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-: Linear Algebra :-

BCS203035

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Chap # 1

Exercise 1.1

Question 1:-

$$x_1 + 5x_2 = 7$$

$$-2x_1 - 7x_2 = -5$$

$$\begin{bmatrix} 1 & 5 & 7 \\ -2 & -7 & -5 \end{bmatrix}$$

Performing $R_2 + 2R_1$

$$\begin{bmatrix} 1 & 5 & 7 \\ 0 & 3 & 9 \end{bmatrix}$$

$$x_1 + 5x_2 = 7 \rightarrow ①$$

$$3x_2 = 9 \rightarrow ②$$

$$② \Rightarrow x_2 = \frac{9}{3} = 3 \quad \text{put in } ①$$

$$① \Rightarrow x_1 + 5(3) = 7$$

$$x_1 + 15 = 7$$

$$x_1 = 7 - 15$$

$$x_1 = -8$$

Solution is $(x_1, x_2) = (-8, 3)$



Question 2:-

$$3x_1 + 6x_2 = -3$$

$$5x_1 + 7x_2 = 10$$

$$\left[\begin{array}{ccc} 3 & 6 & -3 \\ 5 & 7 & 10 \end{array} \right]$$

Divide 3 with R₁.

$$\left[\begin{array}{ccc} 1 & 2 & -1 \\ 5 & 7 & 10 \end{array} \right]$$

Performing R₂ - 5R₁,

$$\left[\begin{array}{ccc} 1 & 2 & -1 \\ 0 & -3 & 15 \end{array} \right]$$

$$x_1 + 2x_2 = -1 \rightarrow ①$$

$$-3x_2 = 15 \rightarrow ②$$

$$② \Rightarrow x_2 = \frac{15}{-3} = -5 \text{ put in } ①$$

$$① \Rightarrow x_1 + 2(-5) = -1$$

$$x_1 = -1 + 10$$

$$x_1 = 9$$

Solution is $(x_1, x_2) = (9, -5)$

Question 3:

Find (x_1, x_2)

$$x_1 + 2x_2 = 4$$

$$x_1 - x_2 = 1$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 1 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 4 \\ 0 & -3 & -3 \end{bmatrix} \quad \text{Performing } R_2 - R_1$$

$$x_1 + 2x_2 = 4 \rightarrow ①$$

$$-3x_2 = -3 \rightarrow ②$$

equ ② becomes

$$\begin{array}{rcl} x_2 = -3 & = 1 \\ & -3 & \end{array}$$

Now put value of x_2 in ①

$$x_1 + 2(1) = 4$$

$$x_1 = 4 - 2 = 2$$

So, the point of intersection is $(x_1, x_2) = (2, 1)$



Question #4:

$$x_1 + 2x_2 = -13$$

$$3x_1 - 2x_2 = 1$$

$$\left[\begin{array}{ccc} 1 & 2 & -13 \\ 3 & -2 & 1 \end{array} \right]$$

Performing $R_2 - 3R_1$,

$$\left[\begin{array}{ccc} 1 & 2 & -13 \\ 0 & -8 & 40 \end{array} \right]$$

$$x_1 + 2x_2 = -13 \rightarrow ①$$

$$-8x_2 = 40 \rightarrow ②$$

$$② \Rightarrow x_2 = \frac{40}{-8} = -5$$

Now put $x_2 = -5$ in ①

$$x_1 + 2(-5) = -13$$

$$x_1 - 10 = -13$$

$$x_1 = -13 + 10$$

$$x_1 = -3$$

Point of intersection is $(x_1, x_2) = (-3, -5)$



Question 5:-

$$\left[\begin{array}{cccccc} 1 & -4 & -3 & 0 & 0 & 7 \\ 0 & 1 & 4 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & -5 \end{array} \right]$$

$$x_1 - 4x_2 - 3x_3 = 7 \rightarrow ①$$

$$x_2 + 4x_3 = 6 \rightarrow ②$$

$$x_3 = 2 \rightarrow ③$$

$$x_4 = -5 \rightarrow ④$$

Put equ ③ in equ ②

$$x_2 + 4(2) = 6$$

$$x_2 + 8 = 6$$

$$x_2 = 6 - 8$$

$$x_2 = -2$$

Put ~~equ~~ $x_2 = -2$ & $x_3 = 2$ in ①

$$x_1 - 4(-2) - 3(2) = 7$$

$$x_1 + 8 - 6 = 7$$

$$x_1 + 2 = 7$$

$$x_1 = 7 - 2$$

$$x_1 = 5$$

$$(x_1, x_2, x_3, x_4) = (5, -2, 2, -5)$$



Question # 6:-

$$\left[\begin{array}{cccc|c} 1 & -6 & 4 & 0 & -1 \\ 0 & 2 & -7 & 0 & -4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 4 & 1 & 2 \end{array} \right]$$

Performing $R_4 - 4R_3$

$$\left| \begin{array}{ccccc} 1 & -6 & 4 & 0 & -4 \\ 0 & 2 & -7 & 0 & 4 \\ 0 & 0 & 1 & 2 & -3 \\ 0 & 0 & 0 & -7 & 14 \end{array} \right|$$

$$x_1 - 6x_2 + 4x_3 = -4 \rightarrow ①$$

$$2x_2 - 7x_3 = 4 \rightarrow ②$$

$$x_3 + 2x_4 = -3 \rightarrow ③$$

$$-7x_4 = 14 \rightarrow ④$$

$$x_4 = 14$$

$$-7$$

$$\boxed{x_4 = -2} \quad \text{use in } ③$$

$$③ \Rightarrow x_3 + 2(-2) = -3$$

$$x_3 - 4 = -3$$

$$x_3 = -3 + 4$$

$$\boxed{x_3 = 1}$$

use value of x_3 and x_4 in equ ②

$$2(x_2) - 7(1) = 4$$

$$2x_2 = 4 + 7$$

$$\boxed{x_2 = \frac{13}{2}}$$

Now equ ① is

$$x_1 - 6\left(\frac{13}{2}\right) + 4(1) = -4$$

$$x_1 - 39 + 4 = -4$$

$$x_1 = 39 - 4 - 4$$

$$\boxed{x_1 = 31}$$

$$(x_1, x_2, x_3, x_4) = \left(31, \frac{13}{2}, 1, -2\right)$$

Question 7:

$$\left[\begin{array}{cccc} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right]$$

Replace R_3 with R_4

$$\left[\begin{array}{cccc} 1 & 7 & 3 & -4 \\ 0 & 1 & -1 & 3 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 1 \end{array} \right]$$

$$0 \neq 1$$

so,

There is no solution.

Question 8:

$$\left[\begin{array}{ccccc} 1 & -5 & 4 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 \end{array} \right]$$

$$x_1 - 5x_2 + 4x_3 = 0 \rightarrow ①$$

$$x_2 + x_4 = 0 \rightarrow ②$$

$$x_3 = 0 \rightarrow ③$$

$$x_4 = 0 \rightarrow ④$$

put ④ in eqn ②

$$x_2 + 0 = 0$$

$$\boxed{x_2 = 0}$$

Put value of x_2 and x_3 in equ ①

$$x_1 - 5(0) + 4(0) = 0$$

$$\boxed{x_1 = 0}$$

The solution is $(0, 0, 0, 0)$.



Question 9:-

$$\left[\begin{array}{ccccc} 1 & -1 & 0 & 0 & -5 \\ 0 & 1 & -2 & 0 & -7 \\ 0 & 0 & 1 & -3 & 2 \\ 0 & 0 & 0 & 1 & 4 \end{array} \right]$$

$$x_1 - x_2 = -5 \rightarrow \textcircled{a}$$

$$x_2 - 2x_3 = -7 \rightarrow \textcircled{b}$$

$$x_3 - 3x_4 = 2 \rightarrow \textcircled{c}$$

$$x_4 = 4 \rightarrow \textcircled{d}$$

use \textcircled{d} in equ \textcircled{c}

$$x_3 - 3(4) = 2$$

$$x_3 - 12 = 2$$

$$x_3 = 2 - 12$$

$$\boxed{x_3 = -10}$$

use in \textcircled{b}

$$\textcircled{b} \Rightarrow x_2 - 2(-10) = -7$$

$$x_2 + 20 = -7$$

$$x_2 = -7 - 20$$

$$\boxed{x_2 = -27}$$

use in \textcircled{a}

$$x_1 - (-27) = -5$$

$$x_1 + 27 = -5$$

$$x_1 = -5 - 27$$

$$\boxed{x_1 = -32}$$

Solution is $(-32, -27, -10, 4)$

Question 10:-

$$\left[\begin{array}{ccccc} 1 & 3 & 0 & -2 & -7 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{array} \right]$$

$$x_1 + 3x_2 - 2x_4 = -7 \rightarrow ①$$

$$x_2 + 3x_4 = 6 \rightarrow ②$$

$$x_3 = 2 \rightarrow ③$$

$$x_4 = 2 \rightarrow ④$$

put ④ in ②

$$x_2 + 3(2) = 6$$

$$x_2 = 6 - 6$$

$$\boxed{x_2 = 0}$$

use $x_2 = 0$ and $x_4 = 2$ in equ ①

$$x_1 + 3(0) - 2(2) = -7$$

$$x_1 + 0 - 4 = -7$$

$$x_1 = -7 + 4$$

$$\boxed{x_1 = -3}$$

The Solution is $(-3, 0, 2, 2)$

Question 11:-

$$x_2 + 5x_3 = -4 \quad \text{---} \rightarrow (1)$$

$$x_1 + 4x_2 + 3x_3 = -2 \quad \text{---} \rightarrow (2)$$

$$2x_1 + 7x_2 + x_3 = -2 \quad \text{---} \rightarrow (3)$$

$$\left[\begin{array}{cccc} 0 & 1 & 5 & -4 \\ 1 & 4 & 3 & -2 \\ 2 & 7 & 1 & -2 \end{array} \right]$$

Exchange R_1 and R_2

$$\left[\begin{array}{cccc} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 2 & 7 & 1 & -2 \end{array} \right]$$

Performing $R_3 - 2R_1$

$$\left[\begin{array}{cccc} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & -1 & -5 & 6 \end{array} \right]$$

Performing $R_3 + R_2$

$$\left[\begin{array}{cccc} 1 & 4 & 3 & 2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & 2 \end{array} \right]$$

The system is inconsistent, because the last row would require $0=2$.

So,

The solution set is empty.

Question 12:-

$$x_1 - 5x_2 + 4x_3 = -3$$

$$2x_1 - 7x_2 + 3x_3 = -2$$

$$-2x_1 + x_2 + 7x_3 = -1$$

$$\left[\begin{array}{cccc} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{array} \right]$$

Performing $R_2 - 2R_1$,

$$\left[\begin{array}{cccc} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ -2 & 1 & 7 & -1 \end{array} \right]$$

Performing $R_3 + 2R_1$,

$$\left[\begin{array}{cccc} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & -9 & 15 & -7 \end{array} \right]$$

Performing $R_3 + 3R_2$

$$\left[\begin{array}{cccc} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & 0 & 0 & -4 \end{array} \right]$$

The system is inconsistent, because the last row would require $0=4$.

So,

The solution set is empty.

Question # 13:-

$$x_1 - 3x_3 = 8$$

$$2x_1 + 2x_2 + 9x_3 = 7$$

$$x_2 + 5x_3 = -2$$

$$\left[\begin{array}{cccc} 1 & 0 & -3 & 8 \\ 2 & 2 & 9 & 7 \\ 0 & 1 & 5 & -2 \end{array} \right]$$

Performing $R_2 - 2R_1$

$$\left[\begin{array}{cccc} 1 & 0 & -3 & 8 \\ 0 & 2 & 15 & -9 \\ 0 & 1 & 5 & -2 \end{array} \right]$$

Performing Exchange R_2 with R_3

$$\left[\begin{array}{cccc} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 2 & 15 & -9 \end{array} \right]$$

Performing $R_3 - 2R_2$

$$\left[\begin{array}{cccc} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 5 & -5 \end{array} \right]$$

Performing $R_3/5$

$$\left[\begin{array}{cccc} 1 & 0 & -3 & 8 \\ 0 & 1 & 5 & -2 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

Performing $R_2 - 5R_3$

$$\left[\begin{array}{cccc} 1 & 0 & -3 & 8 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

Performing $R_1 + 3R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array} \right]$$

$$x_1 = 5$$

$$x_2 = 3$$

$$x_3 = -1$$

So,

The solution is $(5, 3, -1)$.



Question 14:-

$$2x_1 - 6x_3 = -8$$

$$x_2 + 2x_3 = 3$$

$$3x_1 + 6x_2 - 2x_3 = -4$$

$$\left[\begin{array}{ccc|c} 2 & 0 & -6 & -8 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{array} \right]$$

Divide R_1 with 2

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 3 & 6 & -2 & -4 \end{array} \right]$$

Performing $R_3 - 3R_1$

$$\left[\begin{array}{ccc|c} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 6 & 7 & 8 \end{array} \right]$$

Performing $R_3 - 6R_2$

$$\left[\begin{array}{cccc} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & -5 & -10 \end{array} \right]$$

Divide R_3 with (-5)

$$\left[\begin{array}{cccc} 1 & 0 & -3 & -4 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

Performing $R_1 + 3R_3$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

Performing $R_2 - 2R_3$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array} \right]$$

$$x_1 = 2$$

$$x_2 = -1$$

$$x_3 = 2$$

So, solution is $(2, -1, 2)$

Question 15:-

$$x_1 - 6x_2 = 5$$

$$x_2 - 4x_3 + x_4 = 0$$

$$-x_1 + 6x_2 + x_3 + 5x_4 = 3$$

$$-x_2 + 5x_3 + 4x_4 = 0$$

$$\left[\begin{array}{ccccc} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ -1 & 6 & 1 & 5 & 3 \\ 0 & -1 & 5 & 4 & 0 \end{array} \right]$$

Performing $R_3 + R_1$

$$\left[\begin{array}{ccccc} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & -1 & 5 & 4 & 0 \end{array} \right]$$

Performing $R_4 + R_2$

$$\left[\begin{array}{ccccc} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 1 & 5 & \cancel{8} \end{array} \right]$$

Performing $R_3 - R_4$

$$\left[\begin{array}{ccccc} 1 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 0 & 5 & 8 \\ 0 & 0 & 0 & 0 & -8 \end{array} \right]$$

The system is inconsistent because ~~second~~ last row would require $0 = -8$
so,

The solution set is empty.

Question 16:-

$$2x_1 - 4x_4 = -10$$

$$3x_2 + 3x_4 = 0$$

$$x_3 + 4x_4 = -1$$

$$-3x_1 + 2x_2 + 3x_3 + x_4 = 5$$

$$\left| \begin{array}{ccccc} 2 & 0 & 0 & -4 & -10 \\ 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ -3 & 2 & 3 & 1 & 5 \end{array} \right|$$

Divide R₁ with 2

$$\left| \begin{array}{ccccc} 1 & 0 & 0 & -2 & -5 \\ 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ -3 & 2 & 3 & 1 & 5 \end{array} \right|$$

Performing R₄ + 3R₁

$$\left| \begin{array}{ccccc} 1 & 0 & 0 & -2 & -5 \\ 0 & 3 & 0 & 3 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 2 & 3 & -5 & -10 \end{array} \right|$$

Divide R₂ with 3

$$\left| \begin{array}{ccccc} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 2 & 3 & -5 & -10 \end{array} \right|$$

Performing R₄ - 2R₂

$$\left| \begin{array}{ccccc} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 3 & -7 & -10 \end{array} \right|$$

Performing $R_4 - 3R_3$

$$\left| \begin{array}{ccccc} 1 & 0 & 0 & -2 & -5 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & -19 & -7 \end{array} \right|$$

The system is now in echelon form and has a solution.

Question 17 :-

$$2x_1 + 3x_2 = -1$$

$$6x_1 + 5x_2 = 0$$

$$2x_1 - 5x_2 = 7$$

$$\left| \begin{array}{ccc} 2 & 3 & -1 \\ 6 & 5 & 0 \\ 2 & -5 & 7 \end{array} \right|$$

Performing $R_3 - R_1$

$$\left| \begin{array}{ccc} 2 & 3 & -1 \\ 6 & 5 & 0 \\ 0 & -8 & 8 \end{array} \right|$$

Performing $R_2 - 3R_1$

$$\left| \begin{array}{ccc} 2 & 3 & -1 \\ 0 & -4 & 3 \\ 0 & -8 & 8 \end{array} \right|$$

$$\left[\begin{array}{ccc|c} 2 & 3 & -1 & \\ 0 & -4 & 3 & \\ 0 & 0 & 2 & \end{array} \right]$$

Performing $R_3 - 2R_2$

System is inconsistent because $0=2$ in last line. So these equations have no common point of intersection.



Question 18:-

$$2x_1 + 4x_2 + 4x_3 = 4$$

$$x_2 - 2x_3 = -2$$

$$2x_1 + 3x_2 = 0$$

$$\left[\begin{array}{ccc|c} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 2 & 3 & 0 & 0 \end{array} \right]$$

Performing $R_3 - R_1$

$$\left[\begin{array}{ccc|c} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & -1 & -4 & -4 \end{array} \right]$$

Performing $R_3 + R_2$

$$\left[\begin{array}{ccc|c} 2 & 4 & 4 & 4 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & -6 & -8 \end{array} \right]$$

Divide R_1 with 2

$$\left[\begin{array}{cccc} 1 & 2 & 2 & 2 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & -6 & -6 \end{array} \right]$$

Divide R₃ with -6

$$\left[\begin{array}{cccc} 1 & 2 & 2 & 2 \\ 0 & 1 & -2 & -2 \\ 0 & 0 & 1 & 1 \end{array} \right]$$

The system is consistent and there is only one solution. So three planes have only one point in common.



Question 19:-

$$\left[\begin{array}{ccc} 1 & h & 4 \\ 3 & 6 & 8 \end{array} \right]$$

Performing R₂ - 3R₁,

$$\left[\begin{array}{ccc} 1 & h & 4 \\ 0 & 6-3h & -4 \end{array} \right]$$

write c for

$$6 - 3h = 0$$

$$6 = 3h \quad h = 2$$

If h=2 then system has no solution, because 0 ≠ -4.

Question 20:-

$$\begin{bmatrix} 1 & h & -5 \\ 2 & -8 & 6 \end{bmatrix}$$

$$R_2 - 2R_1$$

$$\begin{bmatrix} 1 & h & -5 \\ 0 & -8-2h & 16 \end{bmatrix}$$

write c for

$$-8-2h = 0$$

$$-8 = 2h$$

$$h = -4$$

if $h = -4$ then system has no solution
because $0 \neq 16$.



Question 21:-

$$\begin{bmatrix} 1 & 4 & -2 \\ 3 & h & -6 \end{bmatrix}$$

$$R_2 - 3R_1$$

$$\begin{bmatrix} 1 & 4 & -2 \\ 0 & h-12 & 0 \end{bmatrix}$$

write c for

$$h-12 = 0$$

$$h = 12$$

The second equation $x_2 = 0$ for every value

If c so the system is consistent for all value of h.



Question 22:-

$$\begin{bmatrix} -4 & 12 & h \\ 2 & -6 & -3 \end{bmatrix}$$

$$2R_2 + R_1$$

$$= \begin{bmatrix} -4 & 12 & h \\ 0 & 0 & -6+h \end{bmatrix}$$

Solution is consistent if and only if $-6+h=0$, that is if $h=6$.



Question #25:-

$$\begin{bmatrix} 1 & -4 & 7 & 9 \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$$

$$R_3 - 2R_1$$

$$\begin{bmatrix} 1 & -4 & 7 & 9 \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & k+2g \end{bmatrix}$$

$$R_3 + R_2$$

$$\begin{bmatrix} 1 & -4 & 7 & 9 \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & k+2g+h \end{bmatrix}$$

Original system has solution if & only if $k+2g+h$.