

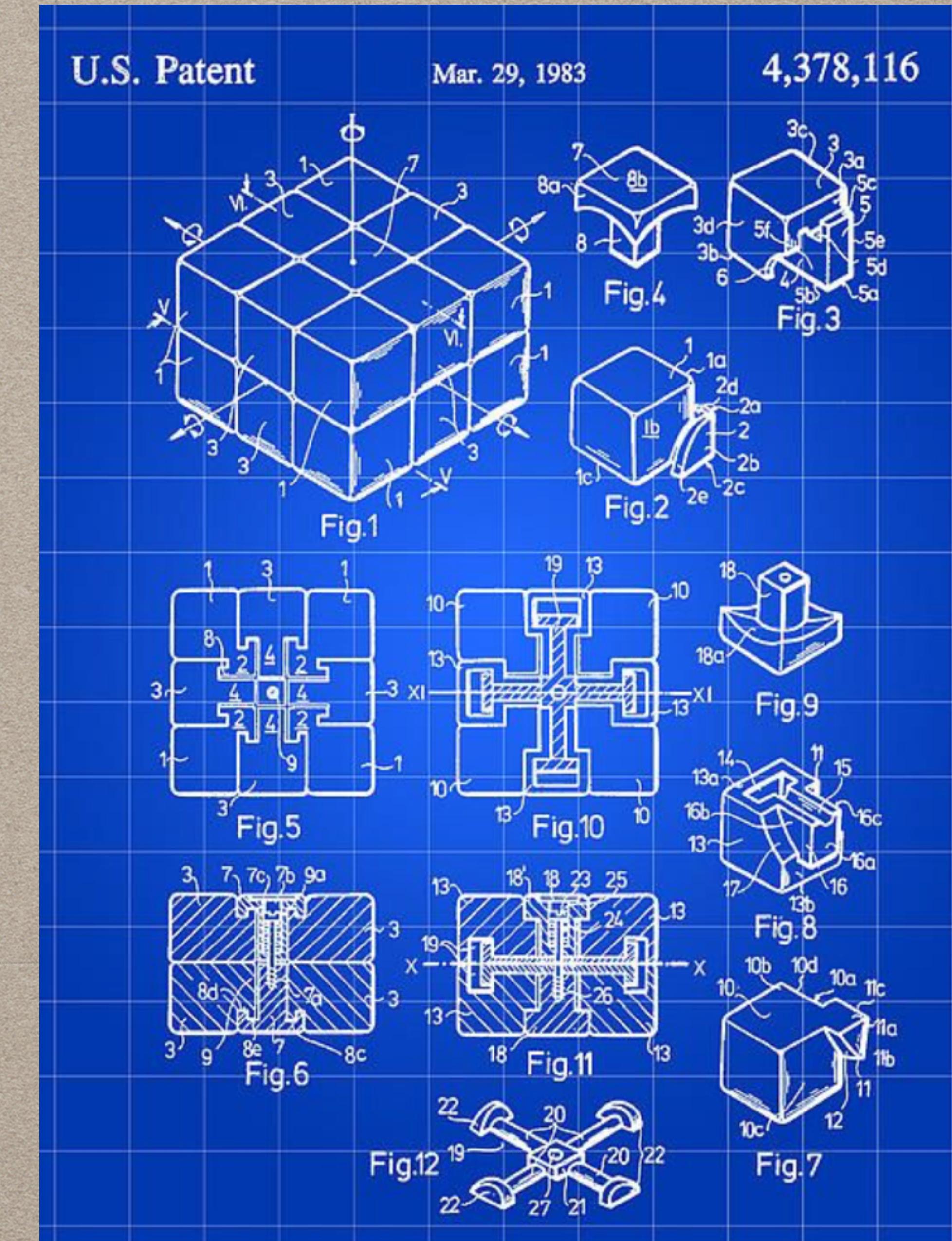
# **IP SOLVER FOR 2\*2\*2 RUBIK'S CUBE**

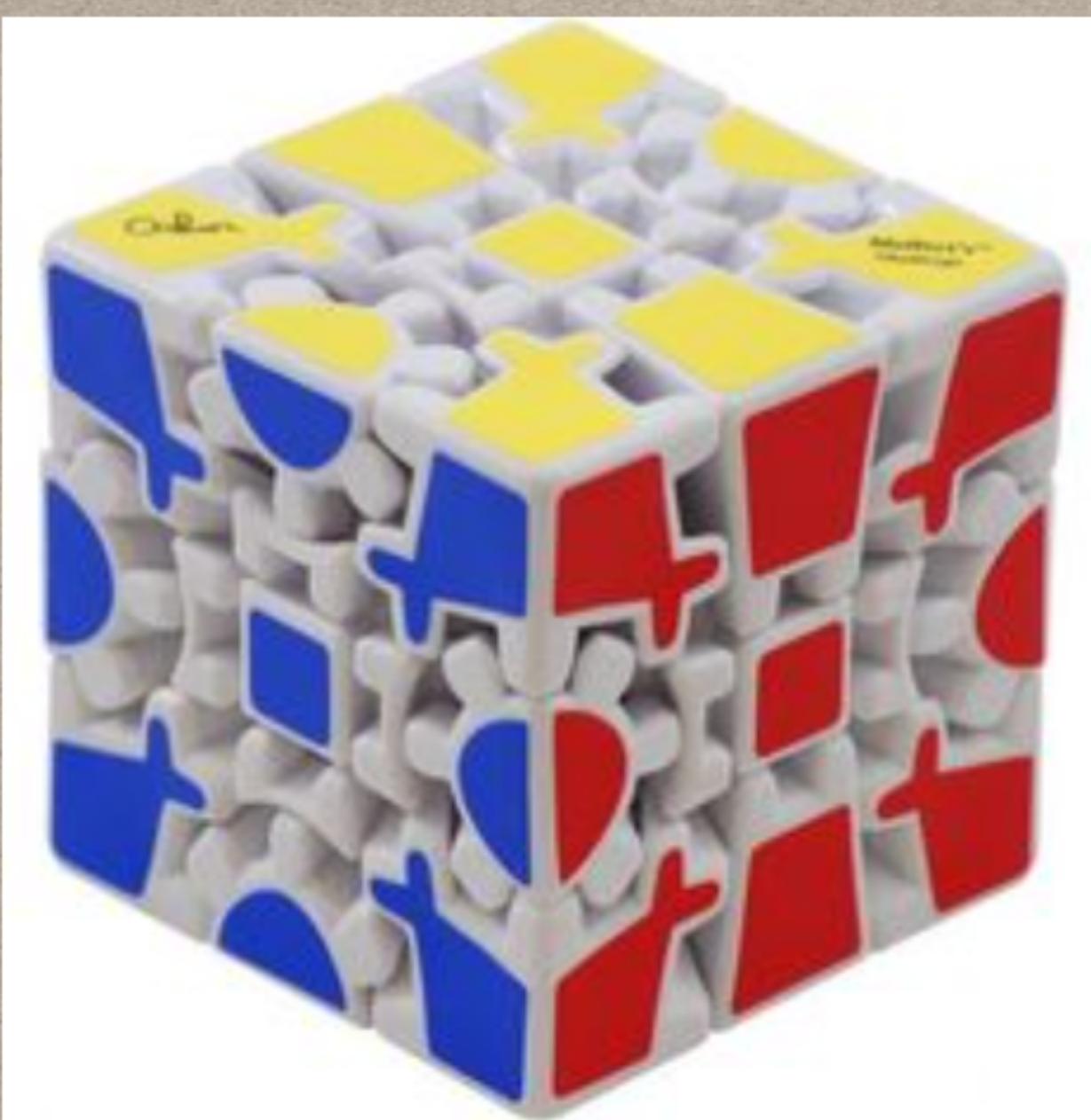
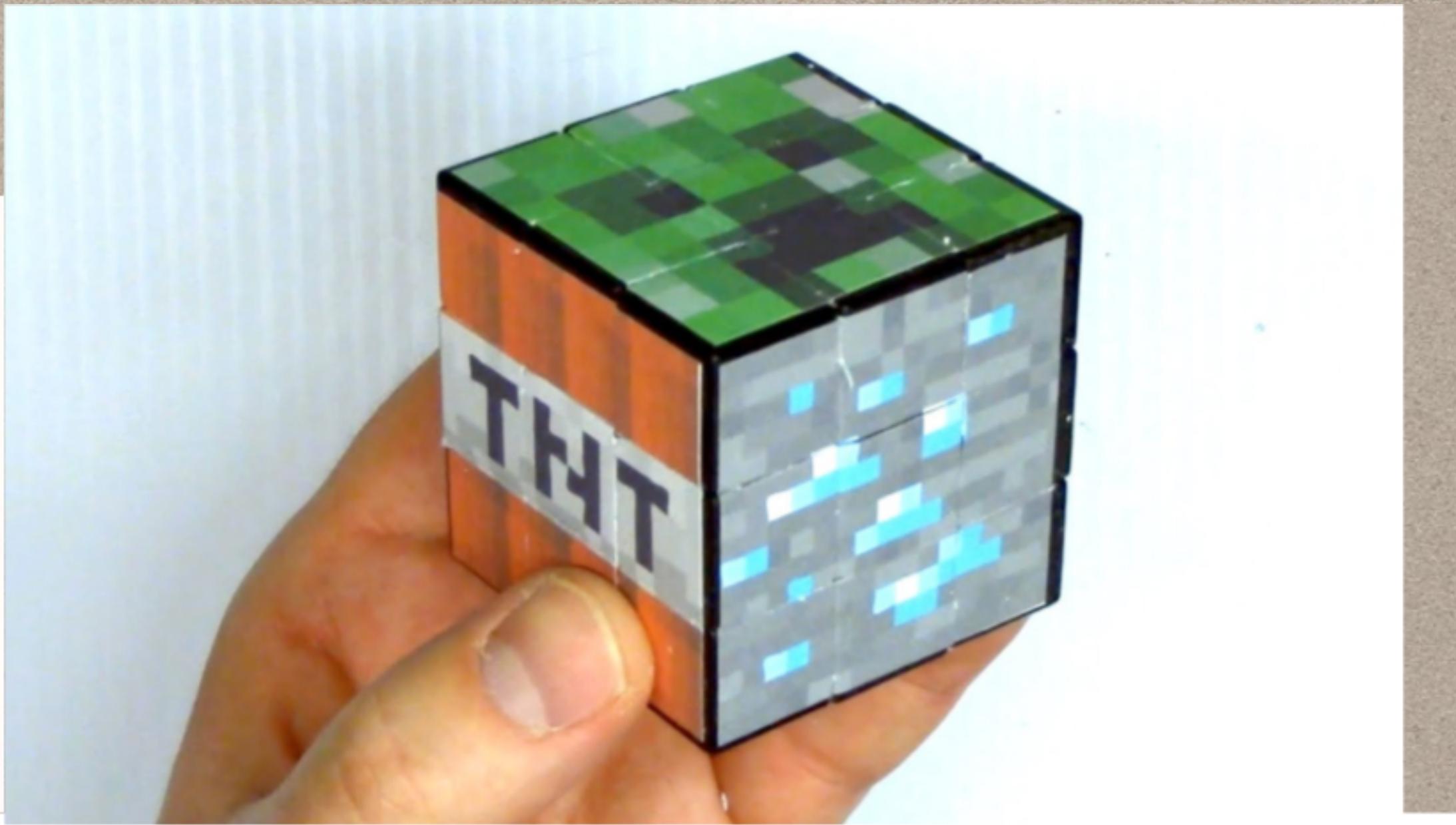
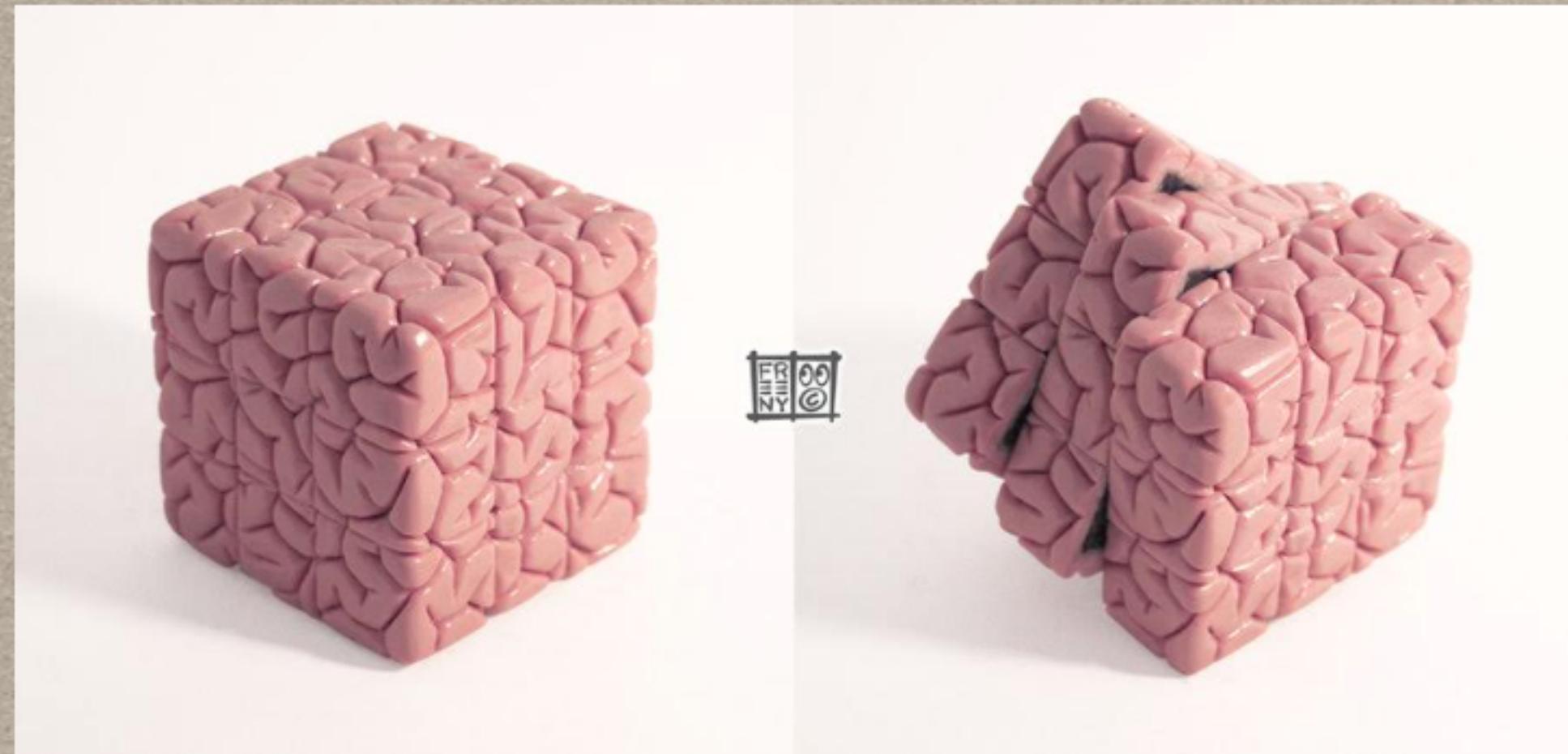
JUN LI  
LUXING ZHANG

**RUBIKS' CUBE**



**ERNO RUBIK**  
HUNGARIAN INVENTOR





# **SETUP FOR IP MODEL**

	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>	<i>N</i>
<i>A</i>			1	2				
<i>B</i>			3	4				
<i>C</i>	5	6	7	8	9	10	11	12
<i>D</i>	13	14	15	16	17	18	19	20
<i>E</i>			21	22				
<i>F</i>			23	24				

# CUBE MAPPING

# DECLARATION OF THE COLORS

<b>INDEX</b>	<b>COLOR</b>
1	RED
2	YELLOW
3	BLUE
4	GREEN
5	BLACK
6	PURPLE

	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>	<i>N</i>
<i>A</i>			5	5				
<i>B</i>			5	5				
<i>C</i>	4	4	1	1	3	3	6	6
<i>D</i>	4	4	1	1	3	3	6	6
<i>E</i>			2	2				
<i>F</i>			2	2				

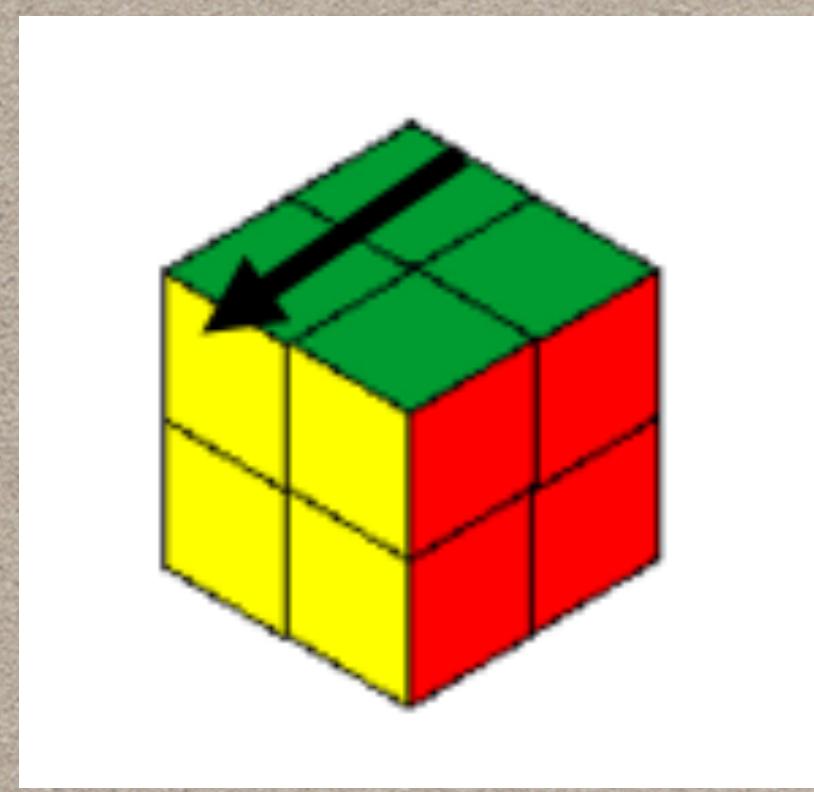
*Final Condition*

# DECLARATION OF THE MOVES

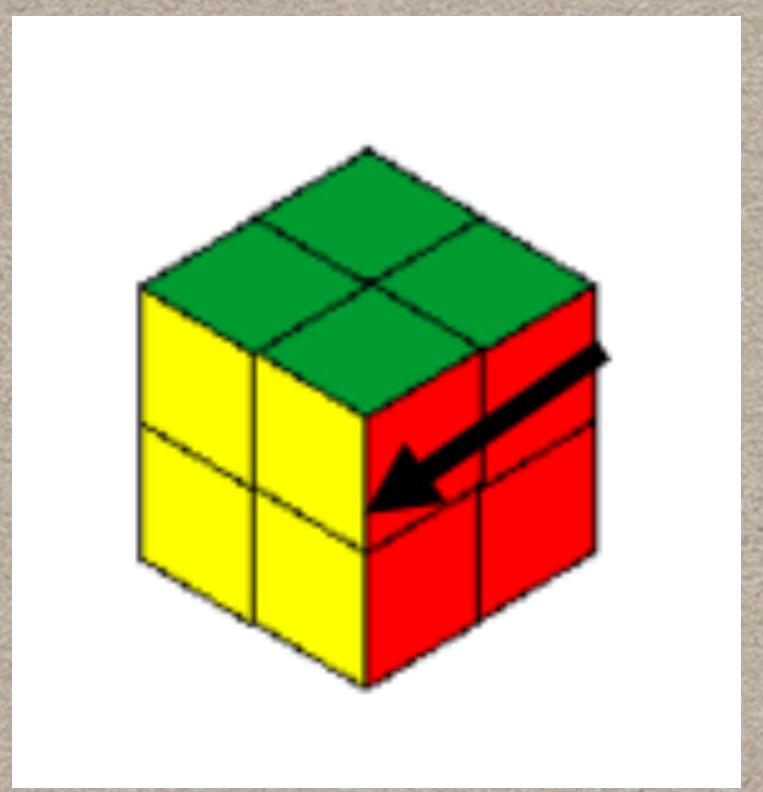
<b>INDEX</b>	<b>MOVE</b>
0	A
1	A'
2	C
3	C'
4	I
5	I'

	G	H	I	J	K	L	M	N
A			1	2				
B			3	4				
C	5	6	7	8	9	10	11	12
D	13	14	15	16	17	18	19	20
E			21	22				
F			23	24				

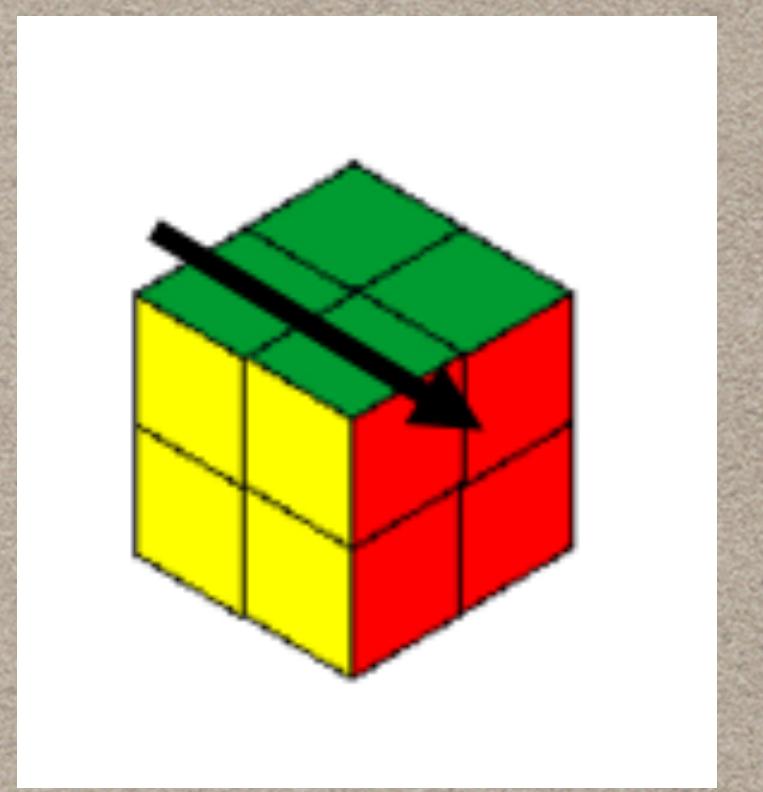
*Position Index*



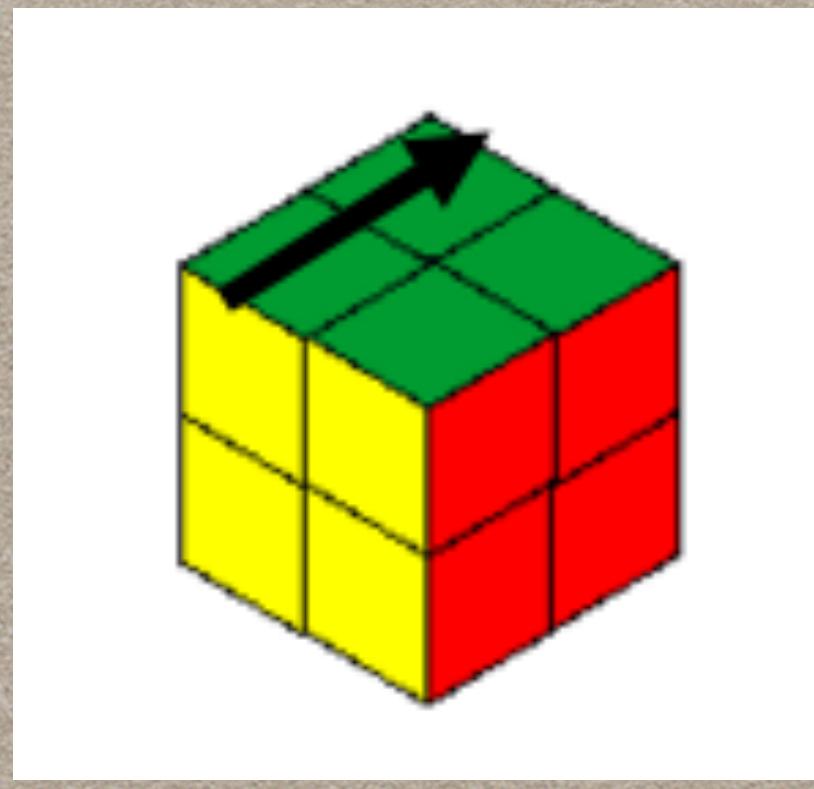
A



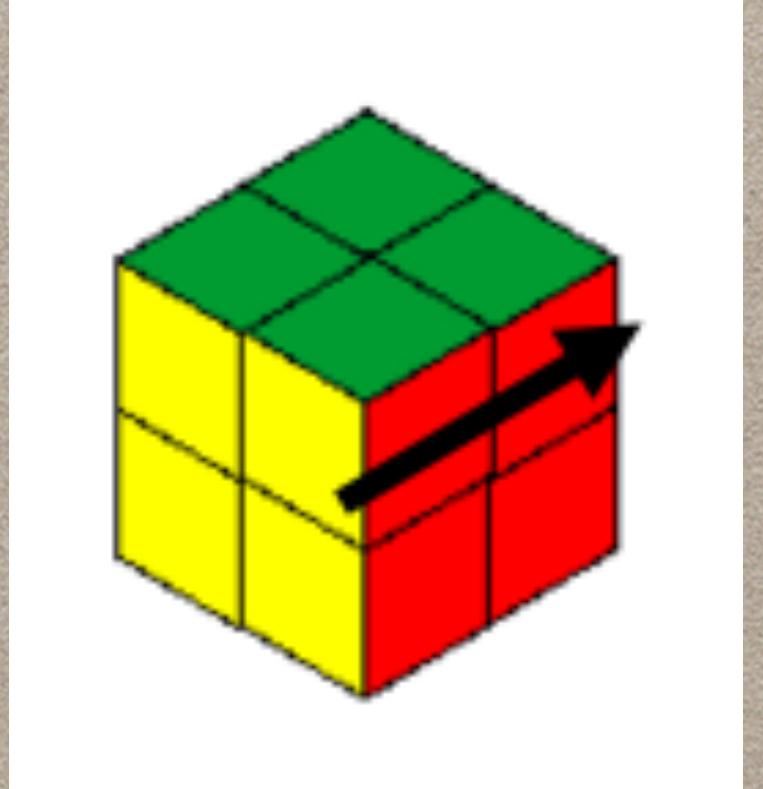
C



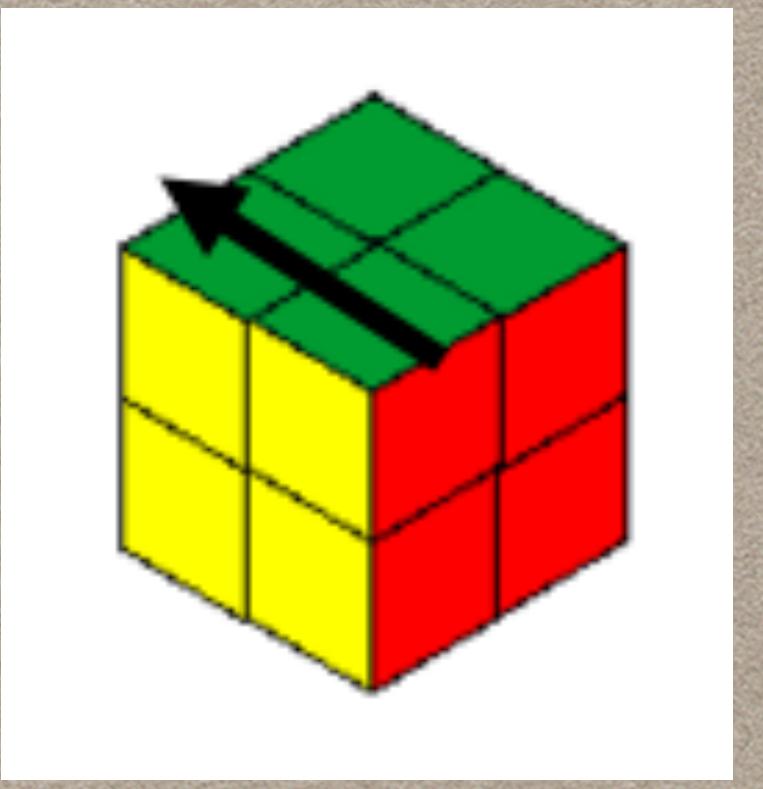
I



A'



C'



I'

	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>	<i>N</i>
<i>A</i>			5	5				
<i>B</i>			5	5				
<i>C</i>	4	4	1	1	3	3	6	6
<i>D</i>	4	4	1	1	3	3	6	6
<i>E</i>			2	2				
<i>F</i>			2	2				

## FIXED PIECE

POSITION 16: 1(RED)

POSITION 17: 3(BLUE)

POSITION 22: 2(YELLOW)

	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>	<i>N</i>
<i>A</i>			1	5				
<i>B</i>			1	5				
<i>C</i>	4	4	2	1	3	3	6	5
<i>D</i>	4	4	2	1	3	3	6	5
<i>E</i>			6	2				
<i>F</i>			6	2				

## INIT.TXT EXAMPLES

- ONE TURN ON I COLUMN
- INITIAL FILE: 1515 44213365 44213365 6262

	<i>G</i>	<i>H</i>	<i>I</i>	<i>J</i>	<i>K</i>	<i>L</i>	<i>M</i>	<i>N</i>
<i>A</i>			4	5				
<i>B</i>			5	3				
<i>C</i>	1	1	3	2	6	6	4	2
<i>D</i>	2	4	5	1	3	5	6	4
<i>E</i>			1	2				
<i>F</i>			6	3				

## INIT.TXT EXAMPLES

- A RANDOM MAP
- INITIAL FILE: 4553 11326642 24513564 1263

# MOVE TUPLES

**$(J, I, I')$  TUPLES FOR MOVE 1 (A):**

$(1, 2, 5)$   $(1, 5, 23)$   $(1, 23, 18)$   $(1, 18, 2)$

$(1, 1, 13)$   $(1, 13, 24)$   $(1, 24, 10)$   
 $(1, 10, 1)$

$(1, 11, 12)$   $(1, 12, 20)$   $(1, 20, 19)$   
 $(1, 19, 11)$

	G	H	I	J	K	L	M	N
A			1	2				
B			3	4				
C	5	6	7	8	9	10	11	12
D	13	14	15	16	17	18	19	20
E			21	22				
F			23	24				

# IP MODEL



# VARIABLES

We assume a 2\*2\*2 Rubik's cube can be solved within 23 turns.

$x_{i,t}$ : color index <1-6>

$y_{j,t}$ : moves <binary>

$i$ : map position <1-24>

$j$ : move type index <0-5>

$t$ : turn number <1-23>

$x_{i,t}$  = the color index at map position i after turn t.

<1-6>

$y_{j,t} = 1$ : if the move at t th turn is index j

= 0: if not <binary>

$x_{1,0}$	$y_{0,1}$
$x_{1,1}$	$y_{0,2}$
$x_{1,2}$	$y_{0,3}$
$x_{1,3}$	$y_{0,4}$
$x_{1,4}$	$y_{0,5}$
$x_{1,5}$	$y_{0,6}$
$x_{1,6}$	$y_{0,7}$
$x_{1,7}$	$y_{0,8}$
$x_{1,8}$	$y_{0,9}$
$x_{1,9}$	$y_{0,10}$
$x_{1,10}$	$y_{0,11}$
$x_{1,11}$	$y_{0,12}$
$x_{1,12}$	$y_{0,13}$
$x_{1,13}$	$y_{0,14}$
$x_{1,14}$	$y_{0,15}$
$x_{1,15}$	$y_{0,16}$
$x_{1,16}$	$y_{0,17}$
$x_{1,17}$	$y_{0,18}$
$x_{1,18}$	$y_{0,19}$
$x_{1,19}$	$y_{0,20}$
$x_{1,20}$	$y_{0,21}$
$x_{1,21}$	$y_{0,22}$
$x_{1,22}$	$y_{0,23}$
$x_{1,23}$	$y_{1,1}$
$x_{2,0}$	$y_{1,2}$
$x_{2,1}$	$y_{1,3}$
$x_{2,2}$	$y_{1,4}$
$x_{2,3}$	

# OBJECTIVE FUNCTION

Minimize  $\sum_{j=1}^6 \sum_{t=1}^{23} t y_{j,t}$

Minimize the number of moves needed to solve a randomly given 2\*2\*2 cube.

$y_{j,t}$ : moves <binary>  
 $j$ : move type index <0-5>  
 $t$ : turn number <1-23>

Minimize  
+ 1 y0,1  
+ 1 y1,1  
+ 1 y2,1  
+ 1 y3,1  
+ 1 y4,1  
+ 1 y5,1  
+ 2 y0,2  
+ 2 y1,2  
+ 2 y2,2  
+ 2 y3,2  
+ 2 y4,2  
+ 2 y5,2  
+ 3 y0,3  
+ 3 y1,3  
+ 3 y2,3  
+ 3 y3,3  
+ 3 y4,3  
+ 3 y5,3  
+ 4 y0,4  
+ 4 y1,4  
+ 4 y2,4  
+ 4 y3,4  
+ 4 y4,4  
+ 4 y5,4

# CONSTRAINTS

- I) INITIAL CONDITION

$x_{i,0} = <1-6>$  (input from init.txt)

$x_{i,t}$ : color index <1-6>

$y_{j,t}$ : moves <binary>

$i$ : map position <1-24>

$j$ : move type index <0-5>

$t$ : turn number <1-23>

Subject To

$x_{1,0} = 4$   
 $x_{2,0} = 5$   
 $x_{3,0} = 5$   
 $x_{4,0} = 3$   
 $x_{5,0} = 1$   
 $x_{6,0} = 1$   
 $x_{7,0} = 3$   
 $x_{8,0} = 2$   
 $x_{9,0} = 6$   
 $x_{10,0} = 6$   
 $x_{11,0} = 4$   
 $x_{12,0} = 2$   
 $x_{13,0} = 2$   
 $x_{14,0} = 4$   
 $x_{15,0} = 5$   
 $x_{16,0} = 1$   
 $x_{17,0} = 3$   
 $x_{18,0} = 5$   
 $x_{19,0} = 6$   
 $x_{20,0} = 4$   
 $x_{21,0} = 1$   
 $x_{22,0} = 2$   
 $x_{23,0} = 6$   
 $x_{24,0} = 3$

# CONSTRAINTS

- II) FINAL CONDITION

$$x_{7,23} = x_{8,23} = x_{15,23} = x_{16,23} = 1$$

$$x_{1,23} = x_{2,23} = x_{3,23} = x_{4,23} = 5 \dots$$

	G	H	I	J	K	L	M	N
A			1	5				
B			1	5				
C	4	4	2	1	3	3	6	5
D	4	4	2	1	3	3	6	5
E			6	2				
F			6	2				

$x_{i,t}$ : color index <1-6>

$y_{j,t}$ : moves <binary>

$i$ : map position <1-24>

$j$ : move type index <0-5>

$t$ : turn number <1-23>

$x_{7,23} = 1$
$x_{8,23} = 1$
$x_{15,23} = 1$
$x_{16,23} = 1$
$x_{5,23} = 4$
$x_{6,23} = 4$
$x_{13,23} = 4$
$x_{14,23} = 4$
$x_{9,23} = 3$
$x_{10,23} = 3$
$x_{17,23} = 3$
$x_{18,23} = 3$
$x_{11,23} = 6$
$x_{12,23} = 6$
$x_{19,23} = 6$
$x_{20,23} = 6$
$x_{1,23} = 5$
$x_{2,23} = 5$
$x_{3,23} = 5$
$x_{4,23} = 5$
$x_{21,23} = 2$
$x_{22,23} = 2$
$x_{23,23} = 2$
$x_{24,23} = 2$

$x_{16,0} - x_{16,1} = 0$
$x_{17,0} - x_{17,1} = 0$
$x_{22,0} - x_{22,1} = 0$
$x_{16,0} - x_{16,2} = 0$
$x_{17,0} - x_{17,2} = 0$
$x_{22,0} - x_{22,2} = 0$
$x_{16,0} - x_{16,3} = 0$
$x_{17,0} - x_{17,3} = 0$
$x_{22,0} - x_{22,3} = 0$
$x_{16,0} - x_{16,4} = 0$
$x_{17,0} - x_{17,4} = 0$
$x_{22,0} - x_{22,4} = 0$
$x_{16,0} - x_{16,5} = 0$
$x_{17,0} - x_{17,5} = 0$
$x_{22,0} - x_{22,5} = 0$
$x_{16,0} - x_{16,6} = 0$

# CONSTRAINTS

- III) AT MOST ONE MOVE PER TURN

$$\sum_{j=1}^6 y_{j,t} \leq 1 \quad \forall 1 \leq t \leq 23$$

$y_{j,t}$ : moves <binary>  
 $j$ : move type index <0-5>  
 $t$ : turn number <1-23>

+  $y_0, 0$   
+  $y_1, 0$   
+  $y_2, 0$   
+  $y_3, 0$   
+  $y_4, 0$   
+  $y_5, 0$   
 $\leq 1$

+  $y_0, 1$   
+  $y_1, 1$   
+  $y_2, 1$   
+  $y_3, 1$   
+  $y_4, 1$   
+  $y_5, 1$   
 $\leq 1$

+  $y_0, 2$   
+  $y_1, 2$   
+  $y_2, 2$   
+  $y_3, 2$   
+  $y_4, 2$   
+  $y_5, 2$   
 $\leq 1$

+  $y_0, 3$   
+  $y_1, 3$   
+  $y_2, 3$   
+  $y_3, 3$   
+  $y_4, 3$   
+  $y_5, 3$   
 $\leq 1$

# CONSTRAINTS

## • IV) MOVED SQUARES

$\forall (j, i, i')$ :

$$x_{i,t} - 6 * (1 - y_{j,t}) \leq x_{i',t+1} \leq x_{i,t} + 6 * (1 - y_{j,t+1})$$

If a move is applied at a turn

then the corresponding squares are moved  
at this turn

```
moves [0] [0] = 2;  
moves [0] [1] = 5;  
moves [0] [2] = 23;  
moves [0] [3] = 18;  
moves [0] [4] = 1;  
moves [0] [5] = 13;  
moves [0] [6] = 24;  
moves [0] [7] = 10;  
moves [0] [8] = 11;  
moves [0] [9] = 12;  
moves [0] [10] = 20;  
moves [0] [11] = 19;  
moves [0] [12] = '\0';
```

```
moves [1] [0] = 18;  
moves [1] [1] = 23;  
moves [1] [2] = 5;  
moves [1] [3] = 2;
```

$x_{i,t}$ : color index <1-6>  
 $y_{j,t}$ : moves <binary>  
 $i$ : map position <1-24>  
 $j$ : move type index <0-5>  
 $t$ : turn number <1-23>

```
x2,0 + 6 y0,1 - x5,1 <= 6  
x5,1 - x2,0 - 6 y0,1 <= 6  
x5,0 + 6 y0,1 - x23,1 <= 6  
x23,1 - x5,0 - 6 y0,1 <= 6  
x23,0 + 6 y0,1 - x18,1 <= 6  
x18,1 - x23,0 - 6 y0,1 <= 6  
x18,0 + 6 y0,1 - x2,1 <= 6  
x2,1 - x18,0 - 6 y0,1 <= 6  
x1,0 + 6 y0,1 - x13,1 <= 6  
x13,1 - x1,0 - 6 y0,1 <= 6  
x13,0 + 6 y0,1 - x24,1 <= 6  
x24,1 - x13,0 - 6 y0,1 <= 6  
x24,0 + 6 y0,1 - x10,1 <= 6  
x10,1 - x24,0 - 6 y0,1 <= 6  
x10,0 + 6 y0,1 - x1,1 <= 6  
x1,1 - x10,0 - 6 y0,1 <= 6  
x11,0 + 6 y0,1 - x12,1 <= 6  
x12,1 - x11,0 - 6 y0,1 <= 6  
x12,0 + 6 y0,1 - x20,1 <= 6  
x20,1 - x12,0 - 6 y0,1 <= 6  
x20,0 + 6 y0,1 - x19,1 <= 6  
x19,1 - x20,0 - 6 y0,1 <= 6  
x19,0 + 6 y0,1 - x11,1 <= 6  
x11,1 - x19,0 - 6 y0,1 <= 6  
x18,0 + 6 y1,1 - x23,1 <= 6  
x23,1 - x18,0 - 6 y1,1 <= 6  
x23,0 + 6 y1,1 - x5,1 <= 6  
x5,1 - x23,0 - 6 y1,1 <= 6  
x5,0 + 6 y1,1 - x2,1 <= 6  
x2,1 - x5,0 - 6 y1,1 <= 6
```

# CONSTRAINTS

- V) UNMOVED SQUARES

If there's no turn involving a square at turn t

then the square remains the same

$x_{i,t}$ : color index<1-6>

$y_{j,t}$ : moves <binary>

i: map position<1-24>

j: move type index <0-5>

t: turn number <1-23>

```
unmove[0][0] = '0';
unmove[0][1] = '1';
unmove[0][2] = '2';
unmove[0][3] = '3';
unmove[0][4] = '4';
unmove[0][5] = '5';
unmove[0][6] = '\0';
unmove[1][0] = '0';
unmove[1][1] = '1';
unmove[1][2] = '2';
unmove[1][3] = '3';
unmove[1][4] = '\0';
unmove[2][0] = '2';
unmove[2][1] = '3';
unmove[2][2] = '4';
unmove[2][3] = '5';
unmove[2][4] = '\0';
unmove[3][0] = '2';
```

$$\begin{aligned}
&x_{1,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} - x_{1,1} \leq 0 \\
&x_{1,1} - x_{1,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} \leq 0 \\
&x_{2,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} - x_{2,1} \leq 0 \\
&x_{2,1} - x_{2,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} \leq 0 \\
&x_{3,0} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} - x_{3,1} \leq 0 \\
&x_{3,1} - x_{3,0} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} \leq 0 \\
&x_{4,0} - 6 y_{2,1} - 6 y_{3,1} - x_{4,1} \leq 0 \\
&x_{4,1} - x_{4,0} - 6 y_{2,1} - 6 y_{3,1} \leq 0 \\
&x_{5,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} - x_{5,1} \leq 0 \\
&x_{5,1} - x_{5,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} \leq 0 \\
&x_{6,0} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} - x_{6,1} \leq 0 \\
&x_{6,1} - x_{6,0} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} \leq 0 \\
&x_{7,0} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} - x_{7,1} \leq 0 \\
&x_{7,1} - x_{7,0} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} \leq 0 \\
&x_{8,0} - 6 y_{2,1} - 6 y_{3,1} - x_{8,1} \leq 0 \\
&x_{8,1} - x_{8,0} - 6 y_{2,1} - 6 y_{3,1} \leq 0 \\
&x_{9,0} - 6 y_{2,1} - 6 y_{3,1} - x_{9,1} \leq 0 \\
&x_{9,1} - x_{9,0} - 6 y_{2,1} - 6 y_{3,1} \leq 0 \\
&x_{10,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} - x_{10,1} \leq 0 \\
&x_{10,1} - x_{10,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} \leq 0 \\
&x_{11,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} - x_{11,1} \leq 0 \\
&x_{11,1} - x_{11,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} \leq 0 \\
&x_{12,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} - x_{12,1} \leq 0 \\
&x_{12,1} - x_{12,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{2,1} - 6 y_{3,1} - 6 y_{4,1} - 6 y_{5,1} \leq 0 \\
&x_{13,0} - 6 y_{0,1} - 6 y_{1,1} - 6 y_{4,1} - 6 y_{5,1} - x_{13,1} \leq 0
\end{aligned}$$

**GUROBI**

# FOR ONE TURN

**INDEX**

0	A
1	A'
2	C
3	C'
4	I
5	I'

**MOVE**

	G	H	I	J	K	L	M	N
A			1	5				
B			1	5				
C	4	4	2	1	3	3	6	5
D	4	4	2	1	3	3	6	5
E			6	2				
F			6	2				

```

Root relaxation: objective 1.000000e+00, 462 iterations, 0.02 seconds

      Nodes    |      Current Node    |      Objective Bounds      |      Work
      Expl Unexpl |  Obj  Depth IntInf | Incumbent    BestBd   Gap | It/Node Time
*       0       0            0      1.0000000      1.00000   0.00%      -     0s

Explored 0 nodes (462 simplex iterations) in 0.10 seconds
Thread count was 4 (of 4 available processors)

Optimal solution found (tolerance 1.00e-04)
Best objective 1.000000000000e+00, best bound 1.000000000000e+00, gap 0.0%

Wrote result file 'cube.sol'

```

```

# Objective value = 1
y0,1 0
y1,1 0
y2,1 0
y3,1 0
y4,1 1
y5,1 0
y0,2 0
y1,2 0
y2,2 0
y3,2 0
y4,2 0
y5,2 0
y0,3 0
y1,3 0
y2,3 0
y3,3 0
y4,3 0

```

# FOR RANDOM TURNS

	G	H	I	J	K	L	M	N
A			4	5				
B			5	3				
C	1	1	3	2	6	6	4	2
D	2	4	5	1	3	5	6	4
E			1	2				
F			6	3				

H328304	188422				72.0000000	15.69333	78.2%	24.4	475s
H328331	184293				70.0000000	15.69333	77.6%	24.4	475s
H328412	129404				49.0000000	15.69333	68.0%	24.4	476s
H328520	126145				48.0000000	15.69333	67.3%	24.4	477s
329967	127228	20.10000	63	20	48.00000	15.72895	67.2%	24.4	480s
334415	130490	26.20000	50	3	48.00000	15.80000	67.1%	24.3	485s
338807	133739	24.32500	59	145	48.00000	15.88909	66.9%	24.3	490s
H341531	124558				45.0000000	15.93333	64.6%	24.2	494s
341533	124562	37.00000	60	4	45.00000	15.93333	64.6%	24.2	495s
344506	126674	17.40000	45	37	45.00000	16.00000	64.4%	24.2	500s
349346	130196	22.27742	48	165	45.00000	16.05172	64.3%	24.1	505s
353391	133176	32.20000	50	30	45.00000	16.11628	64.2%	24.1	510s
358046	136413	41.05714	72	77	45.00000	16.18849	64.0%	24.0	515s
362415	139596	21.24000	58	155	45.00000	16.22600	63.9%	23.9	520s
367443	143169	21.30000	66	118	45.00000	16.30619	63.8%	23.9	525s
371936	146356	19.62000	38	22	45.00000	16.37857	63.6%	23.8	530s

31006974	8053381	40.00825	38	138	45.00000	36.40000	19.1%	17.3	35965s
31008074	8053114	38.40000	37	5	45.00000	36.40000	19.1%	17.3	35970s
31009340	8052870	38.40000	32	3	45.00000	36.40000	19.1%	17.3	35975s
31010573	8052658	cutoff	74		45.00000	36.40000	19.1%	17.3	35980s
31011915	8052403	cutoff	44		45.00000	36.40000	19.1%	17.3	35985s
31013330	8052202	cutoff	57		45.00000	36.40000	19.1%	17.3	35990s
31014279	8051977	cutoff	57		45.00000	36.40000	19.1%	17.3	35995s
31015307	8051790	cutoff	65		45.00000	36.40000	19.1%	17.3	36000s
31016597	8051570	cutoff	55		45.00000	36.40000	19.1%	17.3	36005s
31017744	8051403	cutoff	90		45.00000	36.40000	19.1%	17.3	36010s
31019124	8051178	43.20000	44	4	45.00000	36.40000	19.1%	17.3	36015s
31020362	8051069	cutoff	60		45.00000	36.40000	19.1%	17.3	36020s
31021778	8050912	cutoff	67		45.00000	36.40000	19.1%	17.3	36025s
31022823	8050646	cutoff	73		45.00000	36.40000	19.1%	17.3	36030s
31024409	8050575	cutoff	45		45.00000	36.40000	19.1%	17.3	36035s
31025968	8050443	39.92000	42	70	45.00000	36.40000	19.1%	17.3	36040s
31027482	8050282	40.20000	57	100	45.00000	36.40000	19.1%	17.3	36045s

# **FUTURE IMPROVEMENT**

- TRY SITUATIONS WITH FEWER OPTIMAL MOVES
- TRY TO STOP IN THE MIDDLE
- CONSIDER A CP MODEL
- BETTER CONSTRAINTS

Thanks!

*-Jun, Luxing*