

# 2023 UCPC team note

개는 훌륭하다

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## 1 Data Structure

### 1.1 Segment Tree

```
// Segtree
#include <bits/stdc++.h>
#define INF 1000000007
using namespace std;

int seg[1<<18];

void build_seg(int idx, int l, int r){
    if(l==r){
        // seg[idx]=
        return;
    }
    int mid=(l+r)>>1;
    build_seg(2*idx,l,mid);
    build_seg(2*idx+1,mid+1,r);
    //seg[idx]=min(seg[2*idx],seg[2*idx+1]);
    // Some other operation
}
```

// 원소를 찾으면 disable도 같이 해주면 될 거 같음

```

void update_seg(int idx, int l, int r, int t_idx){
    if(l==r){
        seg[idx]=INF;
        return;
    }
    int mid=(l+r)>>1;
    if(t_idx<=mid) update_seg(2*idx,l,mid,t_idx);
    else update_seg(2*idx+1,mid+1,r,t_idx);
    //seg[idx]=min(seg[2*idx],seg[2*idx+1]);
}

int find_seg(int idx, int l, int r, int t_l, int t_r){
    if(t_l<=l && r<=t_r) return seg[idx];
    int mid=(l+r)>>1, ans=INF;
    if(t_l<=mid) //ans=min(ans,find_seg(2*idx,l,mid,t_l,t_r));
    if(t_r>mid) //ans=min(ans,find_seg(2*idx+1,mid+1,r,t_l,t_r));
    return ans;
}

int main()
{
    ios_base::sync_with_stdio(false); cin.tie(NULL);
    return 0;
}

```

## 1.2 Lazy Segment Tree

```

// Lazy seg
#include <bits/stdc++.h>
#define INF 1000000007
using namespace std;

class node{
public:
    // Do something!
    node(){}
    node(){
        // Do something!
    }
    node operator+(node b){
        node result;
        // Do something!
        return result;
    }
    // int val, lazy_val;
};

class lazy_seg{

```

```

public:
    node seg[1<<18];

    lazy_seg(){}
    lazy_seg(int n){
        //fill(seg,seg+n,node());
    }

    void build_seg(int idx, int l, int r){
        if(l==r){
            // Do something!
        }
        else{
            int mid=(l+r)>>1;
            build_seg(2*idx,l,mid);
            build_seg(2*idx+1,mid+1,r);
            seg[idx]=seg[2*idx]+seg[2*idx+1];
        }
    }

    void update_down(int idx, int l, int r){
        if(l==r) return;
        int mid=(l+r)>>1;
        // seg[2*idx], seg[2*idx+1]

        // seg[idx]의 lazy_val초기화

    }

    void update_seg(int idx, int l, int r, int t_l, int t_r,
                    int val){
        update_down(idx, l, r);
        if(t_l<=l && r<=t_r){
            // seg[idx].val과 seg[idx].lazy_val업데이트
            return;
        }
        int mid=(l+r)>>1;
        if(t_l<=mid) update_seg(2*idx,l,mid,t_l,t_r,val);
        if(t_r>mid) update_seg(2*idx+1,mid+1,r,t_l,t_r,val);
        seg[idx]=seg[2*idx]+seg[2*idx+1];
    }

    node find_seg(int idx, int l, int r, int t_l, int t_r){
        update_down(idx, l, r);
        if(t_l<=l && r<=t_r){
            return seg[idx];
        }
    }

```

```

    node result;
    int mid=(l+r)>>1;
    if(t_l<=mid)
        result=result+find_seg(2*idx,l,mid,t_l,t_r);
    if(t_r>mid)
        result=result+find_seg(2*idx+1,mid+1,r,t_l,t_r);
    return result;
}
};

```

lazy\_seg s;

### 1.3 Lazy Count Tree

// Lazy count tree. Update down이 필요없는 아주 단순한 형태의 lazy\_seg

```

#include <iostream>
using namespace std;

```

```
long long seg[800000];
```

```

void update_seg(int idx, int l, int r, int t_l, int t_r,
                long long val){
    if(t_l<=l && r<=t_r){
        seg[idx]+=val;
        return;
    }
    int mid=(l+r)>>1;
    if(t_l<=mid) update_seg(2*idx,l,mid,t_l,t_r,val);
    if(t_r>mid) update_seg(2*idx+1,mid+1,r,t_l,t_r,val);
}

```

```

long long find_seg(int idx, int l, int r, int t_idx){
    if(l==r) return seg[idx];
    long long res=seg[idx];
    int mid=(l+r)>>1;
    if(t_idx<=mid) res=res+find_seg(2*idx,l,mid,t_idx);
    if(t_idx>mid) res=res+find_seg(2*idx+1,mid+1,r,t_idx);
    return res;
}

```

### 1.4 Persistent Segment Tree

```

// PST
#include <bits/stdc++.h>
using namespace std;

#define N_node 2000000 // V log/Y_RANGE/ scale
#define NX 100001

```

```
#define Y_RANGE 100001
```

```

class node{
public:
    node(){}
    node operator+(node b){
        node res;
        // Implement. 예시는 단순히 개수 세는 것.
        // 왼쪽에 더해지는 node의 l_idx, r_idx를 보존한다.
        res.l_idx=l_idx, res.r_idx=r_idx;
        res.cnt=cnt+b.cnt;
        return res;
    }
    node operator-(node b){
        node res;
        // Implement
        res.l_idx=l_idx, res.r_idx=r_idx;
        res.cnt=cnt-b.cnt;
        return res;
    }
    int l_idx=-1, r_idx=-1; // 여기서 idx는 본인의 idx를 말한다.
    long long cnt=0;
};

class PST{
public:
    int seg_root[NX]; // X-wise root idx.
    node seg[N_node]; // All node datas
    int cur_x=0, s_cnt=0; // s_cnt는 현재까지 사용한 node의 개수
    PST(){}
    void push(long long y){
        // Pusing one element generate another root.
        // Regardless of the val x.
        seg_root[cur_x]=s_cnt++;
        update_seg(seg_root[cur_x],(cur_x==0?-1:seg_root[cur_x-1]),
                    0,Y_RANGE-1,y,1);
        cur_x++;
    }
    node query(long long x1, long long x2,
               long long y1, long long y2){
        if(x1==0) return find_seg(seg_root[x2],0,Y_RANGE-1,y1,y2);
        return find_seg(seg_root[x2],0,Y_RANGE-1,y1,y2)
            -find_seg(seg_root[x1-1],0,Y_RANGE-1,y1,y2);
    }
    void clear(){
        for(int i=0; i<s_cnt; i++) seg[i]=node();
        cur_x=s_cnt=0;
    }
};

```

```

    }

private:
    void update_seg(int idx, int prev_idx, int l, int r, int t_idx,
                    long long val){
        // previous segtree의 idx도 같이 관리. (첫 번째 layer의 경우는 -1)
        if(l==r){
            // Implement. 과거의 데이터에 덮어씌우는 것은 이부분이 유일.
            seg[idx].cnt=val;
            if(prev_idx!=-1) seg[idx]=seg[idx]+seg[prev_idx];
            return;
        }
        int mid=(l+r)>>1;
        if(t_idx<=mid){
            seg[idx].l_idx=s_cnt++;
            seg[idx].r_idx=(prev_idx==-1?-1:seg[prev_idx].r_idx);
            update_seg(seg[idx].l_idx, (prev_idx==-1?-1
                :seg[prev_idx].l_idx), l, mid, t_idx, val);
        }
        else{
            seg[idx].r_idx=s_cnt++;
            seg[idx].l_idx=(prev_idx==-1?-1:seg[prev_idx].l_idx);
            update_seg(seg[idx].r_idx, (prev_idx==-1?-1
                :seg[prev_idx].r_idx), mid+1, r, t_idx, val);
        }
        // 아래의 덧셈연산은 l_idx와 r_idx를 보존하기 위한 방법
        if(seg[idx].l_idx!=-1) seg[idx]=seg[idx]+seg[seg[idx].l_idx];
        if(seg[idx].r_idx!=-1) seg[idx]=seg[idx]+seg[seg[idx].r_idx];
    }
    node find_seg(int idx, int l, int r, int t_l, int t_r){
        if(t_l<=l && r<=t_r) return seg[idx];
        int mid=(l+r)>>1;
        node res;
        if(t_l<=mid && seg[idx].l_idx!=-1)
            res=res+find_seg(seg[idx].l_idx, l, mid, t_l, t_r);
        if(t_r>mid && seg[idx].r_idx!=-1)
            res=res+find_seg(seg[idx].r_idx, mid+1, r, t_l, t_r);
        return res;
    }
};

```

PST p;

## 1.5 Union Find

```

// union_find
#include <bits/stdc++.h>
using namespace std;

```

```

int parent[100000];
int find(int idx){
    if(idx==parent[idx]) return idx;
    return parent[idx]=find(parent[idx]);
}
void c_union(int a, int b){
    a=find(a), b=find(b);
    parent[b]=a;
}

```

## 1.6 Mo's algorithm

```

#include <bits/stdc++.h>
#define endl '\n'
#define cedu(a,b) ((a)%(b)==0?((a)/(b)):((a)/(b))+1)
#define fi first
#define se second
#define pb push_back

using namespace std;

typedef long long ll;
typedef unsigned int ui;
typedef unsigned long long ull;

template<typename T>
inline T umax(T &u, T v){return u = max(u, v);}
template<typename T>
inline T umin(T &u, T v){return u = min(u, v);}
#define INF 1000000007

int sqrtN;

class Query{
public:
    int idx, s, e;
    bool operator < (Query &x){
        if(s/sqrtN != x.s/sqrtN) return s/sqrtN < x.s/sqrtN;
        return e < x.e;
    }
};

Query qs[1000000];
int ans[1000000];

void go(int idx){
}

```

```

void back(int idx){
}

int main(){
    ios_base::sync_with_stdio(false); cin.tie(NULL);
    int q; // number of q
    int s = qs[0].s, e = qs[0].e;
    for(int i=s; i<=e; i++){
        go(i);
    }
    //ans[qs[0].idx] = ;

    for(int i=1; i<q; i++){
        while(s < qs[i].s) back(s++);
        while(s > qs[i].s) go(--s);
        while(e < qs[i].e) go(++e);
        while(e > qs[i].e) back(e--);
        //ans[qs[i].idx] = ;
    }
    for(int i=0; i<q; i++) cout << ans[i] << endl;

    return 0;
}

```

## 1.7 HLD

```

// HLD
#include <bits/stdc++.h>
#define INF 1000000007
#define N 200000

using namespace std;

vector<int> edges[N], child[N];
int parent[N], depth[N], in[N], top[N];
int sz[N], inv_in[N]; // HLD내부적으로 필요.
// inv_in은 seg의 초기화에 필요.
int n, cnt;

void DFS(int idx, int p){
    child[idx].reserve(edges[idx].size()-1);
    for(int i=0; i<edges[idx].size(); i++){
        int next=edges[idx][i];
        if(next!=p){
            child[idx].push_back(next);
            DFS(next,idx);
        }
    }
}

```

```

    }
}

// parent[루트], top[루트], depth[루트]업데이트 해줘야 함.
void DFS1(int idx=0){
    sz[idx]=1;
    for(int i=0; i<child[idx].size(); i++){
        int next=child[idx][i];
        parent[next]=idx;
        depth[next]=depth[idx]+1;
        DFS1(next);
        sz[idx]+=sz[next];
        if(sz[next]>sz[child[idx][0]]) swap(child[idx][0],child[idx][i]);
    }
}

void DFS2(int idx=0){
    in[idx]=cnt;
    for(int i=0; i<child[idx].size(); i++){
        int next=child[idx][i];
        top[next]=(i==0?top[idx]:next);
        cnt++;
        DFS2(next);
    }
}

class node{
public:
    // Do something!
    node(){}
    node(){
    }
    node operator+(node b){
        node result;
        // Do something!
        return result;
    }
    // int val, lazy_val;
};

class lazy_seg{
public:
    node seg[1<<18];

    lazy_seg(){}
}

```

```

lazy_seg(int n){
    //fill(seg,seg+n,node());
}

void build_seg(int idx, int l, int r){
    if(l==r){
        // Do something!
    }
    else{
        int mid=(l+r)>>1;
        build_seg(2*idx,l,mid);
        build_seg(2*idx+1,mid+1,r);
        seg[idx]=seg[2*idx]+seg[2*idx+1];
    }
}

void update_down(int idx, int l, int r){
    if(l==r) return;
    int mid=(l+r)>>1;
    // seg[2*idx], seg[2*idx+1]

    // seg[idx]의 lazy_val 초기화

}

void update_seg(int idx, int l, int r, int t_l, int t_r,
                int val){
    if(t_l<=l && r<=t_r){
        // seg[idx].val과 seg[idx].lazy_val 업데이트
        return;
    }
    update_down(idx, l, r);
    int mid=(l+r)>>1;
    if(t_l<=mid) update_seg(2*idx,l,mid,t_l,t_r,val);
    if(t_r>mid) update_seg(2*idx+1,mid+1,r,t_l,t_r,val);
    seg[idx]=seg[2*idx]+seg[2*idx+1];
}

node find_seg(int idx, int l, int r, int t_l, int t_r){
    if(t_l<=l && r<=t_r){
        return seg[idx];
    }
    update_down(idx, l, r);
    node result;
    int mid=(l+r)>>1;
    if(t_l<=mid) result=result+find_seg(2*idx,l,mid,t_l,t_r);
    if(t_r>mid) result=result+find_seg(2*idx+1,mid+1,r,t_l,t_r);
}

```

```

        return result;
    }
};

lazy_seg s;

int get_LCA(int a,int b){
    while(top[a]!=top[b]){
        if(depth[top[a]]<depth[top[b]]) swap(a,b);
        a=parent[top[a]];
    }
    return (in[a]<in[b]?a:b);
}

node query(int a, int b){
    node result;
    while(top[a]!=top[b]){
        if(depth[top[a]]<depth[top[b]]) swap(a,b);
        result=result+s.find_seg(1,0,n-1,in[top[a]],in[a]);
        // 더하는 순서가 중요하다면 잘 확인할 것
        a=parent[top[a]];
    }
    result=result+s.find_seg(1,0,n-1,
                             min(in[a],in[b]),max(in[a],in[b]));
    return result;
}

void update_query(int a, int b, int val){
    while(top[a]!=top[b]){
        if(depth[top[a]]<depth[top[b]]) swap(a,b);
        s.update_seg(1,0,n-1,in[top[a]],in[a],val);
        a=parent[top[a]];
    }
    s.update_seg(1,0,n-1,
                 min(in[a],in[b]),max(in[a],in[b]),val);
}

/*
int main(){
    // initialize n
    cnt=0;
    parent[0]=-1, top[0]=depth[0]=0;
    DFS(0,-1);
    DFS1(0);
    DFS2(0);
    s.build_seg(1,0,n-1);
}

```

```

    return 0;
}
*/

```

## 1.8 HLD 금광

```

// 금광 + HLD아니냐
#include <bits/stdc++.h>
#define endl '\n'
#define fi first
#define se second
#define pb push_back

using namespace std;

typedef long long ll;

#define N 200020

vector<pair<pair<int,int>,int>> qs;
int datas[N];

vector<int> edges[N], child[N];
int parent[N], depth[N], in[N], top[N];
int sz[N], inv_in[N]; // HLD내부적으로 필요. inv_in은 seg의 초기화에 필요.
int n, cnt;

void DFS(int idx, int p){
    //child[idx].reserve(edges[idx].size()-1);
    for(int i=0; i<edges[idx].size(); i++){
        int next=edges[idx][i];
        if(next!=p){
            child[idx].push_back(next);
            DFS(next,idx);
        }
    }
}

// parent[루트], top[루트], depth[루트]업데이트 해줘야 함.
void DFS1(int idx=0){
    sz[idx]=1;
    for(int i=0; i<child[idx].size(); i++){
        int next=child[idx][i];
        parent[next]=idx;
        depth[next]=depth[idx]+1;
        DFS1(next);
        sz[idx]+=sz[next];
    }
}

```

```

        if(sz[next]>sz[child[idx][0]])
            swap(child[idx][0],child[idx][i]);
    }
}

void DFS2(int idx=0){
    in[idx]=cnt;
    inv_in[cnt]=idx;
    for(int i=0; i<child[idx].size(); i++){
        int next=child[idx][i];
        top[next]=(i==0?top[idx]:next);
        cnt++;
        DFS2(next);
    }
}

class node{
public:
    // Do something!
    node(){}
    node(int val){
        l_mn=r_mn=t_mn=min(0,val);
        l_mx=r_mx=t_mx=max(0,val);
        sum=val;
    }
    node operator+(node b){
        node result;
        result.l_mn=min(l_mn,sum+b.l_mn);
        result.l_mx=max(l_mx,sum+b.l_mx);
        result.r_mn=min(b.r_mn,b.sum+r_mn);
        result.r_mx=max(b.r_mx,b.sum+r_mx);
        result.t_mn=min(min(t_mn, b.t_mn), r_mn+b.l_mn);
        result.t_mx=max(max(t_mx, b.t_mx), r_mx+b.l_mx);
        result.sum=sum+b.sum;
        return result;
    }
    int l_mn, l_mx, r_mn, r_mx, t_mn, t_mx, sum;
};

class lazy_seg{
public:
    node seg[1<<19];
    lazy_seg(){}

    void build_seg(int idx, int l, int r){
        if(l==r){
            seg[idx]=node(datas[inv_in[l]]);
        }
    }
}

```

```

    }
    else{
        int mid=(l+r)>>1;
        build_seg(2*idx,l,mid);
        build_seg(2*idx+1,mid+1,r);
        seg[idx]=seg[2*idx]+seg[2*idx+1];
    }
}

node find_seg(int idx, int l, int r, int t_l, int t_r){
    if(t_l<=l && r<=t_r){
        return seg[idx];
    }
    node result(0);
    int mid=(l+r)>>1;
    if(t_l<=mid) result=result+find_seg(2*idx,l,mid,t_l,t_r);
    if(t_r>mid) result=result+find_seg(2*idx+1,mid+1,r,t_l,t_r);
    return result;
}

};

lazy_seg s;

int get_LCA(int a,int b){
    while(top[a]!=top[b]){
        if(depth[top[a]]<depth[top[b]]) swap(a,b);
        a=parent[top[a]];
    }
    return (in[a]<in[b]?a:b);
}

}

node query(int a, int b){
    node result_l(0); // 금광세그의 특수성 때문이다.
    node result_r(0);
    while(top[a]!=top[b]){
        if(depth[top[a]]<depth[top[b]]){
            result_r=s.find_seg(1,0,n-1,in[top[b]],in[b])+result_r;
            b=parent[top[b]];
        }
        else{
            result_l=s.find_seg(1,0,n-1,in[top[a]],in[a])+result_l;
            a=parent[top[a]];
        }
    }
    if(in[a]<in[b]) result_r=s.find_seg(1,0,n-1,in[a],in[b])+result_r;
    else result_l=s.find_seg(1,0,n-1,in[b],in[a])+result_l;
    // 반전 해줘야 함.

```

```

    swap(result_l.l_mn,result_l.r_mn);
    swap(result_l.l_mx,result_l.r_mx);
    return result_l+result_r;
}

int main(){
    ios_base::sync_with_stdio(false); cin.tie(NULL);
    int t;
    cin>>t;
    while(t--){
        int a,b,c;
        char mode;
        int nn;
        cin>>nn;

        n=1;
        datas[0]=1;
        for(int i=0; i<nn; i++){
            cin>>mode;
            if(mode=='+'){
                n++;
                cin>>a>>b;
                datas[n-1]=b;
                edges[n-1].push_back(a-1);
                edges[a-1].push_back(n-1);
            }
            else{
                cin>>a>>b>>c;
                if(a<b) swap(a,b);
                qs.push_back({a-1,b-1},c);
            }
        }

        cnt=0;
        parent[0]=-1;
        top[0]=depth[0]=0;
        DFS(0,-1);
        DFS1(0);
        DFS2(0);
        s.build_seg(1,0,n-1);
        //cout<<"n in this : "<<n<<endl;
        for(int i=0; i<qs.size(); i++){
            auto res=query(qs[i].fi.fi,qs[i].fi.se);
            //cout<<"query "<<qs[i].fi.fi<<' '<<qs[i].fi.se<<" : "
            // <<res.t_mn<<' '<<res.t_mx<<endl;
            if(res.t_mn<=qs[i].se && res.t_mx>=qs[i].se)
                cout<<"YES"<<endl;

```



```

        else cout<<"NO"<<endl;
    }
    for(int i=0; i<n; i++) edges[i].clear();
    for(int i=0; i<n; i++) child[i].clear();
    qs.clear();
}
return 0;
}

```

## 2 Graph

### 2.1 Dijkstra

```

// Dijkstra algorithm
#include <bits/stdc++.h>
using namespace std;
#define N 200000

vector<pair<int,int>> edges[N]; // {idx, dist}
int min_dist[N];

void dijkstra(int s){
    priority_queue<pair<int,int>> pq;
    fill(min_dist,min_dist+N,-1);
    pq.push({0,s});
    while(!pq.empty()){
        int dist=-pq.top().first, idx=pq.top().second;
        pq.pop();
        if(min_dist[idx]!=-1) continue;
        min_dist[idx]=dist;
        for(int i=0; i<edges[idx].size(); i++){
            int next=edges[idx][i].first,
                n_dist=dist+edges[idx][i].second;
            if(min_dist[next]==-1) pq.push({-n_dist,next});
        }
    }
}

```

### 2.2 Least Common Ancestor

```

// LCA(sparse table)

#include <bits/stdc++.h>
#define endl '\n'

using namespace std;

```

```

#define N 100100
#define MAX 20 // Should satisfy (1<<(MAX-1)) >= N

vector<int> edges[N];
int sparse[N][MAX];
int level[N];

void DFS_init(int idx, int p){
    sparse[idx][0]=p;
    for(int i=0; i<edges[idx].size(); i++){
        int next=edges[idx][i];
        if(next!=p){
            level[next]=level[idx]+1;
            DFS_init(next,idx);
        }
    }
}

void sparse_init(int s){
    level[s]=0;
    DFS_init(s,-1);
    for(int r=1; r<MAX; r++){
        for(int i=0; i<N; i++){
            if(sparse[i][r-1]==-1) sparse[i][r]=-1;
            else sparse[i][r]=sparse[sparse[i][r-1]][r-1];
        }
    }
}

int get_LCA(int a, int b){
    if(level[a]>level[b]) swap(a,b);
    for(int r=MAX-1; r>=0; r--){
        if(sparse[b][r]!=-1 && level[a]<=level[sparse[b][r]])
            b=sparse[b][r];
    }
    if(a==b) return a;
    for(int r=MAX-1; r>=0; r--){
        if(sparse[a][r]!=-1 && sparse[b][r]!=-1
            && sparse[b][r]!=sparse[a][r]){
            a=sparse[a][r], b=sparse[b][r];
        }
    }
    return sparse[a][0];
}

int main(){
    ios_base::sync_with_stdio(false); cin.tie(NULL);
}

```

```

int n,m,a,b;
cin>>n;
for(int i=0; i<n-1; i++){
    cin>>a>>b;
    a--, b--;
    edges[a].push_back(b);
    edges[b].push_back(a);
}
sparse_init(0);
cin>>m;
for(int i=0; i<m; i++){
    cin>>a>>b;
    cout<<get_LCA(a-1,b-1)+1<<endl;
}
}

```

## 2.3 2-SAT

```

// 2-SAT
#include <bits/stdc++.h>
#define N 200000 // Upper bound for number of nodes in graph

using namespace std;

// For scc
vector<int> edges[N];
vector<int> r_edges[N];
int scc[N];
int order[N];
bool visited[N];
int cnt, scc_cnt; // Number of SCC's
int n; // Number of actual nodes in graph

// For 2-SAT
vector<int> r_scc_edges[N];
// reverse edges connecting sccs (in scc index)
vector<int> scc_element[N]; // Nodes of each scc
bool scc_fixed[N];
// whether the value of scc determined (initially set false)
bool scc_state[N]; // Determined scc value
inline int t_idx(int i){return i<<1;}
inline int f_idx(int i){return (i<<1)+1;}
inline int inv_idx(int i){return i^1;}

// Set n before call init function.
void init_scc(){
    for(int i=0; i<n; i++) {
        edges[i].clear(), r_edges[i].clear();
    }
}

```

```

    }
}

void init_sat(){
    init_scc();
    for(int i=0; i<n; i++) {
        r_scc_edges[i].clear(), scc_element[i].clear();
    }
    fill(scc_fixed, scc_fixed+n, false);
    fill(scc_fixed, scc_fixed+n, true);
}

// Build SAT edge for prop A or B // a=t_idx(A). b=t_idx(B);
// A and B could be same. Which means A is true.
// Manually insert edges[b].push_back(inv_idx(b));
// where B is false and b=t_idx(B)
void sat_edge(int a, int b){
    edges[inv_idx(a)].push_back(b);
    r_edges[b].push_back(inv_idx(a));
    if(a!=b){
        edges[inv_idx(b)].push_back(a);
        r_edges[a].push_back(inv_idx(b));
    }
}

void DFS(int idx){
    visited[idx]=true;
    for(int i=0; i<edges[idx].size(); i++)
        if(!visited[edges[idx][i]]) DFS(edges[idx][i]);
    order[cnt]=idx; cnt++;
}

void r_DFS(int idx){
    visited[idx]=true;
    scc[idx]=cnt;
    for(int i=0; i<r_edges[idx].size(); i++)
        if(!visited[r_edges[idx][i]]) r_DFS(r_edges[idx][i]);
}

// Kosaraju's algorithm.
// Set n, edges, r_edges before call this function.
void get_scc(){
    fill(visited, visited+n, false), cnt=0;
    for(int i=0; i<n; i++) if(!visited[i]) DFS(i);
    fill(visited, visited+n, false), cnt=0;
    for(int i=n-1; i>=0; i--)
        if(!visited[order[i]]) {r_DFS(order[i]); cnt++;}
}

```

```

    scc_cnt=cnt;
}

// Update answers of 2-sat to scc_state.
// fill(visited, visited+scc_cnt, false); before call this function.
// Set scc_cnt, r_scc_edges and scc_element before call this function.
bool r_scc_DFS(int idx, bool state){
    visited[idx]=true;
    bool new_state=false;
    for(int i=0; i<r_scc_edges[idx].size(); i++){
        if(!visited[r_scc_edges[idx][i]]){
            new_state=new_state
            ||r_scc_DFS(r_scc_edges[idx][i], state);
        }
    }
    if(!scc_fixed[idx]){
        scc_fixed[idx]=true;
        scc_state[idx]=new_state;
    }
    else new_state=scc_state[idx];

    for(int i=0; i<scc_element[idx].size(); i++){
        int cur=scc_element[idx][i];
        int inv_scc=scc[inv_idx(cur)];
        scc_fixed[inv_scc]=true;
        scc_state[inv_scc]=!scc_state[idx];
    }
    return new_state;
}

```

## 2.4 SCC

```

// 2-SAT
#include <bits/stdc++.h>
#define N 200000 // Upper bound for number of nodes in graph

using namespace std;

// For scc
vector<int> edges[N];
vector<int> r_edges[N];
int scc[N];
int order[N];
bool visited[N];
int cnt, scc_cnt; // Number of SCC's
int n; // Number of actual nodes in graph

```

```

// Set n before call init function.
void init_scc(){
    for(int i=0; i<n; i++) {edges[i].clear(), r_edges[i].clear();}
}

void DFS(int idx){
    visited[idx]=true;
    for(int i=0; i<edges[idx].size(); i++)
        XZBCVN,if(!visited[edges[idx][i]]) DFS(edges[idx][i]);
    order[cnt]=idx; cnt++;
}

void r_DFS(int idx){
    visited[idx]=true;
    scc[idx]=cnt;
    for(int i=0; i<r_edges[idx].size(); i++)
        if(!visited[r_edges[idx][i]]) r_DFS(r_edges[idx][i]);
}

// Kosaraju's algorithm.
// Set n, edges, r_edges before call this function.
void get_scc(){
    fill(visited, visited+n, false), cnt=0;
    for(int i=0; i<n; i++)
        if(!visited[i]) DFS(i);
    fill(visited, visited+n, false), cnt=0;
    for(int i=n-1; i>=0; i--)
        if(!visited[order[i]]) {r_DFS(order[i]); cnt++;}
    scc_cnt=cnt;
}

```

## 2.5 Centroid

```

// Centroid
#include <bits/stdc++.h>
#define N 200000
using namespace std;

vector<int> edges[N];
bool visited[N];
int sub_sz[N];
long long cnt;

int sz_DFS(int idx, int p){
    int result=1;
    for(int i=0; i<edges[idx].size(); i++){

```

```

        if(edges[idx][i]!=p && !visited[edges[idx][i]]){
            result+=sz_DFS(edges[idx][i], idx);
        }
    }
    return sub_sz[idx]=result;
}

int find_centroid(int idx, int p, int tot_sz){
    int result=idx;
    for(int i=0; i<edges[idx].size(); i++){
        int next=edges[idx][i];
        if(next!=p && !visited[edges[idx][i]]
            && sub_sz[next]>tot_sz/2){
            result=find_centroid(next,idx,tot_sz);
            break;
        }
    }
    return result;
}

void main_DFS(int idx, int p){
    int tot_sz=sz_DFS(idx,p);
    if(tot_sz==1) return;
    int cent=find_centroid(idx,p,tot_sz);
    visited[cent]=true;

    // Do something!

    for(int i=0; i<edges[cent].size(); i++){
        int next=edges[cent][i];
        if(!visited[next]) main_DFS(next,cent);
    }
    return;
}

```

### 3 Flows / Matchings

#### 3.1 Min Cost Max Flow

```

#include <bits/stdc++.h>

#define N 1000
#define INF 1000000007

using namespace std;

vector<pair<int,int>> edges[N]; // next, cost.

```

```

// Must initialize after each testcase
int capa[N][N];
int flow[N][N];
bool inQ[N];
// (inQ[i]) means dist[i] changed ans need to be visited.
int pre[N];
int dist[N];

void add_edge(int a, int b, int w, int f){
    // a->b cost:w capa:f (Reverse edge has negative weight)
    edges[a].push_back({b,w}); capa[a][b]=f; flow[a][b]=0;
    edges[b].push_back({a,-w}); capa[b][a]=0; flow[b][a]=0;
}

void floyd_warshall(int s){
    // Only update pre[], dist[]
    fill(inQ, inQ+N, false);
    fill(dist, dist+N, INF);
    int ff=INF;
    queue<int> q;
    q.push(s);
    inQ[s]=true; // To maintain light queue
    dist[s]=0;
    while(!q.empty()){
        int idx=q.front(); inQ[idx]=false;
        q.pop();
        for(int i=0; i<edges[idx].size(); i++){
            int next=edges[idx][i].first, w=edges[idx][i].second;
            if(capa[idx][next]-flow[idx][next]>0
                && dist[idx]+w<dist[next]){
                dist[next]=dist[idx]+w;
                pre[next]=idx;
                if(!inQ[next]) q.push(next);
                inQ[next]=true;
            }
        }
    }
}

int MCMF(int s, int t){
    // Return minimum cost for maximum flow
    int res=0;
    while(true){
        floyd_warshall(s);
        if(dist[t]==INF) break;
        // Get flow
    }
}

```

```

    int cur=t, ff=INF;
    while(cur!=s){
        ff=min(ff, capa[pre[cur]][cur]-flow[pre[cur]][cur]);
        cur=pre[cur];
    }
    // Flow update
    cur=t;
    while(cur!=s){
        flow[pre[cur]][cur]+=ff;
        flow[cur][pre[cur]]-=ff;
        cur=pre[cur];
    }
    res+=dist[t]*ff;
}
return res;
}

```

### 3.2 Dinic

```

#include <bits/stdc++.h>

using namespace std;

#define INF 1000000007
#define N 10000

vector<int> edges[N];
int flow[N][N]; // Use map if N is larger than 20000
int capa[N][N];
int levels[N];
bool visited[N];
int work[N];

void add_edge(int a, int b, int c){ // edge a->b (c)
    // this is NOT bidirectional!
    edges[a].push_back(b); flow[a][b]=0; capa[a][b]=c;
    edges[b].push_back(a); flow[b][a]=0; capa[b][a]=0;
    // If capa accumulate,
    // edges[a].push_back(b); flow[a][b]=0; capa[a][b]=c;
    // edges[b].push_back(a); flow[b][a]=0; capa[b][a]=0;
}

void level_BFS(int s){
    fill(visited, visited+N, false);
    queue<pair<int, int>> q;
    q.push({s, 0});
    while(!q.empty()){
        int idx=q.front().first;

```

```

        int level=q.front().second;
        q.pop();
        if(visited[idx]) continue;
        levels[idx]=level;
        visited[idx]=true;
        for(int i=0; i<edges[idx].size(); i++){
            int next=edges[idx][i];
            if(!visited[next] && capa[idx][next]>flow[idx][next]){
                q.push({next, level+1});
            }
        }
    }
}

int main_DFS(int idx, int f, int t){
    visited[idx]=true;
    if(idx==t || f==0) return f;
    for(int &i=work[idx]; i<edges[idx].size(); i++){
        int next=edges[idx][i];
        if(!visited[next] && levels[next]==levels[idx]+1
            && capa[idx][next]>flow[idx][next]){
            int f_temp=main_DFS(next,
                min(f, capa[idx][next]-flow[idx][next]), t);
            if(f_temp==0) continue;
            flow[idx][next]+=f_temp;
            flow[next][idx]-=f_temp;
            return f_temp;
        }
    }
    return 0;
}

int DINIC(int s, int t){
    int result=0;
    while(true){
        levels[t]=-1;
        level_BFS(s);
        if(levels[t]==-1) break;

        fill(work, work+N, 0);
        while(true){
            fill(visited, visited+N, false);
            int f_temp=main_DFS(s, INF, t);
            if(f_temp==0) break;
            result+=f_temp;
        }
    }
}

```

```

    return result;
}

// 유량이 남은 모든 정점을 탐색
void cut_DFS(int idx){
    if(visited[idx]) return;
    visited[idx]=true;
    for(int i=0; i<edges[idx].size(); i++){
        int next=edges[idx][i];
        if(!visited[next] && capa[idx][next]>flow[idx][next])
            cut_DFS(next);
    }
}

```

### 3.3 Bipartite matching

```

// Bipartite matching
#include <bits/stdc++.h>
using namespace std;

#define N 100000

vector<int> edges[N];
vector<int> rev_edges[N]; // To find maximum independent set.
int pre[N];
bool visited[N];

bool DFS(int idx){
    // Bipartite matching
    if(visited[idx]) return false;
    visited[idx]=true;
    for(int i=0; i<edges[idx].size(); i++){
        int next=edges[idx][i];
        if(pre[next]==-1
            || (!visited[pre[next]] && DFS(pre[next]))){
            pre[next]=idx;
            return true;
        }
    }
    return false;
}

// fill(visited, visited+n, 0); 을 매번 해줘야 한다.
// N^2이 나오는 게 정상
// if(DFS(i)) result++; 이렇게 유량을 구함.

```

## 4 Strings

### 4.1 Aho Corasic

```

// Aho-corasic
// Upper-case code임에 주의.

#include <bits/stdc++.h>
using namespace std;

#define NUM_ALPHA 26
#define N 10000

class node{
public:
    node* next[NUM_ALPHA]={NULL,};
    node* pphi; // parent of phi ; i.e. 최대 접미사의 가장 마지막 노드.
    // 다음 원소는 직접 찾아야 함.
    int val; // 부모를 통해 찾아줘도 된다. // 0-based idx of character
    bool end_of_snippet=false;
    // End node of snippet. Propagated in BFS phase.
    int length_of_snippet;
    // Only valid if end of snippet is true. Propagated in BFS phase.
    int depth=0; // Actual length of the sequence
    bool end=true; // All next pointers are NULL
    node(){
        node(int v){
            val=v;
        }
    };
};

class aho_trie{
public:
    node* root= new node();
    aho_trie(){
        bool is_s(string &s){
            return is_re(root, 0, s);
        }
        void insert_s(string &s){
            insert_re(root, 0, s);
        }
        void clear(){ // clear_all
            clear_re(root);
            for(int i=0; i<NUM_ALPHA; i++) root->next[i]=NULL;
        }
        void get_phi(){
            queue<pair<node*,node*>> phi_q; // cur_pos, pphi
            phi_q.push({root,root});

```

```

while(!phi_q.empty()){
    pair<node*,node*> temp=phi_q.front();
    phi_q.pop();
    node* cur=temp.first;
    node* ptar=temp.second;
    if(cur->depth>1){ // To prevent phi[self]=self
        while(ptar!=root && (ptar->next[cur->val]==NULL))
            ptar=ptar->pphi;
        if(ptar->next[cur->val]!=NULL)
            ptar=ptar->next[cur->val];
    }
    cur->pphi=ptar;
    if(!cur->end_of_snippet && ptar->end_of_snippet){
        cur->end_of_snippet=true;
        cur->length_of_snippet=ptar->length_of_snippet;
    }

    for(int i=0; i<NUM_ALPHA; i++){
        if((cur->next[i])!=NULL){
            phi_q.push({cur->next[i],ptar});
        }
    }
}

void find_matching(string &s_ref, vector<int> &ans){
    int n=s_ref.size();
    ans.clear();
    node* ptar=root;
    for(int i=0; i<n; i++){
        int cur=s_ref[i]-'A'; // transition function
        while(ptar!=root && ptar->next[cur]==NULL)
            ptar=ptar->pphi;
        if(ptar->next[cur]!=NULL) ptar=ptar->next[cur];
        //cout<<(char)(ptar->val+'A')<<' '<<ptar->cnt<<'
        //<<ptar->depth<<endl;
        if(ptar->end_of_snippet)
            ans.push_back(i+1-(ptar->length_of_snippet));
        if(ptar->end) ptar=ptar->pphi;
    }
}

private:
bool is_re(node *cur, int idx, string &s){
    if(idx==s.size()) return (cur->end_of_snippet);
    if((cur->next[s[idx]-'A'])==NULL) return false;
    return is_re(cur->next[s[idx]-'A'],idx+1,s);
}

```

```

void insert_re(node* cur, int idx, string &s){
    if(idx==s.size()){
        cur->end_of_snippet=true;
        cur->length_of_snippet=s.size();
        return;
    }
    cur->end=false;
    if((cur->next[s[idx]-'A'])==NULL){
        cur->next[s[idx]-'A']=new node(s[idx]-'A');
        cur->next[s[idx]-'A']->depth=idx+1;
    }
    insert_re(cur->next[s[idx]-'A'],idx+1,s);
}

void clear_re(node* cur){
    for(int i=0; i<NUM_ALPHA; i++){
        if((cur->next[i])!=NULL){
            clear_re(cur->next[i]);
            delete cur->next[i];
        }
    }
}

};

aho_trie aho;

```

## 4.2 KMP

```

#include <bits/stdc++.h>
using namespace std;

```

```

#define N 2020202
string s_ref;
string s_sni;
int phi[N];
vector<int> match_res;

```

```

void get_phi(){
    int m=s_sni.size();
    phi[0]=0;
    for(int i=1, j=0; i<m; i++){
        while(j>0 && s_sni[i]!=s_sni[j]) j=phi[j-1];
        if(s_sni[i]==s_sni[j]) j++;
        phi[i]=j;
    }
}

void KMP(){
    int n=s_ref.size(), m=s_sni.size();

```

```

match_res.clear();
for(int i=0, j=0; i<n; i++){
    while(j!=0 && s_ref[i]!=s_sni[j]) j=phi[j-1];
    if(s_ref[i]==s_sni[j]) j++;
    if(j==m) match_res.push_back(i+1-m), j=phi[j-1];
}
}

```

### 4.3 Trie

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
#define NUM_ALPHA 26
```

```

class node{
public:
    node* next[NUM_ALPHA]={NULL,};
    char val; // 대부분 필요없다.
    int cnt=0; // 여기서 끝나는 단어들의 개수
    node(){
    }
    node(char c){
        val=c;
    }
};

```

```

class trie{
public:
    node* root= new node();
    trie(){
    }
    bool is_s(string &s){
        return is_re(root, 0, s);
    }
    void insert_s(string &s){
        insert_re(root, 0, s);
    }
    void clear(){ // clear_all
        clear_re(root);
        for(int i=0; i<NUM_ALPHA; i++) root->next[i]=NULL;
    }

```

```

private:
    bool is_re(node *cur, int idx, string &s){
        if(idx==s.size()) return ((cur->cnt)>=1);
        // 여기까지 왔으면 그냥 true해도 되지 않나?
        // 처음에 들어올 때 때문인가
        if((cur->next[s[idx]-'A'])==NULL) return false;

```

```

        return is_re(cur->next[s[idx]-'A'], idx+1, s);
    }
    void insert_re(node* cur, int idx, string &s){
        if(idx==s.size()){
            cur->cnt++;
            return;
        }
        if((cur->next[s[idx]-'A'])==NULL)
            cur->next[s[idx]-'A']=new node(s[idx]);
        insert_re(cur->next[s[idx]-'A'], idx+1, s);
    }
    void clear_re(node* cur){
        for(int i=0; i<NUM_ALPHA; i++){
            if((cur->next[i])!=NULL){
                clear_re(cur->next[i]);
                delete cur->next[i];
            }
        }
    }
};

```

```
trie tr;
```

### 4.4 Suffix LCP

//  $LCP[1] + \dots + LCP[N-1]$  은 모든 중복되는 substring의 개수

```

// result <- result + max(0, prev-LCP[i])
// prev <- LCP[i]
// 이려면 중복되는 substring의 종류를 얻는다.

```

```
#include <bits/stdc++.h>
```

```
using namespace std;
```

```
#define N 200000
```

```
string s;
```

```
int g[N+1], temp_g[N]; // g[i]는 정렬 전 i번째 접미사가 접근하는 group ordering
```

```
int t; // 전역변수
```

```
int sa[N], order[N]; // inv_sa==order of suffix array
```

```
int LCP[N];
```

```

bool compare(int i, int j){
    if(g[i]==g[j]) return g[i+t]<g[j+t];
    else return g[i]<g[j];
}

```

// SA[i]는 정렬된 suffix중 i번째의 것이 원래의 suffix중 몇 번째냐



// order[i]는 SA[i]의 역함수인데, i번째의 원래 suffix가 정렬되면 몇 번째로 들어가냐.

```
void suffix_sort(const string &s){
    int n=s.size();
    for(int i=0; i<n; i++) sa[i]=i, g[i]=s[i]; // 대소관계만 중요
    g[n]=-1, temp_g[0]=0;
    for(t=1; t<n; t<=<=1){
        sort(sa,sa+n,compare);
        for(int i=1; i<n; i++){
            // sa[i]는 앞에서 2t번째로 정렬된 것을 의미하는 듯 함.
            // temp_g는 지금까지 밝혀진 정보로도 확실히 크다면 +1,
            // 아니면 이전 값과 동일.
            temp_g[sa[i]]=temp_g[sa[i-1]]+compare(sa[i-1],sa[i]);
        }
        for(int i=0; i<n; i++) g[i]=temp_g[i];
    }
    for(int i=0; i<n; i++) order[sa[i]]=i;
}

void get_lcp(const string &s){
    int n=s.size();
    for(int i=0, j=0; i<n; i++){
        if(order[i]!=0){
            // i는 이제 원래 idx임.
            // sa[order[i]]와 sa[order[i]-1]의 최대 일치 -> LCP[order[i]]
            // 원래 idx i의 정렬했을 때 바로 앞의 suffix의 원래 idx가 pre임.
            // LCP는 늘어나는 건 많이 늘어날 수 있지만, LCP가 줄어드는 일은 최대 1만큼만.
            int pre=sa[order[i]-1];
            while(max(i+j,pre+j)<n && s[i+j]==s[pre+j]) j++;
            LCP[order[i]]=j;
            j=max(j-1,0);
        }
    }
}
```

## 5 Mathematics

### 5.1 Prime

```
// 에라토스테네스의 체
#include <bits/stdc++.h>
#define endl '\n'
#define cediv(a,b) ((a)%(b)==0?((a)/(b)):((a)/(b))+1)
#define fi first
#define se second
#define pb push_back
```

```
using namespace std;

typedef long long ll;
typedef unsigned int ui;
typedef unsigned long long ull;

template<typename T>
inline T umax(T &u, T v){return u = max(u, v);}
template<typename T>
inline T umin(T &u, T v){return u = min(u, v);}

#define N_PRIME 1000010

bool is_prime[N_PRIME];
vector<int> primes;

void build_prime(int n){
    // n<N_PRIME
    fill(is_prime+2,is_prime+n+1,true);
    for(int i=2; i<=n; i++){
        if(!is_prime[i]) continue;
        for(int j=2; i*j<=n; j++) is_prime[i*j]=false;
    }
    for(int i=2; i<=n; i++) if(is_prime[i]) primes.push_back(i);
}

// 소인수분해 코드
int datas[100000];
int cnt[100000];
// 1~1e6까지 소수가 약 7만개 수준 1~1e7까지는 70만개 수준

int main(){
    int n;
    cin>>n;
    for(int i=0; i<n; i++) cin>>datas[i];

    // 다음 코드는 datas[0]*datas[1]*...*datas[n-1]의 소인수분해를 한다.
    // behavior를 바꾸고 싶으면 cnt[j]값을 더하는 대신에
    // cnt[i].push_back(j); 등으로 바꾸면 됨.
    for(int i=0; i<n; i++){
        int temp=datas[i];
        for(int j=0; j<primes.size(); j++){
            if(primes[j]>sqrt(temp) || primes[j]>=temp) break;
            while(temp%primes[j] && temp%primes[j]==0){
                temp/=primes[j];
                cnt[j]++;
            }
        }
    }
}
```

```

    }
    // 그럼 이제 지금 temp는 소수인 상황
    int idx=lower_bound(primes.begin(), primes.end(), temp)
        -primes.begin();
    cnt[idx]++;
}
return 0;
}

```

## 5.2 Arithmetic Inverse

```

// Arithmetic inverse
#include <bits/stdc++.h>
using namespace std;

#define MOD 998244353
#define MAX_DIGIT 62

// (ak)*(bk)^(-1) == a*b^(-1) == (ak%P)*(bk%P)^(-1) (mod P)
long long ari_inv(long long num){
    // Calculate num^(MOD-2)
    long long res=1, mult=num;
    for(int i=0; i<MAX_DIGIT; i++){
        if((MOD-2)&(1LL<<i)){
            res=(res*mult)%MOD;
        }
        mult=(mult*mult)%MOD;
    }
    return res;
}

```

## 5.3 Factorization

```

// Factorization
#include <bits/stdc++.h>
#define endl '\n'
#define cediv(a,b) ((a)%b==0?((a)/(b)):((a)/(b))+1)
#define fi first
#define se second
#define pb push_back

using namespace std;

typedef long long ll;
typedef unsigned int ui;
typedef unsigned long long ull;

template<typename T>

```

```

inline T umax(T &u, T v){return u = max(u, v);}
template<typename T>
inline T umin(T &u, T v){return u = min(u, v);}

vector<int> prime;
int prime_cnt[100];

void ppush(int num){
    if(prime.size()==0) prime.push_back(num);
    else if(prime[prime.size()-1]!=num) prime.push_back(num);
    prime_cnt[prime.size()-1]++;
}

void factorize(ll temp){
    while(temp!=1){
        ll end=sqrt(temp)+1;
        bool flag=false;
        for(int i=2; i<=end; i++){
            if(temp%i==0){
                ppush(i);
                temp=temp/i;
                flag=true;
                break;
            }
        }
        if(!flag){
            ppush(temp);
            break;
        }
    }
}

```

## 5.4 GCD

```

// gcd
#include <bits/stdc++.h>
#define endl '\n'
#define cediv(a,b) ((a)%b==0?((a)/(b)):((a)/(b))+1)
#define fi first
#define se second
#define pb push_back

using namespace std;

int gcd(int x, int y){
    if(x<y) swap(x,y);
    if(y==0) return x;
    if(x%y==0) return y;
    return gcd(y,x%y);
}

```

```
}
```

## 5.5 $nCr$

```
// nCr
#include <bits/stdc++.h>
#define MOD 1000000007
#define N 200000

using namespace std;

long long fact[N], f_inv[N];

void fill_fact(){
    fact[0]=1;
    for(int i=1; i<N; i++) fact[i]=fact[i-1]*i%MOD;
    fill(f_inv,f_inv+N,1);
    for(int k=0; k<30; k++){
        for(int i=0; i<N; i++){
            f_inv[i]=f_inv[i]*f_inv[i]%MOD;
            if((MOD-2)&(1<<(29-k))) f_inv[i]=f_inv[i]*fact[i]%MOD;
        }
    }
}

long long nCr(int n, int r){
    return ((fact[n]*f_inv[r])%MOD)*f_inv[n-r]%MOD;
}
```

## 5.6 FFT

```
// FFT
#include <bits/stdc++.h>
using namespace std;

typedef complex<double> cpx;
const int SZ = 1048576;

void FFT(cpx g[], bool inv = false){
    int n = SZ;
    for(int i = 1, j = 0; i < n; ++i){
        int b = n/2;
        while(!((j ^ b) & b)) b /= 2;
        if(i < j) swap(g[i], g[j]);
    }
    for(int k = 1; k < n; k *= 2){
        double a = (inv ? M_PI/k : -M_PI/k);
        cpx w(cos(a), sin(a));
```

```
        for(int i = 0; i < n; i += k*2){
            cpx wp(1, 0);
            for(int j = 0; j < k; ++j){
                cpx x = g[i+j], y = g[i+j+k] * wp;
                g[i+j] = x + y;
                g[i+j+k] = x - y;
                wp *= w;
            }
        }
    }
    if(inv){
        for(int i = 0; i < n; ++i)
            g[i] /= n;
    }
}
```

## 5.7 NTT

```
// NTT
#include <bits/stdc++.h>
#define endl '\n'
using namespace std;

typedef long long ll;

/* MOD = a * 2^b + 1 일 때, [a, b, Primitive root] 순서쌍
N으로 사용하는 값이 2^b보다 크면 사용할 수 없다.
998244353 [119, 23, 3]
2281701377 [17, 27, 3]
2483027969 [37, 26, 3]
2113929217 [63, 25, 5]
104857601 [25, 22, 3]
1092616193 [521, 21, 3]
*/

#define SZ 1<<18
#define MOD 998244353
#define AA 119
#define BB 23
#define PR 3
#define MAX_DIGIT 62

long long ari_inv(long long num){
    long long res=1, mult=num;
    for(ll i=0; i<MAX_DIGIT; i++){
        if((MOD-2)&(1LL<<i)){
            res=(res*mult)%MOD;
        }
    }
}
```

```

        mult=(mult*mult)%MOD;
    }
    return res;
}

// 출력이 음수가 될 수 있음!!
void NTT(ll g[], bool inv = false){
    ll n = SZ;
    ll w_pow[n+1];
    ll bit_cnt=0;
    for(; bit_cnt<=BB; bit_cnt++){
        if((n>>bit_cnt) == 1) break;
    }
    assert ((n>>bit_cnt)==1);

    //  $w^n \% MOD == 1$  이어야 함.
    //  $w = PR^{(p-1) / n} = PR^a$  ( $a = 2^{(b-bit\_cnt)}$ )
    ll w = 1;
    ll pow = AA<<(BB-bit_cnt);
    ll mul = PR;
    for(int i=0; i<MAX_DIGIT; i++){
        if(pow&(1LL<<i)) w=(w*mul)%MOD;
        mul=(mul*mul)%MOD;
    }
    w = (inv ? w : ari_inv(w));

    for(ll i = 1, j = 0; i < n; ++i){
        ll b = n/2;
        while(!((j ^ b) & b)) b /= 2;
        if(i < j) swap(g[i], g[j]);
    }

    ll cnt=0;
    for(ll k = 1; k < n; k *= 2){
        cnt++;
        ll ww = w; //  $ww^{(2*k)} \% MOD == 1$  이어야 함.
        //  $ww = w^{(n/(2*k))} = w^{(2^{(bit\_cnt-cnt)})}$ 
        for(ll i=0; i<bit_cnt-cnt; i++) ww=(ww*ww)%MOD;
        for(ll i=0; i<k; i++) w_pow[i]=(i==0?1:(w_pow[i-1]*ww)%MOD);
        for(ll i = 0; i < n; i += k*2){
            ll wp = 1;
            for(ll j = 0; j < k; ++j){
                ll x = g[i+j], y = g[i+j+k]*w_pow[j];
                //MOD*MOD가 ll로 표현된다면 중간과정에서 %MOD 필요없음.
                g[i+j] = (x + y);
                g[i+j+k] = (x - y);
            }
        }
    }
}

```

```

    }
    }
    for(int i=0; i<n; i++) g[i]%MOD;
    // 위의 for문 동안 한 번만 방문
}
if(inv){
    ll ari_n = ari_inv(n);
    for(ll i = 0; i < n; ++i)
        g[i] = (g[i]*ari_n)%MOD;
}
}

/*
// convolution (size(datas) = SZ >= 2*n 인 2의 거듭제곱)
NTT(datas1);
NTT(datas2);
for(ll i=0; i<SZ; i++) datas1[i]=(datas1[i]*datas2[i])%MOD;
NTT(datas1,true);
*/

```

## 5.8 NTT + Chinese Remainder Thm

```

// NTT + CRT
#include <bits/stdc++.h>
#define endl '\n'
using namespace std;

typedef long long ll;

/* MOD =  $a \cdot 2^b + 1$  일 때,  $[a, b, \text{Primitive root}]$  순서쌍
  N으로 사용하는 값이  $2^b$ 보다 크면 사용할 수 없다.
  998244353 [119, 23, 3]
  2281701377 [17, 27, 3]
  2483027969 [37, 26, 3]
  2113929217 [63, 25, 5]
  104857601 [25, 22, 3]
  1092616193 [521, 21, 3]
  */

ll SZ=1<<18;
#define MOD1 998244353
#define AA1 119
#define BB1 23
#define PR1 3
#define MOD2 2483027969
#define AA2 37
#define BB2 26

```

```
#define PR2 3
#define MAX_DIGIT 62
```

```
long long ari_inv(long long num, bool ver){
    long long res=1, mult=num;
    ll MOD = (ver?MOD1:MOD2);
    for(ll i=0; i<MAX_DIGIT; i++){
        if((MOD-2)&(1LL<<i)){
            res=(res*mult)%MOD;
        }
        mult=(mult*mult)%MOD;
    }
    return res;
}
```

```
void NTT(ll g[], bool ver, bool inv=false){
    ll MOD=(ver?MOD1:MOD2), AA=(ver?AA1:AA2), BB=(ver?BB1:BB2),
        PR=(ver?PR1:PR2);
    ll n = SZ;
    ll w_pow[n+1];
    ll bit_cnt=0;
    for(; bit_cnt<=BB; bit_cnt++){
        if((n>>bit_cnt) == 1) break;
    }
    assert ((n>>bit_cnt)==1);

    //  $w^n \% MOD == 1$  이어야 함.
    //  $w = PR^{((p-1)/n)} = PR^{(a*2^{(b-bit\_cnt)})}$ 
    ll w = 1;
    ll pow = AA<<(BB-bit_cnt);
    ll mul = PR;
    for(int i=0; i<MAX_DIGIT; i++){
        if(pow&(1LL<<i)) w=(w*mul)%MOD;
        mul=(mul*mul)%MOD;
    }
    w = (inv ? w : ari_inv(w,ver));

    for(ll i = 1, j = 0; i < n; ++i){
        ll b = n/2;
        while(!((j ^ b) & b)) b /= 2;
        if(i < j) swap(g[i], g[j]);
    }

    ll cnt=0;
    for(ll k = 1; k < n; k *= 2){
        cnt++;

```

```
        ll ww = w; //  $ww^{(2*k)} \% MOD == 1$  이어야 함.
        //  $ww=w^{(n/(2*k))}=w^{(2^{(bit\_cnt-cnt)})}$ 
        for(ll i=0; i<bit_cnt-cnt; i++) ww=(ww*ww)%MOD;
        for(ll i=0; i<k; i++) w_pow[i]=(i==0?1:(w_pow[i-1]*ww)%MOD);
        for(ll i = 0; i < n; i += k*2){
            ll wp = 1;
            for(ll j = 0; j < k; ++j){
                ll x = g[i+j], y = g[i+j+k]*w_pow[j];
                g[i+j] = (x + y);
                // MOD*MOD+MOD가 ll로 표현된다면 중간과정에서 %MOD 필요없음.
                g[i+j+k] = (x - y);
            }
        }
        for(int i=0; i<n; i++) g[i]%=MOD;
        // 위의 for문 동안 한 번만 방문
    }
    if(inv){
        ll ari_n = ari_inv(n,ver);
        for(ll i = 0; i < n; ++i)
            g[i] = (g[i]*ari_n)%MOD;
    }
}

inline ll large_mul(ll a, ll b){
    // 이것 때문에 TLE가 뜨진 않는다.
    ll MOD=(MOD1*MOD2);
    a=(a%MOD+MOD)%MOD;
    b=(b%MOD+MOD)%MOD;
    ll mul=a;
    ll res=0;
    for(int i=0; i<MAX_DIGIT; i++){
        if(b & (1LL<<i)) res=(res+mul)%MOD;
        mul=(mul*mul)%MOD;
    }
    return res;
}

/*
// for MOD1
NTT(datas1_M1, true, false);
NTT(datas2_M1, true, false);
for(ll i=0; i<SZ; i++) datas1_M1[i]=(datas1_M1[i]*datas2_M1[i])%MOD1;
NTT(datas1_M1, true, true);

// for MOD2
NTT(datas1_M2, false, false);
NTT(datas2_M2, false, false);

```

```
for(ll i=0; i<SZ; i++) datas1_M2[i]=(datas1_M2[i]*datas2_M2[i])%MOD2;
NTT(datas1_M2, false, true);
```

```
// Chinese Theorem
ll n1 = MOD2, n2=MOD1, MOD=(MOD1*MOD2);
ll s1 = ari_inv(MOD2, true), s2=ari_inv(MOD1, false);
for(int i=0; i<2*l; i++){
    datas1_M1[i] = (large_mul(datas1_M1[i]*n1,s1)
        + large_mul(datas1_M2[i]*n2,s2)) % MOD;
}
*/
```

## 5.9 Ternary Search

```
// Ternary search
#include <bits/stdc++.h>
using namespace std;

#define INF 1e18

long long f(long long x){
    long long res;
    // calculate f(x)
    return res;
}

long long ternary(long long s, long long e){
    // Return smallest x if there are multiple minimum values.
    while(3<=e-s){
        long long l=(s+s+e)/3, r=(s+e+e)/3;
        if(f(l)>f(r)) s=l;
        else e=r;
    }
    long long mx=INF, res;
    for(long long i=s; i<=e; i++){
        long long temp=f(i);
        if(mx>temp) mx=temp, res=i;
    }
    return res;
}
```

## 6 Geometry

### 6.1 Basic Operations

```
// Credit : PLEASE OPEN TESTDATA teamnote
inline int diff(double lhs, double rhs) {
```

```
    if (lhs - eps < rhs && rhs < lhs + eps) return 0;
    return (lhs < rhs) ? -1 : 1;
}

inline bool is_between(double check, double a, double b) {
    if (a < b)
        return (a - eps < check && check < b + eps);
    else
        return (b - eps < check && check < a + eps);
}

struct Point {
    double x, y;
    bool operator==(const Point& rhs) const {
        return diff(x, rhs.x) == 0 && diff(y, rhs.y) == 0;
    }
    Point operator+(const Point& rhs) const {
        return Point{ x + rhs.x, y + rhs.y };
    }
    Point operator-(const Point& rhs) const {
        return Point{ x - rhs.x, y - rhs.y };
    }
    Point operator*(double t) const {
        return Point{ x * t, y * t };
    }
};

struct Circle {
    Point center;
    double r;
};

struct Line {
    Point pos, dir;
};

inline double inner(const Point& a, const Point& b) {
    return a.x * b.x + a.y * b.y;
}

inline double outer(const Point& a, const Point& b) {
    return a.x * b.y - a.y * b.x;
}

inline int ccw_line(const Line& line, const Point& point) {
    return diff(outer(line.dir, point - line.pos), 0);
}

inline int ccw(const Point& a, const Point& b, const Point& c) {
    return diff(outer(b - a, c - a), 0);
}
```

```

}
inline double dist(const Point& a, const Point& b) {
    return sqrt(inner(a - b, a - b));
}
inline double dist2(const Point &a, const Point &b) {
    return inner(a - b, a - b);
}

inline double dist(const Line& line, const Point& point, bool segment = false) {
    double c1 = inner(point - line.pos, line.dir);
    if (segment && diff(c1, 0) <= 0) return dist(line.pos, point);
    double c2 = inner(line.dir, line.dir);
    if (segment && diff(c2, c1) <= 0) return dist(line.pos + line.dir, point);
    return dist(line.pos + line.dir * (c1 / c2), point);
}

bool get_cross(const Line& a, const Line& b, Point& ret) {
    double mdet = outer(b.dir, a.dir);
    if (diff(mdet, 0) == 0) return false;
    double t2 = outer(a.dir, b.pos - a.pos) / mdet;
    ret = b.pos + b.dir * t2;
    return true;
}

bool get_segment_cross(const Line& a, const Line& b, Point& ret) {
    double mdet = outer(b.dir, a.dir);
    if (diff(mdet, 0) == 0) return false;
    double t1 = -outer(b.pos - a.pos, b.dir) / mdet;
    double t2 = outer(a.dir, b.pos - a.pos) / mdet;
    if (!is_between(t1, 0, 1) || !is_between(t2, 0, 1)) return false;
    ret = b.pos + b.dir * t2;
    return true;
}

Point inner_center(const Point &a, const Point &b, const Point &c) {
    double wa = dist(b, c), wb = dist(c, a), wc = dist(a, b);
    double w = wa + wb + wc;
    return Point{ (wa * a.x + wb * b.x + wc * c.x) / w, (wa * a.y + wb * b.y + wc * c.y) / w };
}

Point outer_center(const Point &a, const Point &b, const Point &c) {
    Point d1 = b - a, d2 = c - a;
    double area = outer(d1, d2);
    double dx = d1.x * d1.x * d2.y - d2.x * d2.x * d1.y
        + d1.y * d2.y * (d1.y - d2.y);
    double dy = d1.y * d1.y * d2.x - d2.y * d2.y * d1.x
        + d1.x * d2.x * (d1.x - d2.x);
}

```

```

    return Point{ a.x + dx / area / 2.0, a.y - dy / area / 2.0 };
}

vector<Point> circle_line(const Circle& circle, const Line& line) {
    vector<Point> result;
    double a = 2 * inner(line.dir, line.dir);
    double b = 2 * (line.dir.x * (line.pos.x - circle.center.x)
        + line.dir.y * (line.pos.y - circle.center.y));
    double c = inner(line.pos - circle.center, line.pos - circle.center)
        - circle.r * circle.r;
    double det = b * b - 2 * a * c;
    int pred = diff(det, 0);
    if (pred == 0)
        result.push_back(line.pos + line.dir * (-b / a));
    else if (pred > 0) {
        det = sqrt(det);
        result.push_back(line.pos + line.dir * ((-b + det) / a));
        result.push_back(line.pos + line.dir * ((-b - det) / a));
    }
    return result;
}

vector<Point> circle_circle(const Circle& a, const Circle& b) {
    vector<Point> result;
    int pred = diff(dist(a.center, b.center), a.r + b.r);
    if (pred > 0) return result;
    if (pred == 0) {
        result.push_back((a.center * b.r + b.center * a.r) * (1 / (a.r + b.r)));
        return result;
    }
    double aa = a.center.x * a.center.x + a.center.y * a.center.y - a.r * a.r;
    double bb = b.center.x * b.center.x + b.center.y * b.center.y - b.r * b.r;
    double tmp = (bb - aa) / 2.0;
    Point cdiff = b.center - a.center;
    if (diff(cdiff.x, 0) == 0) {
        if (diff(cdiff.y, 0) == 0)
            return result; // if (diff(a.r, b.r) == 0): same circle
        return circle_line(a, Line{ Point{ 0, tmp / cdiff.y }, Point{ 1, 0 } });
    }
    return circle_line(a,
        Line{ Point{ tmp / cdiff.x, 0 }, Point{ -cdiff.y, cdiff.x } });
}

Circle circle_from_3pts(const Point& a, const Point& b, const Point& c) {
    Point ba = b - a, cb = c - b;
    Line p{ (a + b) * 0.5, Point{ ba.y, -ba.x } };
    Line q{ (b + c) * 0.5, Point{ cb.y, -cb.x } };
}

```

```

    Circle circle;
    if (!get_cross(p, q, circle.center))
        circle.r = -1;
    else
        circle.r = dist(circle.center, a);
    return circle;
}

Circle circle_from_2pts_rad(const Point& a, const Point& b, double r) {
    double det = r * r / dist2(a, b) - 0.25;
    Circle circle;
    if (det < 0)
        circle.r = -1;
    else {
        double h = sqrt(det);
        // center is to the left of a->b
        circle.center = (a + b) * 0.5 + Point{ a.y - b.y, b.x - a.x } * h;
        circle.r = r;
    }
    return circle;
}

```

## 6.2 Convex Hull

```

// Convex hull
#include <bits/stdc++.h>
#define fi first
#define se second
using namespace std;
typedef long long ll;
#define N 100000

pair<ll,ll> datas[N]; // Vertices
// Convex hull algorithm never permute initial vertices.
int sorted_idx[N]; // Idx of 'sorted' datas.
int ans[N]; // Elements of convex hull

inline bool CCW(const pair<ll,ll>& a,
               const pair<ll,ll>& b, const pair<ll,ll>& c){
    return (b.fi-a.fi)*(c.se-a.se)-(b.se-a.se)*(c.fi-a.fi)>0;
    // 변 위의 점도 필요하다면 >=
}

inline long long dist(const pair<ll,ll>& a,const pair<ll,ll>& b){
    return (a.fi-b.fi)*(a.fi-b.fi)+(a.se-b.se)*(a.se-b.se);
}

class comp_c{
public:

```

```

    pair<ll,ll> mp;
    comp_c(pair<ll,ll> minpos){
        mp=minpos;
    }
    bool operator()(int a_idx, int b_idx){
        pair<ll,ll> a=datas[a_idx], b=datas[b_idx];
        ll com=(a.fi-mp.fi)*(b.se-mp.se)-(a.se-mp.se)*(b.fi-mp.fi);
        if(com==0) return dist(a,mp)<dist(b,mp);
        else return com>0;
    }
};

int convex_hull(int n){ // # number of vertices
    // Phase 1: Sort
    int m_idx=0;
    for(int i=1; i<n; i++) if(datas[m_idx]>datas[i]) m_idx=i;
    for(int i=0; i<n; i++) sorted_idx[i]=i;
    swap(sorted_idx[0], sorted_idx[m_idx]);
    sort(sorted_idx,sorted_idx+n,comp_c(datas[m_idx]));

    // Phase 2: Get Convexhull
    int st[N]; // Elements of st are the idx of datas
    int cur=0, st_cnt=0;
    // cur : # of visited elements of s,
    // st_cnt : # of elements in stack
    while(cur<n){
        while(st_cnt>=2 && !CCW(datas[st[st_cnt-2]],
                               datas[st[st_cnt-1]],datas[sorted_idx[cur]])) st_cnt--;
        st[st_cnt++]=sorted_idx[cur++];
    }
    // Counter clockwise, starts with leftmost vertices.
    for(int i=0; i<st_cnt; i++) ans[i]=st[i];
    return st_cnt;
}

```

## 6.3 Rotating Calipers

```

// Rotating calipers
#include <bits/stdc++.h>
#define fi first
#define se second

using namespace std;
typedef long long ll;

#define N 100000

pair<ll,ll> datas[N]; // Vertices

```



```
// Convex hull algorithm never permute initial vertices.
int sorted_idx[N]; // Idx of 'sorted' datas.
int ans[N]; // Elements of convex hull

inline bool CCW(const pair<ll,ll>& a, const pair<ll,ll>& b,
               const pair<ll,ll>& c){
    return (b.fi-a.fi)*(c.se-a.se)-(b.se-a.se)*(c.fi-a.fi)>0;
    // 변 위의 점도 필요하다면 >=
}

inline bool CCW_4(const pair<ll,ll>& a,
                 const pair<ll,ll>& na, const pair<ll,ll>& b, pair<ll,ll> &nb){
    return (na.fi-a.fi)*(nb.se-b.se)-(na.se-a.se)*(nb.fi-b.fi)>=0;
}

inline long long dist(const pair<ll,ll>& a, const pair<ll,ll>& b){
    return (a.fi-b.fi)*(a.fi-b.fi)+(a.se-b.se)*(a.se-b.se);
}

class comp_c{
public:
    pair<ll,ll> mp;
    comp_c(pair<ll,ll> minpos){
        mp=minpos;
    }
    bool operator()(int a_idx, int b_idx){
        pair<ll,ll> a=datas[a_idx], b=datas[b_idx];
        ll com=(a.fi-mp.fi)*(b.se-mp.se)-(a.se-mp.se)*(b.fi-mp.fi);
        if(com==0) return dist(a,mp)<dist(b,mp);
        else return com>0;
    }
};

int convex_hull(int n){ // # number of vertices
    // Phase 1: Sort
    int m_idx=0;
    for(int i=1; i<n; i++) if(datas[m_idx]>datas[i]) m_idx=i;
    for(int i=0; i<n; i++) sorted_idx[i]=i;
    swap(sorted_idx[0], sorted_idx[m_idx]);
    sort(sorted_idx,sorted_idx+n,comp_c(datas[m_idx]));

    // Phase 2: Get Convexhull
    int st[N]; // Elements of st are the idx of datas
    int cur=0, st_cnt=0;
    // cur : # of visited elements of s,
    // st_cnt : # of elements in stack
    while(cur<n){
        while(st_cnt>=2 && !CCW(datas[st[st_cnt-2]],
                               datas[st[st_cnt-1]],datas[sorted_idx[cur]])) st_cnt--;

```

```
        st[st_cnt++]=sorted_idx[cur++];
    }
    for(int i=0; i<st_cnt; i++) ans[i]=st[i];
    // Counter clockwise, starts with leftmost vertices.
    return st_cnt;
}

pair<int,int> rotating_calipers(int sz_hull){
    // # of vertices in convex hull.
    // ans[] must be filled before call this function.
    int mx_a, mx_b; // mx_a and mx_b are the idx of two vertices
                    // making the maximum distance.
    long long mx=0;
    for(int a=0, p=0; a<sz_hull; a++){
        while(p+1<sz_hull && CCW_4(datas[ans[a]], datas[ans[a+1]],
                                   datas[ans[p]], datas[ans[p+1]])){
            ll temp=dist(datas[ans[a]], datas[ans[p]]);
            if(mx<temp) mx=temp, mx_a=ans[a], mx_b=ans[p];
            p++;
        }
        ll temp=dist(datas[ans[a]], datas[ans[p]]);
        if(mx<temp) mx=temp, mx_a=ans[a], mx_b=ans[p];
    }
    return {mx_a, mx_b}; // Idx of initial vertices. i.e. datas[]
}

```

## 7 Miscellaneous

### 7.1 Mathematics

**Green's Theorem** Let  $C$  be a positively oriented, piecewise smooth, simple closed curve in a plane. Then

$$\oint_C (Ldx + Mdy) = \iint_D \left( \frac{\partial M}{\partial x} - \frac{\partial L}{\partial y} \right) dxdy,$$

where the path integral is taken counterclockwise.

#### Shoelace Lemma

```
for (int i=0; i<n; i++){
    area += X[i]*Y[(i+1)%len] - X[i]*Y[(i+len-1)%len];
}
return abs(area/2);

```

#### Burnside's Lemma

$$|X/G| = \sum_{g \in G} |X^g|$$