Exercise 1.3

Question # 02:-

$$U = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad V = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$U + V = \begin{bmatrix} 3 \\ 2 \end{bmatrix} + \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$U - 2V = \begin{bmatrix} 3+2 \\ 2-1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

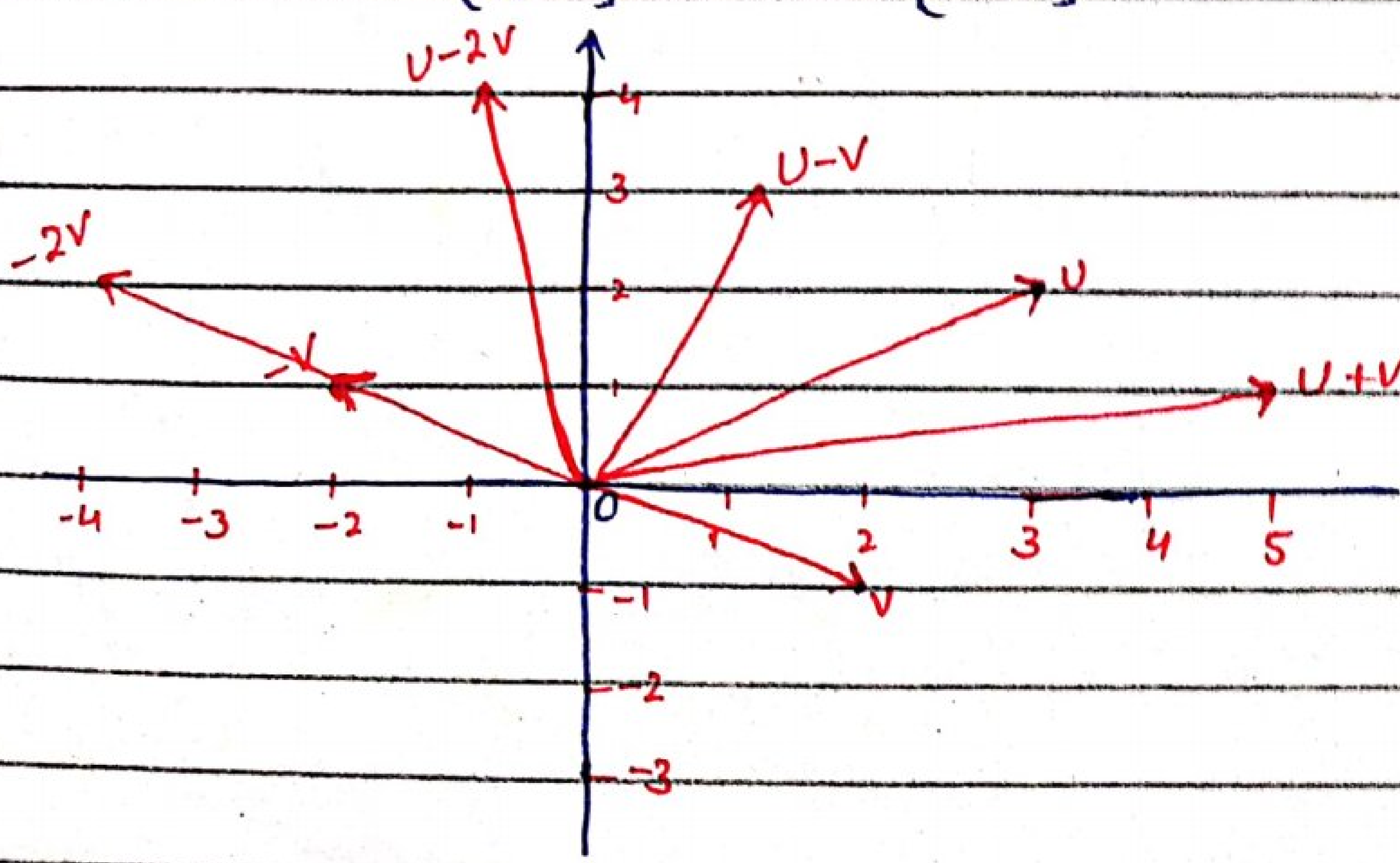
$$U - 2V = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - 2\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$U - 2V = \begin{bmatrix} 3-4 \\ 2+2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

Question No 04:-

$$U = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad V = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$



$$-v = -\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$-2v = -2\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$U+V = \begin{bmatrix} 3 \\ +2 \end{bmatrix} + \begin{bmatrix} +2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3+2 \\ +2-1 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$$

$$U-2V = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - 2\begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

$$= \begin{bmatrix} 3-4 \\ 2+2 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$U-V = \begin{bmatrix} 3 \\ 2 \end{bmatrix} - \begin{bmatrix} 2 \\ -1 \end{bmatrix}$$

$$= \begin{bmatrix} 3-2 \\ 2+1 \end{bmatrix}$$

$$U-V = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

Question #05:-

$$x_1 \begin{bmatrix} 3 \\ -2 \\ 8 \end{bmatrix} + x_2 \begin{bmatrix} 5 \\ 0 \\ -9 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 8 \end{bmatrix}$$

$$3x_1 + 5x_2 = 2$$
$$-2x_1 + 0x_2 = -3$$
$$8x_1 - 9x_2 = 8$$

$$\begin{aligned} 3x_1 + 5x_2 &= 2 \\ -2x_1 \quad &= -3 \\ 8x_1 - 9x_2 &= 8 \end{aligned}$$

Usually the intermediate steps are not displayed

$\longleftrightarrow$

Question #06:-

$$x_1 \begin{bmatrix} 3 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 7 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3x_1 \\ -2x_1 \end{bmatrix} + \begin{bmatrix} 7x_2 \\ 3x_2 \end{bmatrix} + \begin{bmatrix} -2x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$3x_1 + 7x_2 - 2x_3 = 0$$
$$-2x_1 + 3x_2 + x_3 = 0$$

Usually the intermediate steps are not displayed

Question # 7:-

vector a, b, c and d.

To reach a from origion we travel one unit of u direction and -2 unit of v direction then we reaches to a

So,

$$a = u - 2v \longrightarrow (1)$$

$$b = 2u - 3v$$
$$c = 2u - 3.5v$$
$$d = 3u - 4v$$

Question # 8:-

vectors w, x, y and z.

$$w = 2v - u$$
$$x = 2v - 2u$$
$$y = 3.5v - 2u$$
$$z = 4v - 3u$$

Question # 9:-

$$x_2 + 5x_3 = 0$$
$$4x_1 + 6x_2 - x_3 = 0$$
$$-x_1 + 3x_2 - 8x_3 = 0$$

$$\begin{bmatrix} x_2 + 5x \\ 4x_1 + 6x_2 - x_3 \\ -x_1 + 3x_2 - 8x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

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$$\begin{bmatrix} 0x_1 \\ 4x_1 \\ -x_1 \end{bmatrix} + \begin{bmatrix} 1x_2 \\ 6x_2 \\ 3x_2 \end{bmatrix} + \begin{bmatrix} 5x_3 \\ -x_3 \\ -8x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1 \begin{bmatrix} 0 \\ 4 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 6 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -1 \\ -8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Usually intermediate calculations are not displayed.

$\longleftrightarrow$

Question # 11:-

$$a_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}, \ a_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}, \ a_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}, \ b = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

$$a_1x_1 + a_2x_2 + a_3x_3 = b$$

$$x_1 \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x_1 + 0 + 5x_3 \\ -2x_1 + x_2 - 6x_3 \\ 0 + 2x_2 + 8x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix}$$

$R_2 + 2R_1$

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{bmatrix}$$

$R_3 - 2R_2$

$$\begin{bmatrix} 1 & 0 & 5 & 3 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

system is continuous and Infinite. So the system having many solutions.

$\longleftrightarrow$

Question # 12:-

$$a_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, a_2 = \begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix}, a_3 = \begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix}, b = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

$$a_1x_1 + a_2x_2 + a_3x_3 = b$$

$$x_1\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + x_2\begin{bmatrix} -2 \\ 3 \\ -2 \end{bmatrix} + x_3\begin{bmatrix} -6 \\ 7 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x_1 - 2x_2 - 6x_3 \\ 0 + 3x_2 + 7x_3 \\ x_1 - 2x_2 + 5x_3 \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 1 & -2 & 5 & 9 \end{bmatrix}$$

$R_3 - R_1$

$$\begin{bmatrix} 1 & -2 & -6 & 11 \\ 0 & 3 & 7 & -5 \\ 0 & 0 & 11 & -2 \end{bmatrix}$$

The system is consistent and has a solution.

$\longleftrightarrow$

Question 13:-

$$\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ -2 & 8 & -4 & -3 \end{bmatrix}$$

$R_3 + 2R_1$

$$\begin{bmatrix} 1 & -4 & 2 & 3 \\ 0 & 3 & 5 & -7 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

The system for this argumented matrix is inconsistent so b is not the linear combination of the vectors formed from the columns of matrix A.

Question # 14:-

$$A = \begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{bmatrix}, \; b = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ -2 & 1 & -6 & -1 \\ 0 & 2 & 8 & 6 \end{bmatrix}$$

$R_2 + 2R_1$

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 2 & 8 & 6 \end{bmatrix}$$

R_3 divide with 2

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 1 & 4 & 3 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 5 & 2 \\ 0 & 1 & 4 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Linear combination corresponding to this matrix has many solution, so b is a linear combination of column of A.

Question # 15:-

$$\begin{bmatrix} 1 & -5 & 3 \\ 3 & -8 & -5 \\ -1 & 2 & h \end{bmatrix}$$

$R_2 - 3R_1$

$$\begin{bmatrix} 1 & -5 & 3 \\ 0 & 7 & -14 \\ -1 & 2 & h \end{bmatrix}$$

$R_3 + R_1$

$$\begin{bmatrix} 1 & -5 & 3 \\ 0 & 7 & -14 \\ 0 & -3 & h+3 \end{bmatrix}$$

R_2 divide with 7

$$\begin{bmatrix} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & -3 & h+3 \end{bmatrix}$$

$R_3 + 3R_2$

$$\begin{bmatrix} 1 & -5 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & h-3 \end{bmatrix}$$

$h-3=0$

$h=3$

Vector b is in span $\{a_1, a_2\}$ when $h-3$ is 0.

16- let $v_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$ and $y = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$

$$\begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ -2 & 7 & -5 \end{bmatrix}$$

$R_3 + 2R_1$

$$\begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ 0 & 3 & -5+2h \end{bmatrix}$$

$R_3 - 3R_2$

$$\begin{bmatrix} 1 & -2 & h \\ 0 & 1 & -3 \\ 0 & 0 & 2h+4 \end{bmatrix}$$

$$2h+4=0$$
$$2h=-4$$
$$h=-2$$

vector y is in span $\{v_1, v_2\}$ when $4+2h=0$ when $h=-2$.

17- $v_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix}$, $v_2 = \begin{bmatrix} -4 \\ 0 \\ 1 \end{bmatrix}$

$$1 \cdot v_1 + 1 \cdot v_2 = \begin{bmatrix} -1 \\ 1 \\ 3 \end{bmatrix}$$

$$1 \cdot V_1 - 1 \cdot V_2 = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$$

$$2 \cdot V_1 + 2 \cdot V_2 = \begin{bmatrix} -2 \\ 2 \\ 6 \end{bmatrix}$$

$$0 \cdot V_1 + 1 \cdot V_2 = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$$

18- $V_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$, $V_2 = \begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}$

$$1 \cdot V_1 - 1 \cdot V_2 = \begin{bmatrix} 3 \\ -2 \\ -2 \end{bmatrix}$$

$$1 \cdot V_1 + 1 \cdot V_2 = \begin{bmatrix} -1 \\ 4 \\ -2 \end{bmatrix}$$

$$0 \cdot V_1 - 2 \cdot V_2 = \begin{bmatrix} 4 \\ -6 \\ 0 \end{bmatrix}$$

$$1 \cdot V_1 - 0 \cdot V_2 = \begin{bmatrix} 1 \\ +1 \\ -2 \end{bmatrix}$$

Question 25 :-

$$A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix} \quad \text{is } b \text{ in } W?$$

$$= \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ -2 & 6 & 3 & -4 \end{bmatrix}$$

$R_3 + 2R_1$

$$= \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 6 & -5 & 4 \end{bmatrix}$$

$R_3 - 2R_2$

$$= \begin{bmatrix} 1 & 0 & -4 & 4 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -1 & 2 \end{bmatrix}$$

The system for this argumented matrix is consistent so, b is in W.