

$$\text{设 } \mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N)^T = \begin{pmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \vdots \\ \mathbf{x}_N^T \end{pmatrix}$$

则:

$$\sum_{i=1}^N \mathbf{x}_i \mathbf{x}_i^T = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_N) \begin{pmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \vdots \\ \mathbf{x}_N^T \end{pmatrix} = \mathbf{X}^T \mathbf{X}$$

$$\sum_{i=1}^N \mathbf{x}_i^T \mathbf{x}_i = (\mathbf{x}_1^T, \mathbf{x}_2^T, \dots, \mathbf{x}_N^T) \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \vdots \\ \mathbf{x}_N \end{pmatrix} = \text{trace}(\mathbf{X}^T \mathbf{X})$$

$$f(\mathbf{x}) = \mathbf{x}^T \mathbf{A} \mathbf{x} \quad \rightarrow \quad f'(\mathbf{x}) = \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = (\mathbf{A} + \mathbf{A}^T) \mathbf{x}$$

$$f(\mathbf{x}) = \mathbf{A} \mathbf{x} \quad \rightarrow \quad f'(\mathbf{x}) = \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \mathbf{A}^T$$

$$f(\mathbf{x}) = \mathbf{x}^T \mathbf{A} \quad \rightarrow \quad f'(\mathbf{x}) = \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} = \mathbf{A}$$

$$f(\mathbf{x}) = C \quad \rightarrow \quad f'(\mathbf{x}) = 0$$

$$f(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T \mathbf{A}^T \mathbf{y} \quad \rightarrow \quad f'(\mathbf{x}, \mathbf{y}) = \frac{\partial f}{\partial \mathbf{x}} = \mathbf{A}^T \mathbf{y}$$

$$f(\mathbf{x}, \mathbf{y}) = \mathbf{y}^T \mathbf{A} \mathbf{x} \quad \rightarrow \quad f'(\mathbf{x}, \mathbf{y}) = \frac{\partial f}{\partial \mathbf{x}} = \mathbf{A}^T \mathbf{y}$$

$$F(\mathbf{x}) = f(\mathbf{x})g(\mathbf{x}) \quad \rightarrow \quad F'(\mathbf{x}) = \frac{\partial [f(\mathbf{x})g(\mathbf{x})]}{\partial \mathbf{x}} = g(\mathbf{x}) \frac{\partial f(\mathbf{x})}{\partial \mathbf{x}} + f(\mathbf{x}) \frac{\partial g(\mathbf{x})}{\partial \mathbf{x}}$$

$$F(\mathbf{x}) = f(g(\mathbf{x})) \quad \rightarrow \quad F'(\mathbf{x}) = \frac{\partial [f(g(\mathbf{x}))]}{\partial \mathbf{x}} = \frac{\partial g^T}{\partial \mathbf{x}} \frac{\partial f}{\partial g}$$