Exercise 1.7

① $\begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}$

$$\begin{bmatrix} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & -6 & -8 & 0 \end{bmatrix}$$

$R_3 + 3R_2$

$$\begin{bmatrix} 5 & 7 & 9 & 0 \\ 0 & 2 & 4 & 0 \\ 0 & 0 & 4 & 0 \end{bmatrix}$$

$R_2 - R_3$

$$\begin{bmatrix} 5 & 7 & 9 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 4 & 0 \end{bmatrix}$$

$4x_3 = 0$

$\boxed{x_3 = 0}$

$2x_2 = 0$

$\boxed{x_2 = 0}$

$5x_1 + 7x_2 + 9x_3 = 0$

$5x_1 + 7(0) + 9(0) = 0$

$5x_1 + 0 + 0 = 0$

$\boxed{x_1 = 0}$

linerly independent.

(2) $\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -8 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}$

$$\begin{bmatrix} 0 & 0 & -1 & 0 \\ 2 & 0 & 3 & 0 \\ 3 & -8 & 1 & 0 \end{bmatrix}$$

Interchange R_1 & R_3

$$\begin{bmatrix} 3 & -8 & 1 & 0 \\ 2 & 0 & 3 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

Interchang R_1 & R_2

$$\begin{bmatrix} 2 & 0 & 3 & 0 \\ 3 & -8 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$2R_2 - 3R_1$

$$\begin{bmatrix} 2 & 0 & 3 & 0 \\ 0 & -16 & -7 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$2x_1 + 3x_3 = 0 \longrightarrow (1)$

$-16x_2 - 7x_3 = 0 \longrightarrow (2)$

$-x_3 = 0 \longrightarrow (3)$

(3) $\Rightarrow$ $\boxed{x_3 = 0}$

(2) $\Rightarrow$ $-16x_2 - 7(0) = 0$

$-16x_2 - 0 = 0$

$\boxed{x_2 = 0}$

(1) $\Rightarrow$ $2x_1 + 3(0) = 0$

$2x_1 + 0 = 0$

$\boxed{x_1 = 0}$

linearly independent.

(3) $\begin{bmatrix} 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -4 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} 2 & -4 \\ -3 & 6 \end{bmatrix}$

Taking determinent

$= (2 \times 6) - (-3 \times -4)$

$= 12 - 12$

$= 0$

linearly dependent. because $|A| = 0$ & it is a scalar product $V_1 = V_2$

(4) $\begin{bmatrix} -1 \\ 3 \end{bmatrix}, \begin{bmatrix} -3 \\ -9 \end{bmatrix}$

$= \begin{bmatrix} -1 & -3 \\ 3 & -9 \end{bmatrix}$

Taking determinent

$= (-1 \times -9) - (3 \times -3)$

$= 9 - (-9)$

$= 9 + 9$

$= 18 \neq 0$

linearly independent because $|A| \neq 0$ & it is a scalar product $V_1 \neq V_2$

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(5) $\begin{bmatrix} 0 & -3 & 9 & 0 \\ 2 & 1 & -7 & 0 \\ -1 & 4 & -5 & 0 \\ 1 & -4 & -2 & 0 \end{bmatrix}$

R_1 interchang R_2

$$\begin{bmatrix} 2 & 1 & -7 & 0 \\ 0 & -3 & 9 & 0 \\ -1 & 4 & -5 & 0 \\ 1 & -4 & -2 & 0 \end{bmatrix}$$

$R_4 + R_3$

$$\begin{bmatrix} 2 & 1 & -7 & 0 \\ 0 & -3 & 9 & 0 \\ -1 & 4 & -5 & 0 \\ 0 & 0 & -7 & 0 \end{bmatrix}$$

R_2 interchange R_3

$$\begin{bmatrix} 2 & 1 & -7 & 0 \\ -1 & 4 & -5 & 0 \\ 0 & -3 & 9 & 0 \\ 0 & 0 & -7 & 0 \end{bmatrix}$$

$2R_2 + R_1$

$$\begin{bmatrix} 2 & 1 & -7 & 0 \\ 0 & 9 & -17 & 0 \\ 0 & -3 & 9 & 0 \\ 0 & 0 & -7 & 0 \end{bmatrix}$$

$$2x_1 + x_2 - 7x_3 = 0 \rightarrow (1)$$
$$9x_2 - 17x_3 = 0 \rightarrow (2)$$
$$-3x_2 + 9x_3 = 0 \rightarrow (3)$$
$$-7x_3 = 0 \rightarrow (4)$$

x_4 is free.
So it is non-trival and linearly dependent.

(7) $\begin{bmatrix} 1 & 4 & -3 & 0 & 0 \\ -2 & -7 & 5 & 1 & 0 \\ -4 & -5 & 7 & 5 & 0 \end{bmatrix}$

$R_3 + 4R_1$

$\begin{bmatrix} 1 & 4 & -3 & 0 & 0 \\ -2 & -7 & 5 & 1 & 0 \\ 0 & 11 & -5 & 5 & 0 \end{bmatrix}$

x_4 is free.
So it is non-trivial and linearly dependent.

(9) $\begin{bmatrix} 1 & -3 & 5 & 0 \\ -3 & 9 & -7 & 0 \\ 2 & -6 & h & 0 \end{bmatrix}$

$R_2 + 3R_1$

$\begin{bmatrix} 1 & -3 & 5 & 0 \\ 0 & 0 & 8 & 0 \\ 2 & -6 & h & 0 \end{bmatrix}$

$R_3 - 2R_1$

$\begin{bmatrix} 1 & -3 & 5 & 0 \\ 0 & 0 & 8 & 0 \\ 0 & 0 & h-10 & 0 \end{bmatrix}$

(a) Solution is inconsistent, v_3 is not span in v_1 and v_2

(b) It is non-trivial solution so linearly dependent.

(10) $$\begin{bmatrix} 1 & -3 & 2 \\ -3 & 9 & -5 \\ -5 & 15 & h \end{bmatrix}$$

$$R_2 + 3R_1$$

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 0 & 1 \\ -5 & 15 & h \end{bmatrix}$$

$$R_3 + 5R_1$$

$$\begin{bmatrix} 1 & -3 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & h-10 \end{bmatrix}$$

(a) Solution is inconsistent so v_3 is not span in v_1 and v_2.

(b) The solution is non-trivial so it is linearly dependent.

(11) $$\begin{bmatrix} 2 & 4 & -2 \\ -2 & -6 & 2 \\ 4 & 7 & h \end{bmatrix}$$

$$R_2 + R_1$$

$$\begin{bmatrix} 2 & 4 & -2 \\ 0 & -2 & 0 \\ 4 & 7 & h \end{bmatrix}$$

R_1 divide with 2

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 4 & 7 & h \end{bmatrix}$$

$$R_3 - 4R_1$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & -2 & 0 \\ 0 & -1 & h+4 \end{bmatrix}$$

R_2 divide with -2

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & -1 & h+4 \end{bmatrix}$$

$$R_3 + R_2$$

$$\begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & h+4 \end{bmatrix}$$

$$h+4=0$$

$$h=4$$

So, it is linearly dependent.

(13) $$\begin{bmatrix} 1 & -2 & 3 \\ 5 & -9 & h \\ -3 & 6 & -9 \end{bmatrix}$$

$R_2 - 5R_1$

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & h-15 \\ -3 & 6 & -9 \end{bmatrix}$$

$R_3 + 3R_1$

$$\begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & h-15 \\ 0 & 0 & 0 \end{bmatrix}$$

R_3 is free.
So it is linearly independent.

(15) $$\begin{bmatrix} 5 & 2 & 1 & -1 \\ 1 & 8 & 3 & 7 \end{bmatrix}$$

$5R_2 - R_1$

$$\begin{bmatrix} 5 & 2 & 1 & -1 \\ 0 & 38 & 14 & 36 \end{bmatrix}$$

Here x_3 and x_4 are free.
so, it is a trivial solution and linearly independent.

(17) $$\begin{bmatrix} 5 & 0 & -7 \\ -3 & 0 & 2 \\ -1 & 0 & 4 \end{bmatrix}$$

In this question C_2 is 0 so x_2 is free. It is a trivial and linearly independent.

(19) $\begin{bmatrix} -8 & 2 \\ 12 & -3 \\ -4 & -1 \end{bmatrix}$

$R_2 + 3R_3 \to \cancel{R_1}$

$\begin{bmatrix} 8 & 2 \\ 0 & -6 \\ -4 & -1 \end{bmatrix}$

x_3 is free so it is a trivial solution and linearly independent

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