National University of Sciences and Technology (NUST)
School of Electrical Engineering and Computer Science

ME-100: Engineering Mechanics

PROJECT REPORT

Class: BEE-11D

Submitted to: Dr. Kamran Zeb

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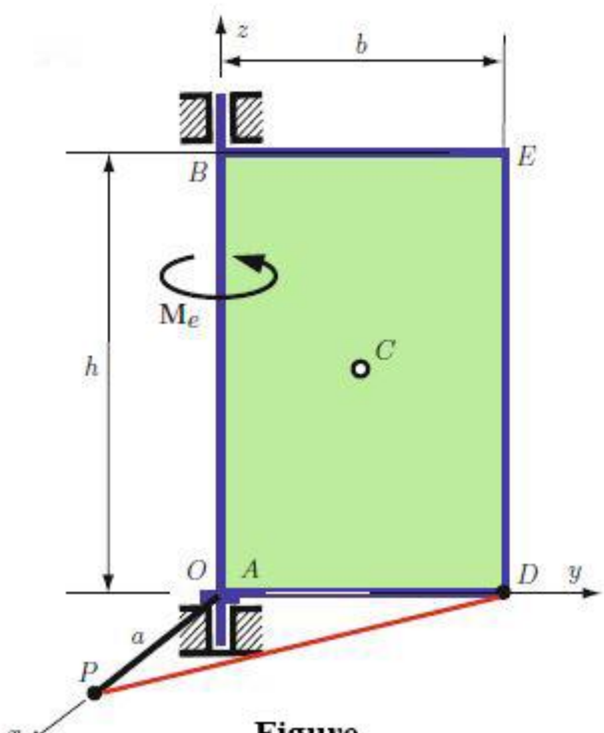
Concepts Used:

- Conditions of Equilibrium in Three-Dimensional force system
- Free body Diagrams
- Position Vector and force vectors along a line
- Principal of moments and moment about specific axis
- Cross product and Resolution of forces

Implemented Software: MATLAB®

Problem Statement:



Figure

The vertical shaft ***AB***, as shown in figure, is mounted through bearings at ***A*** and ***B***, and is supporting a uniform rectangular plate ***ABED*** with mass ***m*** and edges length

AB **=** ***DE*** **=** ***h*** and ***AD*** **=** ***BE*** **=** ***b***. The mass of the shaft is negligible, and the mass of the plate is ***m***. The distance between the upper bearing located at ***B*** and the lower bearing located at ***A*** is equal with ***h***. The bearing at ***A*** supports the entire vertical load. A moment of magnitude ***Me*** is applied to shaft in the vertically upward direction. The plate is constrained from rotating about the vertical axis by the action of a cable attached to outside corner of the plate denoted by ***D***. The other end of the cable is attached to a fixed support point ***P*** that is in a perpendicular line ***PA*** to the plate. The perpendicular distance from the cable attachment point, ***P***, to the plate is equal to ***PA*** **=** ***a***. Find the bearing reaction forces and the tension in the cable ***PD***. For the numerical application Use:

h **= 0.8m**
b **= 0.6m**
a **= 0.4m**

m **= 80 kg**
Me **= 100Nm**
g **= 9.81 m/s2**

Solution:

- We are required to find the tension in cable ***BD*** when the force is applied vertical to shaft.
- We are also required to find the reaction forces due to bearings.

Bearings At point ***A*** and ***B*** exert force in the perpendicular direction to the force along the shaft. This means they do not provide any opposing force to the moment along the shaft, hence no couple forms.

The weight of the plate attached to the shaft is vertically downward and is F=mg.

We start by making our reference axis. With origin at ***A***. By using standard notation for axis representation, the shaft will be along ***z-axis*** and the cable attached will be in the ***xy-plane***.

Considering the mechanical system to be the shaft-plate combination, the bearing forces of interest at ***A*** and ***B,*** and the weight of the plate ***D*** acts on the mechanical system.

In addition to the two unknown horizontal force components at the bearing ***B***,the cable force on the plate also acts on the system along with the three bearing reactions at ***A*** (the vertical support on the shaft plus the two horizontal bearing components).

To determine vertical and horizontal components of the reaction force, the following equations will be written.

F = **F** + **F**B + **G** + **T** = **0**, (Sum of all forces acting on system is zero)
or equivalent
F = FAx ı + FAy j + FAzk + FBx ı + FBy j + T r/|r| − mg k = 0,

Resolving into equations of x,y,z:
Fx = FAx + FBx + Tx = 0,
Fy = FAy + FBy + Ty = 0,
Fz = FAz − mg = 0,
Where

For X component of tension in cable ***PD***:
Tx = T(xP-xD)/((xP-xD)^2+(yP-Yd)^2)^1/2)

Similarly, for y component of cable ***PD***:
Ty = T(yP-yD)/((xP-xD)^2+(yP-Yd)^2)^1/2)

The sum of moments of all forces about point ***A*** is zero, that is
MA = Me + rAB × FB + rAD × T + rAC × G = 0

Where we apply cross product and add to get moments about **A**.

MATLAB CODE:

```
clear all;
clc;
close all
h=0.8;
b=0.6;
a=0.4;
m = 80.;
g = 9.81;
Me = 100.;
fprintf('Given dimensions of the shaft are as follows:\n\t a = 0.4m, b
= 0.6m, h = 0.8m, \n\t Mass of shaft = 80kg')
fprintf('\n\t Moment about point E in the vertical direction is Me =
100Nm\n')


Ax=0; Ay=0; Az=0; % m
Bx=0; By=0; Bz=h; % m
Dx=0; Dy=b; Dz=0; % m
Ex=0; Ey=b; Ez=h; % m
Cx=0; Cy=b/2;Cz=h/2; % m
Px=a; Py=0; Pz=0; % m
fprintf('\t Let us assume point A of the shaft is at origin
(0,0,0).\n\t Now we calculate the position vectors of');
fprintf(' each point')
fprintf('\n\t rA = 0i + 0j + 0k');
fprintf('\n\t rB = 0i + 0j + 0.8k');
fprintf('\n\t rC = 0i + 0.3j + 0.4k');
```

```
fprintf('\n\t rD = 0i + 0.6j + 0k');

fprintf('\n\t rE = 0i + 0.6j + 0.4k');

fprintf('\n\t rP = 0.4i + 0j + 0k');

fprintf('\n\t rG = 0i + 0j + -784.8k\n\t ');

rA=[Ax Ay Az];

rB=[Bx By Bz];

rD=[Dx Dy Dz];

rE=[Ex Ey Ez];

rC=[Cx Cy Cz];

rP=[Px Py Pz];

G = [0 0 -m*g];


fprintf('Now we find the unit vector in direction of the tension in
cable PD.\n\t ');

fprintf('u = rPD / ||rPD||\n\t ');

fprintf('Next, we find the tension as\n\t T = ||T||.u\n\t ');

fprintf('The reaction forces at the bearings FA and FB are given as
\n\t ');

fprintf('\n\t FA = FAxi + FAyj + Fzk');

fprintf('\n\t FB = FAxi + FAyj + 0k\n\t ');

syms FAx FAy FAz

FA = [FAx,FAy,FAz];

syms FBx FBy

FB = [FBx,FBy,0];


fprintf('\nEvaluating the equations of equilibrium of the system as
follows\n\t ');

fprintf('\nSum of Forces = 0 \n\t ');

fprintf('\nFA + FB + G + T = 0 \n\t ');
```

```
fprintf('\nNow we can resolve the equations of equilibrium into their respective axial components\n and solve for Fx, Fy, and Fz');


u=(rP-rD)/norm(rP-rD);

syms T

T = T*u;

SF = FA + FB + G + T;

Me=[0,0,Me];

SMA = cross(rC, G) + cross(rD, T) + cross(rB, FB)+ Me;

Fx=vpa(SF(1),3); %this function rounds the figure off to 3 decimal places

fprintf('\n\nSum of forces in x-direction = Fx : %s = 0 \n\t ',char(Fx))

Fy=vpa(SF(2),3);

fprintf('\nSum of forces in y-direction = Fy : %s = 0 \n\t ',char(Fy))

Fz=vpa(SF(3),3);

fprintf('\nSum of forces in z-direction = Fz : %s = 0 \n\t ',char(Fz))

fprintf('\nSimilarly, we find the moments about each axis as well.\n\t ');


Mx=vpa(SMA(1),3);

fprintf('\n\t Moment about x-axis = Mx : %s = 0 \n\t ',char(Mx))

My=vpa(SMA(2),3);

fprintf('Moment about y-axis = My : %s = 0 \n\t ',char(My))

Mz=vpa(SMA(3),3);

fprintf('Moment about z-axis = Mz : %s = 0 \n\t ',char(Mz))

sol=solve(...

SF(1) , SF(2), SF(3),...

SMA(1),SMA(2),SMA(3));
```

```
FAxs=eval(sol.FAx);

FAys=eval(sol.FAy);

FAzs=eval(sol.FAz);

FBxs=eval(sol.FBx);

FBys=eval(sol.FBy);

Ts=eval(sol.T);

fprintf('\n\t\t\t\t\t\t From the above equations of equilibrium we
obtained the following results.\n')

fprintf('\t FAx= %5.3f N \n',FAxs)

fprintf('\t FAy= %5.3f N \n',FAys)

fprintf('\t FAz= %5.3f N \n',FAzs)

fprintf('\t FBx= %5.3f N \n',FBxs)

fprintf('\t FBy= %5.3f N \n',FBys)

fprintf('\t T= %5.3f   N \n',Ts)

FAs = [FAxs FAys FAzs];

FBs = [FBxs FBys 0];

Ts = Ts*u;

axis_value=1;

hold on

axis([-axis_value axis_value -axis_value axis_value -axis_value
axis_value])

grid on

view(140,20);

quiver3(0,0,0,axis_value+0.1,0,0,...

'Color','b','LineWidth',1.0);

text(axis_value+0.1,0,0,' x',...

'fontsize',12,'fontweight','b');

quiver3(0,0,0,0,axis_value+0.1,0,...

'Color','b','LineWidth',1.0);
```

```
text(0,axis_value+0.1,0,' y',...
'fontsize',12,'fontweight','b');
quiver3(0,0,0,0,0,axis_value+0.1,'Color','b','LineWidth',1.0);
text(0,0,axis_value+0.1,' z',...
'fontsize',12,'fontweight','b');

scatter3(Ax,Ay,Az,60,'k','filled')
scatter3(Bx,By,Bz,60,'k','filled')
scatter3(Dx,Dy,Dz,30,'b','filled')
scatter3(Ex,Ey,Ez,30,'b','filled')
scatter3(Cx,Cy,Cz,30,'r','filled')
scatter3(Px,Py,Pz,30,'b','filled')
text(Ax,Ay,Az,' A','fontsize',12);
text(Bx,By,Bz+0.05,' B','fontsize',12);
text(Dx,Dy,Dz,' D','fontsize',12);
text(Ex,Ey,Ez,' E','fontsize',12);
text(Cx,Cy,Cz,' C','fontsize',12);
text(Px,Py,Pz+0.05,' P','fontsize',12);

vert = [Ax Ay Az; Bx By Bz; Ex Ey Ez; Dx Dy Dz];

fac = [1 2 3 4];

prism=patch('Faces',fac,'Vertices',vert,'FaceColor','g');
line([Px Dx],[Py Dy],[Pz Dz],'LineStyle','--
','Color','k','LineWidth',1)
fs = 1000;
quiver3(Dx,Dy,Dz,Ts(1)/fs,Ts(2)/fs,Ts(3)/fs,'Color','k','LineWidth',2)
;
```

```
text(Dx+Ts(1)/fs,Dy+Ts(2)/fs,Dz+0.03+Ts(3)/fs,'T');
quiver3(Ax,Ay,Az,FAs(1)/fs,FAs(2)/fs,FAs(3)/fs,'Color','k','LineWidth'
,2);
text(Ax+FAs(1)/fs,Ay+FAs(2)/fs,Az-0.1+FAs(3)/fs,'F_A');
quiver3(Bx,By,Bz,FBs(1)/fs,FBs(2)/fs,FBs(3)/fs,'Color','k','LineWidth'
,2);
text(Bx+FBs(1)/fs,By+FBs(2)/fs,Bz+0.05+FBs(3)/fs,'F_B');
quiver3(Cx,Cy,Cz,G(1)/fs,G(2)/fs,G(3)/fs,'Color','k','LineWidth',2);
text(Cx+G(1)/fs,Cy+G(2)/fs,Cz+G(3)/fs,'G');
quiver3(Bx,By,Bz,Me(1)/fs,Me(2)/fs,Me(3)/fs,'Color','k','LineWidth',2)
;
text(Bx+Me(1)/fs,By+0.05+Me(2)/fs,Bz+0.05+Me(3)/fs,'M_e');
alpha(prism,0.3);
xlabel('x(m)'); ylabel('y(m)');zlabel('z(m)');
%THE END%
```

OUTPUT:

Given dimensions of the shaft are as follows:

a = 0.4m, b = 0.6m, h = 0.8m,

Mass of shaft = 80kg

Moment about point E in the vertical direction is Me = 100Nm

Let us assume point A of the shaft is at origin (0,0,0).

Now we calculate the position vectors of each point

rA = 0i + 0j + 0k

rB = 0i + 0j + 0.8k

rC = 0i + 0.3j + 0.4k

rD = 0i + 0.6j + 0k

rE = 0i + 0.6j + 0.4k

rP = 0.4i + 0j + 0k

rG = 0i + 0j + -784.8k

Now we find the unit vector in direction of the tension in cable PD.

u = rPD / ||rPD||

Next, we find the tension as

T = ||T||.u

The reaction forces at the bearings FA and FB are given as

FA = FAxi + FAyj + Fzk

FB = FAxi + FAyj + 0k

Evaluating the equations of equilibrium of the system as follows

Sum of Forces = 0

FA + FB + G + T = 0

Now we can resolve the equations of equilibrium into their respective axial components and solve for Fx, Fy, and Fz

Sum of forces in x-direction = Fx : FAx + FBx + 0.555*T = 0

Sum of forces in y-direction = Fy : FAy + FBy - 0.832*T = 0

Sum of forces in z-direction = Fz : FAz - 785.0 = 0

Similarly, we find the moments about each axis as well.

Moment about x-axis = Mx : - 0.8*FBy - 235.0 = 0

Moment about y-axis = My : 0.8*FBx = 0

Moment about z-axis = Mz : 100.0 - 0.333*T = 0

From the above equations of equilibrium we obtained the following results.

FAx= -166.667 N

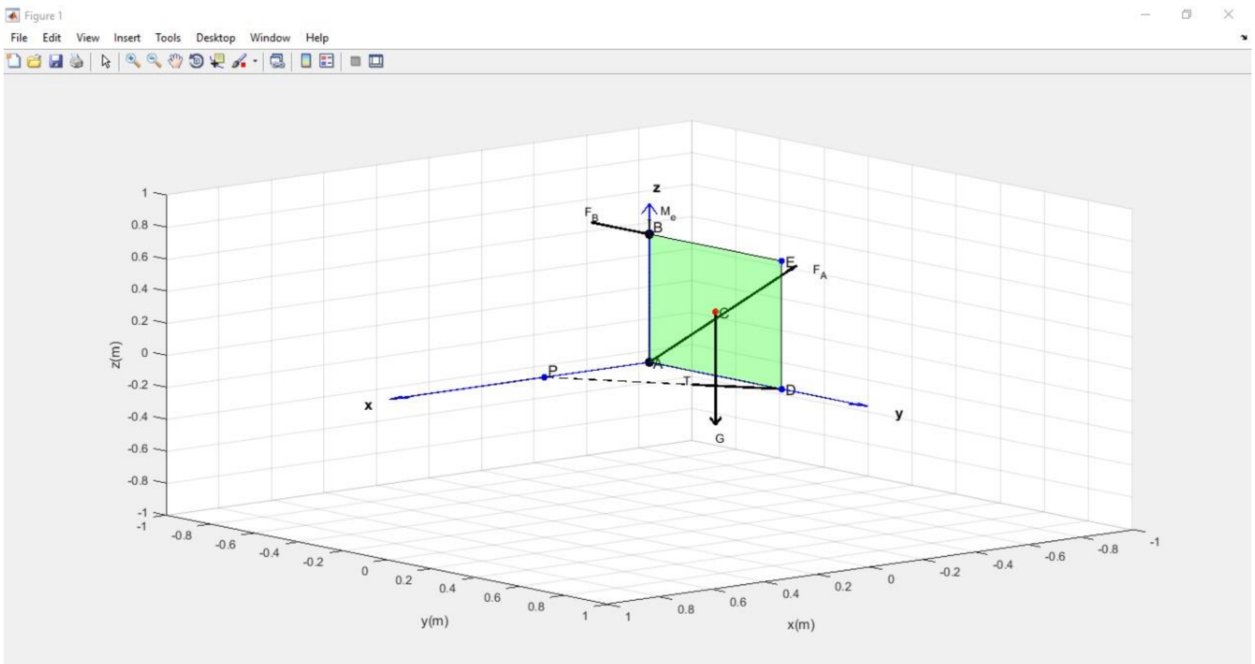
FAy= 544.300 N

FAz= 784.800 N

FBx= 0.000 N

FBy= -294.300 N

T= 300.463   N



Learning outcomes:

We Used and implemented Moment in 3D systems, applied equilibrium equations and calculated moments about a point or axis using vector or scalar approach.

We also prepared a mathematical model on MATLAB® of the mechanical system proposed and the solution obtained was verified through the experimental/theoretical results obtained.