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Subject: Linear Algebra

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<u>Assignment # 02</u>

Q No 1:

Prove that $\lambda^3 - \text{trace}\,\lambda^2 + \text{Sum}(A) - |A| = 0$ for:

$$A = \begin{bmatrix} -4 & 1 & 1 \\ 2 & -3 & 2 \\ 3 & 3 & -2 \end{bmatrix}$$

$$A = |A - \lambda I| = 0$$

$$= \begin{bmatrix} -4 & 1 & 1 \\ 2 & -3 & 2 \\ 3 & 3 & -2 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4-\lambda & 1 & 1 \\ 2 & -3-\lambda & 2 \\ 3 & 3 & -2-\lambda \end{bmatrix}$$

$$= (-4-\lambda)\begin{vmatrix} -3-\lambda & 2 \\ 3 & -2-\lambda \end{vmatrix} - 1\begin{vmatrix} 2 & 2 \\ 3 & -2-\lambda \end{vmatrix} + 1\begin{vmatrix} 2 & -3-\lambda \\ 3 & 3 \end{vmatrix}$$

$$= (-4-\lambda)\left((-3-\lambda)(-2-\lambda)-6\right) - 1\left(2(-2-\lambda)-6\right) + 1\left(6-3(-3-\lambda)\right)$$

$$= (-4-\lambda)(6+3\lambda+2\lambda+\lambda^2-6) - 1(-4-2\lambda-6) + 1(6+9+3\lambda)$$

$$= (-4-\lambda)(\lambda^2+5\lambda) - 1(-2\lambda-10) + 1(15+3\lambda)$$

$$= -4\lambda^2 - 20\lambda - \lambda^3 - 5\lambda^2 + 2\lambda + 10 + 15 + 3\lambda$$

$$= -\lambda^3 - 9\lambda^2 - 15\lambda + 25$$

$$= \lambda^3 + 9\lambda^2 + 15\lambda - 25$$

for Trace sum of Diagonal

$$\text{Trace} = -4-3-2 = -9$$

for $|A|$:

$$|A| = \begin{vmatrix} -4 & 1 & 1 \\ 2 & -3 & 2 \\ 3 & 3 & -2 \end{vmatrix}$$

$$= -4\begin{vmatrix} 3 & 2 \\ 3 & -2 \end{vmatrix} - 1\begin{vmatrix} 2 & 2 \\ 3 & -2 \end{vmatrix} + 1\begin{vmatrix} 2 & -3 \\ 3 & 3 \end{vmatrix}$$

$$= -4(6-6) - 1(-4-6) + 1(6+9)$$

$$= 0 + 10 + 15$$

$$|A| = +25$$

for $\text{Sum}|A|$:

$$\begin{vmatrix} -4 & 1 & 1 \\ 2 & -3 & 2 \\ 3 & 3 & -2 \end{vmatrix}$$

$$= 12 + 8 + 6 - 2 - 3 - 6$$

$$= 26 - 11$$

$$= 15 \quad \text{Hence Proved}$$

$$\lambda^3 - \text{trace}(A) + \text{Sum}|A| - |A| = \lambda^3 + 9\lambda^2 + 15\lambda - 25$$

QNO2 : Use LU Decomposition to solve the Matrix :

$$A = \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix}$$

$$A = LU$$

for upper triangular Matrix :

$$= \begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix} \begin{matrix} R_2 \Rightarrow R_2 + 2R_1 \\ R_3 \Rightarrow R_3 + R_1 \end{matrix}$$

$$= \begin{bmatrix} -1 & 0 & 3 \\ 0 & 1 & 9 \\ 0 & 1 & 5 \end{bmatrix} R_2 \Rightarrow R_2 - R_3$$

$$= \begin{bmatrix} -1 & 0 & 3 \\ 0 & 1 & 9 \\ 0 & 0 & -4 \end{bmatrix}$$

Now for lower triangular Matrix :

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

Eq. becomes $\Rightarrow$

$$\begin{bmatrix} -1 & 0 & 3 \\ 2 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 3 \\ 0 & 1 & 9 \\ 0 & 0 & -4 \end{bmatrix}$$