Lec # 16:-

Reflexive Closure.

$R = \{(1,1), (1,2), (2,1), (3,2)\}.$ $\quad A = \{1, 2, 3\}.$

$\Delta = \{(a,a) \mid a \in A\}.$

$\Delta = \{(1,1), (2,2), (3,3)\}.$

$R \cup \Delta = \{(1,1), (1,2), (2,1), (3,2)\} \cup \{(1,1), (2,2), (3,3)\}.$

$= \{(1,1), (1,2), (2,1), (3,2), (2,2), (3,3)\}.$

Ex1:- $R = \{(a,b) \mid a < b\}$ $\quad A = \mathbb{Z}$

P483 find the closure of R.

$\Delta = \{(a,a) \mid a \in \mathbb{Z}\}. = \{(a,a) \mid a = a\}.$

$R \cup \Delta = \{(a,b) \mid a < b \lor a = b\}. = \{(a,b) \mid a \leq b\}.$

Symmetric Closure.

$R \cup R^{-1}.$

Ex2:- $R = \{(a,b) \mid a > b\}.$ $\quad A = \mathbb{Z}.$

P483 find Symmetric Closure.

$R^{-1} = \{(b,a) \mid (a,b) \in R.\}.$

$= \{(b,a) \mid a > b\}.$

$= \{(a,b) \mid b > a\}.$ $\quad b > a \Rightarrow a < b.$

$R \cup R^{-1} = \{(a,b) \mid a > b \lor a < b\}.$

$R \cup R^{-1} = \{(a,b) \mid a \neq b\}.$

Transitive Closure:-

→ Problem

a b. $\quad$ b c.

→ Problem

$\begin{array}{c} a \ b \\ \downarrow \downarrow \end{array}$ $\qquad\qquad\qquad\qquad\qquad$ $\begin{array}{c} b \ \ c \\ \downarrow \ \uparrow \end{array}$

$R = \{(1,3), (1,4), (2,1), (3,2)\}$. $\qquad$ $A = \{1,2,3,4\}$.

$(1,2), (2,3), (2,4), (3,1)$.

$\forall a,b,c \in A$ if $(a,b) \in R \land (b,c) \in R \rightarrow (a,c) \in R$.

$R = \{(1,3), (1,4), (2,1), (3,2)\} \cup \{(1,2), (2,3), (2,4), (3,1)\}$.

$R = \{(1,3), (\underset{b}{1},\underset{c}{4}), (2,1), (3,2), (1,2), (2,3), (2,4), (\underset{a}{3},\underset{b}{1})\}$.

$(3,4)$,

loop: {
Add. missing elements. }.

Paths in A Directed Graph.

Ex3
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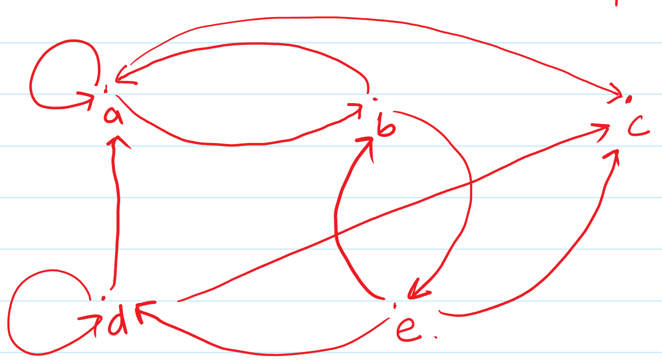


a, e
$(a,b)(b,e)$. $= 2$.
a,b,e $= 3-1 = 2$.
a,a,b,e

path: a path from a to b exist if there is a sequence of edges. $(a,x_1),(x_1,x_2),(x_2,x_3),\ldots(x_{n-1},x_n)(x_n,b)$.

theorem: let R be defined on A.
$\exists$ a path of length $n > 0, n \in \mathbb{Z}^+$ from a to b iff $(a,b) \in R^n$.

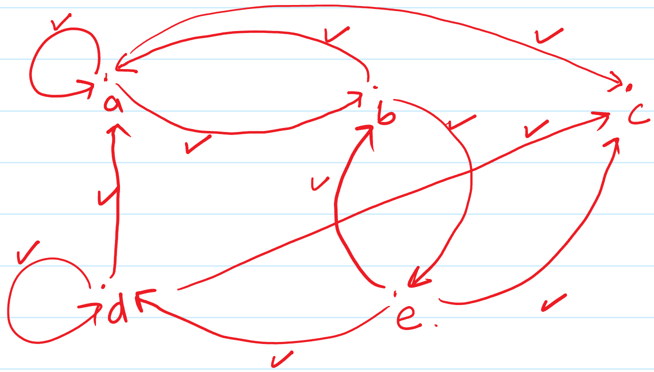
$R^1 \circ R^1 = R^2$
$R^2 \circ R^1 = R^3$

$R \circ R = R$

$R^2 \circ R^1 = R^3$

$\vdots$

$R^{n-1} \circ R = R^n$.



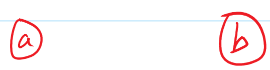
$R^1 = \{(a,a), (a,b), (b,a), (a,c), (b,c), (e,b), (d,d), (d,a), (d,c), (e,c), (e,d)\}$.

$(a,a) \in R^2$

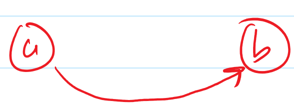
Connectivity Relation. Let R be defined on A.

$(a,b) \in R^*$ iff $\exists$ atleast one path from a to b.

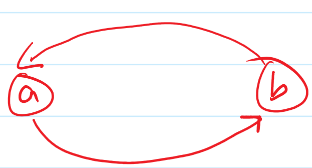
$$R^* = \bigcup_{i=1}^{\infty} R^i.$$



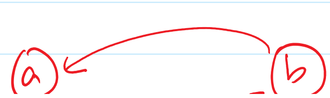
$R = \{\}. \rightarrow R^* = \{\}.$

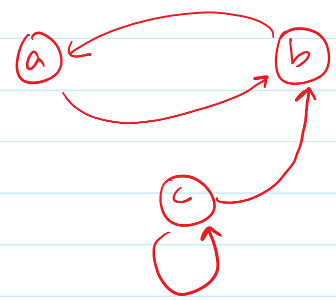


$R = \{(a,b)\}$.



$R^* = \{(a,a), (a,b), (b,a), (b,b)\}$.





Ex 4 / 485 :- $R = \{(a,b) \mid a$ has met $b\}$. $A =$ Set of people in the world.

R^n, R^*.

$R^2 = R^1 \circ R^1$.

$= $ if $\exists\, x$ Such that a met x & x met b.

(a,c) $S \circ R$, if $(a,b) \in R \wedge (b,c) \in S$

(a,b) R $A \times B$.

(b,c). S $B \times C$.

$R^3 =$. if $\exists\, x$ two persons x_1, x_2 Such That
a met x_1
x_1 met x_2
x_2 " b.

(a,b) $R \circ R$.
if $(a,x) \in R \wedge (x,b) \in R$.
if a has met x $\wedge$ x has met b.

$R^n =$ a met x_1
x_1 " x_2
x_2 " x_3
⋮
x_{n-1} " x_n
x_n " b.

$R^* =$ a and b if $\exists$ any number of persons in the middle.

$R = \{(a,b) \mid a$ shares border with $b\}$.

$A =$ Set of Countries.

Ex 5.6
p 486.

The transitive closure of a relation $= R^*$.

EQUIVALENCE RELATION.