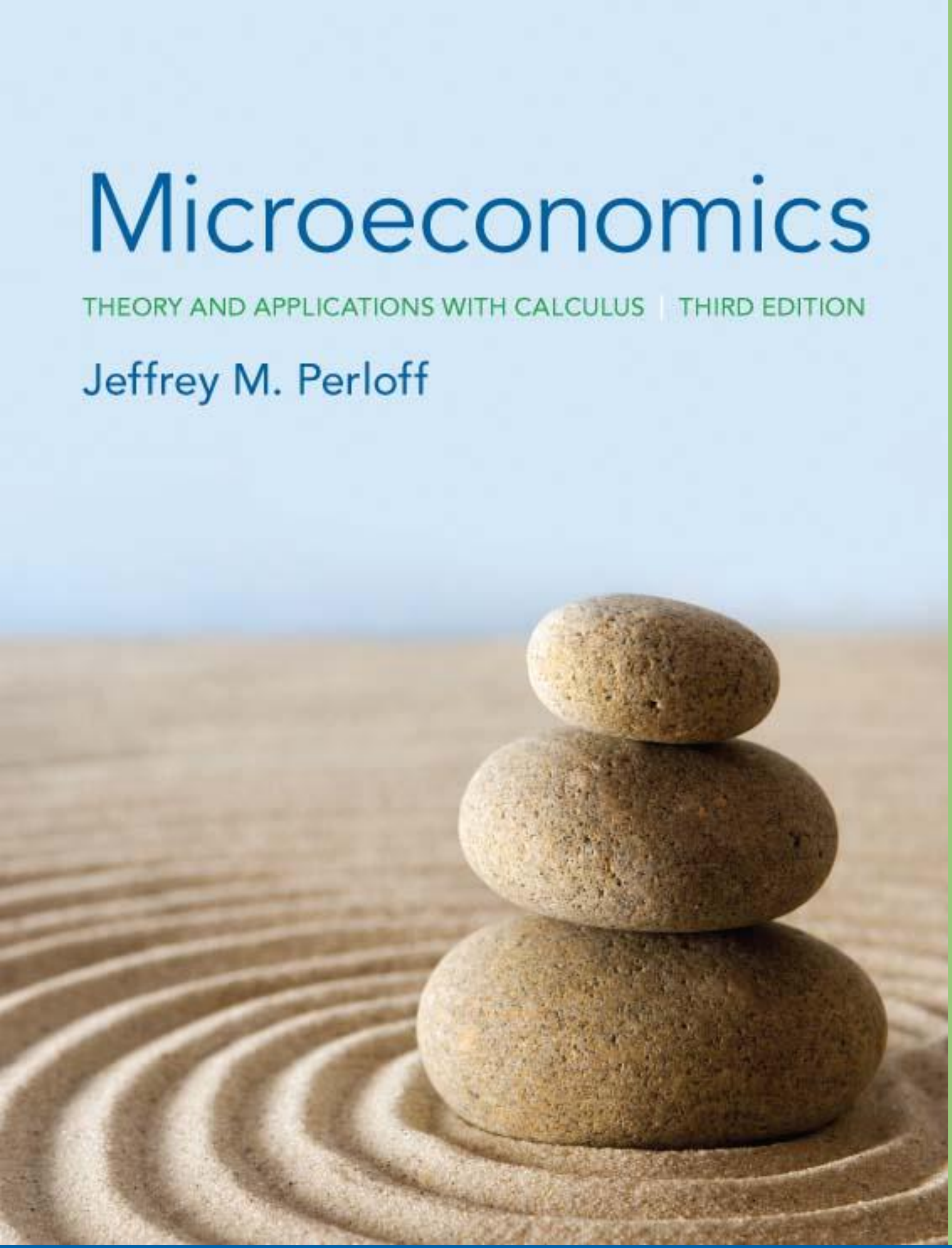


If this is coffee, please bring me some tea; but if this is tea, please bring me some coffee.

Abraham Lincoln

Intermediate Microeconomics
by Saba Bukhari

Recommended textbooks

- Microeconomic analysis by Hal Varian
- Microeconomics: Theory and applications with calculus by Perloff

Chapter 1. Introduction

- **Economics**

It is the study of how to make use of limited resources to fulfil unlimited human wants.

- it is the study of allocation of scare resources

Microeconomics vs Macroeconomics

- Microeconomics

→The branch of economics that studies decision-making by a single individual, household, firm, industry, or level of government

- Macroeco

→The branc economic studies de making fo economy whole

What is microeconomics?

- The study of the allocation of scarce resources
  - Scarcity is the mother of economics

Three players of Microeconomics

- Consumer
  - Maximize individual happiness (utility)
  - Choose that bundle of goods and services that will max their utility st their given income
- Producer
  - Maximize profit, minimize cost
- Government
  - Maximize public happiness (utility)

BASIC CONCEPTS:

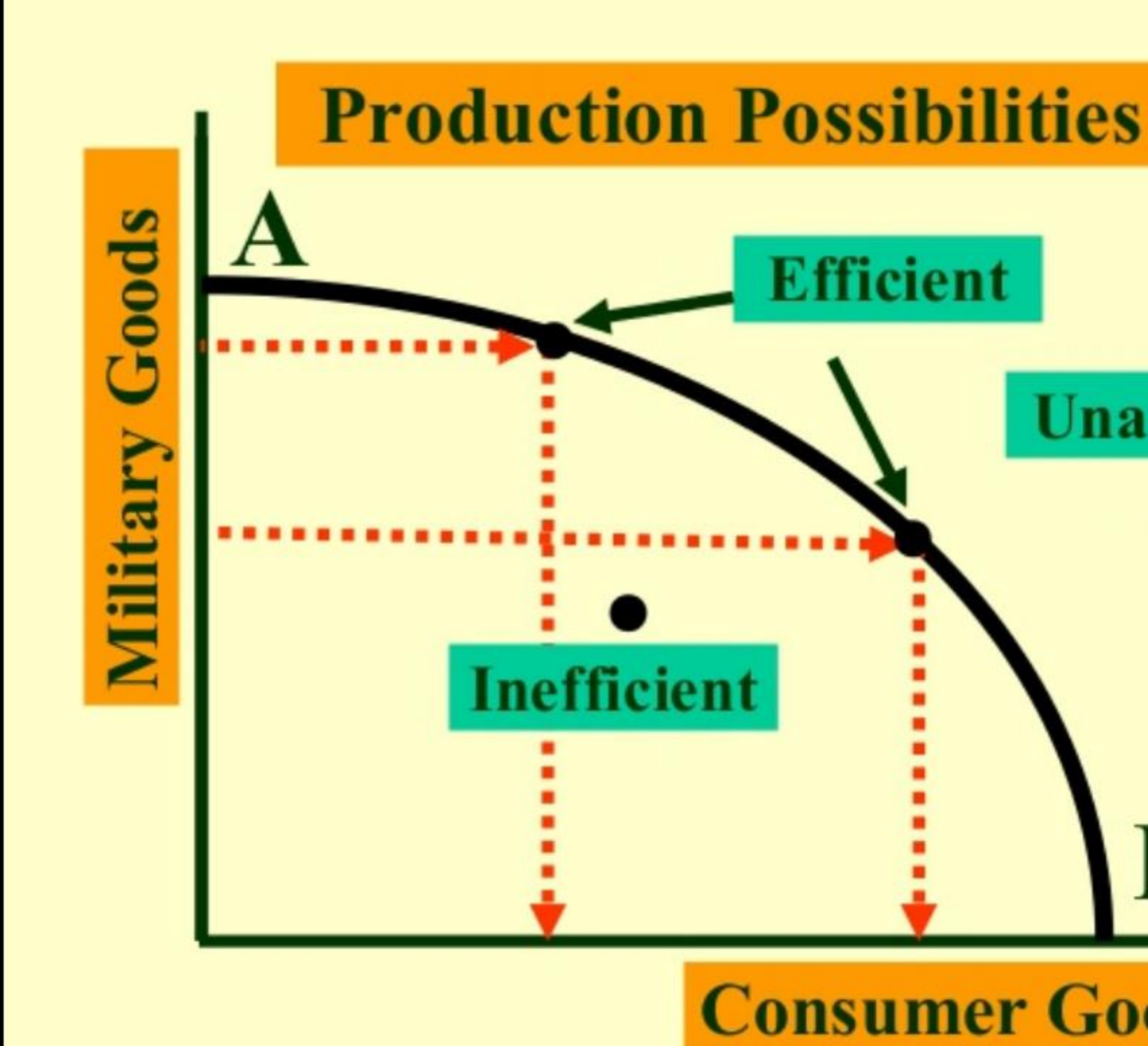
- Scarcity - the fundamental economic problem that human wants exceed the availability of time, goods, and resources.
- Choice – Because individuals and society can never have everything they desire, they therefore are forced to make choices
- Opportunity cost – the second best alternative foregone for a chosen option.

5

Clip slide

Production Possibil
Curve:

- A curve that shows th
maximum combinatio
two outputs that an
economy can produc
given its available
resources and techno



Production Possibilities
Military Goods
A
Efficient
Una
Inefficient
Consumer Go

Theories and Models

- Economic theory is the presentation of set of r/s governing the behavior of individual consumers, producers and the govt.
- Economics is concerned with explanation of observed phenomena
  - Theories are used to explain observed phenomena in terms of a set of basic rules and assumptions:
    - The Theory of the Firm
    - The Theory of Consumer Behavior

How are economic theories formulated & economic models constructed?

1. Define the problem and phenomena to be investigated.
2. Formulate a hypothesis about the relationships among the relevant variables.
3. Determine testable predictions from the hypothesis.
4. Test the accuracy of the predictions using real world data.
5. Accept or revise the theory on the basis of the tests conducted.

Static, comparative and dynamic equilibrium

- **Static:** it is the branch of economics which studies the prosperities of equilibrium at one point in time.
- **Comparative:** we compare two equilibrium positions
- Compare the new with previous one
- **Dynamic:** means over the time adjustments
- **Comparative static:** shocking the equilibrium and then compare them e.g. if the prices of substitutes changes it effects prices and quantity both

Partial and General Equilibrium

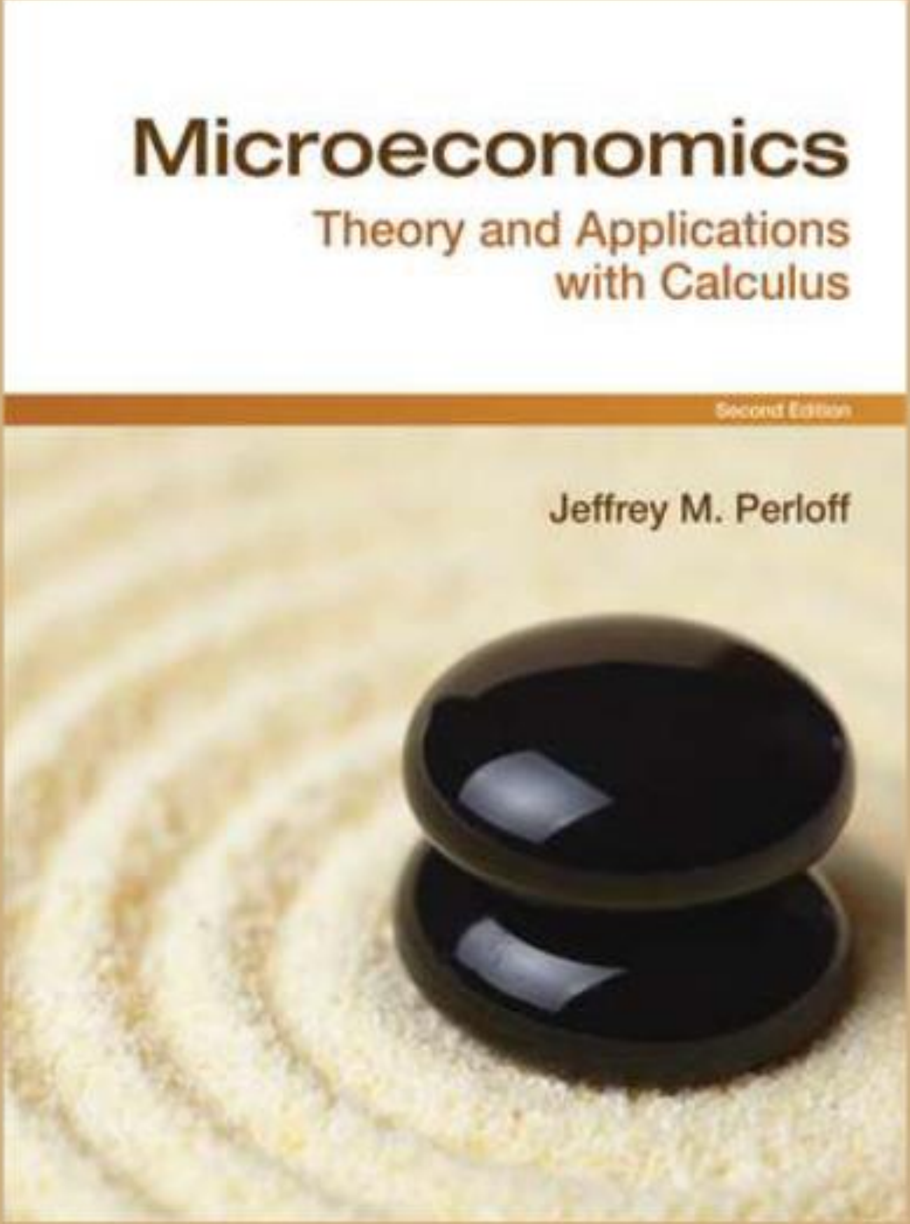
- **P.E**

It is used for ceteris paribus assumption i.e. other things remaining the same how a change in one variable will cause a change in another e.g. The effect of strike on sugar industry

- **G.E**

It is concerned when all the markets are in equilibrium simultaneously. E.g. the effect of transport strike on sugar, rubber, aluminum industries

Chapter 2



Supply and Demand

Talk is cheap because supply exceeds demand.

Addison-Wesley is an imprint of

PEARSON

Chapter 2 Outline

2.1 Demand

- The quantity of a good or service that consumers demand depends on price and other factors such as consumers' incomes and the prices of related goods.
- The ***demand function*** describes the mathematical relationship between quantity demanded (Q_d), price (p) and other factors that influence purchases:

$$Q = D(p, p_s, p_c, Y)$$

- p = per unit price of the good or service
- p_s = per unit price of a substitute good
- p_c = per unit price of a complementary good
- Y = consumers' income

2.1 Demand

- We often work with a linear demand function.
- Example: estimated demand function for pork in Canada

$$Q = 171 - 20p + 20p_b + 3p_c + 2Y$$

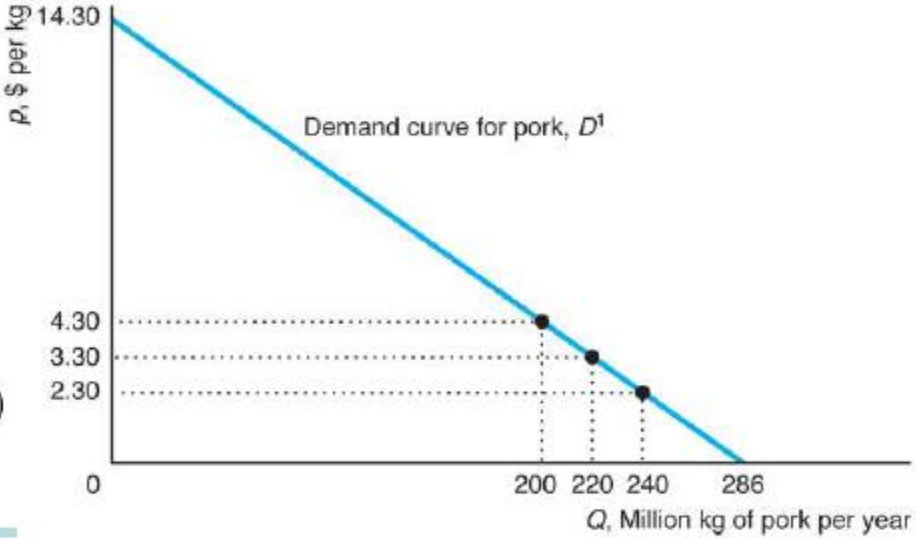
  - Q_d = quantity of pork demanded (million kg per year)
  - p = price of pork (in Canadian dollars per kg)
  - p_b = price of beef, a substitute good (in Canadian dollars per kg)
  - p_c = price of chicken, another substitute (in Canadian dollars per kg)
  - Y = consumers' income (in Canadian dollars per year)
- Graphically, we can only depict the relationship between Q_d and p, so we hold the other factors constant.

2.1 Demand Example: Canadian Pork

Assumptions about p_b, p_c, and Y to simplify equation

- p_b = \$4/kg
- p_c = \$3.33/kg
- Y = \$12.5 thousand

$$
\begin{aligned}
Q &= 171 - 20p + 20p_b + 3p_c + 2Y \\
&= 171 - 20p + (20 \times 4) + \left(3 \times 3\frac{1}{3}\right) + (2 \times 12.5) \\
&= 286 - 20p = D(p)
\end{aligned}
$$

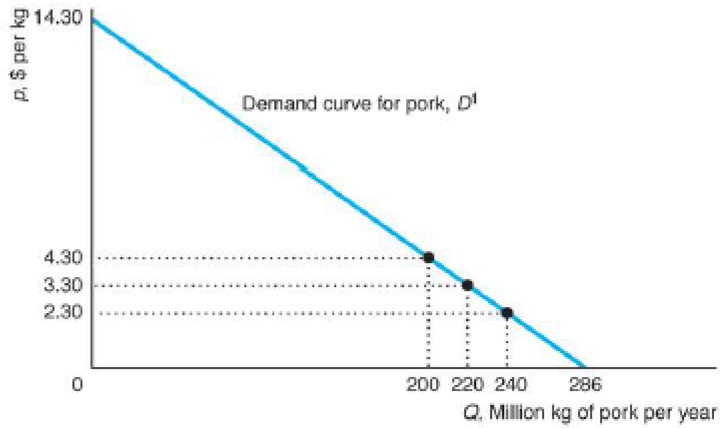


$$
\frac{dQ}{dp} = -20 \Longrightarrow \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta p}{\Delta Q} = \frac{\$1 \text{ per kg}}{-20 \text{ million kg per year}} = -\$0.05 \text{ per million kg per year}
$$

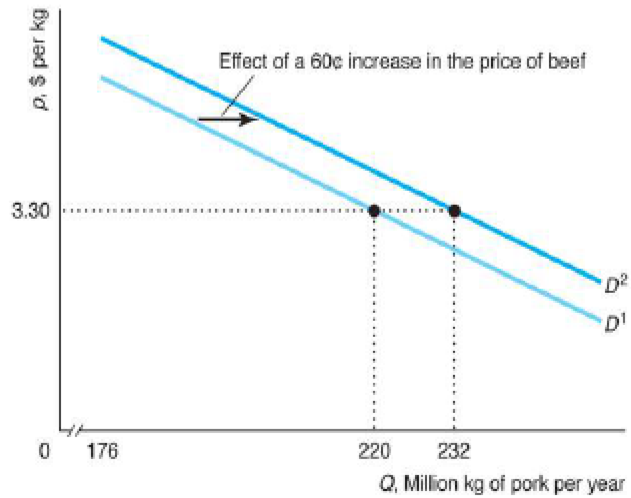
2.1 Demand Example: Canadian Pork

- Changing the own-price of pork simply moves us along an existing demand curve.

$$Q = 298 - 20\,p$$



- Changing one of the things held constant (e.g. p_b, p_c, and Y) shifts the entire demand curve.
- $p_b \uparrow$ to \$4.60 /kg



2.2 Supply

- The quantity of a good or service that firms supply depends on price and other factors such as the cost of inputs that firms use to produce the good or service.
- The ***supply function*** describes the mathematical relationship between quantity supplied (Q_s), price (p) and other factors that influence the number of units offered for sale:

$$Q = S(p, p_h)$$

  - p = per unit price of the good or service
  - p_h = per unit price of other production factors

2.2 Supply

- We often work with a linear supply function.
- Example: estimated supply function for pork in Canada.

$$Q = 178 + 40p - 60p_h$$

  - Q_s = quantity of pork supplied (million kg per year)
  - p = price of pork (in Canadian dollars per kg)
  - p_h = price of hogs, an input (in Canadian dollars per kg)

- Graphically, we can only depict the relationship between Q_s and p, so we hold the other factors constant.

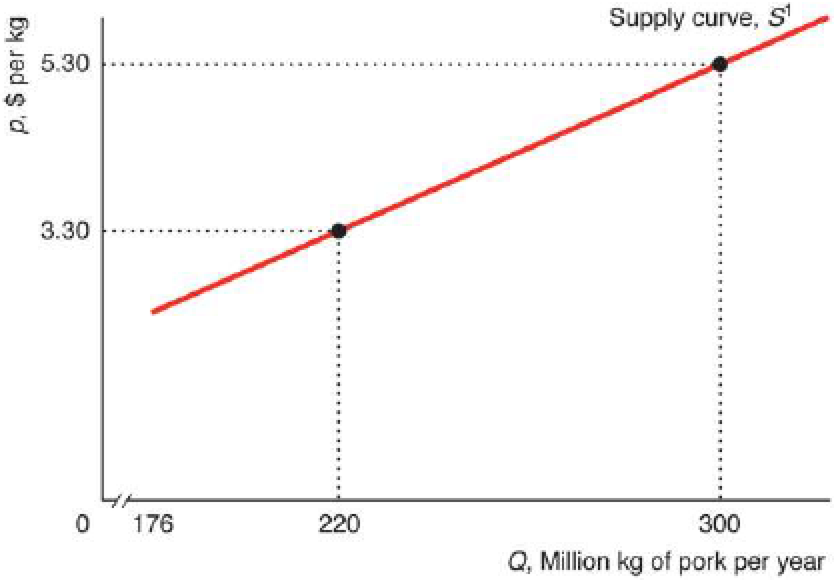
2.2 Supply Example: Canadian Pork

- Assumption about p_h to simplify equation
- p_h = \$1.50/kg

$$Q = 178 + 40p - 60p_h$$

$$\Downarrow$$

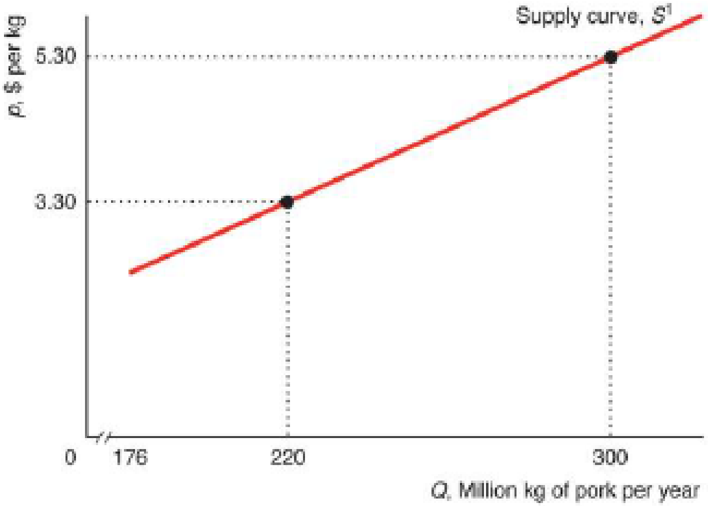
$$Q = 88 + 40p$$



$$\frac{dQ_s}{dp} = 40 \Longrightarrow \frac{dp}{dQ_s} = \frac{1}{40} = slope$$

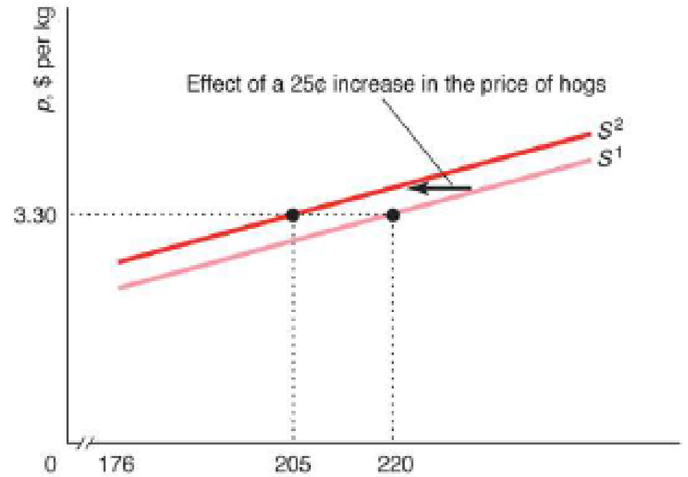
2.2 Supply Example: Canadian Pork

- Changing the own-price of pork simply moves us along an existing supply curve.



- Changing one of the things held constant (e.g. p_h) shifts the entire supply curve.
- p_h ↑ to \$4.60 /kg

$$Q = 73 + 40p$$



2-1

2.3 Market Equilibrium

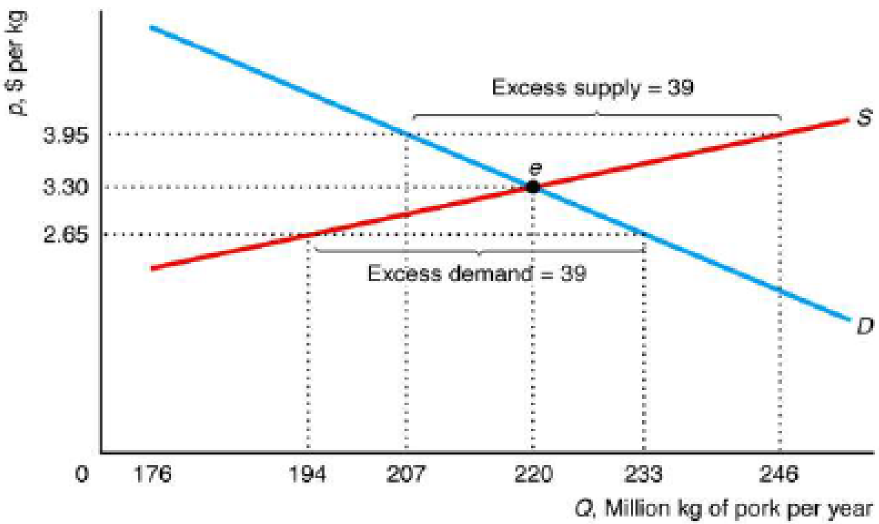
- The interaction between consumers' demand curve and firms' supply curve determines the market price and quantity of a good or service that is bought and sold.
- Mathematically, we find the price that equates the quantity demanded, Q_d, and the quantity supplied, Q_s:
  - Given $Q_d = 286 - 20p$ and $Q_s = 88 + 40p$ find p such that $Q_d = Q_s$: $286 - 20p = 88 + 40p$

$p = \$3.30$

$$Q_d = Q_s$$
$$286 - (20 \times 3.30) = 88 + (40 \times 3.30)$$
$$220 = 220$$

2.3 Market Equilibrium

- Graphically, market equilibrium occurs where the demand and supply curves intersect.
  - At any other price, excess supply or excess demand results.
  - Natural market forces push toward equilibrium Q and p.

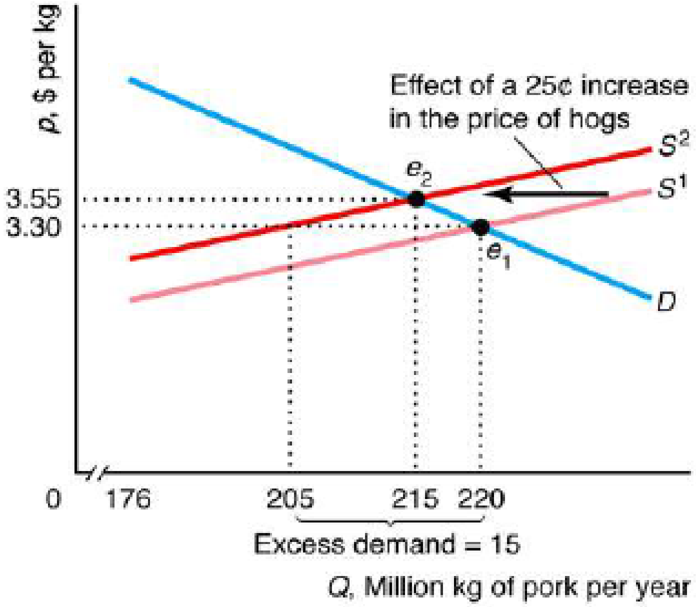


2.4 Shocking the Equilibrium: Comparative Statics

- Changes in a factor that affects demand, supply, or a new government policy alters the market price and quantity of a good or service.
- Changes in demand and supply factors can be analyzed graphically and/or mathematically.
  - Graphical analysis should be familiar from your introductory microeconomics course.
  - Mathematical analysis simply utilizes demand and supply functions to solve for a new market equilibrium.
- Changes in demand and supply factors can be large or small.
  - Small changes are analyzed with Calculus.

2.4 Shocking the Equilibrium: Comparative Statics with Discrete (large) Changes

- Graphically analyzing the effect of an increase in the price of hogs
  - When an input gets more expensive, producers supply less pork at every price.



2.4 Shocking the Equilibrium: Comparative Statics with Discrete (large) Changes

- Mathematically analyzing the effect of an increase in the price of hogs $Q_s = 73 + 40p$
  - If p_h increases by \$0.25, new p_h = \$1.75 and

$$Q_d = Q_s$$
$$286 - 20p = 73 + 40p$$
$$p = \$3.55$$

$\Longrightarrow$

$$Q_d = 286 - 20(3.55) = 215$$
$$Q_s = 73 + 40(3.55) = 215$$

2.4 Shocking the Equilibrium: Comparative Statics with Small Changes

- Demand and supply functions are written as general functions of the price of the good, holding all else constant. $Q = D(p)$
  - Supply is also a function of some exogenous (not in firms' control) variable, *a*. $Q = S(p, a)$
- Because the intersection of demand and supply determines the price, *p*, we can write the price as an implicit function of the supply-shifter, *a*: $Q = D(p(a))$ $Q_s = S(p(a), a)$
- In equilibrium: $D(p(a)) = S(p(a), a)$

2.4 Shocking the Equilibrium: Comparative Statics with Small Changes

- Given the equilibrium condition $D(p(a)) = S(p(a), a)$, we differentiate with respect to *a* using the chain rule to determine how equilibrium is affected by a small change in *a*:

$$\frac{\mathrm{d}D(p(a))}{\mathrm{d}p}\frac{\mathrm{d}p}{\mathrm{d}a} = \frac{\partial S(p(a), a)}{\partial p}\frac{\mathrm{d}p}{\mathrm{d}a} + \frac{\partial S(p(a), a)}{\partial a}$$

- Rearranging:

$$\frac{\mathrm{d}p}{\mathrm{d}a} = \frac{\dfrac{\partial S}{\partial a}}{\dfrac{\mathrm{d}D}{\mathrm{d}p} - \dfrac{\partial S}{\partial p}}$$

Elasticities of Supply and Demand

Price Elasticity of Demand

- Measures the sensitivity of quantity demanded to price changes.
  - It measures the % change in the quantity demanded for a good or service that results from a one percent change in the price.
  - The price elasticity of demand is:

$$E_P = (\%\Delta Q)/(\%\Delta P)$$

Elasticities of Supply and Demand

Price Elasticity of Demand

- The % change in a variable is the absolute change in the variable divided by the original level of the variable. So the price elasticity of demand is also:

$$E_P = \frac{\Delta Q/Q}{\Delta P/P} = \frac{P}{Q}\frac{\Delta Q}{\Delta P}$$

Elasticities of Supply and Demand

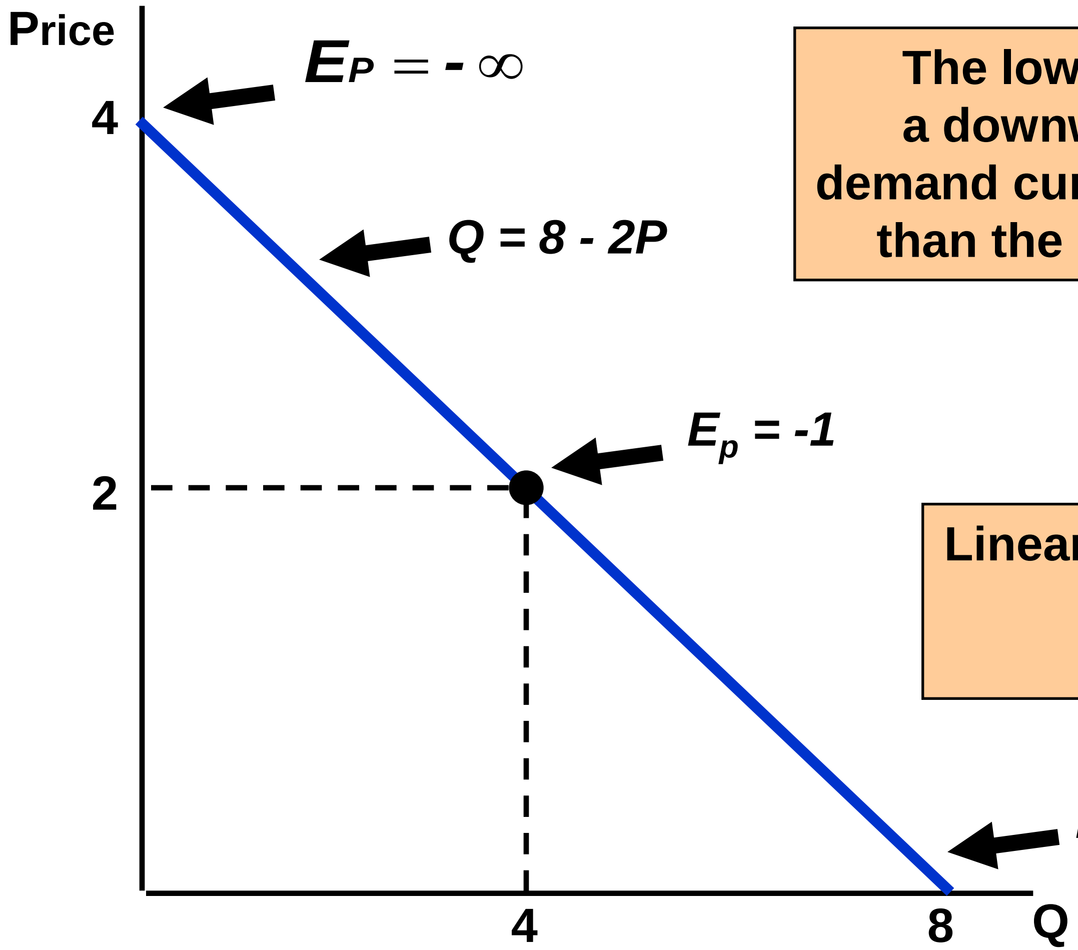
- Interpreting Price Elasticity of Demand Values

1) Because of the inverse relationship between P and Q; E_P is negative.

2) If $|E_P| > 1$, the % change in quantity demanded is greater than the % change in price. We say *demand is price elastic*.

3) If $|E_P| < 1$, the % change in quantity demanded is less than the % change in price. We say *demand is price inelastic*.

Price Elasticities of Demand



The lower portion of a downward sloping demand curve is less elastic than the upper portion.

Linear Demand Curve
Q = a - bP
Q = 8 - 2P

Elasticities of Supply and Demand

Other Demand Elasticities

- Income elasticity of demand measures the % change in quantity demanded resulting from a one percent change in income. The income elasticity of demand is:

$$E_I = \frac{\Delta Q/Q}{\Delta I/I} = \frac{I}{Q}\frac{\Delta Q}{\Delta I}$$

Elasticities of Supply and Demand

Other Demand Elasticities

- Cross price elasticity of demand = the % change in the quantity demanded of one good that results from a one percent change in the price of another good.

- The cross price elasticity for substitutes is positive, while that for complements is negative. For example, consider the substitute goods, butter and margarine.

$$E_{Q_bP_m} = \frac{\Delta Q_b/Q_b}{\Delta P_m/P_m} = \frac{P_m}{Q_b}\frac{\Delta Q_b}{\Delta P_m}$$

Elasticities of Supply and Demand

Elasticities of Supply

- Price elasticity of supply measures the % change in quantity supplied resulting from a 1% change in price.
- The elasticity is usually positive because price and quantity supplied are positively related (Higher price gives producers an incentive to increase output)
- We can refer to elasticity of supply with respect to interest rates, wage rates, and the cost of raw materials.

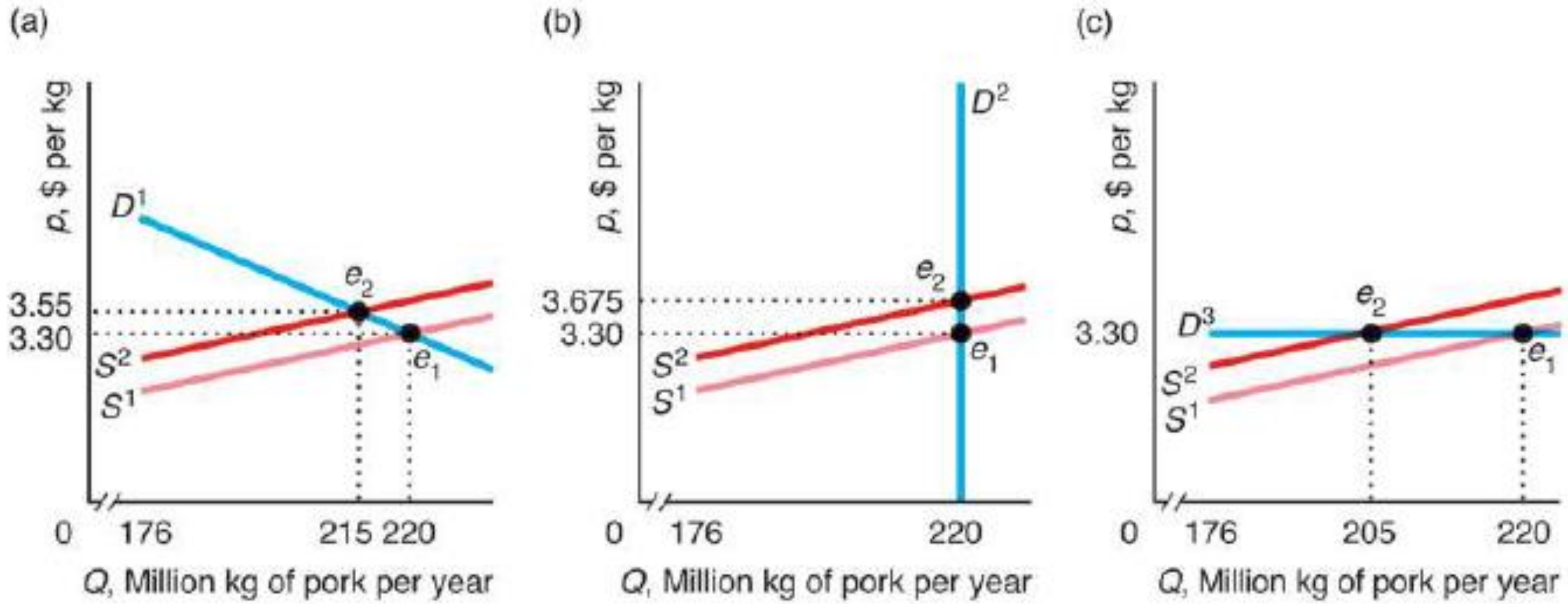
SR Versus LR Elasticities

Price Elasticity of Demand

- Price elasticity of demand varies with the amount of time consumers have to respond to a price.
- Most goods and services:
  - Short-run elasticity is less than long-run elasticity (e.g. gasoline). People tend to drive smaller and more fuel efficient cars in the long-run
- Other Goods (durables):
  - Short-run elasticity is greater than long-run elasticity (e.g. automobiles). People may put off immediate consumption, but eventually older cars must be replaced.

2.5 Elasticities

- The shape of demand and supply curves influence how much shifts in demand or supply affect market equilibrium.
  - Shape is best summarized by elasticity.



2.5 Example: Elasticity of Demand

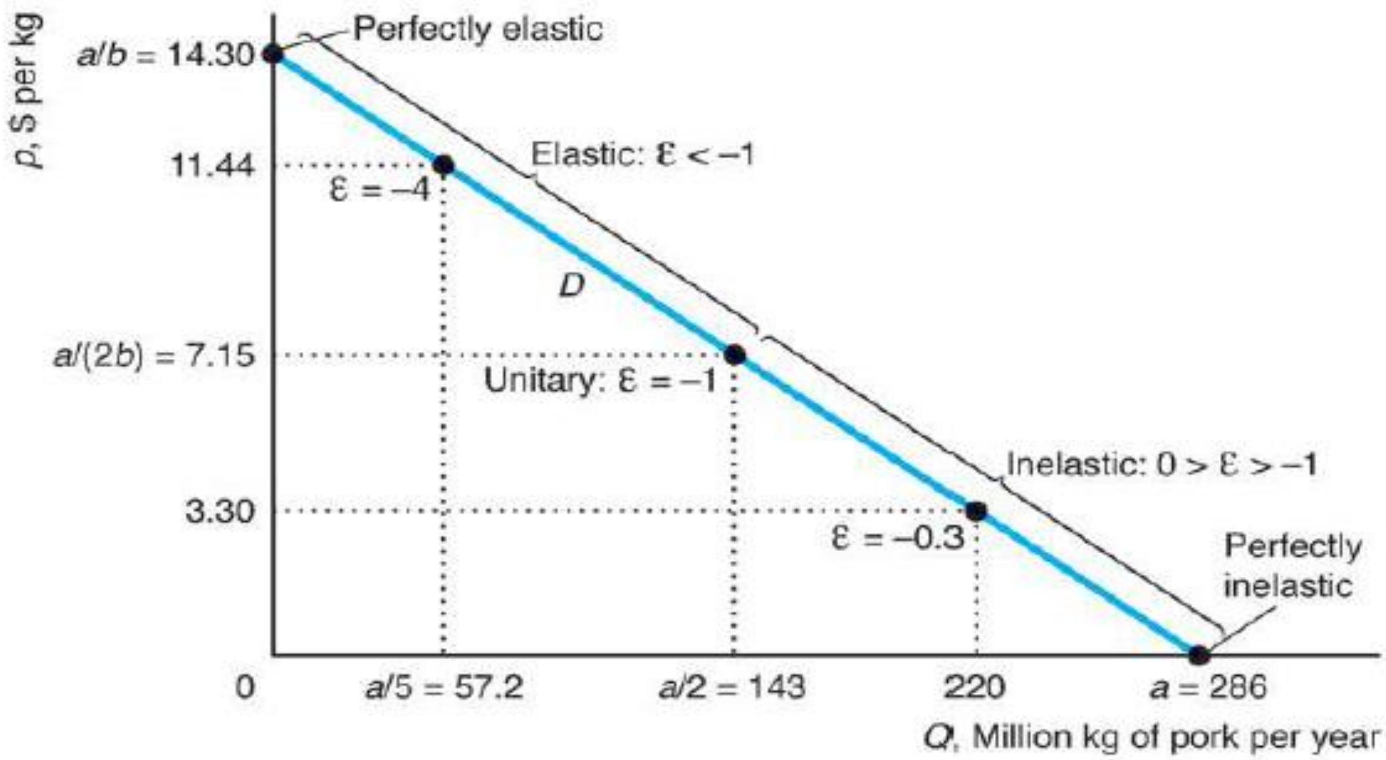
- Previous pork demand was $Q_d = 286 - 20p$
- Calculating price elasticity of demand at equilibrium (p=\$3.30 and Q=220):

$$\varepsilon = b\frac{p}{Q} = -20 \times \frac{3.30}{220} = -0.3$$

- Interpretation:
  - negative sign consistent with downward-sloping demand
  - a 1% increase in the price of pork leads to a 0.3% decrease in quantity of pork demanded

2.5 Elasticity of Demand

- Elasticity of demand varies along a linear demand curve



2.5 Elasticities

- There are other common elasticities that are used to gauge responsiveness.
  - income elasticity of demand

$$\xi = \frac{\text{percentage change in quantity demanded}}{\text{percentage change in income}} = \frac{\Delta Q / Q}{\Delta Y / Y} = \frac{\partial Q}{\partial Y}\frac{Y}{Q}$$

  - cross-price elasticity of demand

$$\frac{\text{percentage change in quantity demanded}}{\text{percentage change in price of another good}} = \frac{\Delta Q / Q}{\Delta p_o / p_o} = \frac{\partial Q}{\partial p_o}\frac{p_o}{Q}$$

  - elasticity of supply

$$\eta = \frac{\text{percentage change in quantity supplied}}{\text{percentage change in price}} = \frac{\Delta Q / Q}{\Delta p / p} = \frac{\partial Q}{\partial p}\frac{p}{Q}$$

Price elasticities

- Necessities : Inelastic
- Luxuries: Elastic
- Many close substitutes: elastic

Income elasticity

- food: inelastic
- Necessities : inelastic
- Luxuries: elastic
- Inferior good : negative
- Normal goods: positive

Cross price elasticity

- Compliments: negative
- Substitutes : positive

Supply elasticity

- Perishable items: inelastic

CES Supply curve

- Elasticity do not vary along the supply curve
- $Q = BP^{\phi}$

Continued…

- Elasticity of demand changes as we move along a linear demand curve but its slope remains the same.
- Special case of demand curve in which elasticity of demand remains the same at every point along the curve is CES dc. It is expressed by an exponential function or log linear DC e.g

$$E_P = \frac{\Delta Q/Q}{\Delta P/P} = \frac{P}{Q}\frac{\Delta Q}{\Delta P}$$

- $Q = AP^{\epsilon}$

$\ln Q = \ln A + \epsilon \ln P$

$= \epsilon A P^{\epsilon - 1} * P/Q$ as $Q = AP^{\epsilon}$

$= \epsilon$

Chapter 3 Outline

3.1 Preferences
3.2 Utility
3.3 Budget Constraint
3.4 Constrained Consumer Choice
3.5 Behavioral Economics
Challenge Solution

Chapter 3: Model of Consumer Behavior

- Premises of the model:

1. Individual ***tastes*** or ***preferences*** determine the amount of pleasure people derive from the goods and services they consume.
2. Consumers face ***constraints***, or limits, on their choices.
3. Consumers ***maximize*** their well-being or pleasure from consumption subject to the budget and other constraints they face.

3.1 Preferences

- To explain consumer behavior, economists assume that consumers have a set of tastes or preferences that they use to guide them in choosing between goods.
- Goods are ranked according to how much pleasure a consumer gets from consuming each.
  - Preference relations summarize a consumer's ranking
  - $\succ$ is used to convey strict preference (e.g. $a \succ b$)
  - $\succsim$ is used to convey weak preference (e.g. $a \succsim b$)
  - $\sim$ is used to convey indifference (e.g. $a \sim b$)

3.1 Preferences

it shows how a consumer prefers one bundle over another

- **Properties of preferences**:
  1. **Completeness**
     - We compare two bundles of goods (e.g. *a* and *b*), a consumer can rank them so that either $a \succ b$, $b \succ a$, or $a \sim b$. consumer is indifferent b/w two bundles
  2. **Transitivity**
     - Consumers' rankings are logically consistent in the sense that if $a \succ b$ and $b \succ c$, then $a \succ c$.
  3. **Non satiation: More is Better**
     - All else the same, more of a commodity is better than less.
     - **Good**: more is preferred over less
     - **Bad**: Less is preferred to more e.g. pollution

4. Continuity-

if a consumer prefers Bundle *a* to Bundle *b*, then the consumer prefers Bundle *c* to *b* if *c* is very close to *a*.

5. **Strict Convexity-**

means that consumers prefer averages to extremes, i.e. more balanced baskets that have some of each good.

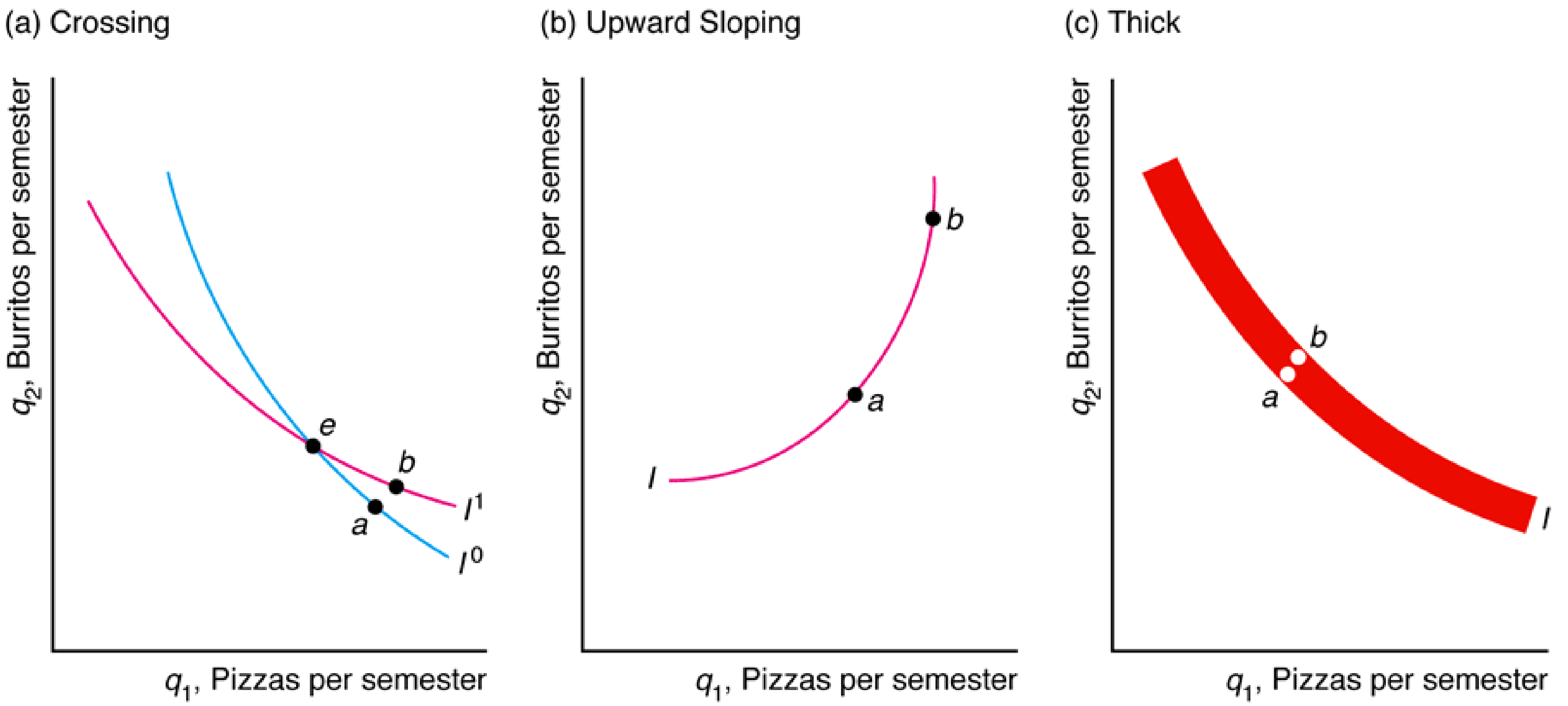
– For example if Bundle *a* and Bundle *b* are distinct bundles and the consumer prefers both of these bundles to Bundle *c*, then the consumer prefers a weighted average of *a* and *b*, $\beta a+(1-\beta)b$, to Bundle *c*.

3.1 Indifference Curves

- The set of all bundles of goods that a consumer views as being equally desirable can be traced out as an ***indifference curve***.
- **important properties of indifference curves**:
  1. Ic’s are convex to origin and downward sloping
  2. Bundles on indifference curves farther from the origin are preferred to those on indifference curves closer to the origin.
  3. Higher ic shows a higher level of satisfaction
  4. Indifference curves cannot cross.
  5. Indifference curves slope downward.
  6. Indifference curves cannot be thick.

3.1 Indifference Curves

- Impossible indifference curves:



Therefore, indifference curves must slope downward to the right

- If they sloped upward, they would violate the assumption that more is preferred to less
    - Some points that had more of both goods would be indifferent to a basket with less of both goods

    - To consume more food, the consumer is willing to give up some units of clothing consumption and still get the same utility. In other words, if food consumption is decreased, the consumer needs to obtain more units of clothing to keep the same level of utility.

3.2 Utility

- The ***utility function*** shows the level of satisfaction that a consumer derives by the consumption of goods.
  - Given a specific utility function, you can graph a specific indifference curve and determine exactly how much utility is gained from specific consumption choices.
  - Example: q_1 = pizza and q_2 = burritos
    - Bundle x contains 16 pizzas and 9 burritos: $U(x) = 12$
    - Bundle y contains 13 pizzas and 13 burritos: $U(y) = 13$
    - Thus, $y \succ x$

$$U = \sqrt{q_1 q_2}$$

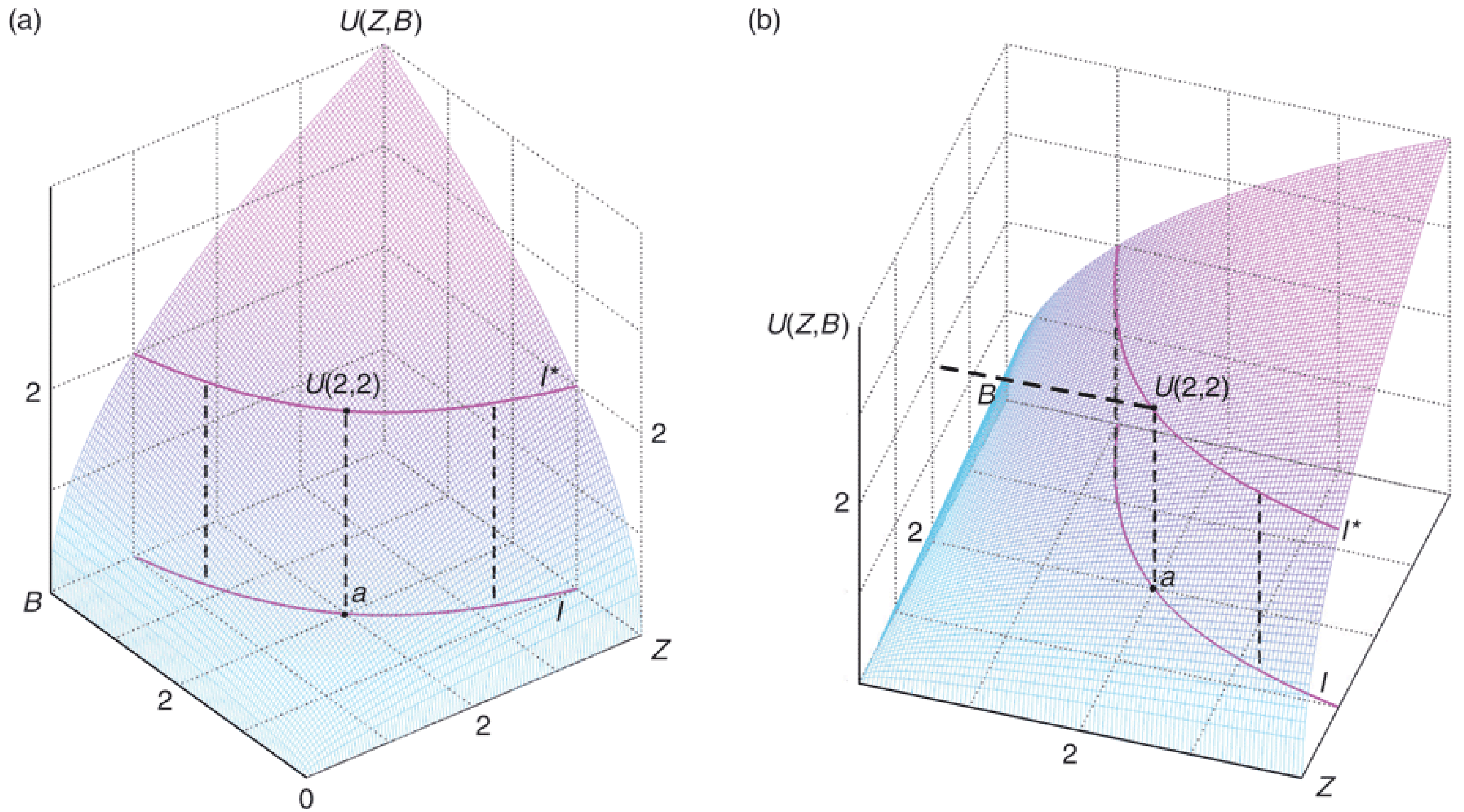
3.2 Utility

- **Utility is an *ordinal* measure rather than a *cardinal* one**.
  - Utility tells us the relative ranking of two things but not how much more one rank is valued than another.

- A utility function can be transformed into another utility function in such a way that preferences are maintained.
  - ***Positive monotonic transformation***

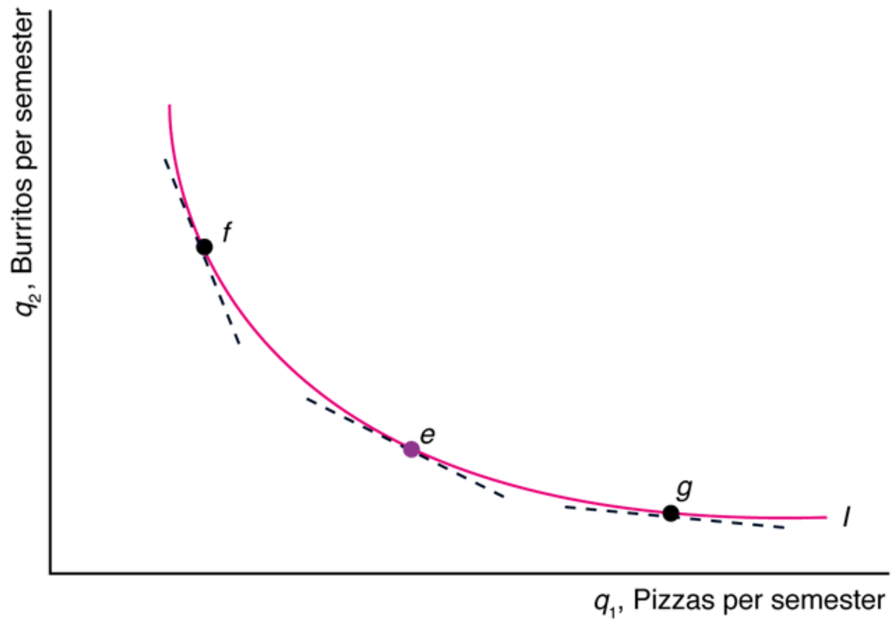
3.2 Utility and Indifference Curves

- The general utility function (for q_1 = pizza and q_2 = burritos) is $\overline{U} = U(q_1, q_2)$



3.2 Willingness to Substitute Between Goods

- ***Marginal Rate of Substitution*** **(MRS)** is the maximum amount of one good that a consumer will sacrifice (trade) to obtain one more unit of another good.
  - It is the slope at a particular point on the indifference curve
  - MRS = dq_2 / dq_1



3.2 Marginal Utilities and Marginal Rate of Substitution for Five Utility Functions

Utility Function	$U(q_1, q_2)$	$U_1 = \dfrac{\partial U(q_1, q_2)}{\partial q_1}$	$U_2 = \dfrac{\partial U(q_1, q_2)}{\partial q_2}$	$MRS = -\dfrac{U_1}{U_2}$
Perfect substitutes	$iq_1 + jq_2$	i	j	$-\dfrac{i}{j}$
Perfect complements	$\min(iq_1, jq_2)$	0	0	0
Cobb-Douglas	$q_1^a q_2^{1-a}$	$a\dfrac{U(q_1, q_2)}{q_1}$	$(1-a)\dfrac{U(q_1, q_2)}{q_2}$	$-\dfrac{a}{1-a}\dfrac{q_2}{q_1}$
Constant Elasticity of Substitution (CES)	$(q_1^\rho + q_2^\rho)^{1/\rho}$	$(q_1^\rho + q_2^\rho)^{(1-\rho)/\rho} q_1^{\rho-1}$	$(q_1^\rho + q_2^\rho)^{(1-\rho)/\rho} q_2^{\rho-1}$	$-\left(\dfrac{q_1}{q_2}\right)^{\rho-1}$
Quasilinear	$u(q_1) + q_2$	$\dfrac{du(q_1)}{dq_1}$	1	$-\dfrac{du(q_1)}{dq_1}$

Notes: $i > 0$, $j > 0$, $0 < a < 1$, $\rho \neq 0$, and $\rho < 1$. We are evaluating the prefect complements' indifference curve at its right-angle corner, where it is not differentiable, hence the formula $MRS = -U_1/U_2$ is not well-defined. We arbitrarily say that the $MRS = 0$ because no substitution is possible.

3.2 Marginal Utility and MRS

- The MRS depends on how much extra utility a consumer gets from a little more of each good.
  - ***Marginal utility*** is the extra utility that a consumer gets from consuming the last unit of a good, holding the consumption of other goods constant.

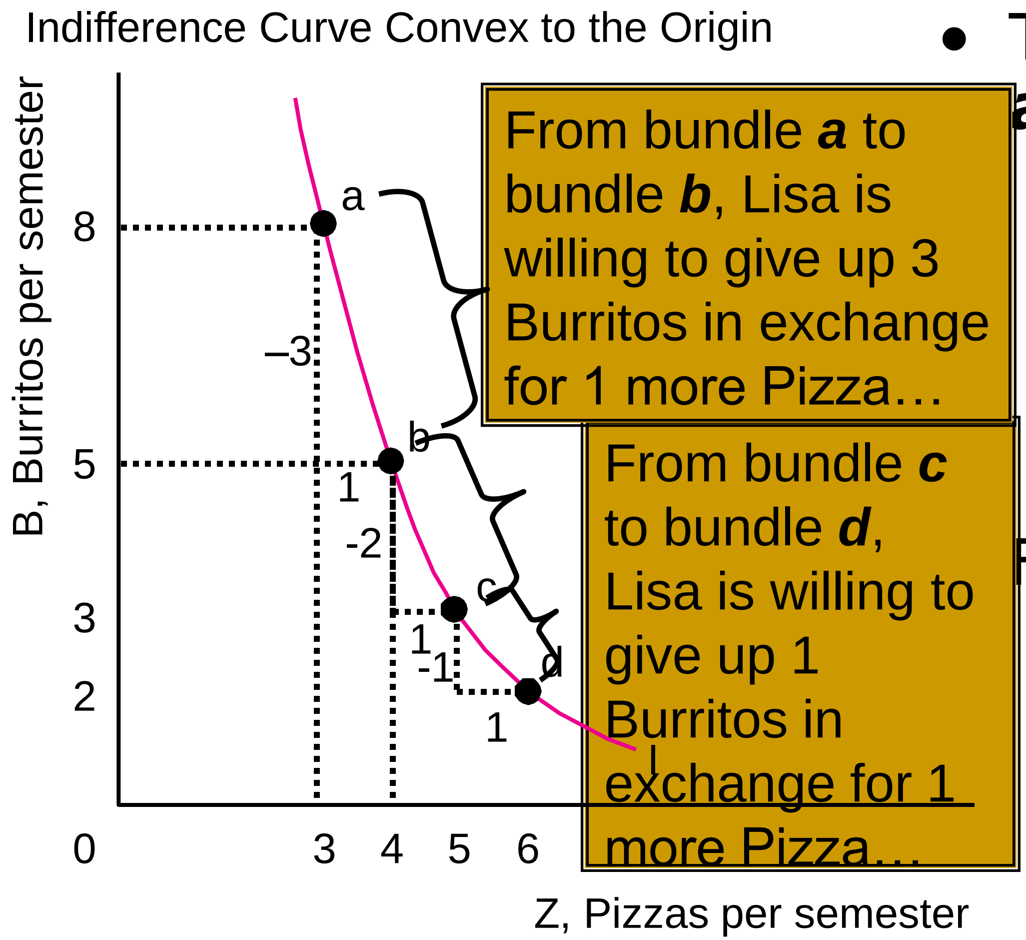
$$\text{marginal utility of pizza} = \frac{\partial U}{\partial q_1} = U_1$$

- Using calculus to calculate the MRS:

$$MRS = \frac{\mathrm{d}q_2}{\mathrm{d}q_1} = -\frac{\partial U / \partial q_1}{\partial U / \partial q_2} = -\frac{U_1}{U_2}$$

(a) MRS along an Indifference curve

4-63



- The MRS from bundle ***a*** to bundle ***b*** is -3.
  - This is the same as the slope of the indifference curve between those two points.
- From ***b*** to ***c***,
  - MRS = -2.
  - This is the same as the slope of the indifference curve between those two points.

Marginal Rate of Substitution

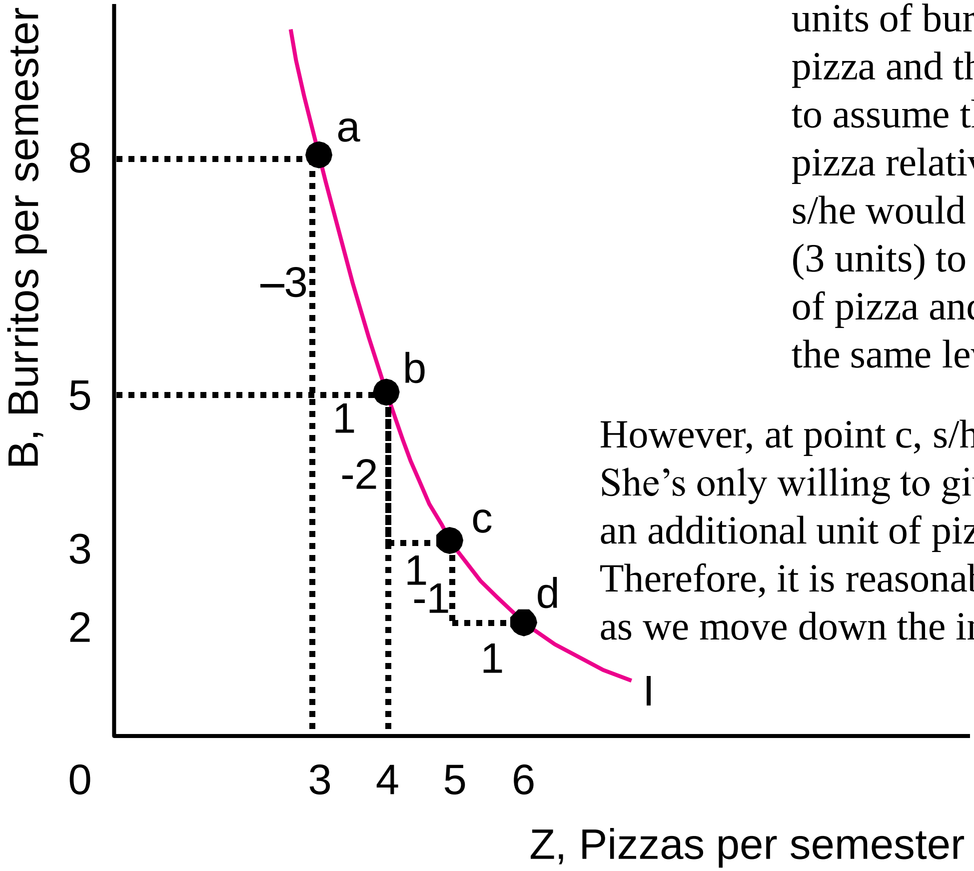
- Indifference curves are typically convex (bowed inward).
  – As more of one good is consumed, a consumer would prefer to give up fewer units of a second good to get additional units of the first one (property of diminishing MRS)
- In other words, consumers generally prefer a *balanced* market basket

Marginal Rate of Substitution

- The MRS decreases as we move down the indifference curve
  - Along an indifference curve there is a *diminishing marginal rate of substitution*.
  - The MRS went from 3 to 2 to 1

Diminishing MRS along an Indifference curve

Indifference Curve Convex to the Origin



At point a, the consumer has many units of burritos and few units of pizza and therefore, it is reasonable to assume that s/he may value pizza relatively more than burritos: s/he would give up a lot of burritos (3 units) to obtain 1 additional unit of pizza and still be able to keep the same level of utility.

However, at point c, s/he has few burritos and a lot of pizza. She's only willing to give up a small amount of burritos for an additional unit of pizza, making the slope very flat. Therefore, it is reasonable to assume that MRS diminishes as we move down the indifference curve.

Willingness to Substitute between goods

Suppose more generally we have an indifference curve: $\overline{U} = U(q_1, q_2)$

$$MRS = \frac{dq_2}{dq_1} = -\frac{\partial U/\partial q_1}{\partial U/\partial q_2} = -\frac{U_1}{U_2}$$

Where U1 is MU of good 1

Marginal Utility and Consumer Choice

- It must be the case along an indifference curve that

$$\boxed{0 = MU_F(\Delta F) + MU_C(\Delta C)}$$

No change in total utility along an indifference curve.
Trade off of one good to the other leaves the consumer just as well off.

Solved Problem 3.2

4-69

Suppose that Jackie has what is known as a Cobb-Douglas utility function:

$$U = q_1^a q_2^{1-a}$$

where a is a positive constant, q1 is the number of CDs she buys a year and q2 is the number of movie DVDs she buys. What is her MRS?

Answer:

$$MRS = \frac{dq_2}{dq_1} = -\frac{U_1}{U_2} = -\frac{aU/q_1}{(1-a)U/q_2} = -\frac{a}{1-a}\frac{q_2}{q_1}$$

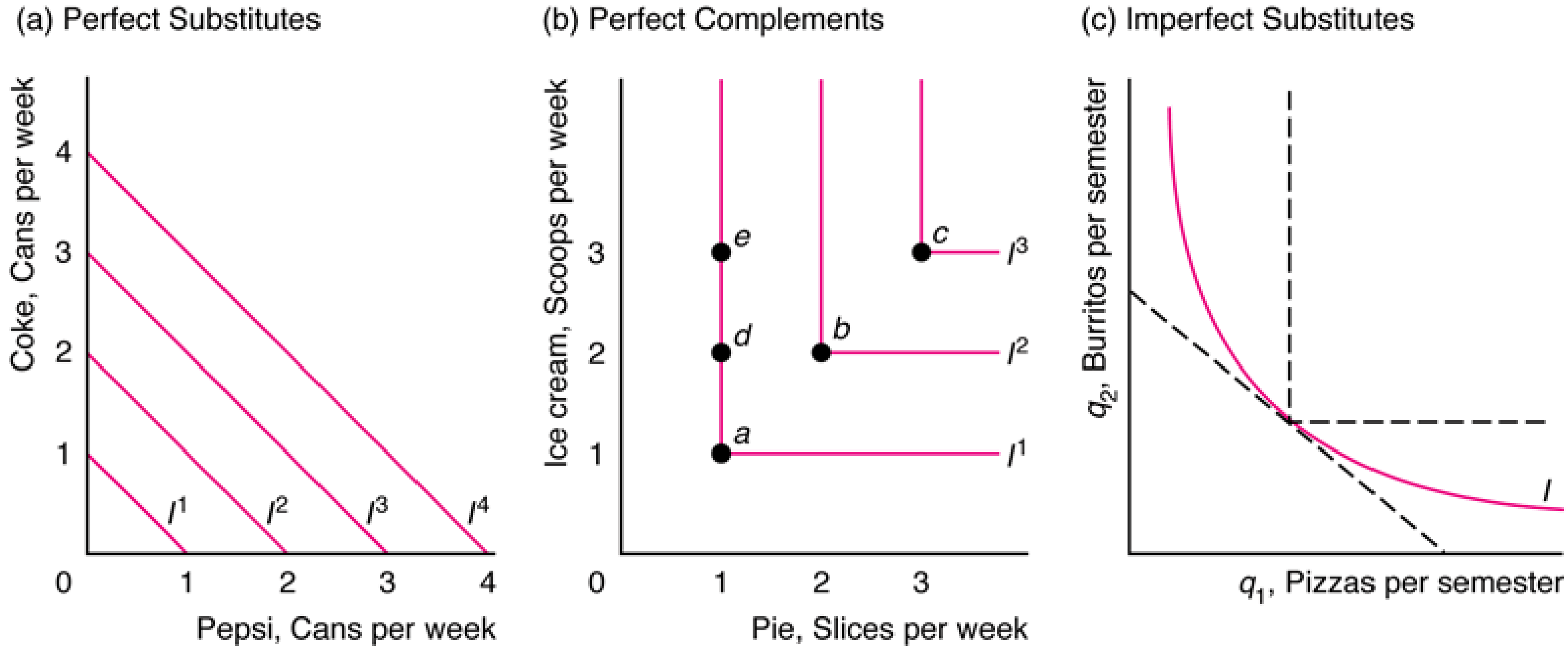
Curvature of Indifference Curves.

- Casual observation suggests that most people's indifference curves are **convex.**

- **Exceptions:**
  - **Perfect substitutes -** goods that a consumer is completely indifferent as to which to consume.
    $$U(C, G) = iC + jG$$
  - **Perfect complements -** goods that a consumer is interested in consuming only in fixed proportions $U(A, V) = \min(iA, jV)$

3.2 Curvature of Indifference Curves

- MRS (willingness to trade) diminishes along many typical indifference curves that are concave to the origin.
- Different utility functions generate different indifference curves:



3.2 Curvature of Indifference Curves

- Perfect Substitutes
  - Goods that a consumer is completely indifferent between
  - Example: Clorox (C) and Generic Bleach (G)

$$U(C,\, G) = iC + jG$$

  - MRS = -2 (constant)

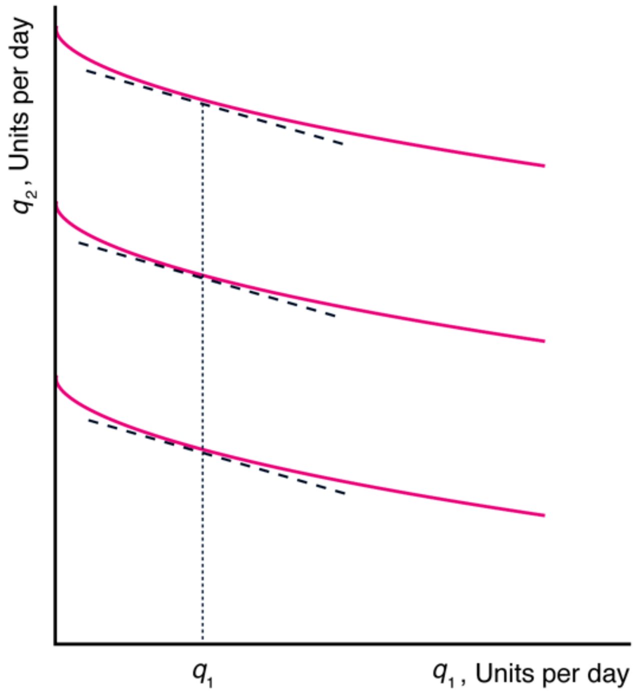
- Perfect Complements
  - Goods that are consumed in fixed proportions
  - Example: Apple pie (A) and Ice cream (I)

$$U(A,\, V) = \min\,(iA,\, jV)$$

  - MRS is undefined

3.2 Curvature of Indifference Curves

- Imperfect Substitutes
  - Between extreme examples of perfect substitutes and perfect complements are standard-shaped, convex indifference curves.
  - Cobb-Douglas utility function (e.g. $U = q_1^a q_2^{1-a}$) indifference curves never hit the axes.
  - Quasilinear utility function (e.g. $U(q_1, q_2) = u(q_1) + q_2$) indifference curves hit one of the axes.



Budget Constraint

4-74

- **budget line** (or *budget constraint*) - the bundles of goods that can be bought if the entire budget is spent on those goods at given prices.

- **opportunity set -** all the bundles a consumer can buy, including all the bundles inside the budget constraint and on the budget constraint

3.3 Budget Constraint

- Consumers maximize utility subject to constraints.
- If we assume consumers can't save and borrow, current period income determines a consumer's budget.
- Given prices of pizza (p_1) and burritos (p_2), and income Y, the ***budget line*** is

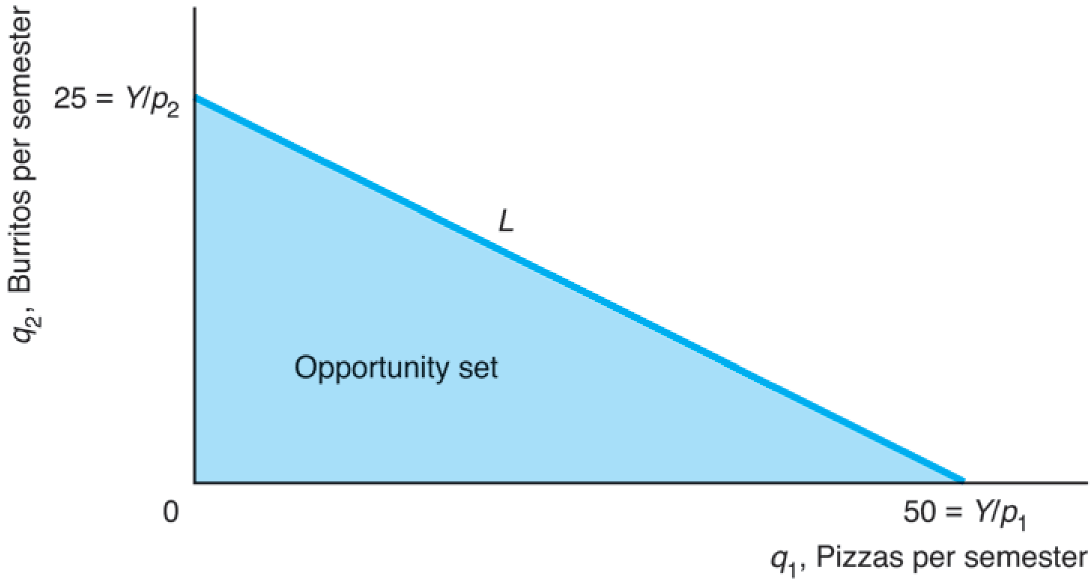
$$p_1 q_1 + p_2 q_2 = Y$$

- Example:
  - Assume p_1 = \$1, p_2 = \$2 and Y = \$50
  - Rewrite the budget line equation for easier graphing ($y=mx+b$ form):

$$q_2 = \frac{\$50 - (\$1 \times q_1)}{\$2} = 25 - \frac{1}{2} q_1$$

3.3 Budget Constraint

- ***Marginal Rate of Transformation*** (MRT) is how the market allows consumers to trade one good for another.
  - It is the slope of the budget line: $$MRT = \frac{dq_2}{dq_1} = -\frac{p_1}{p_2}$$



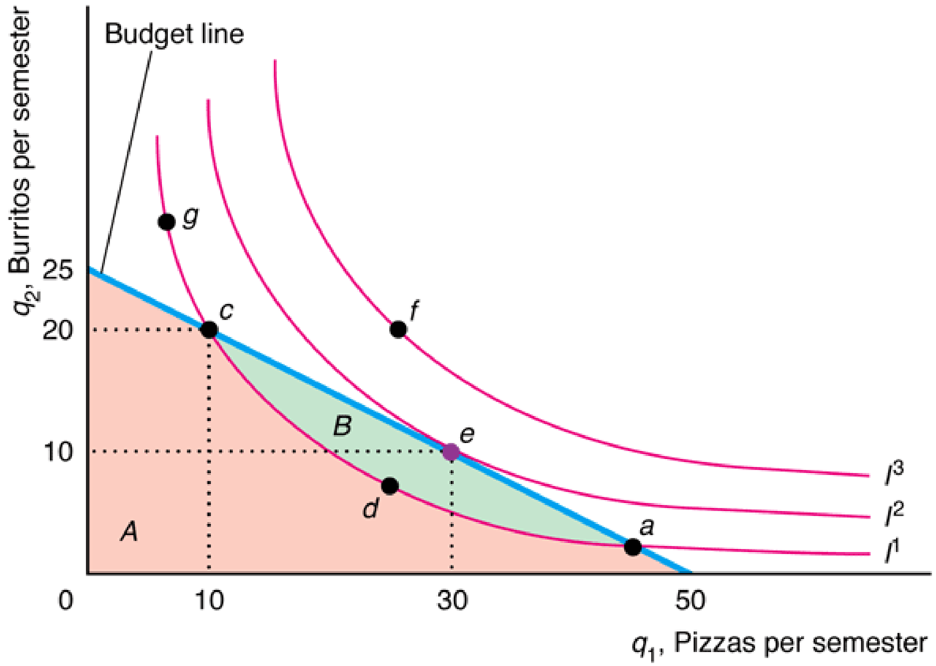
3.4 Constrained Consumer Choice

- Consumers maximize their well-being (utility) subject to their budget constraint.
- The highest indifference curve attainable given the budget is the consumer's ***optimal bundle***.
- When the optimal bundle occurs at a point of tangency between the indifference curve and budget line, this is called an ***interior solution***.
  - Mathematically, $$MRS = -\frac{U_1}{U_2} = -\frac{p_1}{p_2} = MRT$$
  - Rearranging, we can see that the marginal utility per dollar is equated across goods at the optimum:

$$\frac{U_1}{p_1} = \frac{U_2}{p_2}$$

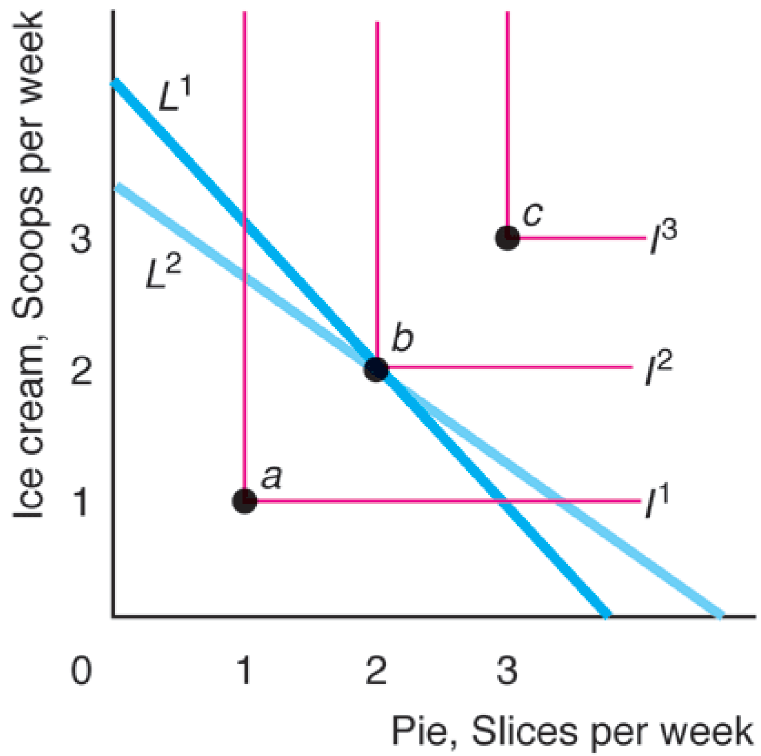
3.4 Constrained Consumer Choice

- The interior solution that maximizes utility without going beyond the budget constraint is Bundle *e*.
- The interior optimum is where $MRS = -\dfrac{U_1}{U_2} = -\dfrac{p_1}{p_2} = MRT$



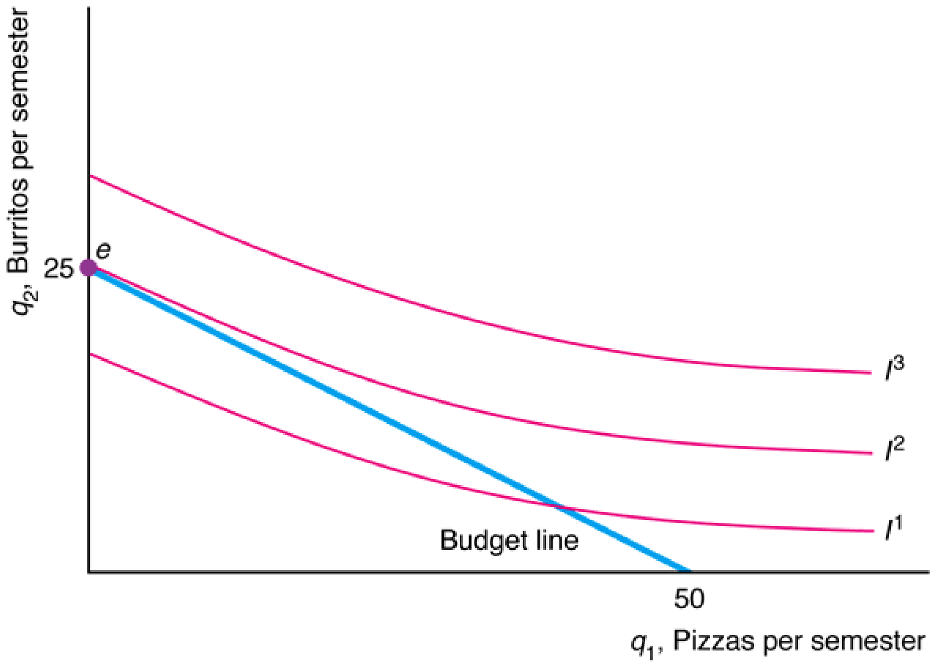
3.4 Constrained Consumer Choice with Perfect Complements

- The optimal bundle is on the budget line and at the right angle (i.e. vertex) of an indifference curve.



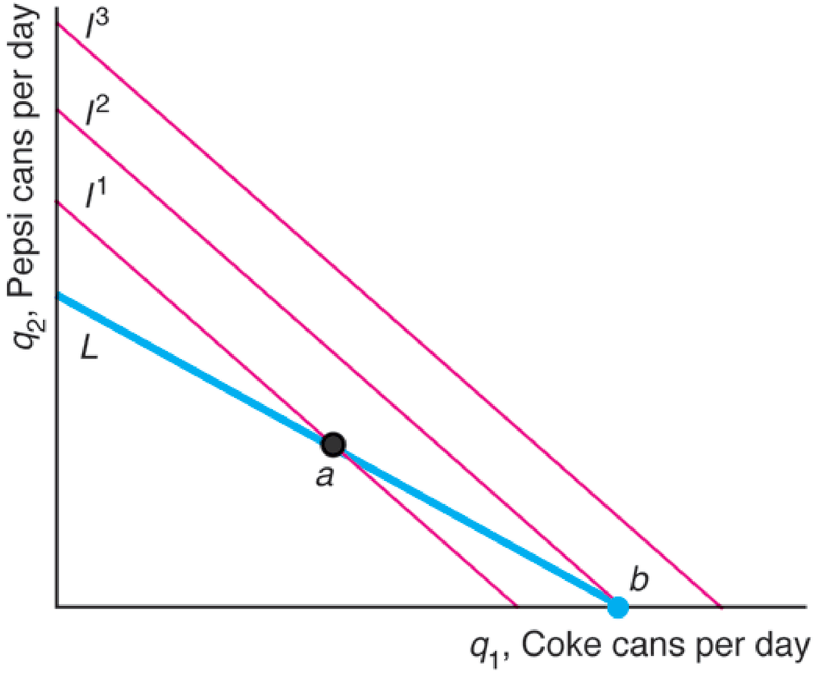
3.4 Constrained Consumer Choice with Quasilinear Preferences

- If the relative price of one good is too high and preferences are quasilinear, the indifference curve will not be tangent to the budget line and the consumer's optimal bundle occurs at a ***corner solution***.



3.4 Constrained Consumer Choice with Perfect Substitutes

- With perfect substitutes, if the marginal rate of substitution does not equal the marginal rate of transformations, then the consumer's optimal bundle occurs at a ***corner solution***, bundle b.



3.4 Consumer Choice with Calculus

- Our graphical analysis of consumers' constrained choices can be stated mathematically:

$$\max_{q_1, q_2} U(q_1, q_2)$$

$$\text{s.t.} \quad Y = p_1 q_1 + p_2 q_2$$

- The optimum is still expressed as in the graphical analysis:

$$MRS = -\frac{U_1}{U_2} = -\frac{p_1}{p_2} = MRT$$

- These conditions hold if the utility function is quasi-concave, which implies indifference curves are convex to the origin.
- Solution reveals utility-maximizing values of q_1 and q_2 as functions of prices, p_1 and p_2, and income, Y.

3.4 Consumer Choice with Calculus

- Example (Solved Problem 3.5):

$$U(q_1, q_2) = \left(q_1^{\rho} + q_2^{\rho}\right)^{\frac{1}{\rho}}, \text{ where } 0 \neq \rho \leq 1.^{13}$$

$$\max_{q_1,q_2} U(q_1,q_2) = \left(q_1^{\rho} + q_2^{\rho}\right)^{\frac{1}{\rho}}$$

$$\text{s.t.}\quad Y = p_1q_1 + p_2q_2$$

$$\max_{q_1} U\left(q_1, \frac{Y - p_1q_1}{p_2}\right) = \left(q_1^{\rho} + \left[\frac{Y - p_1q_1}{p_2}\right]^{\rho}\right)^{1/\rho}$$

$$\frac{1}{\rho}\left(q_1^{\rho} + \left[\frac{Y - p_1q_1}{p_2}\right]^{\rho}\right)^{\frac{1-\rho}{\rho}}\left(\rho q_1^{\rho-1} + \rho\left[\frac{Y - p_1q_1}{p_2}\right]^{\rho-1}\left[-\frac{p_1}{p_2}\right]\right) = 0$$

$$\frac{1}{\rho}\left(q_1^{\rho} + \left[\frac{Y - p_1q_1}{p_2}\right]^{\rho}\right)^{\frac{1-\rho}{\rho}}\left(\rho q_1^{\rho-1} + \rho\left[\frac{Y - p_1q_1}{p_2}\right]^{\rho-1}\left[-\frac{p_1}{p_2}\right]\right) = 0$$

$$q_1 = \frac{Yp_1^{z-1}}{p_1^{z} + p_2^{z}},$$

$$q_2 = \frac{Yp_2^{z-1}}{p_1^{z} + p_2^{z}}$$

3.4 Consumer Choice with Calculus

- A second approach to solving constrained utility maximization problems is the Lagrangian method:

$$\max_{q_1, q_2, \lambda} \mathcal{L} = U(q_1, q_2) + \lambda(Y - p_1 q_1 - p_2 q_2)$$

- The critical value of $\mathcal{L}$ is found through first-order conditions:

$$\frac{\partial \mathcal{L}}{\partial q_1} = \frac{\partial U}{\partial q_1} - \lambda p_1 = U_1 - \lambda p_1 = 0 \qquad \frac{\partial \mathcal{L}}{\partial \lambda} = Y - p_1 q_1 - p_2 q_2 = 0$$

$$\frac{\partial \mathcal{L}}{\partial q_2} = U_2 - \lambda p_2 = 0$$

- Equating the first two of these equations yields:

$$\lambda = \frac{U_1}{p_1} = \frac{U_2}{p_2}$$

Maximizing utility subject to a constraint using calculus

- Lisa's objective is to maximize her utility, $U(q_1, q_2)$, subject to (s.t.) her budget constraint:

$$\max_{q_1, q_2} U(q_1, q_2)$$

$$\text{s.t. } Y = p_1 q_1 + p_2 q_2$$

- Here the control variables are q_1 and q_2.
- Lisa has no control over the prices she faces, p_1 and p_2, or her income, Y.
- Two methods of solving the problem. We'll focus on the substitution method only.

Substitution

- First, we can substitute the budget constraint into the utility function. Using algebra we can rewrite the budget constraint as $q_1 = (Y - p_2 q_2)/p_1$ and substitute this expression for q_1 in the utility function

$$\max_{q_2} U\left(\frac{Y - p_2 q_2}{p_1}, q_2\right)$$

- Using standard maximization techniques we can solve this problem. (i.e. take the first derivative of the utility function with respect to q_2 and set it equal to zero)

Substitution

4-87

$$\frac{dU}{dq_2} = \frac{\partial U}{\partial q_1}\frac{dq_1}{dq_2} + \frac{\partial U}{\partial q_2} = \left(-\frac{p_2}{p_1}\right)\frac{\partial U}{\partial q_1} + \frac{\partial U}{\partial q_2} = \left(-\frac{p_2}{p_1}\right)U_1 + U_2 = 0$$

- By rearranging terms in the above equation, we get the same condition for an optimum that we obtained using a graphical approach.

$$MRS = -\frac{U_1}{U_2} = -\frac{p_1}{p_2} = MRT$$

- When we combine the MRS=MRT condition with the budget constraint we have two equations in two unknowns , q_1 and q_2. So we can solve for the optimal q_1 and q_2 as function of prices, p_1 and p_2, and income

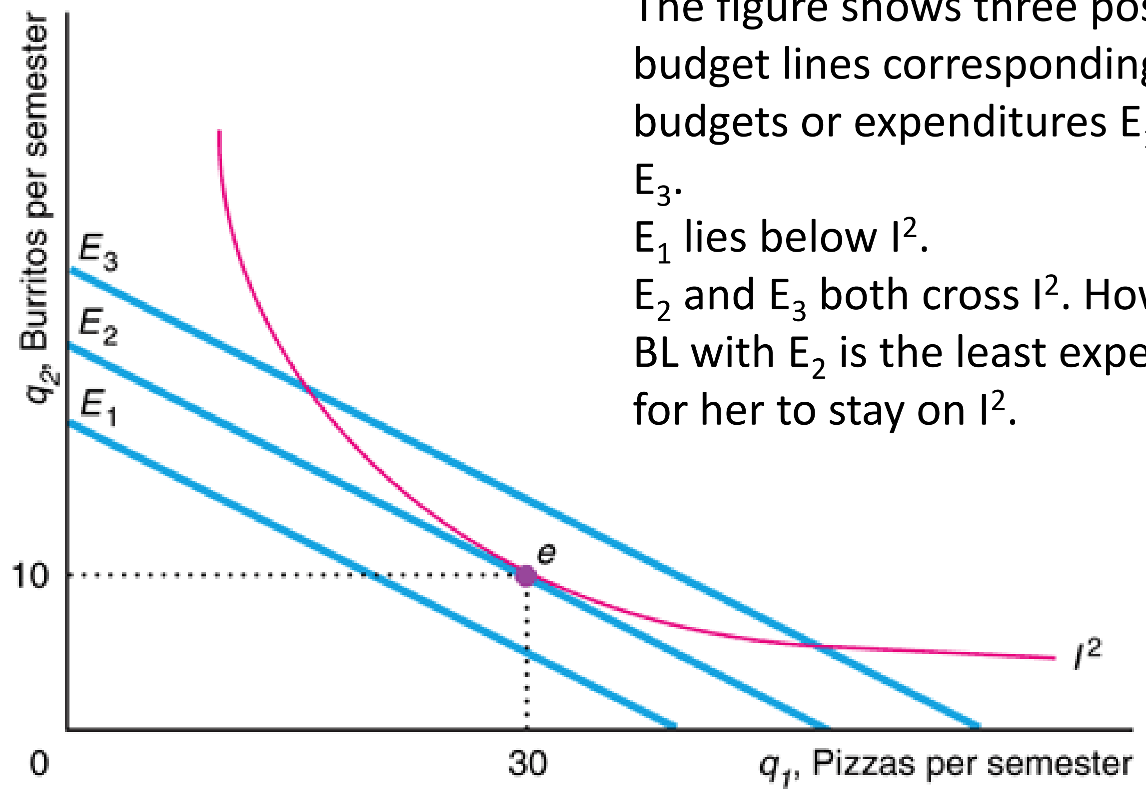
Minimizing Expenditure

4-88

- We have shown how Lisa chooses quantities of goods so as to maximize her utility subject to a budget constraint.
- There is a related or *dual* constrained maximization problem where she finds the combination of goods that achieve a particular level of utility for the least expenditure.
- Earlier we showed that Lisa maximized her utility by picking a bundle of $q_1 = 30$ and $q_2 = 10$ at the indifference curve I^2
- Now we see: How can Lisa make the lowest possible expenditure to maintain her utility at a particular level which corresponds to indifference curve I^2?

Fig. 3.8 Minimizing Expenditure

4-89



The figure shows three possible budget lines corresponding to budgets or expenditures E_1, E_2 and E_3.
E_1 lies below I^2.
E_2 and E_3 both cross I^2. However the BL with E_2 is the least expensive way for her to stay on I^2.

The rule for minimizing expenditure while achieving a given level of utility is to choose the lowest expenditure such that the budget line touches-is tangent to-the relevant indifference curve

Minimizing Expenditure

- Solving either of the two problems yields the same optimal values.
- More useful to use the expenditure minimizing approach
- We can use calculus to solve the expenditure minimizing problem

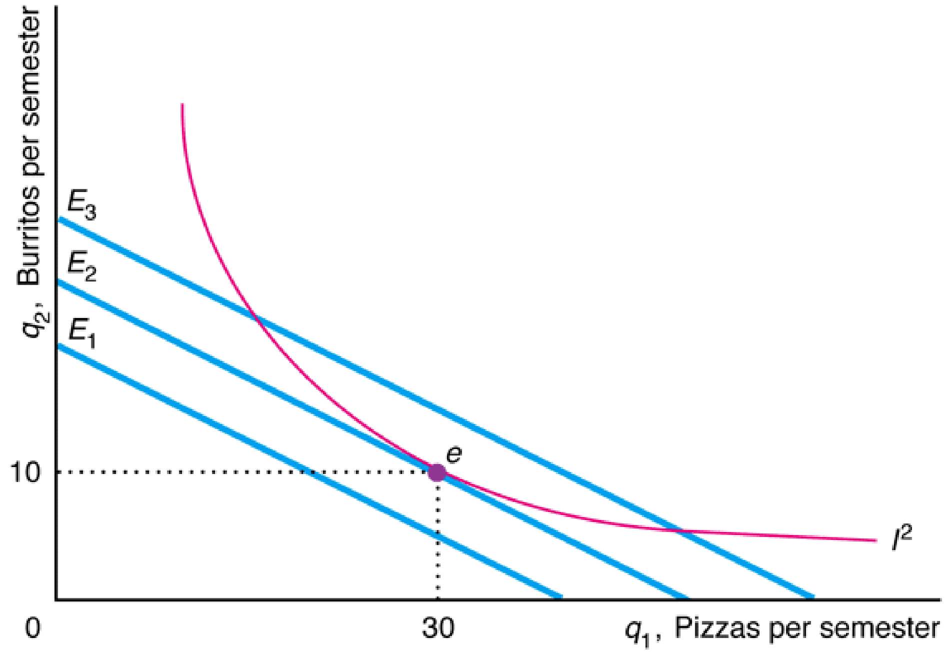
$$\min_{q_1, q_2} E = p_1 q_1 + p_2 q_2$$
$$\text{s.t. } \overline{U} = U(q_1, q_2)$$

- The solution of this problem gives us the expenditure function: the relationship showing the minimal expenditures necessary to achieve a specific utility level for a given set of prices.

$$E = E(p_1, p_2, \overline{U})$$

3.4 Minimizing Expenditure

- Utility maximization has a dual problem in which the consumer seeks the combination of goods that achieves a particular level of utility for the least expenditure.



3.4 Type of Solution for Five Utility Functions

Utility Function	$U(q_1, q_2)$	Type of Solution
Perfect complements	$\min(iq_1, jq_2)$	interior
Cobb-Douglas	$q_1^a q_2^{1-a}$	interior
Constant Elasticity of Substitution	$(q_1^\rho + q_2^\rho)^{1/\rho}$	interior
Perfect substitutes	$iq_1 + jq_2$	interior or corner
Quasilinear	$u(q_1) + q_2$	interior or corner

Notes: $i > 0$, $j > 0$, $0 < a < 1$, $\rho \neq 0$, and $\rho < 1$.

3.4 Expenditure Minimization with Calculus

- Minimize expenditure, *E*, subject to the constraint of holding utility constant:

$$\min_{q_1, q_2} E = p_1 q_1 + p_2 q_2$$

$$\text{s.t. } \overline{U} = U(q_1, q_2)$$

- The solution of this problem, the ***expenditure function***, shows the minimum expenditure necessary to achieve a specified utility level for a given set of prices:

$$E = E\left(p_1, p_2, \overline{U}\right)$$

3.5 Behavioral Economics

- What if consumers are not rational, maximizing individuals?
  - ***Behavioral economics*** adds insights from psychology and empirical research on cognition and emotional biases to the rational economic model.
    - **Tests of transitivity**: evidence supports transitivity assumption for adults, but not necessarily for children.
    - **Endowment effect:** some evidence that endowments of goods influence indifference maps, which is not the assumption of economic models.
    - **Salience:** evidence that consumers are more sensitive to increases in pre-tax prices than post-tax price increases from higher ad valorem taxes.
      - **Bounded rationality** suggests that calculating post-tax prices is "costly" so some people don't bother to do it, but they would use the information if it were provided.

Challenge Solution

- Max, a German, and Bob, a Yank, have the same preferences – perfect substititues. The U.S. relative after-tax price of e-books is lower than the German relative after-tax price. Due to the relative price differences, Max reads printed books and Bob reads e-books.

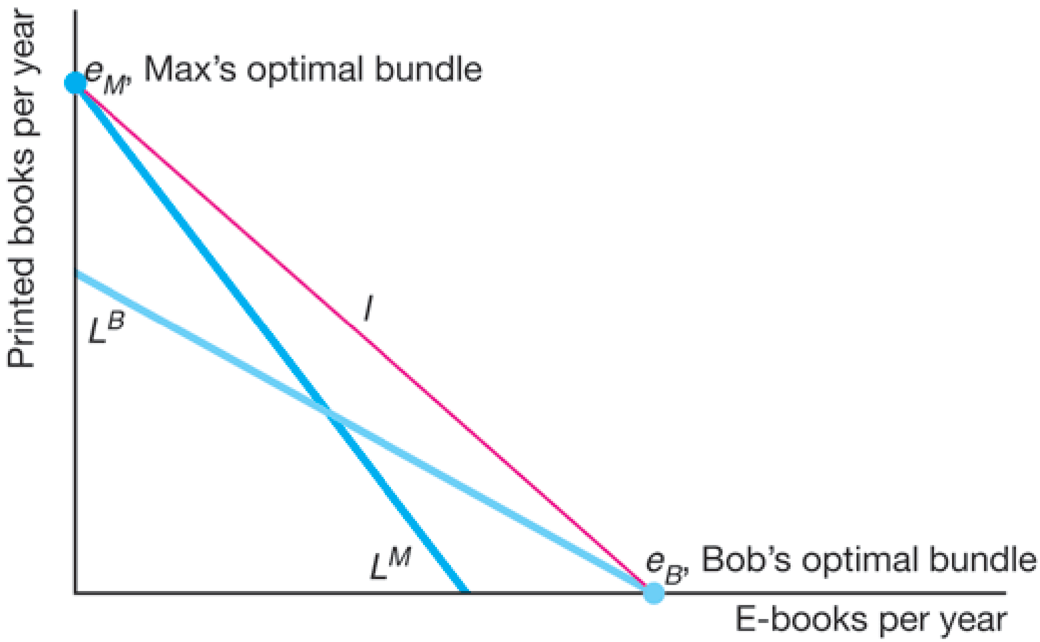


Figure 3.11 Optimal Bundles on Convex Sections of Indifference Curves