Scientific Computing (COMP3407)
Programming Assignment 3

Start: April 3, 2019 Due: 22:00, April 12, 2019

1 Convergence Region (60 marks)

Consider the unconstrained optimization problem

$$\min f(x,y) = \min\{-\cos(x)\cos(y/5)\}$$

(a) (10 marks) There is a portion of the problem region within which the Hessian matrix of $f(x,y)$ is positive definite. Identify this region by analytical analysis.
(b) (15 marks) One way to adaptively chose the step size of steepest descent method is to use backtracking line search.

- Firstly, set fixed parameters $0 < \beta < 1$ and $0 < \gamma \leq 1/2$,
- Then at each iteration, after obtaining the current gradient, start with $\alpha = 1$, let

$$\alpha = \beta\alpha$$

  until

$$f(x - \alpha\nabla f(x)) \leq f(x) - \gamma\alpha||\nabla f(x)||^2$$

- Finally, we get the step size α.

Implement the steepest descent method with line search for this problem, choose $\beta = \gamma = 1/2$. The input should be the initial guess of (x,y), the returned value should be a Boolean variable indicating whether the method converges.
(c) (15 marks) Implement Newton's method for this problem. Use the same input and returned value as in the previous question.
(d) (20 marks)) Randomly choose initial (x,y) within $[-\frac{\pi}{2}, \frac{\pi}{2}] \times [-\frac{5\pi}{2}, \frac{5\pi}{2}]$ and apply the two methods in (b) and (c) for various initial guesses. Draw dots with two different colors at the initial points. Their colors are decided by whether the method converges starting from this initialization. Compare the convergence regions of the two methods.

2 Nonlinear Least-Squares Fitting (40 marks)

We will consider here fitting an exponentially-damped sinusoidal curve with four unknown parameters (amplitude, decay, period, and phase, respectively),

$$f(x;\mathbf{c}) = c_1 e^{c_2 x}\sin(c_3 x + c_4)$$

(a) (10 marks) **Data generation:** Generate $m = 100$ points $\{x_i\}_{i=1}^m$ randomly and uniformly distributed in the interval $[0, 10]$ using the **rand** function. Compute the actual function

$$\tilde{f}(x) = e^{-x/2} \sin(2x)$$

At the data points x_i, add perturbations with absolute value on the order of 10^{-2} to the corresponding function values $\tilde{f}(x_i)$ to generate data y_i (use the **randn** function). Display the synthetic data and the actual function on the same plot.
(b) (25 marks) **Data fitting:** Let the loss function take the form

$$L(\mathbf{c}) = \frac{1}{2} \sum_{i=1}^{m} (y_i - f(x_i, \mathbf{c}))^2,$$

where $\{(x_i, y_i)\}_{i=1}^m$ is the data set generated in (a). Implement the Gauss-Newton's algorithm to minimize this loss function and find the optimal parameter $\mathbf{c}$, using the initial guess $\mathbf{c}_0 = (1.1, -0.4, 1.9, 0)$ which is known to be near the correct values.
(c) (5 marks) If you start with $\mathbf{c}_0 = (1, 1, 1, 1)$, does the method still converge to the correct answer?