

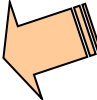
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# **Data Mining**

**Know your Data**

# Getting to Know Your Data

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- Data Objects and Attribute Types 
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary

# Data Objects

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- Data sets are made up of data objects.
- A **data object** represents an entity.
- Examples:
  - sales database: customers, store items, sales
  - medical database: patients, treatments
  - university database: students, professors, courses
- Also called *samples*, *examples*, *instances*, *data points*, *objects*, *tuples*.
- Data objects are described by **attributes**.
- Database rows -> data objects; columns -> attributes.

# Attributes

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- **Attribute (or dimensions, features, variables):**  
a data field, representing a characteristic or feature of a data object.
  - *E.g., customer\_ID, name, address*
- Types:
  - Nominal
  - Binary
  - Numeric: quantitative
    - Interval-scaled
    - Ratio-scaled

# Attribute Types

- **Nominal:** categories, states, or “names of things”
  - $Hair\_color = \{black, blond, brown, grey, red, white\}\{0,1,2...\}$
  - marital status, occupation, ID numbers (numbers but math operations can't be done on them), zip codes
  - No mean and median but mode can be used
- **Binary**
  - Nominal attribute with only 2 states (0 and 1)
  - Symmetric binary: both outcomes equally important
    - e.g., gender
  - Asymmetric binary: outcomes not equally important.
    - e.g., medical test (positive vs. negative)
    - Convention: assign 1 to most important outcome (e.g., HIV positive)
- **Ordinal**
  - Values have a meaningful order (ranking) but magnitude between successive values is not known.
  - $Size = \{small, medium, large\}$ , grades, army rankings

# Numeric Attribute Types

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- Quantitative (integer or real-valued). Can do math on them i.e. mean, median and mode, etc.
- **Interval Scaled**
  - Measured on a scale of **equal-sized units**
  - Values have order
    - E.g., *temperature in  $C^{\circ}$  or  $F^{\circ}$ , calendar dates*
  - No true zero-point.  $0^{\circ} C$  is not showing "no temp"
- **Ratio Scaled**
  - Inherent **zero-point**
    - e.g., *length, counts, monetary quantities, years of experience, word counts, weight, height etc.*

# Discrete vs. Continuous Attributes

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## ■ Discrete Attribute

- Has only a finite or countably infinite set of values
  - E.g., zip codes, profession, or the set of words in a collection of documents
- Attributes Hair\_Color, Smoker, Med Test, Drink\_Size each have a finite number of values, thus are discrete.
- Discrete attributes may have numeric values 0 and 1 for binary attributes
- Age have values from 0 to 110
- Customer\_ID is countably infinite
- Zip codes

# Discrete vs. Continuous Attributes

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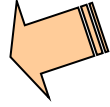
- **Continuous Attribute**

- Has real numbers as attribute values
  - E.g., temperature, height, or weight
- Practically, real values can only be measured and represented using a finite number of digits
- Continuous attributes are typically represented as floating-point variables



# Chapter 2: Getting to Know Your Data

---

- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data 
- Data Visualization
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# Basic Statistical Descriptions of Data

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- Motivation
  - To better understand the data: central tendency, variation and spread
- Data dispersion characteristics
  - median, max, min, quantiles, outliers, variance, etc.
- Numerical dimensions correspond to sorted intervals
  - Data dispersion: analyzed with multiple granularities of precision
  - Boxplot or quantile analysis on sorted intervals

# Measuring the Central Tendency

- Mean (algebraic measure) (sample vs. population):

Note:  $n$  is sample size and  $N$  is population size.

$$\bar{x} = \frac{1}{N} \sum_{i=1}^n x_i$$

- Weighted arithmetic mean:

- Trimmed mean: chopping extreme values

$$\bar{x} = \frac{\sum_{i=1}^N w_i x_i}{\sum_{i=1}^N w_i}$$

- Median:

- Middle value if odd number of values, or average of the middle two values otherwise

- Mode

- Value that occurs most frequently in the data

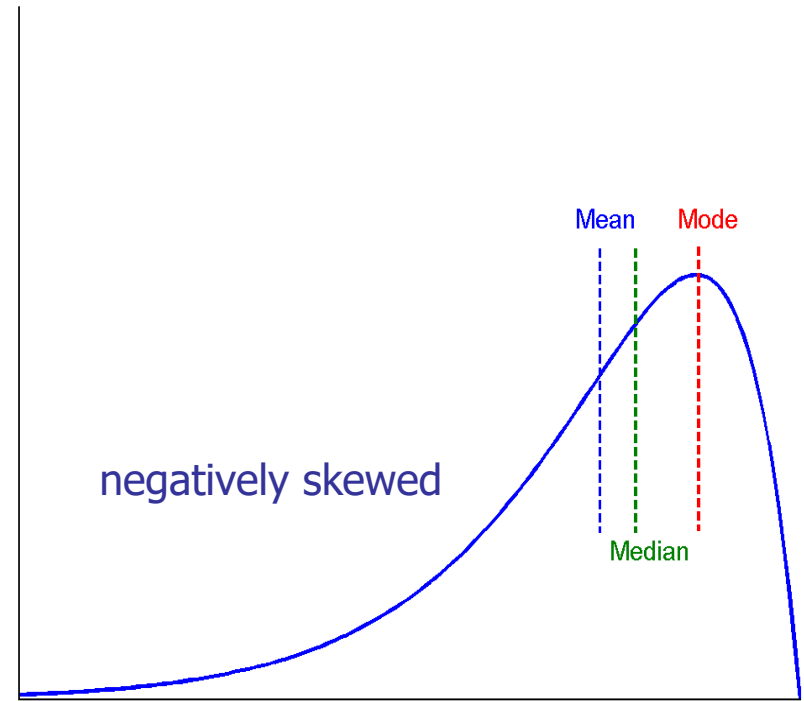
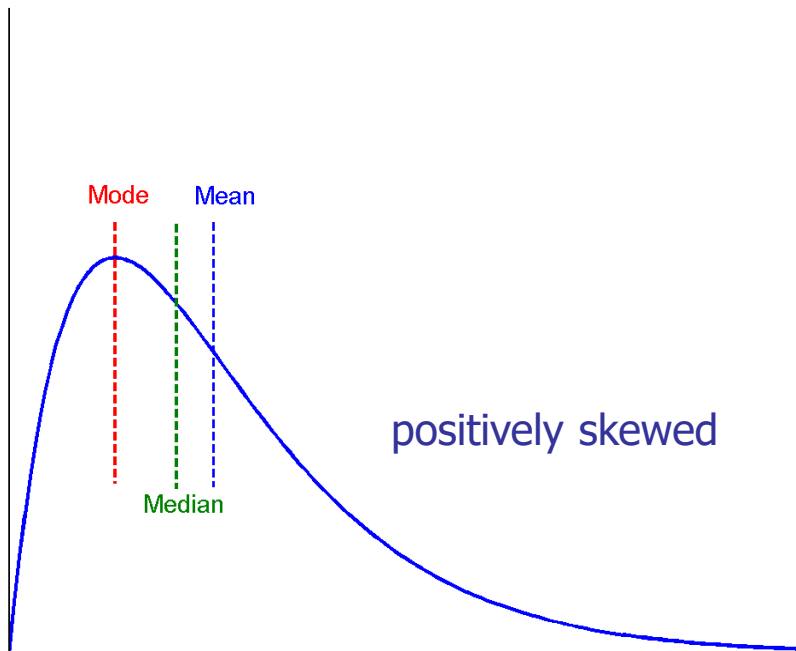
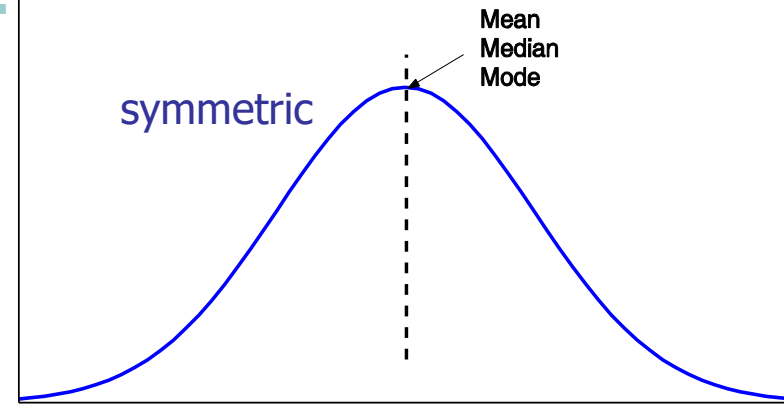
- Unimodal, bimodal, trimodal

- Empirical formula:  $mean - mode \approx 3 \times (mean - median)$

- Mode for unimodal frequencies can be approximated if mean and median values are known

# Symmetric vs. Skewed Data

- Median, mean and mode of symmetric, positively and negatively skewed data

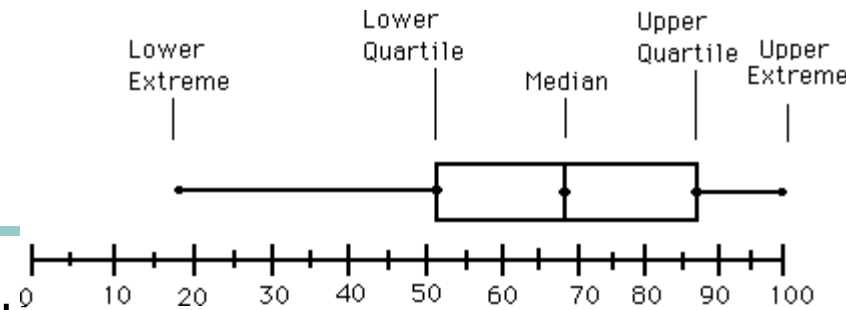


# Measuring the Dispersion of Data

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- Quartiles, outliers and boxplots
  - **Quartiles:**  $Q_1$  (25<sup>th</sup> percentile),  $Q_3$  (75<sup>th</sup> percentile)
  - **Inter-quartile range:**  $IQR = Q_3 - Q_1$
  - **Five number summary:** min,  $Q_1$ , median,  $Q_3$ , max
  - **Boxplot:** ends of the box are the quartiles; median is marked; add whiskers, and plot outliers individually
  - **Outlier:** usually, a value lower/higher
    - than  $o = (1.5 \times IQR)$  of  $(Q_1 - o)/(Q_3 + o)$

# Boxplot Analysis

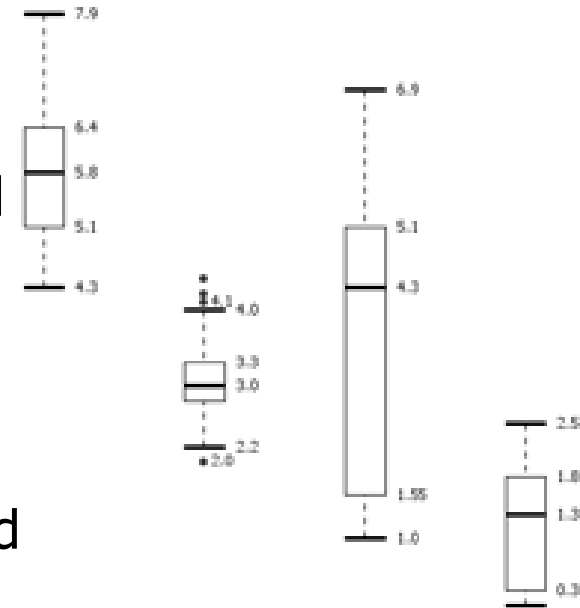


- **Five-number summary** of a distribution

- Minimum, Q1, Median, Q3, Maximum

- **Boxplot**

- Data is represented with a box
- The ends of the box are at the first and third quartiles, i.e., the height of the box is IQR
- The median is marked by a line within the box
- Whiskers: two lines outside the box extended to Minimum and Maximum
- Outliers: points beyond a specified outlier threshold, plotted individually



# Exercise 1

---

- Given the following data
- 7, 8, 7, 10, 7, 2, 8, 2, 7, 30
- Median =
- Q1 (index=2.5~3) =
- Q3 (index=7.5~8) =
- IQR =  $Q3 - Q1$
- 5 number summary =
- Outliers based on IQR =

# Exercise 1 – Solution

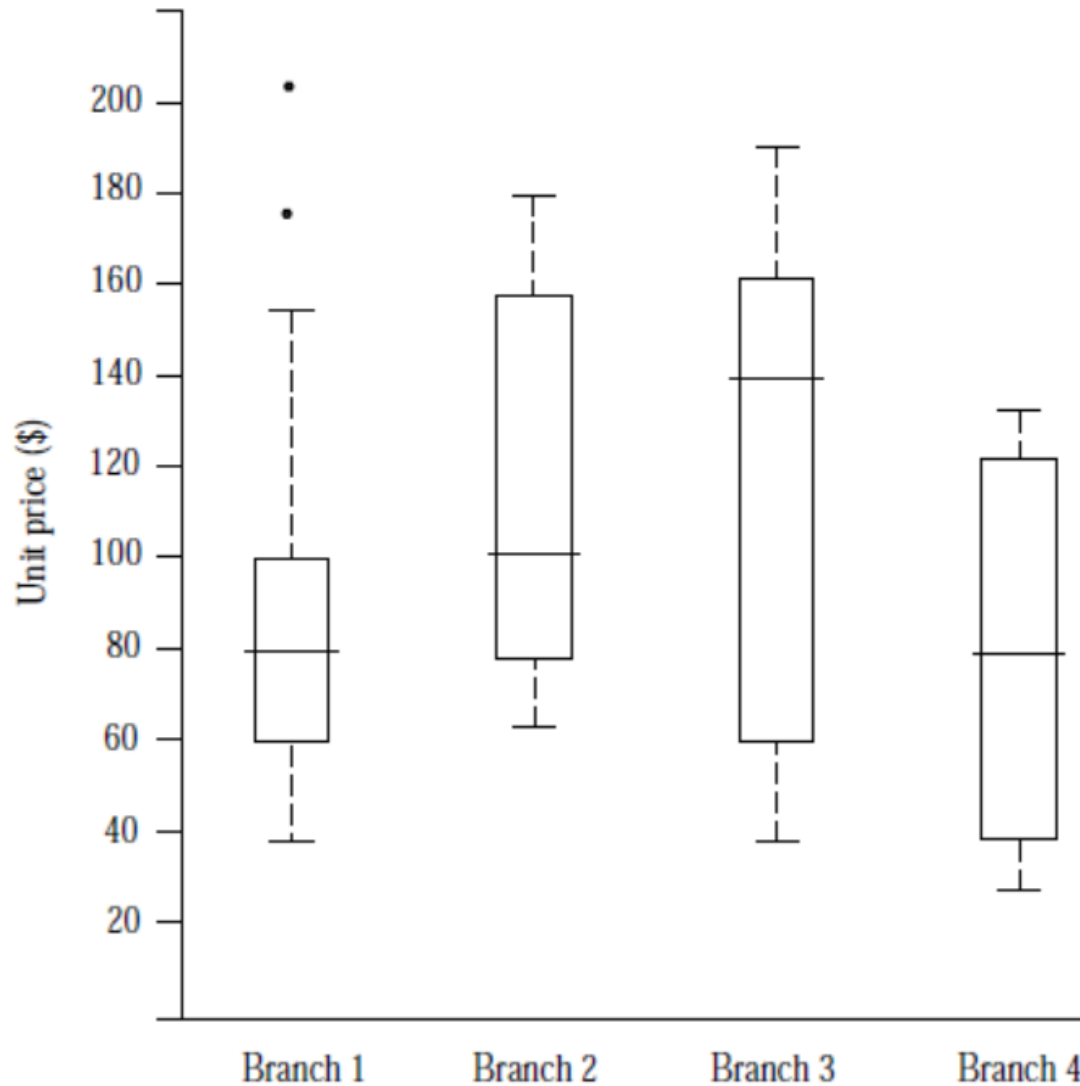
- Given the following data
- 7, 8, 7, 10, 7, 2, 8, 2, 7, 30
- Sort the data 2,2,7,7,7,7,8,8,10,30
- Median = 7
- $Q_1$  (index =  $10 * .25 = 2.5 \sim 3$ ) = 7
- $Q_3$  (index =  $10 * .75 = 7.5 \sim 8$ ) = 8
- $IQR = Q_3 - Q_1 = 8 - 7 = 1$
- **Five number summary:** min,  $Q_1$ , median,  $Q_3$ , max
- 5 number summary = 2, 7, 7, 8, 30
- Outliers based on IQR =
  - $IQR * 1.5 = 1 * 1.5 = 1.5 = x$
  - Values which are less than ( $Q_1 - x$ ) Or greater than ( $Q_3 + x$ )
  - $Q_1 - 1.5 = 7 - 1.5 = 5.5$
  - $Q_3 + 1.5 = 8 + 1.5 = 9.5$
  - Outliers = 2, 10, 30



## Exercise 2

- Find the outliers in the following using IQR
  - 2,2,3,4,7,7,8,9,10,30
- Find the outliers based on IQR
- $Q1 = 3$
- $Q3 = 9$
- $IQR = Q3 - Q1 = 9 - 3 = 6$
- $x = 1.5 * IQR = 9$
- $\text{Outliers} < Q1 - x = 3 - 9 = -6$
- $\text{Outliers} > Q3 + x = 9 + 9 = 18$
- $\text{Outliers} = 30$

# Boxplot Analysis



# Lab Task 1

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- Import data sales\_data.csv
- See the metadata view to check the attribute type, statistics (mean, mode, etc.), range and no of missing values.
- Create Boxplot for suitable fields

# Python Hint

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- #Import Basic Libraries
- `import numpy as np` # linear algebra
- `import pandas as pd` # data processing, CSV file I/O (e.g. `pd.read_csv`)
- `import matplotlib.pyplot as plt` #data visualization
- `import seaborn as sns` #data visualization

# Variance and Standard Deviation

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- Variance and standard deviation
- **Variance:** (algebraic, scalable computation)

$$\sigma^2 = \frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 = \left( \frac{1}{N} \sum_{i=1}^N x_i^2 \right) - \bar{x}^2,$$

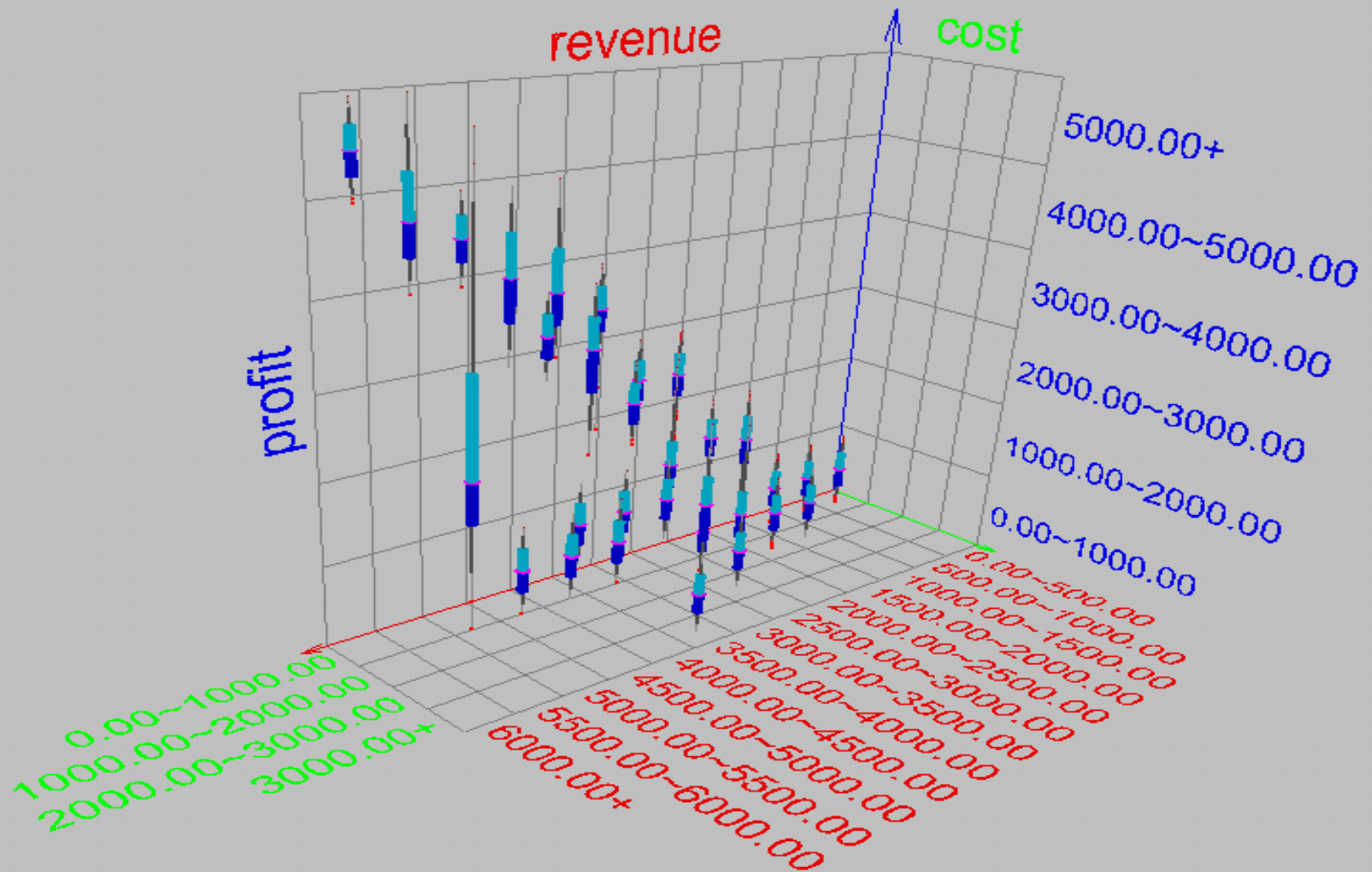
- **Standard deviation**  $s$  (or  $\sigma$ ) is the square root of variance  $s^2$  (or  $\sigma^2$ )
- Variance and standard deviation are measures of data dispersion.
- Low SD means observations are close to the mean
- High SD means the data are spread out over a large range of values

## Exercise 3: Find Variance and SD

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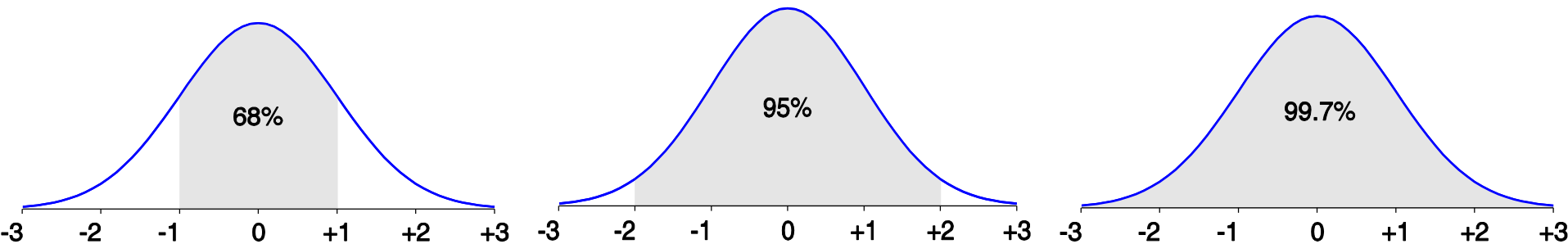
- 5, 10, 15
- $N = 3$
- $\text{Mean} = (5+10+15)/3 = 10$
- $\text{Var} = ((5-10)^2 + (10-10)^2 + (15-10)^2) / N$
- $= (-5^2 + 0^2 + 5^2) / 3$
- $= (25 + 0 + 25) / 3 = 50/3 = 16.7$
- $\text{StDev} = \text{sqrt}(\text{Var}) = \text{sqrt}(16.7) \sim 4$

# Visualization of Data Dispersion: 3-D Boxplots



# Properties of Normal Distribution Curve

- The normal (distribution) curve
  - From  $\mu - \sigma$  to  $\mu + \sigma$ : contains about 68% of the measurements ( $\mu$ : mean,  $\sigma$ : standard deviation)
  - From  $\mu - 2\sigma$  to  $\mu + 2\sigma$ : contains about 95% of it
  - From  $\mu - 3\sigma$  to  $\mu + 3\sigma$ : contains about 99.7% of it





# Lab Task2

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- Find normal distribution curve of usable attributes from your sales dataset

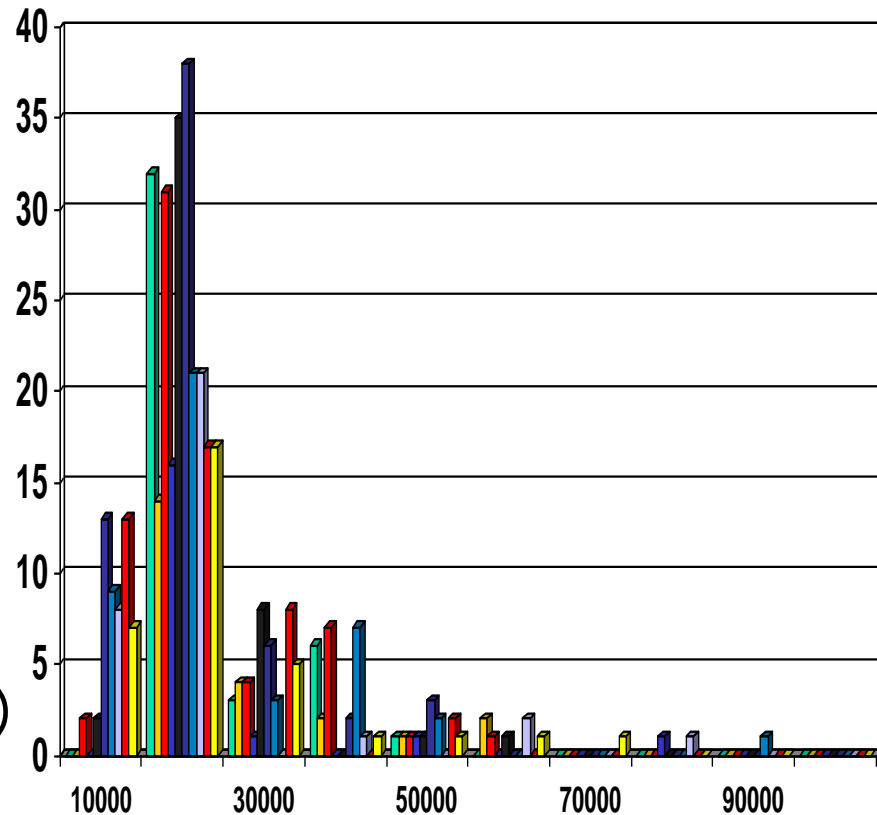
# Graphic Displays of Basic Statistical Descriptions

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- **Boxplot:** graphic display of five-number summary
- **Histogram:** x-axis are values, y-axis repres. frequencies
- **Scatter plot:** each pair of values is a pair of coordinates and plotted as points in the plane

# Histogram Analysis

- Histogram: Graph display of tabulated frequencies, shown as bars
- It shows what proportion of cases fall into each of several categories
- Bar chart represents categorical data while histogram represents quantitative data
- The categories are usually specified as non-overlapping intervals of some variable. The categories (bars) must be adjacent



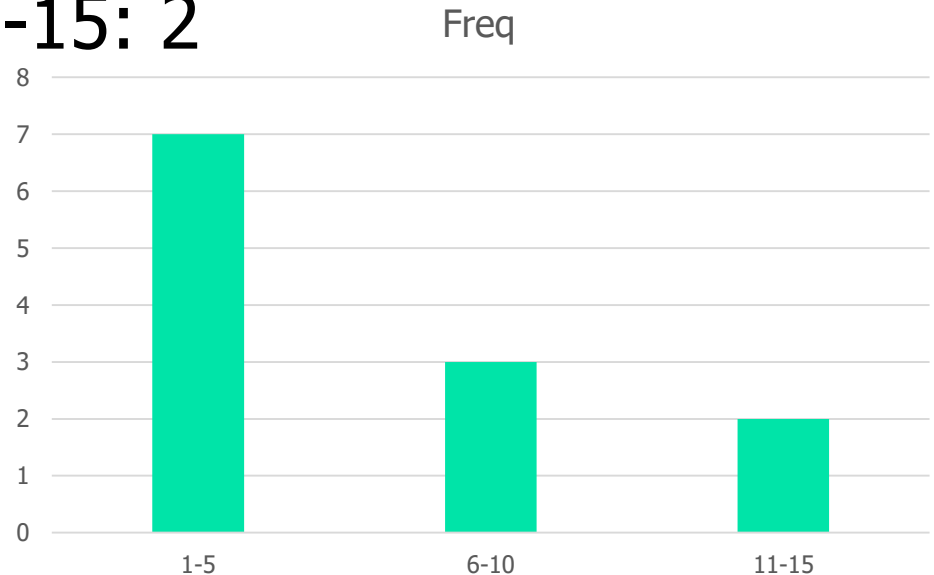
## Exercise 4

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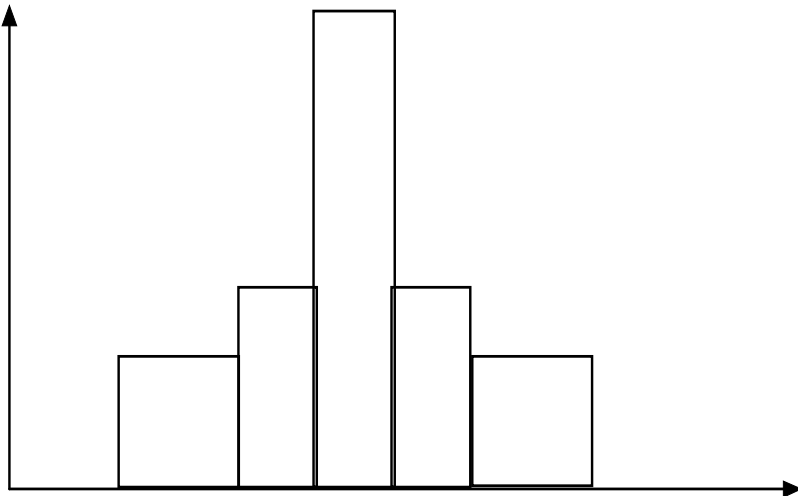
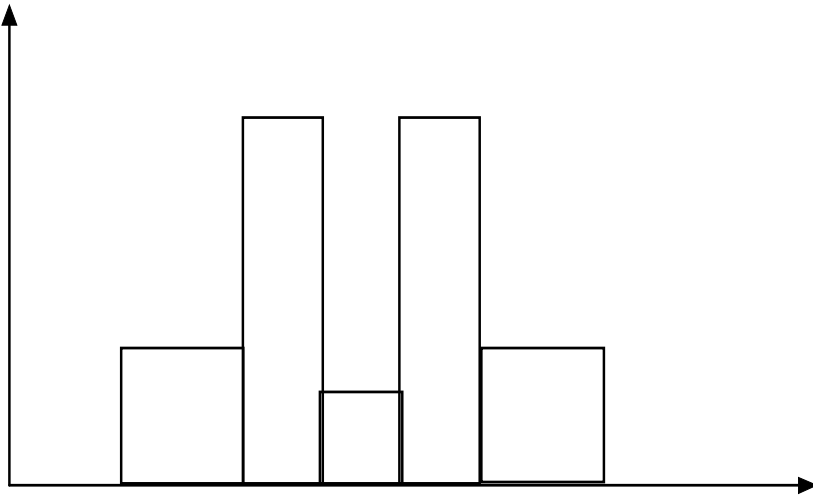
- Given the ages of the children:
  - 9,7,12,10,5,4,8,2,4,3,1,2,8,14
- Convert the data into ranges
- Count the frequency for each range
- Create a histogram

## Exercise 4

- Given the ages of the children:
  - 9,7,12,10,5,4,8,2,4,3,1,2,8,14
- Convert the data into ranges
  - 1-5,6-10,11-15
- Count the frequency for each range
  - 1-5: 7, 6-10: 3, 11-15: 2
- Create a barchart



# Histograms Often Tell More than Boxplots



- The two histograms shown in the left may have the same boxplot representation
  - The same values for: min, Q1, median, Q3, max
- But they have rather different data distributions

# Lab work

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- Create a histogram of amount
- Create a histogram of single\_price
- Create a bar chart of product\_category and single\_price

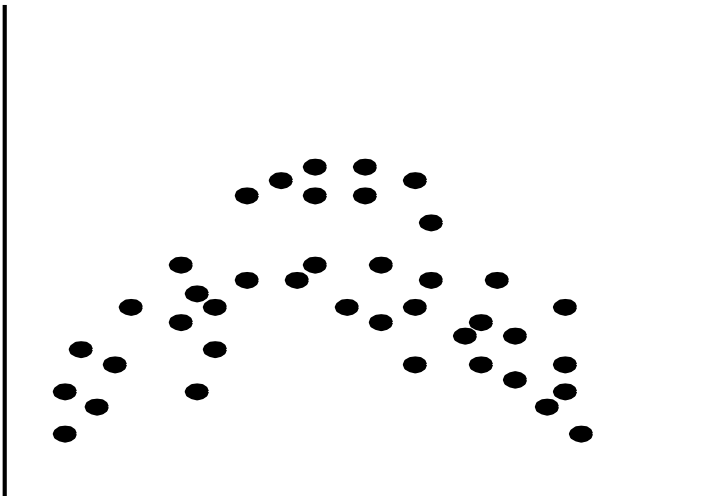
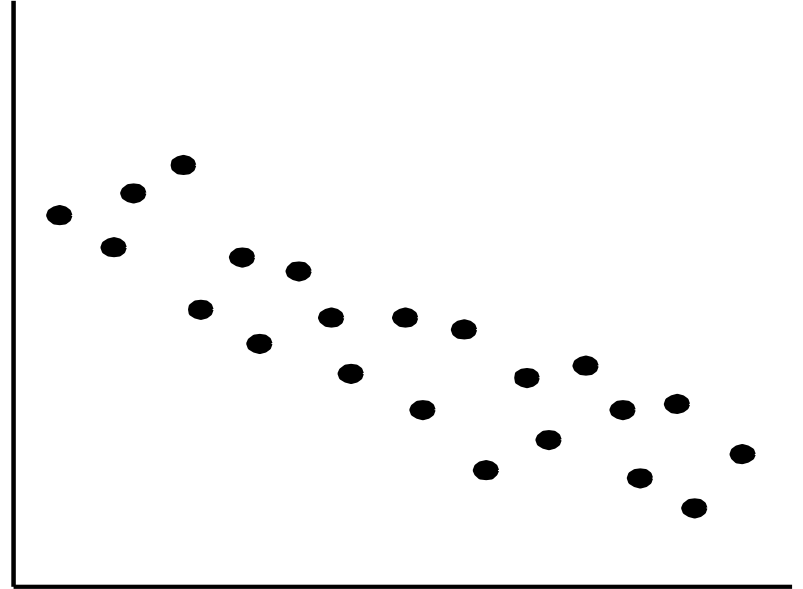
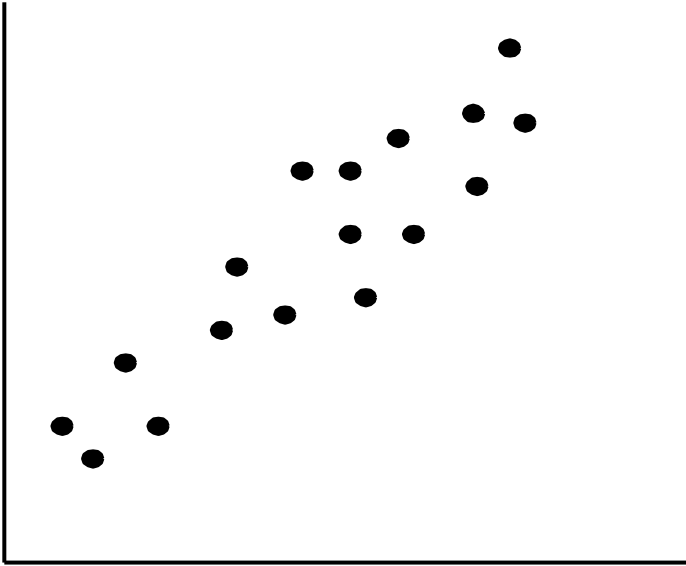
# Scatter plot

- Provides a first look at bivariate data (involving two attributes) to see clusters of points, outliers, etc
- Each pair of values is treated as a pair of coordinates and plotted as points in the plane
- Helps in finding if there is a correlation between two attributes





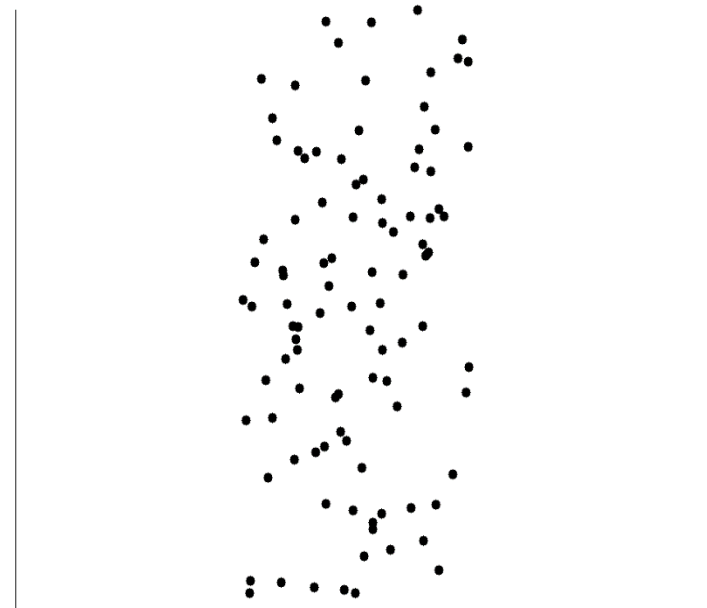
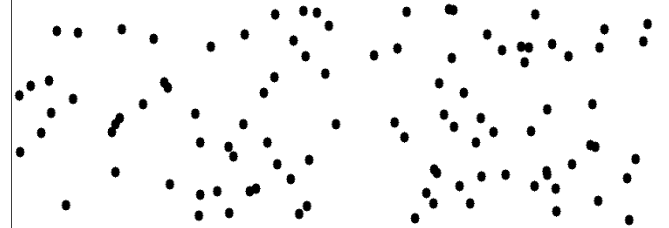
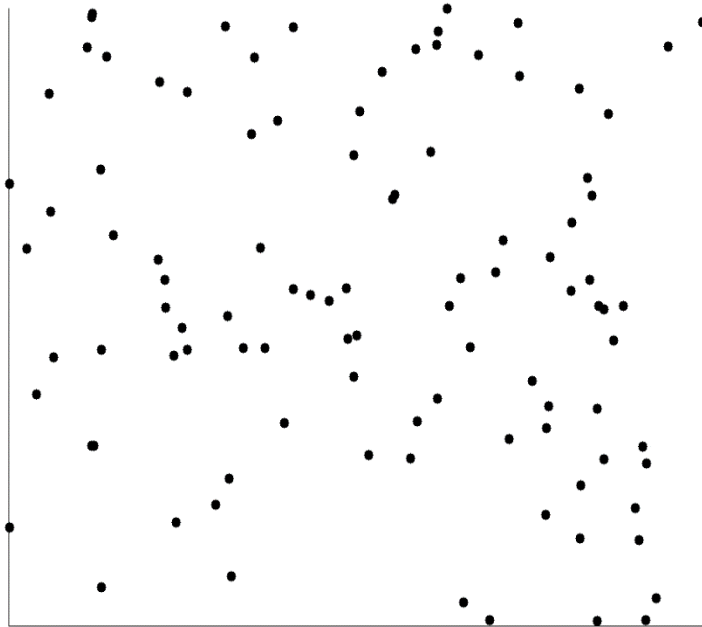
# Positively and Negatively Correlated Data



- The left half fragment is positively correlated
- The right half is negative correlated

# Uncorrelated Data

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## Exercise 6

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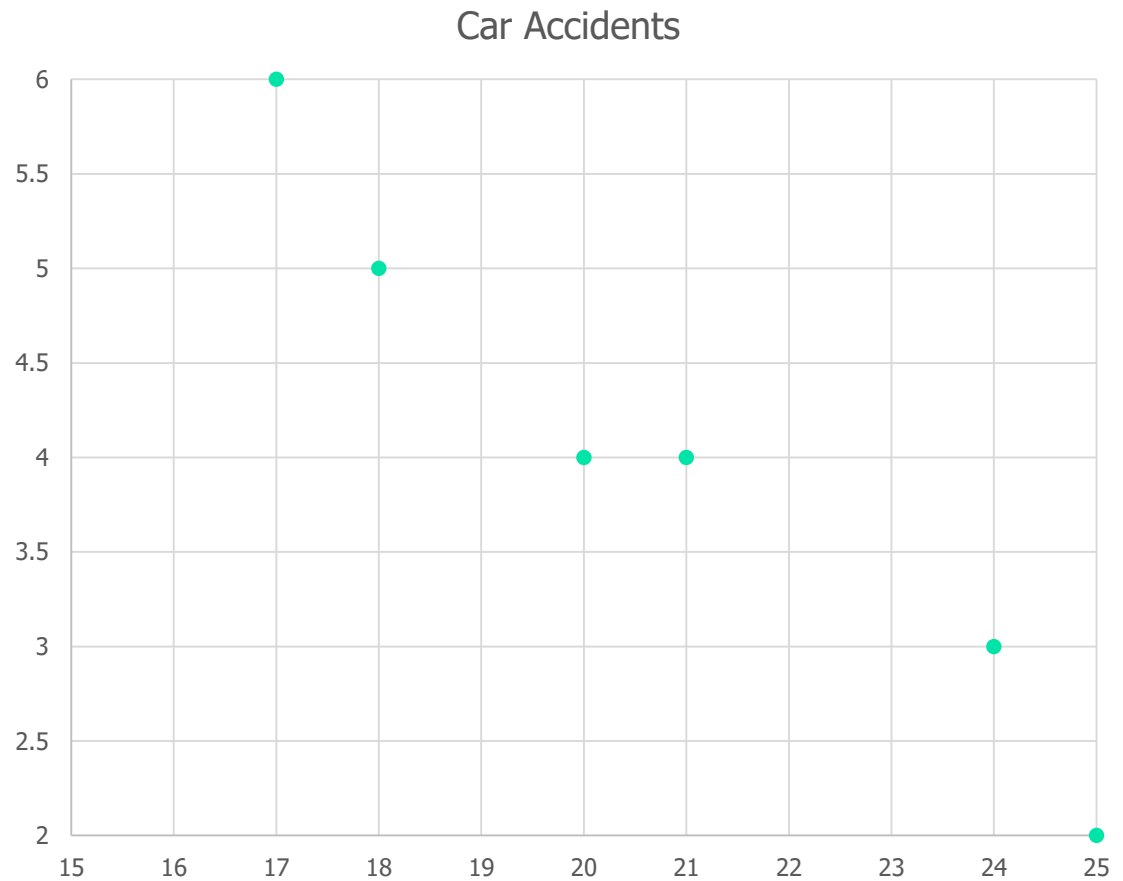
- Create a scatter plot for the following data

Age	Car Accidents
17	6
21	4
18	5
25	2
20	4
24	3

# Exercise 6


- Create a scatter plot for the following data

Age	Car Accidents
17	6
21	4
18	5
25	2
20	4
24	3



# Chapter 2: Getting to Know Your Data

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- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization 
- Measuring Data Similarity and Dissimilarity
- Summary

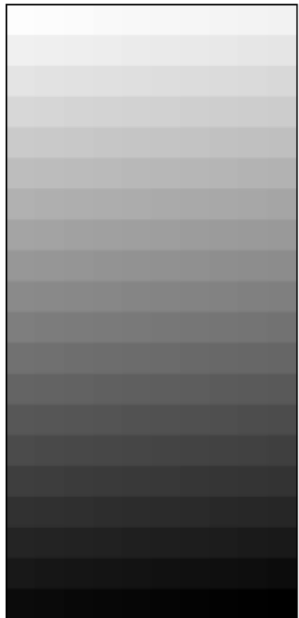
# Data Visualization

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- Why data visualization?
  - Gain insight into an information space by mapping data onto graphical primitives
  - Provide qualitative overview of large data sets
  - Search for patterns, trends, structure, irregularities, relationships among data
  - Help find interesting regions and suitable parameters for further quantitative analysis
- Categorization of visualization methods:
  - Pixel-oriented visualization techniques
  - Icon-based visualization techniques
  - Hierarchical visualization techniques
  - Visualizing complex data and relations

# Pixel-Oriented Visualization Techniques

- For a data set of  $m$  dimensions, create  $m$  windows on the screen, one for each dimension
- The  $m$  dimension values of a record are mapped to  $m$  pixels at the corresponding positions in the windows
- The colors of the pixels reflect the corresponding values



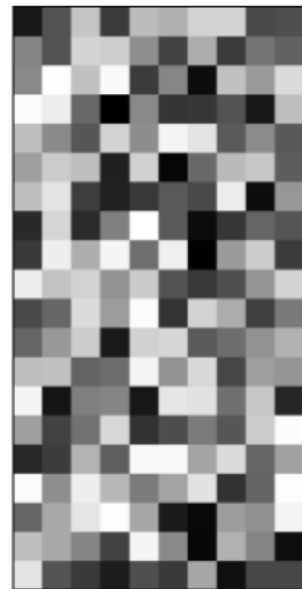
(a) Income



(b) Credit Limit



(c) transaction volume



(d) age

# Icon-Based Visualization Techniques

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- Visualization of the data values as features of icons
- Typical visualization methods
  - Chernoff Faces
  - Stick Figures
- General techniques
  - Shape coding: Use shape to represent certain information encoding

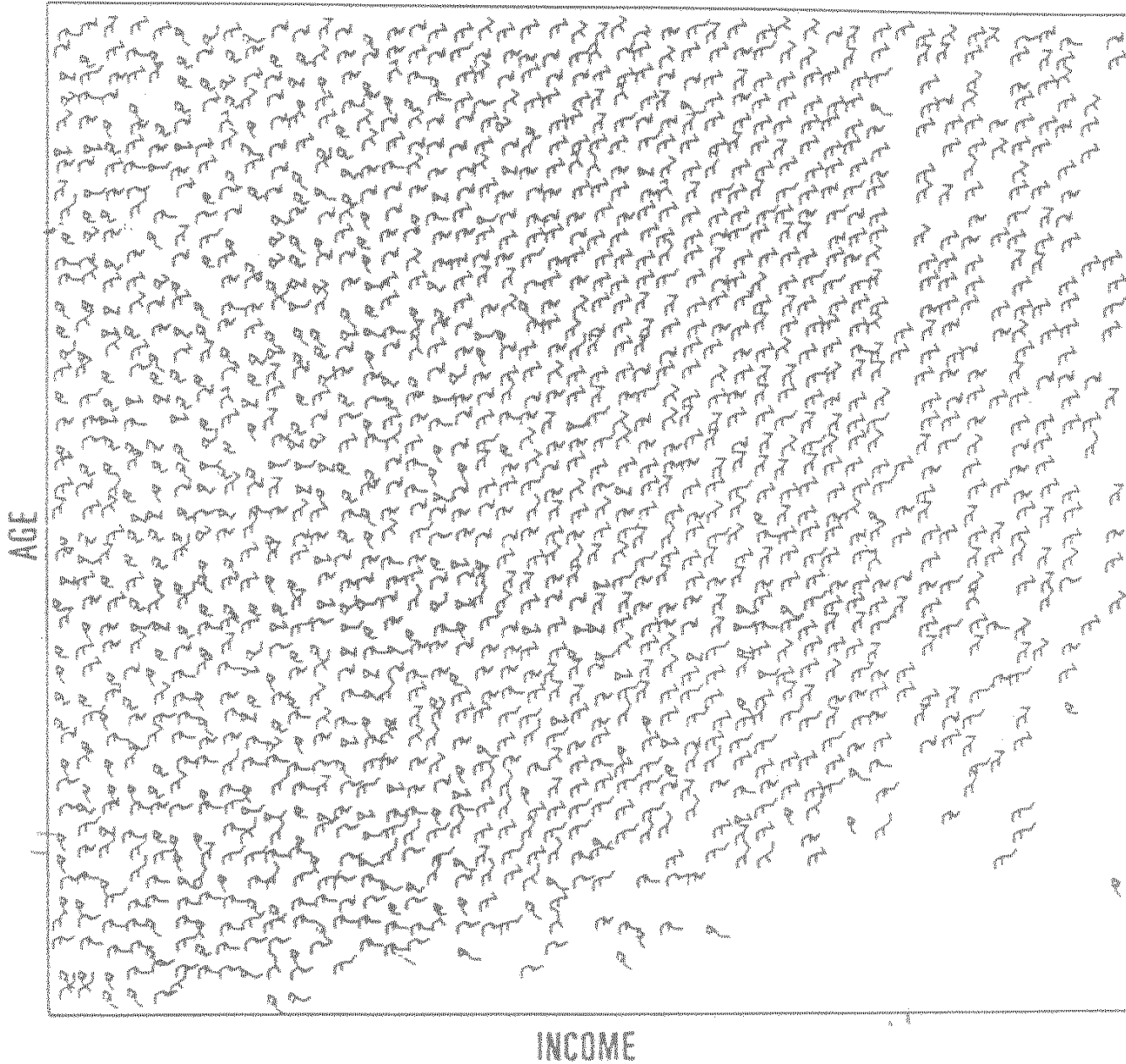


# Chernoff Faces

- A way to display variables on a two-dimensional surface, e.g., let  $x$  be eyebrow slant,  $y$  be eye size,  $z$  be nose length, etc.
- The figure shows faces produced using 10 characteristics--head eccentricity, eye size, eye spacing, eye eccentricity, pupil size, eyebrow slant, nose size, mouth shape, mouth size, and mouth opening): Each assigned one of 10 possible values, generated using Mathematica (S. Dickson)
- REFERENCE: Gonick, L. and Smith, W. The Cartoon Guide to Statistics. New York: Harper Perennial, p. 212, 1993
- Weisstein, Eric W. "Chernoff Face." From *MathWorld*--A Wolfram Web Resource. [mathworld.wolfram.com/ChernoffFace.html](http://mathworld.wolfram.com/ChernoffFace.html)



# Stick Figure



A census data figure showing age, income, gender, education, etc.

A 5-piece stick figure (1 body and 4 limbs w. different angle/length)

# Hierarchical Visualization Techniques

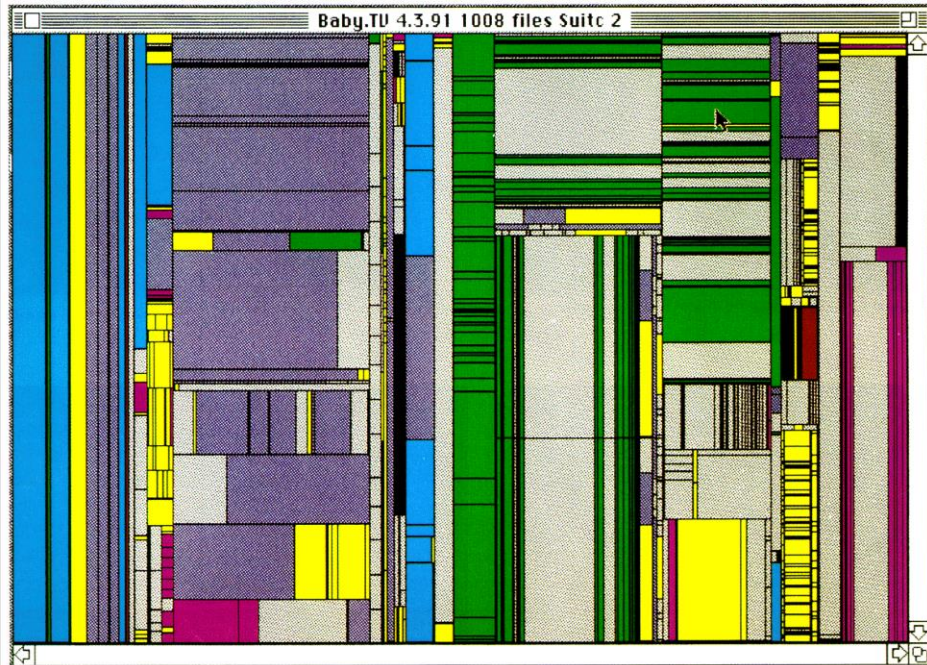
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- Visualization of the data using a hierarchical partitioning into subspaces
- Methods
  - Tree-Map

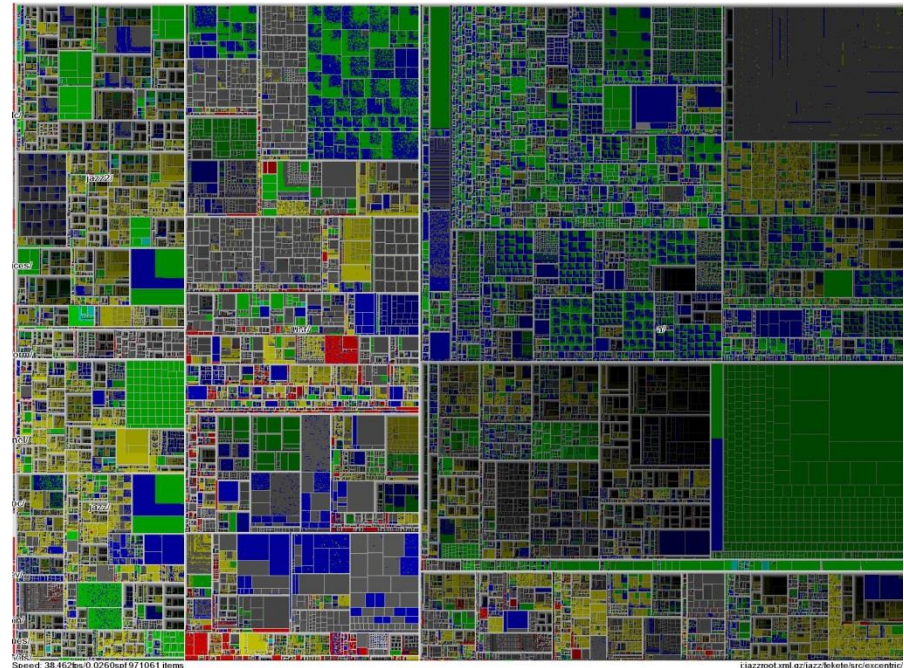


# Tree-Map

- Screen-filling method which uses a hierarchical partitioning of the screen into regions depending on the attribute values



Schneiderman@UMD: Tree-Map of a File System



Schneiderman@UMD: Tree-Map to support large data sets of a million items



- Visualizing non-numerical data: text and social networks
- Tag cloud: visualizing user-generated tags
  - The importance of tag is represented by font size/color
- Besides text data, there are also methods to visualize relationships, such as visualizing social networks



## Newsmap: Google News Stories in 2005

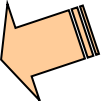
# Assignment 1

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- Find appropriate datasets from kaggle for the following visualization techniques and apply them
  - Pixel-oriented visualization
  - Chernoff faces
  - Stick figures
  - Tree map

# Chapter 2: Getting to Know Your Data

---

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# Similarity and Dissimilarity

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- **Similarity**

- Numerical measure of how alike two data objects are
- Value is higher when objects are more alike
- Often falls in the range  $[0,1]$

- **Dissimilarity** (e.g., distance)

- Numerical measure of how different two data objects are
- Lower when objects are more alike
- Minimum dissimilarity is often 0

- **Proximity** refers to a similarity or dissimilarity



# Data Matrix and Dissimilarity Matrix

## ■ Data matrix

- n data points with p dimensions
- Two modes

$$\begin{bmatrix} x_{11} & \dots & x_{1f} & \dots & x_{1p} \\ \dots & \dots & \dots & \dots & \dots \\ x_{i1} & \dots & x_{if} & \dots & x_{ip} \\ \dots & \dots & \dots & \dots & \dots \\ x_{n1} & \dots & x_{nf} & \dots & x_{np} \end{bmatrix}$$

## ■ Dissimilarity matrix

- n data points, but registers only the distance
- A triangular matrix
- Single mode

$$\begin{bmatrix} 0 & & & & \\ d(2,1) & 0 & & & \\ d(3,1) & d(3,2) & 0 & & \\ \vdots & \vdots & \vdots & & \\ d(n,1) & d(n,2) & \dots & \dots & 0 \end{bmatrix}$$

# Dissimilarity Measure for Nominal Attributes

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- Can take 2 or more states, e.g., red, yellow, blue, green (generalization of a binary attribute)
- Method 1: Simple matching
  - $m$ : # of matches,  $p$ : total # of variables

$$d(i, j) = \frac{p - m}{p}$$

- Method 2: Use a large number of binary attributes
  - creating a new binary attribute for each of the  $M$  nominal states. E.g. to encode nominal attr 'color', a binary attr can be created for each of the colors listed. Yellow will have 1, others 0

## Exercise 7

- Find the distance based on the attributes “Favorite Color” and “Favorite Food” between
  - Ali and Bilal
  - Ali and Faris

	<b>Favorite Color</b>	<b>Favorite Food</b>	<b>Plays Chess</b>	<b>Plays Football</b>	<b>Salary Age (1000s)</b>	<b>Grade</b>
<i>Ali</i>	Blue	Cake	Yes	Yes	20	34C
<i>Bilal</i>	Yellow	Cake	Yes	Yes	25	25B
<i>Ehsan</i>	Yellow	Pasta	Yes	No	20	25C
<i>Faris</i>	Yellow	Burger	No	No	20	25A

## Exercise 7

$$d(i, j) = \frac{p - m}{p}$$

- Find the distance based on the attributes “Favorite Color” and “Favorite Food” between
  - Ali and Bilal
  - $(2-1)/2 = 0.5$
  - Ali and Faris
  - $(2-0)/2 = 1$

	<b>Favorite Color</b>	<b>Favorite Food</b>	<b>Plays Chess</b>	<b>Plays Football</b>	<b>Salary Age (1000s)</b>	<b>Grade</b>
<i>Ali</i>	Blue	Cake	Yes	Yes	20	34C
<i>Bilal</i>	Yellow	Cake	Yes	Yes	25	25B
<i>Ehsan</i>	Yellow	Pasta	Yes	No	20	25C
<i>Faris</i>	Yellow	Burger	No	No	20	25A

# Proximity Measure for Binary Attributes

- A contingency table for binary data

		Object $j$		
		1	0	sum
Object $i$	1	$q$	$r$	$q + r$
	0	$s$	$t$	$s + t$
sum		$q + s$	$r + t$	$p$

- Dissimilarity measure for symmetric binary variables:

$$d(i, j) = \frac{r + s}{q + r + s + t}$$

- Dissimilarity measure for asymmetric binary variables:

$$d(i, j) = \frac{r + s}{q + r + s}$$

- Jaccard coefficient (*similarity* measure for *asymmetric* binary variables):

$$sim_{Jaccard}(i, j) = \frac{q}{q + r + s}$$

# Dissimilarity between Binary Variables

## ■ Example

Name	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	Y	N	P	N	N	N
Mary	Y	N	P	N	P	N
Jim	Y	P	N	N	N	N

	1	0	sum
1	$q$	$r$	$q+r$
0	$s$	$t$	$s+t$
sum	$q+s$	$r+t$	$p$

- $d(\text{jack}, \text{mary}), d(\text{jack}, \text{jim})$
- $d(\text{jim}, \text{mary})$
- Let the values Y and P be 1, and the value N 0

$$d(i, j) = \frac{r + s}{q + r + s}$$

# Dissimilarity between Binary Variables

## Example

Name	Fever	Cough	Test-1	Test-2	Test-3	Test-4
Jack	Y	N	P	N	N	N
Mary	Y	N	P	N	P	N
Jim	Y	P	N	N	N	N

- Let the values Y and P be 1, and the value N 0

$$d(i, j) = \frac{r + s}{q + r + s}$$

$$d(jack, mary) = \frac{0 + 1}{2 + 0 + 1} = 0.33$$

$$d(jack, jim) = \frac{1 + 1}{1 + 1 + 1} = 0.67$$

$$d(jim, mary) = \frac{1 + 2}{1 + 1 + 2} = 0.75$$

		Mary		
		1	0	$\Sigma_{row}$
Jack	1	2	0	2
	0	1	3	4
	$\Sigma_{col}$	3	3	6

		Jim		
		1	0	$\Sigma_{row}$
Jack	1	1	1	2
	0	1	3	4
	$\Sigma_{col}$	2	4	6

		Mary		
		1	0	$\Sigma_{row}$
Jim	1	1	1	2
	0	2	2	4
	$\Sigma_{col}$	3	3	6

	1	0	sum
1	$q$	$r$	$q + r$
0	$s$	$t$	$s + t$
sum	$q + s$	$r + t$	$p$

## Exercise 8

- Find the distance based on the “symmetric” attributes “Plays Chess” and “Plays Football” between
  - Ali and Bilal
  - Ali and Faris

	<b>Favorite Color</b>	<b>Favorite Food</b>	<b>Plays Chess</b>	<b>Plays Football</b>	<b>Age</b>	<b>Salary (1000s)</b>	<b>Grade</b>
<i>Ali</i>	Blue	Cake	Yes	Yes	20	34	C
<i>Bilal</i>	Yellow	Cake	Yes	Yes	25	25	B
<i>Ehsan</i>	Yellow	Pasta	Yes	No	20	25	C
<i>Faris</i>	Yellow	Burger	No	No	20	25	A



	1	0	sum
1	$q$	$r$	$q+r$
0	$s$	$t$	$s+t$
sum	$q+s$	$r+t$	$p$

## Exercise 8

$$d(i, j) = \frac{r + s}{q + r + s + t}$$

- Find the distance based on the “symmetric” attributes “Plays Chess” and “Plays Football” between
  - Ali and Bilal
    - $\text{dist}(\text{Ali}, \text{Bilal}) = 0/2 = 0$
  - Ali and Faris
    - $\text{dist}(\text{Ali}, \text{Faris}) = 2/2 = 1$

	<b>Favorite Color</b>	<b>Favorite Food</b>	<b>Plays Chess</b>	<b>Plays Football</b>	<b>Age</b>	<b>Salary (1000s)</b>	<b>Grade</b>
<i>Ali</i>	Blue	Cake	Yes	Yes	20	34	C
<i>Bilal</i>	Yellow	Cake	Yes	Yes	25	25	B
<i>Ehsan</i>	Yellow	Pasta	Yes	No	20	25	C
<i>Faris</i>	Yellow	Burger	No	No	20	25	A

# Distance on Numeric Data: Minkowski Distance

- *Minkowski distance*: A popular distance measure

$$d(i, j) = \sqrt[h]{|x_{i1} - x_{j1}|^h + |x_{i2} - x_{j2}|^h + \cdots + |x_{ip} - x_{jp}|^h}$$

where  $i = (x_{i1}, x_{i2}, \dots, x_{ip})$  and  $j = (x_{j1}, x_{j2}, \dots, x_{jp})$  are two  $p$ -dimensional data objects, and  $h$  is the order (the distance so is also called L- $h$  norm)

# Special Cases of Minkowski Distance

- $h = 1$ : **Manhattan** ( $L_1$  norm) **distance**

$$d(i, j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \dots + |x_{ip} - x_{jp}|$$

- $h = 2$ : ( $L_2$  norm) **Euclidean** distance

$$d(i, j) = \sqrt{(|x_{i1} - x_{j1}|^2 + |x_{i2} - x_{j2}|^2 + \dots + |x_{ip} - x_{jp}|^2)}$$

- $h \rightarrow \infty$  **“supremum”** ( $L_{\max}$  norm,  $L_{\infty}$  norm) distance.

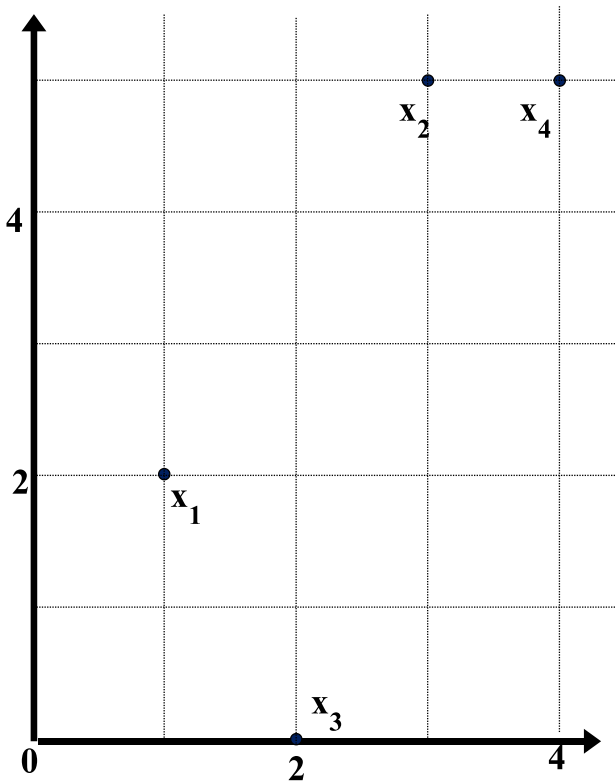
- This is the maximum difference between any component (attribute) of the vectors

$$d(i, j) = \lim_{h \rightarrow \infty} \left( \sum_{f=1}^p |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_f |x_{if} - x_{jf}|$$

# Example: Minkowski Distance

## Dissimilarity Matrices

point	attribute 1	attribute 2
x1	1	2
x2	3	5
x3	2	0
x4	4	5



## Manhattan ( $L_1$ )

L	x1	x2	x3	x4
x1	0			
x2	5	0		
x3	3	6	0	
x4	6	1	7	0

## Euclidean ( $L_2$ )

L2	x1	x2	x3	x4
x1	0			
x2	3.61	0		
x3	2.24	5.1	0	
x4	4.24	1	5.39	0

## Supremum

$L_\infty$	x1	x2	x3	x4
x1	0			
x2	3	0		
x3	2	5	0	
x4	3	1	5	0

## Exercise 9

- Find the distance based on the attributes “Age” and “Salary” between
  - Ali and Bilal (Euclidean)
  - Ali and Bilal (Supremum)

	<b>Favorite Color</b>	<b>Favorite Food</b>	<b>Plays Chess</b>	<b>Plays Football</b>	<b>Salary Age (1000s)</b>	<b>Grade</b>
<i>Ali</i>	Blue	Cake	Yes	Yes	20	34C
<i>Bilal</i>	Yellow	Cake	Yes	Yes	25	25B
<i>Ehsan</i>	Yellow	Pasta	Yes	No	20	25C
<i>Faris</i>	Yellow	Burger	No	No	20	25A

## Exercise 9

$$d(i, j) = \sqrt{(|x_{i1} - x_{j1}|^2 + |x_{i2} - x_{j2}|^2 + \dots + |x_{ip} - x_{jp}|^2)}$$

$$d(i, j) = \lim_{h \rightarrow \infty} \left( \sum_{f=1}^p |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_f |x_{if} - x_{jf}|$$

- Find the distance based on the attributes “Age” and “Salary” between
  - Ali and Bilal (Euclidean)
    - $\text{dist}(\text{Ali}, \text{Bilal}) = \sqrt{5^2 + 9^2} = \sqrt{106}$
  - Ali and Bilal (Supremum)
    - $\text{dist}(\text{Ali}, \text{Bilal}) = \max(5, 9) = 9$

	<b>Favorite Color</b>	<b>Favorite Food</b>	<b>Plays Chess</b>	<b>Plays Football</b>	<b>Salary Age (1000s)</b>	<b>Grade</b>
<i>Ali</i>	Blue	Cake	Yes	Yes	20	34C
<i>Bilal</i>	Yellow	Cake	Yes	Yes	25	25B
<i>Ehsan</i>	Yellow	Pasta	Yes	No	20	25C
<i>Faris</i>	Yellow	Burger	No	No	20	25A

# Ordinal Variables

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- An ordinal variable can be discrete or continuous
- Order is important, e.g., rank
- Can be treated like interval-scaled
  - replace  $x_{if}$  by their rank  $r_{if} \in \{1, \dots, M_f\}$
  - map the range of each variable onto  $[0, 1]$  by replacing  $i$ -th object in the  $f$ -th attribute by

$$z_{if} = \frac{r_{if} - 1}{M_f - 1}$$

- compute the dissimilarity using methods for interval-scaled variables.

## Exercise 10

- Find the distance based on the attributes “Age”, “Salary”, and “Grade” between
  - Ali and Bilal (Euclidean)
  - Ali and Bilal (Supremum)
  - Set  $A=1$ ,  $B=2$ ,  $C=3$

	<b>Favorite Color</b>	<b>Favorite Food</b>	<b>Plays Chess</b>	<b>Plays Football</b>	<b>Salary Age (1000s)</b>	<b>Grade</b>
<i>Ali</i>	Blue	Cake	Yes	Yes	20	34C
<i>Bilal</i>	Yellow	Cake	Yes	Yes	25	25B
<i>Ehsan</i>	Yellow	Pasta	Yes	No	20	25C
<i>Faris</i>	Yellow	Burger	No	No	20	25A



## Exercise 10

$$d(i, j) = |x_{i1} - x_{j1}| + |x_{i2} - x_{j2}| + \dots + |x_{ip} - x_{jp}|$$

$$d(i, j) = \lim_{h \rightarrow \infty} \left( \sum_{f=1}^p |x_{if} - x_{jf}|^h \right)^{\frac{1}{h}} = \max_f^p |x_{if} - x_{jf}|$$

- Find the distance based on the attributes "Age", "Salary", and "Grade" between

- Ali and Bilal (Manhattan)

- $\text{dist}(\text{Ali}, \text{Bilal}) = |20-25| + |34-25| + |1-0.5| = 14.5$

- Ali and Bilal (Supremum)

- $\text{dist}(\text{Ali}, \text{Bilal}) = \max(|20-25|, |34-25|, |1-0.5|) = 9$

	Favorite Color	Favorite Food	Plays Chess	Plays Football	Age	Salary (1000s)	Grade	Grade (N)
Ali	Blue	Cake	Yes	Yes	20	34	C	1
Bilal	Yellow	Cake	Yes	Yes	25	25	B	0.5
Ehsan	Yellow	Pasta	Yes	No	20	25	C	1
Faris	Yellow	Burger	No	No	20	25	A	0

# Attributes of Mixed Type

- A database may contain all attribute types
  - Nominal, symmetric binary, asymmetric binary, numeric, ordinal
- One may use a weighted formula to combine their effects

$$d(i, j) = \frac{\sum_{f=1}^p \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^p \delta_{ij}^{(f)}}$$

- $f$  is binary or nominal:
  - $d_{ij}^{(f)} = 0$  if  $x_{if} = x_{jf}$ , or  $d_{ij}^{(f)} = 1$  otherwise
- $f$  is numeric: use the normalized distance
- $f$  is ordinal
  - Compute ranks  $r_{if}$  and  $z_{if} = \frac{r_{if} - 1}{M_f - 1}$
  - Treat  $z_{if}$  as interval-scaled
- $\delta_{ij}^{(f)} = 0$ 
  - if  $x_{if}$  or  $x_{jf}$  is missing
  - or  $x_{if} = x_{jf} = 0$  for binary asymmetric attributes

# Exercise 11

- Find the distance based on all the attributes (mixed type)
  - Ali and Bilal
  - Ali and Faris

	<b>Favorite Color</b>	<b>Favorite Food</b>	<b>Plays Chess</b>	<b>Plays Football</b>	<b>Age</b>	<b>Salary (1000s)</b>	<b>Grade</b>
<i>Ali</i>	Blue	Cake	Yes	Yes	20	34	C
<i>Bilal</i>	Yellow	Cake	Yes		25	25	B
<i>Ehsan</i>	Yellow	Pasta	Yes	No	20	25	C
<i>Faris</i>	Yellow	Burger	No	No	20	25	A

- $f$  is binary or nominal:  
 $d_{ij}^{(f)} = 0$  if  $x_{if} = x_{jf}$ , or  $d_{ij}^{(f)} = 1$  otherwise
- $f$  is numeric: use the normalized distance
- $f$  is ordinal
  - Compute ranks  $r_{if}$  and
  - Treat  $z_{if}$  as interval-scaled

## Exercise 11

$$d(i, j) = \frac{\sum_{f=1}^p \delta_{ij}^{(f)} d_{ij}^{(f)}}{\sum_{f=1}^p \delta_{ij}^{(f)}}$$

- if  $x_{if}$  or  $x_{jf}$  is missing
- or  $x_{if} = x_{jf} = 0$  for binary asymmetric attributes

- Find the distance based on all the attributes (mixed type). For numeric attributes, use Manhattan distance.

### ■ Ali and Bilal

- $\text{dist}(\text{Ali}, \text{Bilal}) =$   
 $(1*1 + 1*0 + 1*0 + 0*1 + 1*5 + 1*9 + 1*0.5) / (1 + 1 + 1 + 0 + 1 + 1 + 1) = 15.5 / 6 = 2.58$

### ■ Ali and Faris

- $\text{dist}(\text{Ali}, \text{Faris}) =$   
 $(1*1 + 1*1 + 1*1 + 1*1 + 1*0 + 1*9 + 1*1) / (1 + 1 + 1 + 1 + 1 + 1 + 1) = 14 / 7 = 2$

	Favorite Color	Favorite Food	Plays Chess	Plays Football	Age	Salary (1000s)	Grade
Ali	Blue	Cake	Yes	Yes	20	34	1
Bilal	Yellow	Cake	Yes		25	25	0.5
Ehsan	Yellow	Pasta	Yes	No	20	25	1
Faris	Yellow	Burger	No	No	20	25	0

# Cosine Similarity

- A **document** can be represented by thousands of attributes, each recording the *frequency* of a particular word (such as keywords) or phrase in the document.

Document	teamcoach		hockey	baseball	soccer	penalty	score	win	loss	season
Document1	5	0	3	0	2	0	0	2	0	0
Document2	3	0	2	0	1	1	0	1	0	1
Document3	0	7	0	2	1	0	0	3	0	0
Document4	0	1	0	0	1	2	2	0	3	0

- Other vector objects: gene features in micro-arrays, ...
- Applications: information retrieval, biologic taxonomy, gene feature mapping, ...
- Cosine measure: If  $d_1$  and  $d_2$  are two vectors (e.g., term-frequency vectors), then

$$\cos(d_1, d_2) = \frac{d_1 \bullet d_2}{\|d_1\| \times \|d_2\|}$$

where  $\bullet$  indicates vector dot product,  $\|d\|$ : Euclidean norm of vector d.

# Example: Calculating Cosine Similarity

- Calculating Cosine Similarity:

$$\cos(d_1, d_2) = \frac{d_1 \bullet d_2}{\|d_1\| \times \|d_2\|}$$

where  $\bullet$  indicates vector dot product,  $\|d\|$ : Euclidean norm of vector d.

- Ex: Find the **similarity** between documents 1 and 2.

$$d_1 = (5, 0, 3, 0, 2, 0, 0, 2, 0, 0) \quad d_2 = (3, 0, 2, 0, 1, 1, 0, 1, 0, 1)$$

- First, calculate vector dot product

$$d_1 \bullet d_2 = 5 \times 3 + 0 \times 0 + 3 \times 2 + 0 \times 0 + 2 \times 1 + 0 \times 1 + 0 \times 1 + 2 \times 1 + 0 \times 0 + 0 \times 1 = 25$$

- Then, calculate  $\|d_1\|$  and  $\|d_2\|$

$$\|d_1\| = \sqrt{5 \times 5 + 0 \times 0 + 3 \times 3 + 0 \times 0 + 2 \times 2 + 0 \times 0 + 0 \times 0 + 2 \times 2 + 0 \times 0 + 0 \times 0} = 6.481$$

$$\|d_2\| = \sqrt{3 \times 3 + 0 \times 0 + 2 \times 2 + 0 \times 0 + 1 \times 1 + 1 \times 1 + 0 \times 0 + 1 \times 1 + 0 \times 0 + 1 \times 1} = 4.12$$

- Calculate cosine similarity:  $\cos(d_1, d_2) = 25 / (6.481 \times 4.12) = 0.94$

# Example

- D1: **A** red apple
- D2: I like red apple
- D3: Apple computers **are** good
- D4: Red apple red apple red apple red apple

Term by Document Matrix

	This	Red	Apple	I	Like	Comp	Good
D1	0	1	1	0	0	0	0
D2	0	1	1	1	1	0	0
D3	0	0	1	0	0	1	1
D4	0	4	4	0	0	0	0

Cosine(d1,d4) =

$$(0*0+1*4+1*4+0*0+0*0+0*0+0*0)/(\text{sqrt}(2)*\text{sqrt}(32))=8/8 = 1$$

## Exercise 12

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- Create a term by document matrix for the following documents (only consider green terms)
  - D1: I like to eat red apples
  - D2: Red apples are sweet
  - D3: Apple computers are easy to use computers
- Find the distance cosine similarity between
  - D1 and D2
  - D1 and D3



# Exercise 12

- Create a term by document matrix for the following documents

- D1: I like to eat red apples
- D2: Red apples are sweet
- D3: Apple computers are easy to use computers

	Like								
D1									
D2									
D3									

- Find the distance cosine similarity between
  - D1 and D2
  - D1 and D3

# Exercise 12

- Create a term by document matrix for the following documents

- D1: I like to eat red apples
- D2: Red apples are sweet
- D3: Apple computers are easy to use computers

	like	eat	red	apples	sweet	computers	easy	use
D1	1	1	1	1	0	0	0	0
D2	0	0	1	1	1	0	0	0
D3	0	0	0	1	0	2	1	1

- Find the distance cosine similarity between

$$||d1|| = \sqrt{1^2 + 1^2 + 1^2 + 1^2} = \sqrt{4}$$

$$||d2|| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$$

$$||d3|| = \sqrt{1^2 + 2^2 + 1^2 + 1^2} = \sqrt{7}$$

- D1 and D2


- $\text{sim}(d1, d2) = (1+1)/(\sqrt{4}*\sqrt{3}) = 2/\sqrt{12}$

- D1 and D3

- $\text{sim}(d1, d3) = 1/(\sqrt{4}*\sqrt{7}) = 1/\sqrt{28}$

# Chapter 2: Getting to Know Your Data

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- Data Objects and Attribute Types
- Basic Statistical Descriptions of Data
- Data Visualization
- Measuring Data Similarity and Dissimilarity
- Summary 

# Summary

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- Data attribute types: nominal, binary, ordinal, interval-scaled, ratio-scaled
- Many types of data sets, e.g., numerical, text, graph, Web, image.
- Gain insight into the data by:
  - Basic statistical data description: central tendency, dispersion, graphical displays
  - Data visualization: map data onto graphical primitives
  - Measure data similarity
- Above steps are the beginning of data preprocessing.
- Many methods have been developed but still an active area of research.

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