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(Exercise # 1.8)

Q no.1).

Let $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$ define $T : R^2 \to R^2$ by image of $u = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $v = \begin{bmatrix} a \\ b \end{bmatrix}$ = ?

$T(x) = Ax$

$T(u) = Au$

$\rightarrow T(u) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} 2+0 \\ 0-6 \end{bmatrix} = \begin{bmatrix} 2 \\ -6 \end{bmatrix}$

$\begin{bmatrix} 2 \\ -6 \end{bmatrix}$ is image of 'u' under 'T.'

$\rightarrow T(v) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2a+0 \\ 0+2b \end{bmatrix} = \begin{bmatrix} 2a \\ 2b \end{bmatrix}$

$\begin{bmatrix} 2a \\ 2b \end{bmatrix}$ is image of 'v' under 'T'

Q no.2) $A = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}$, $u = \begin{bmatrix} 3 \\ 6 \\ -9 \end{bmatrix}$, $v = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

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$T : R^3 \rightarrow R^3$

$T(u) = ?$, $T(v) = ?$

$\rightarrow T(u) = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix} \begin{bmatrix} 3 \\ 6 \\ -9 \end{bmatrix}$

$= \begin{bmatrix} 1/3(3) + 0 + 0 \\ 0 + (1/3)(6) + 0 \\ 0 + 0 + (1/3)(-9) \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ is image of 'u' under 'T'

$\rightarrow T(v) = \begin{bmatrix} 1/3 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/3 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$

$= \begin{bmatrix} 1/3(a) + 0 + 0 \\ 0 + 1/3(b) + 0 \\ 0 + 0 + 1/3(c) \end{bmatrix}$

$= \begin{bmatrix} a/3 \\ b/3 \\ c/3 \end{bmatrix}$ is image of 'v' under 'T'

Qno.3) $A = \begin{bmatrix} 1 & 0 & -3 \\ -3 & 1 & 6 \\ 2 & -2 & -1 \end{bmatrix}$, $b = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 & -3 \\ -3 & 1 & 6 \\ 2 & -2 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ -1 \end{bmatrix}$

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The matrix equation is

$$Ax = b$$

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ -3 & 1 & 6 & 3 \\ 2 & -2 & -1 & -1 \end{bmatrix}$$

$\rightarrow R_2 + 3R_1$:

$\rightarrow R_3 - 2R_1$

$$= \begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & -3 & -3 \\ 0 & -2 & 5 & 3 \end{bmatrix}$$

$\rightarrow R_3 + 2R_2$

$$= \begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & -1 & -3 \end{bmatrix}$$

$\rightarrow \boxed{x_3 = 3}$

$\rightarrow x_2 - 3x_3 = -3$

$x_2 - 3(3) = -3$

$x_2 - 9 = -3$

$x_2 = -3 + 9$

$\boxed{x_2 = 6}$

$\rightarrow x_1 - 3x_3 = -2$

$x_1 - 3(3) = -2$

$x_1 - 9 = -2$

$x_1 = -2 + 9$

$\boxed{x_1 = 7}$

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hence $= X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \\ 3 \end{bmatrix}$

As number of pivot element is three and we get three values of x. hence, this x is unique.

Q no.7)

A 6X5 has the form.

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} X & X & X & X & X \\ X & X & X & X & X \\ X & X & X & X & X \\ X & X & X & X & X \\ X & X & X & X & X \\ X & X & X & X & X \end{bmatrix} \end{array} \begin{bmatrix} X \\ X \\ X \\ X \\ X \end{bmatrix}$$

$$= \begin{bmatrix} X \\ X \\ X \\ X \\ X \\ X \end{bmatrix} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array}$$

$R^a \longrightarrow R^b \Rightarrow R^5 \longrightarrow R^6.$

Hence $a = 5$

$b = 6.$

Q no. 8)

no. of columns : 5
no. of rows : 7
matrix : 7×5.

$R^5 \to R^7$.

Q no. 9)

$$A = \begin{bmatrix} 1 & -3 & 5 & -5 \\ 0 & 1 & -3 & 5 \\ 2 & -4 & 4 & -4 \end{bmatrix}$$

The homogeneous system is

$Ax = 0$

$$\begin{bmatrix} 1 & -3 & 5 & -5 \\ 0 & 1 & -3 & 5 \\ 2 & -4 & 4 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The augmented matrix is ;

$$\begin{bmatrix} 1 & -3 & 5 & -5 & 0 \\ 0 & 1 & -3 & 5 & 0 \\ 2 & -4 & 4 & -4 & 0 \end{bmatrix}$$

$\to R_3 - 2R_1$

$$= \begin{bmatrix} 1 & -3 & 5 & -5 & 0 \\ 0 & 1 & -3 & 5 & 0 \\ 0 & 2 & -6 & 6 & 0 \end{bmatrix}$$

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$\rightarrow R_3 - 2R_2$

$$= \begin{bmatrix} 1 & -3 & 5 & -5 & 0 \\ 0 & 1 & -3 & -5 & 0 \\ 0 & 0 & 0 & -4 & 0 \end{bmatrix}$$

$\rightarrow x_4 = 0$

$\rightarrow x_2 - 3x_3 - 5x_4 = 0$

$\rightarrow x_1 - 3x_2 + 5x_3 - 5x_4 = 0.$

Here x_3 is free variable

$\rightarrow x_4 = 0$

$\rightarrow x_2 = 3x_3$

$\rightarrow x_1 = 3x_2 - 5x_3$

$x_1 = 9x_3 - 5x_3$

$\rightarrow x_1 = 4x_3$

solution set : $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 4x_3 \\ 3x_3 \\ x_3 \\ 0 \end{bmatrix}$ Or $= x_3 \begin{bmatrix} 4 \\ 3 \\ 1 \\ 0 \end{bmatrix}$

Qno.11) $A = \begin{bmatrix} 1 & -3 & 5 & -5 \\ 0 & 1 & -3 & 5 \\ 2 & -4 & 4 & 4 \end{bmatrix}$

$b = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$, is b in range of $x \rightarrow Ax$

Finally BingO!

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To prove $x \to Ax$
then $Ax = b$ or $[A \; b]$
must be consistent.

$$[A \; b] = \begin{bmatrix} 1 & -3 & 5 & -5 & -1 \\ 0 & 1 & -3 & 5 & 1 \\ 2 & -4 & 4 & 4 & 0 \end{bmatrix}$$

$R_3 - 2R_1$

$$= \begin{bmatrix} 1 & -3 & 5 & -5 & -1 \\ 0 & 1 & -3 & 5 & 1 \\ 0 & 2 & -6 & 6 & 2 \end{bmatrix}$$

$R_3 - 2R_2$

$$= \begin{bmatrix} 1 & -3 & 5 & -5 & -1 \\ 0 & 1 & -3 & 5 & 1 \\ 0 & 0 & 0 & -4 & 0 \end{bmatrix}$$

→ Here $x_4 = 0$
and x_3 is free variable
→ $x_2 = 3x_3 - 5x_4 - 1$
→ $x_1 = 3x_2 - 5x_3 + 5x_4 + 1$

The system gives consistent solution so b is in range of transformation $x \to Ax$.

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Q no.13) $u = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$, $v = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$

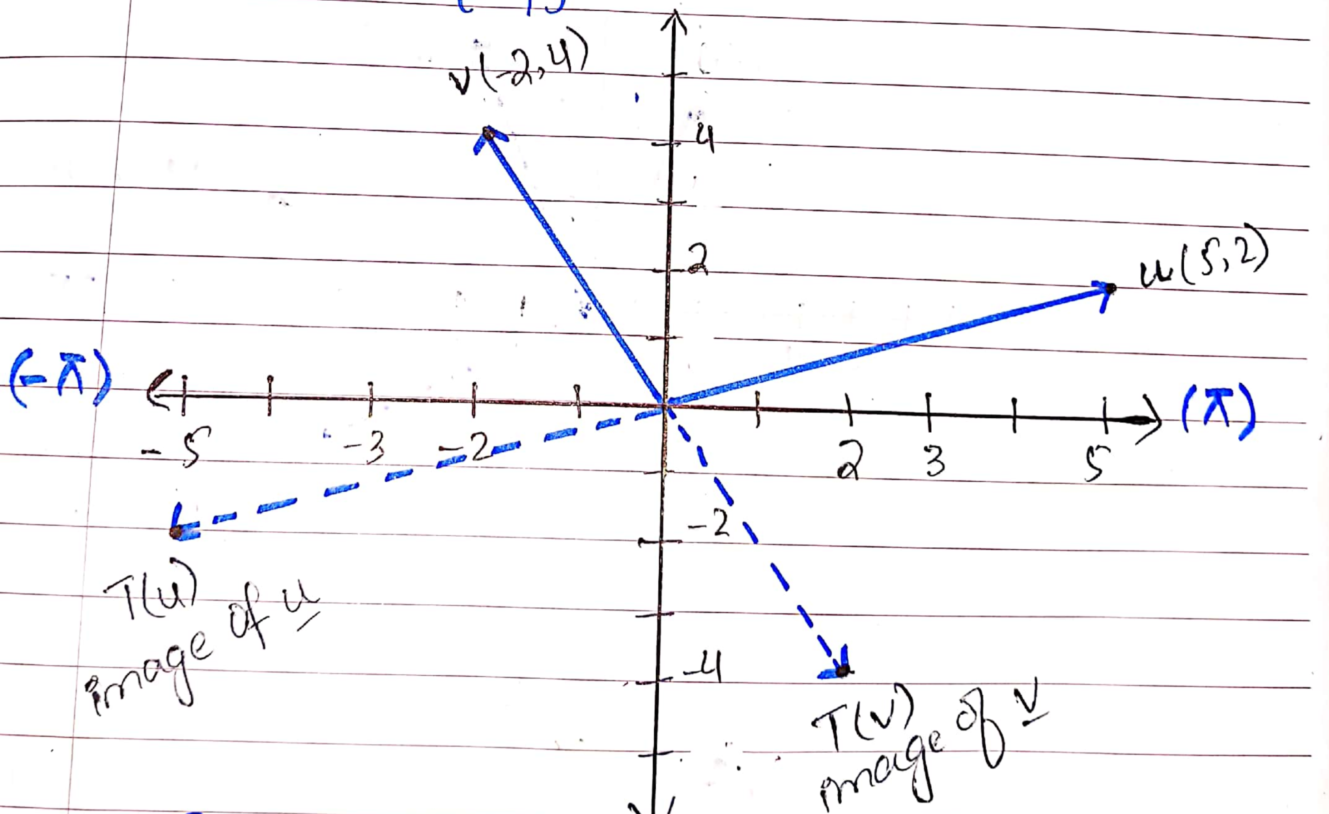
$$T(x) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$\rightarrow$ $T(u) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} -5+0 \\ 0-2 \end{bmatrix}$

$= \begin{bmatrix} -5 \\ -2 \end{bmatrix}$

$\rightarrow$ $T(v) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 2+0 \\ 0-4 \end{bmatrix}$

$= \begin{bmatrix} 2 \\ -4 \end{bmatrix}$.



This transformation can be described as the rotation of π radians about the

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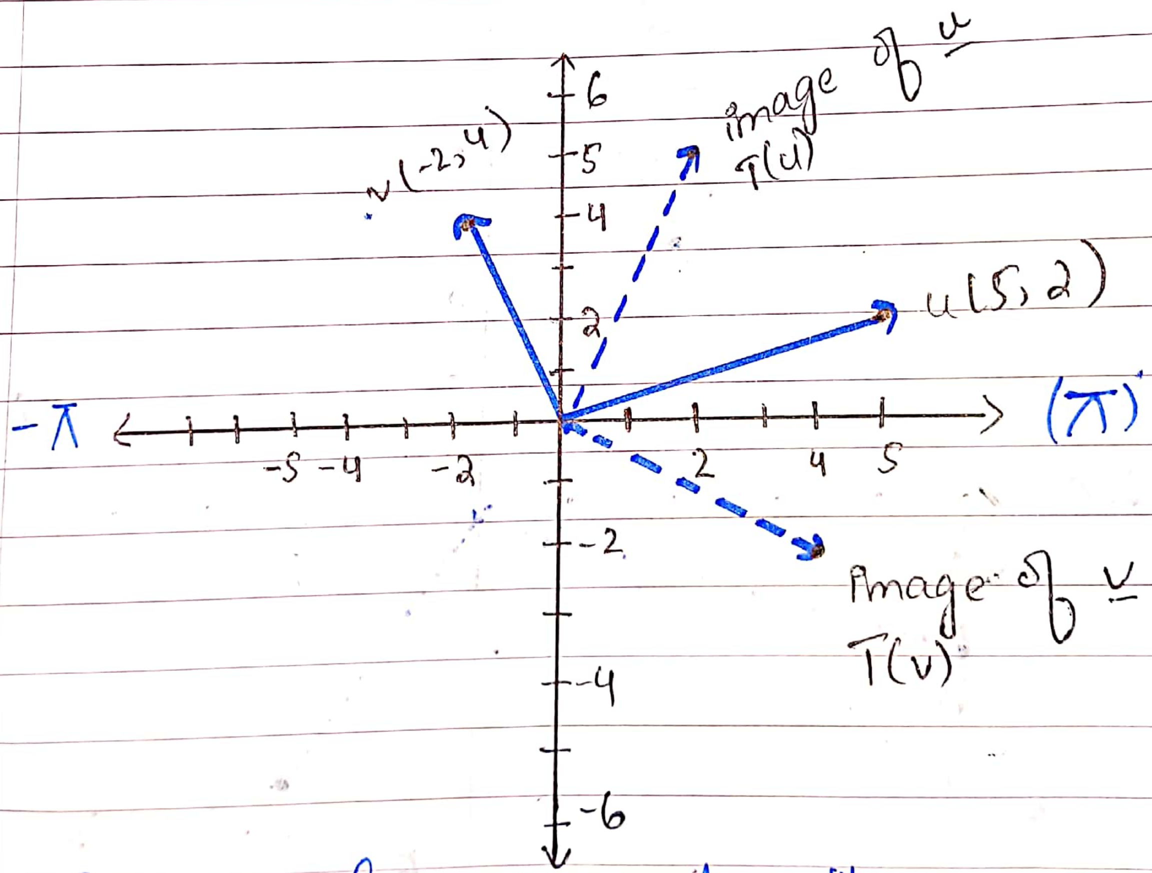
origin or the rotation of π radians about the origin.

Q no. 15) $T(x) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$T(u) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$

$T(v) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$



This transformation describes rotation of π to $-\pi$ radians about origin.

Bingo! Finally

Scanned with CamScanner

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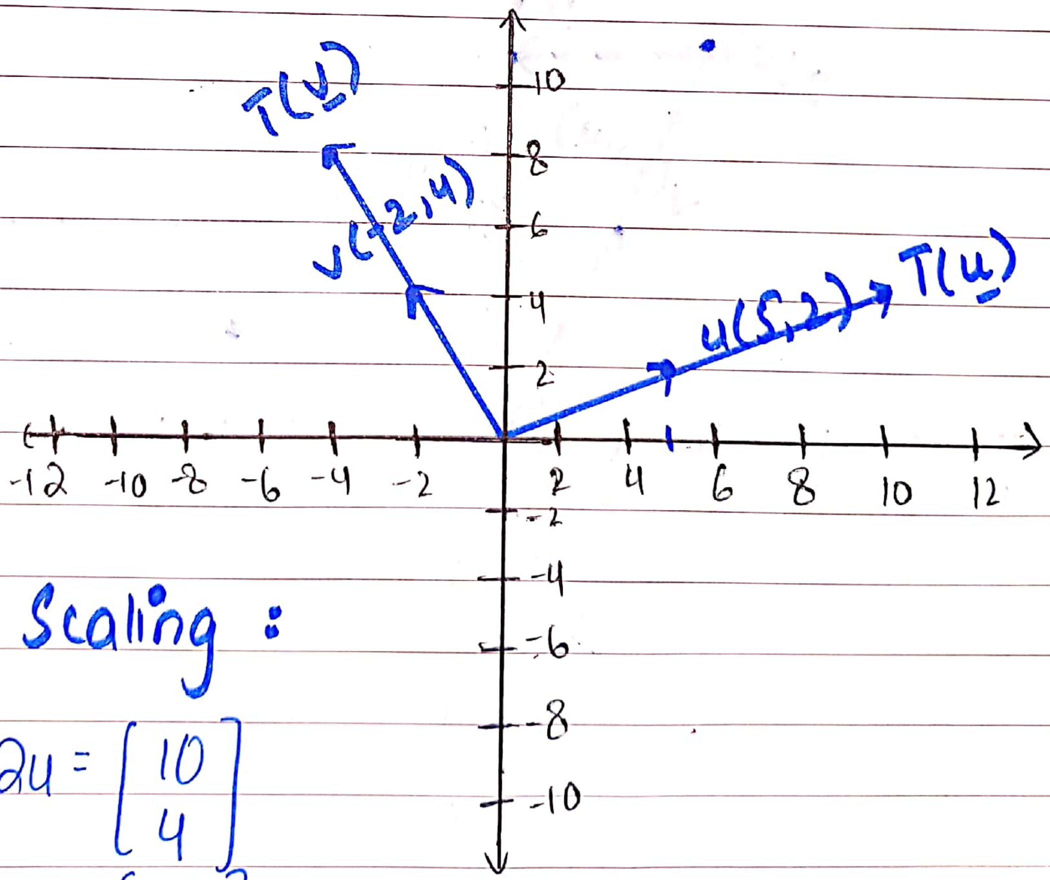
Q no. 14) $T(x) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$\rightarrow T(u) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 4 \end{bmatrix}$

$\rightarrow T(v) = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} -4 \\ 8 \end{bmatrix}$

Scaling :

→ The image is far away from the original vectors. That's why we will do scaling



Scaling :

$2u = \begin{bmatrix} 10 \\ 4 \end{bmatrix}$

$2v = \begin{bmatrix} -4 \\ 8 \end{bmatrix}$

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Q no. 16) $T(x) = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

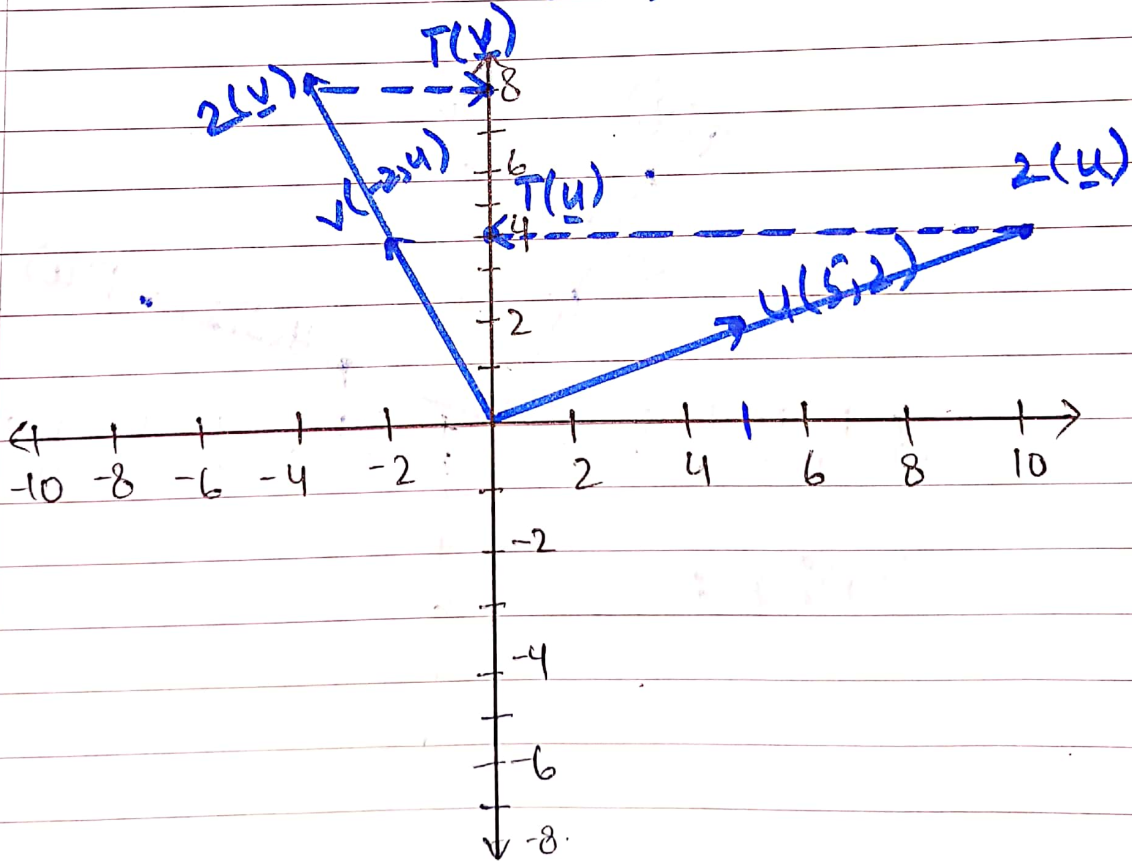
$\rightarrow T(u) = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 4 \end{bmatrix}$

$\rightarrow T(v) = \begin{bmatrix} 0 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} -2 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \end{bmatrix}$

Scaling :

$2(u) = \begin{bmatrix} 10 \\ 4 \end{bmatrix}$

$2(v) = \begin{bmatrix} -4 \\ 8 \end{bmatrix}$.



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Q no.17) Let $T : R^2 \to R^2$

$u = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$, $T(u) = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$v = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$, $T(v) = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

Find $2u$, $3v$ and $2u + 3v$

Properties of linearty of T :

(i) $T(u+v) = T(u) + T(v)$

(ii) $T(cu) = cT(u)$.

→ Applying properties ;

- $T(2u) = 2T(u)$

$$= 2\begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 \\ 2 \end{bmatrix}.$$

- $T(3v) = 3T(v)$

$$= 3\begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 9 \end{bmatrix}.$$

- $T(2u+3v) = 2T(u) + 3T(v)$

$$= \begin{bmatrix} 8 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

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$T(2u+3v) = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$

Q no. 19) $e_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $e_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$y_1 = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $y_2 \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

(e_1 maps y_1 $\Rightarrow$ image of e_1 is y_1)

$R^2 \longrightarrow R^2$

$T(e_1) = y_1$ and $T(e_2) = y_2$.

So, $T(e_1) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, $T(e_2) = \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

Image of $\begin{bmatrix} 5 \\ -3 \end{bmatrix}$:

It can be written as ;

$\begin{bmatrix} 5 \\ -3 \end{bmatrix} = 5e_1 + (-3e_2)$.

$= 5\begin{bmatrix} 1 \\ 0 \end{bmatrix} - 3\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

→ Since T is a linear transformation

$T\begin{bmatrix} 5 \\ -3 \end{bmatrix} = T(5e_1 - 3e_2)$.

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$$= 5T(e_1) - 3T(e_2)$$

$$= 5\begin{bmatrix} 2 \\ 5 \end{bmatrix} - 3\begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$= \begin{bmatrix} 10 + 3 \\ 25 - 18 \end{bmatrix}$$

$$T\begin{bmatrix} 5 \\ -3 \end{bmatrix} = \begin{bmatrix} 13 \\ 7 \end{bmatrix}.$$

Image of $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 e_1 + x_2 e_2.$$

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = T(x_1 e_1 + x_2 e_2)$$

$$= x_1 T(e_1) + x_2 T(e_2).$$

$$= x_1 \begin{bmatrix} 2 \\ 5 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 6 \end{bmatrix}$$

$$T\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2x_1 - x_2 \\ 5x_1 + 6x_2 \end{bmatrix}$$