ sketched go through

$(0,0)$, $(0,1)$, $(0,-1)$



25. (a) Program a calculator or computer to use Euler's method to compute $y(1)$, where $y(x)$ is the solution of the initial-value problem

$$\frac{dy}{dx} + 3x^2y = 6x^2 \qquad y(0) = 3$$

(i) $h = 1$ (ii) $h = 0.1$
(iii) $h = 0.01$ (iv) $h = 0.001$

(b) Verify that $y = 2 + e^{-x^3}$ is the exact solution of the differential equation.

(c) Find the errors in using Euler's method to compute $y(1)$ with the step sizes in part (a). What happens to the error when the step size is divided by 10?

a) Write a program

b) $y = 2 + e^{-x^3} \rightarrow y' = -3x^2 e^{-x^3}$

$LHS = y' + 3x^2 y = -3x^2 e^{-x^3} + 3x^2(2 + e^{-x^3}) = -3x^2 e^{-x^3} + 6x^2 + 3x^2 e^{-x^3} = 6x^2 = RHS$

$y(0) = 2 + e^{-0} = 2 + 1 = 3$

c) The exact value of $y(1)$ is $2 + e^{-1^3} = 2 + e^{-1}$

(i) For $h = 1$: (ex value) − (approximate value) $= 2 + e^{-1} - 3 \approx -0.6321$

(ii) For $h = 0.1$: (ex value) − (approximate value) $= 2 + e^{-1} - 2.3929 \approx -0.0249$

(iii) For $h = 0.01$: (ex value) − (approximate value) $= 2 + e^{-1} - 2.3701 \approx -0.0022$

(iv) For $h = 0.001$: (ex value) − (approximate value) $= 2 + e^{-1} - 2.3681 \approx -0.0002$

In (ii) − (iv), it seems that when the step size is divided by 10, the error estimate is also divided by 10

2. $\dfrac{dy}{dx} = \dfrac{x}{y^4}$

(สมการแบบแยกตัวแปรได้)

$$y^4\,dy = x\,dx$$

$$\int y^4\,dy = \int x\,dx$$

$$\frac{y^5}{5} = \frac{x^2}{2} + c$$

$$y^5 = \frac{5x^2}{2} + \underbrace{5c}_{\to C}$$

$$y = \sqrt[5]{\frac{5x^2}{2} + C}$$

15. $\overset{\to A'}{\dfrac{dA}{dr}} = Ab^2 \cos br, \quad A(0) = b^3 \longrightarrow r = 0$

$A = b^3$

$$\frac{1}{A}\,dA = b^2 \cos(br)\,dr$$

$$\int \frac{1}{A}\,dA = \int b^2 \cos(br)\,dr$$

$$\ln|A| = b^2 \sin(br) + c$$

$$\left.\begin{aligned} \ln|b^3| &= b^2 \sin(0) + c \\ \ln|b^3| &= c \end{aligned}\right\} \text{หาค่า } c$$

$$\ln|A| = b^2 \sin(br) + \ln|b^3|$$

$$A = \pm e^{b^2 \sin(br) + \ln|b^3|}$$