Exercise 1.2

Question # 3:-

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 0 & -2 & -8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

$R_3 - 3R_1$

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 0 & -2 & -8 \\ 0 & 0 & -3 & -12 \end{bmatrix}$$

R_2 divide with (-2)

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & -3 & -12 \end{bmatrix}$$

R_3 divide with (-3)

$$\begin{bmatrix} 1 & 2 & 4 & 8 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} \textcircled{1} & 2 & 4 & 8 \\ 0 & 0 & \textcircled{1} & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \textcircled{1} & 2 & 4 & 8 \\ 2 & 4 & \textcircled{6} & 8 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

Pivot columns are 1 and 3.

Question # 04:-

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 2 & 4 & 5 & 4 \\ 4 & 5 & 4 & 2 \end{bmatrix}$$

$R_2 - 2R_1$

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 0 & -3 & -6 \\ 4 & 5 & 4 & 2 \end{bmatrix}$$

$R_3 - 4R_1$

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 0 & -3 & -6 \\ 0 & -3 & -12 & -18 \end{bmatrix}$$

R_2 divide with (-3)

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & -3 & -12 & -18 \end{bmatrix}$$

R_3 divide with (-3)

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 4 & 6 \end{bmatrix}$$

Interchange R_2 and R_3

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 - 2R_2$

$$\begin{bmatrix} 1 & 0 & -4 & 5 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_1 + 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 13 \\ 0 & 1 & 4 & 6 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_2 - 4R_3$

$$\begin{bmatrix} \textcircled{1} & 0 & 0 & 13 \\ 0 & \textcircled{1} & 0 & -2 \\ 0 & 0 & \textcircled{1} & 2 \end{bmatrix}$$

$$\begin{bmatrix} \textcircled{1} & 2 & 4 & 5 \\ 2 & \textcircled{4} & 5 & 4 \\ 4 & 5 & \textcircled{4} & 2 \end{bmatrix}$$

Pivot columns are 1, 2 and 3.

Question # 7:-

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$$

$R_2 - 3R_1$

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{bmatrix}$$

R_2 divide with (-5)

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$R_1 - 4R_2$

$$\begin{bmatrix} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$x_1 + 3x_2 = -5$

$x_1 = -3x_2 - 5$

x_2 is free

$x_3 = 3$

Question # 8:-

$$\begin{bmatrix} 1 & -3 & 0 & -5 \\ -3 & 7 & 0 & 9 \end{bmatrix}$$

$R_2 + 3R_1$

$$\begin{bmatrix} 1 & -3 & 0 & -5 \\ 0 & -2 & 0 & -6 \end{bmatrix}$$

R_2 divide with -2

$$\begin{bmatrix} 1 & -3 & 0 & -5 \\ 0 & +1 & 0 & 3 \end{bmatrix}$$

$R_1 - 3R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 3 \end{bmatrix}$$

$x_1 = 4$

$x_2 = 3$

x_3 is free

Question #9 :-

$$\begin{bmatrix} 0 & 1 & -2 & 3 \\ 1 & -3 & 4 & -6 \end{bmatrix}$$

Interchange R_1 and R_2

$$\begin{bmatrix} 1 & -3 & 4 & -6 \\ 0 & 1 & -2 & 3 \end{bmatrix}$$

$$R_1 + 3R_2$$

$$\begin{bmatrix} 1 & 0 & -2 & 3 \\ 0 & 1 & -2 & 3 \end{bmatrix}$$

$x_1 - 2x_3 = 3$

$x_1 = 3 + 2x_3$

$x_2 - 2x_3 = 3$

$x_2 = 3 + 2x_3$

x_3 is free

Question # 10 :-

$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ -2 & 4 & -5 & 6 \end{bmatrix}$$

$$R_2 + 2R_1$$

$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ 0 & 0 & -7 & 14 \end{bmatrix}$$

R_2 divide with (-7)

$$\begin{bmatrix} 1 & -2 & -1 & 4 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$R_1 + R_2$$

$$\begin{bmatrix} 1 & -2 & 0 & 2 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

$$x_1 - 2x_2 = 2$$

$$x_1 = 2 + 2x_2$$

$$x_3 = -2$$

x_2 is free.

Question # 11 :-

$$\begin{bmatrix} 3 & -2 & 4 & 0 \\ 9 & -6 & 12 & 0 \\ 6 & -4 & 8 & 0 \end{bmatrix}$$

R_2 divide with 3

$$\begin{bmatrix} 3 & -2 & 4 & 0 \\ 3 & -2 & 4 & 0 \\ 6 & -4 & 8 & 0 \end{bmatrix}$$

R_3 divide with 2

$$\begin{bmatrix} 3 & -2 & 4 & 0 \\ 3 & -2 & 4 & 0 \\ 3 & -2 & 4 & 0 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 3 & -2 & 4 & 0 \\ 3 & -2 & 4 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_2 - R_1$

$$\begin{bmatrix} 3 & -2 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$3x_1 - 2x_2 + 4x_3 = 0$$

$$3x_1 = 2x_2 + 4x_3$$

$$x_1 = \frac{2}{3}x_2 + \frac{4}{3}x_3$$

x_2 is free.

x_3 is free.

Question # 13:-

$$\begin{bmatrix} 1 & -3 & 0 & -1 & 0 & -2 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 + 3R_2$

$$\begin{bmatrix} 1 & 0 & 0 & -1 & -12 & 1 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$R_1 + R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -3 & 1 \\ 0 & 1 & 0 & 0 & -4 & 1 \\ 0 & 0 & 0 & 1 & 9 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$x_1 - 3x_5 = 1 \rightarrow (1)$

$x_2 - 4x_5 = 1 \rightarrow (2)$

$x_4 + 9x_5 = 4 \rightarrow (3)$

(1) $\Rightarrow$ $x_1 = 3x_5 + 1$

$x_2 = 4x_5 + 1$

x_3 is free

$x_4 = -9x_5 + 4$

x_5 is free.

Question # 12:-

$$\left[\begin{array}{cccc|c} 1 & 0 & -9 & 0 & 4 \\ 0 & 1 & 3 & 0 & -1 \\ 0 & 0 & 0 & 1 & -7 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$$

In this matrix last row is equal to $0 = 1$ so the system is inconsistent and the system has no solution.

Question # 14:-

$$\left[\begin{array}{cccccc} \textcircled{1} & 0 & -5 & 0 & -8 & 3 \\ 0 & \textcircled{1} & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & \textcircled{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_1 + 8R_3$

$$\left[\begin{array}{cccccc} \textcircled{1} & 0 & -5 & 0 & 0 & 3 \\ 0 & \textcircled{1} & 4 & -1 & 0 & 6 \\ 0 & 0 & 0 & 0 & \textcircled{1} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$$x_1 - 5x_3 = 3 \longrightarrow (1)$$

$$x_2 + 4x_3 - x_4 = 6 \longrightarrow (2)$$

$$x_5 = 0 \longrightarrow (3)$$

$x_1 = 5x_3 + 3$

$x_2 = -4x_3 + x_4 + 6$

x_3 is free.

x_4 is free.

$x_5 = 0$

$$3h + 6 = 0$$
$$3h = -6$$
$$\boxed{h = -2}$$

If $3h+6=0$ then $h=-2$ So the system is inconsistent and system has many solution.

Question # 19:-

$$x_1 + hx_2 = 2$$
$$4x_1 + 8x_2 = K$$

$$\begin{bmatrix} 1 & h & 2 \\ 4 & 8 & K \end{bmatrix}$$

$$R_2 - 4R_1$$

$$\begin{bmatrix} 1 & h & 2 \\ 0 & 8-4h & K-8 \end{bmatrix}$$

$$8 - 4h = 0$$
$$4h = 8$$
$$h = 2$$

Case I:-

When $h=2$ and $K \neq 8$

$$\begin{bmatrix} 1 & h & 2 \\ 0 & 0 & * \end{bmatrix}$$

System is inconsistent and system has no solution.

Case II :-

When $h = 2$ and $K = 8$

$$\begin{bmatrix} 1 & h & 2 \\ 0 & 0 & 0 \end{bmatrix}$$

System is consistent and have many solutions.

Case III :-

When $h \neq 2$ then

$$\begin{bmatrix} 1 & h & 2 \\ 0 & * & * \end{bmatrix}$$

Now the system has unique solution.

Question # 20:-

$$x_1 - 3x_2 = 1$$
$$2x_1 + hx_2 = K$$

$$\begin{bmatrix} 1 & -3 & 1 \\ 2 & h & K \end{bmatrix}$$

$$R_2 - 2R_1$$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & h+6 & K-2 \end{bmatrix}$$

$$h + 6 = 0$$
$$h = -6$$

Case I :-

When $h = -6$ and $k \neq 2$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 0 & * \end{bmatrix}$$

System is inconsistent and having no solution.

Case II :-

When $h = -6$ and $k = 2$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

System is consistent and having many solutions.

Case III :-

When $h \neq -6$ then

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & * & * \end{bmatrix}$$

Then the system have a unique and exact solution. There are no free variables.

Question # 17:-

$$\begin{bmatrix} 1 & -1 & 4 \\ -2 & 3 & h \end{bmatrix}$$

$$R_2 + 2R_1$$

$$\begin{bmatrix} 1 & -1 & 4 \\ 0 & 1 & h+8 \end{bmatrix}$$

$$R_1 + R_2$$

$$\begin{bmatrix} \textcircled{1} & 0 & h+12 \\ 0 & \textcircled{1} & h+8 \end{bmatrix}$$

The system has solution since the argumented or solution column cannot be a pivot column as 'h' is only involved in the solution column.

Question # 18:-

$$\begin{bmatrix} 1 & -3 & 1 \\ h & 6 & -2 \end{bmatrix}$$

$$R_2 - hR_1$$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 6+3h & -2-h \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 1 \\ 0 & 3h+6 & -h-2 \end{bmatrix}$$