Fuzzy System: An Overview

A **fuzzy system** is a decision-making system that uses "fuzzy logic" to handle uncertainty and ambiguity. Unlike classical binary logic, where variables can have only two values (true or false, 0 or 1), fuzzy logic allows variables to have degrees of truth, which can take any value between 0 and 1.

Key Concepts in a Fuzzy System

1. **Fuzzification**: Converting crisp (precise) inputs into fuzzy values using membership functions.
2. **Fuzzy Rules**: "IF-THEN" rules that define how fuzzy inputs relate to fuzzy outputs.
3. **Inference**: Applying fuzzy logic operators (like AND, OR, MIN, MAX) to the fuzzy rules to generate fuzzy output.
4. **Defuzzification**: Converting fuzzy output back into a crisp (precise) value.

Membership Functions in Fuzzy Logic

A **Membership Function (MF)** defines how each point in the input space (the universe of discourse) is mapped to a membership value (or degree of truth) between 0 and 1. It represents the degree to which a given input belongs to a fuzzy set.

Why Use Membership Functions?

Membership functions help us deal with the imprecision and vagueness of linguistic terms like "Hot", "Cold", "Tall", "Short", etc. For example, the temperature **30°C** may be "partly warm" and "partly hot." Membership functions allow for this partial membership by assigning degrees like 0.6 for "Warm" and 0.4 for "Hot."

Key Concepts

1. **Universe of Discourse**: The range of all possible values for the input variable.
    - Example: If the input variable is "temperature," the universe of discourse could be $[0, 100]$ °C.
2. **Fuzzy Set**: A set with no sharp boundary. Instead of being a simple "Yes" (1) or "No" (0), elements in the set have a degree of belonging.
    - Example: The set of "Warm temperatures" might include temperatures from 20°C to 35°C with different degrees of membership.
3. **Degree of Membership**: The degree (between 0 and 1) that indicates how strongly a given input belongs to a fuzzy set.
    - Example: For the fuzzy set **Warm** and input **30°C**, the membership degree may be 0.6 (partially Warm).

Example of a Fuzzy System

Problem: Controlling the speed of a fan based on **temperature** and **humidity**.

- Inputs: Temperature (in °C) and Humidity (in %)
- Output: Speed of the fan (in RPM)

1. Fuzzification

Convert crisp inputs (like 30°C and 70% humidity) into fuzzy values using membership functions.

- **Temperature** is classified as **Cold, Warm, or Hot**.
- **Humidity** is classified as **Low, Medium, or High**.

Membership Functions (Example)

- **Temperature (T)**:
  - Cold (0°C to 20°C)
  - Warm (15°C to 30°C)
  - Hot (25°C to 40°C)
- **Humidity (H)**:
  - Low (0% to 40%)
  - Medium (30% to 70%)
  - High (60% to 100%)

Suppose the temperature is **30°C** and humidity is **70%**.

- Temperature (30°C) can be classified as:
  - **Warm**: 0.2 (since 30°C is close to the boundary of Warm-Hot)
  - **Hot**: 0.8 (since 30°C is near the start of the Hot range)
- Humidity (70%) can be classified as:
  - **Medium:** 0.2 (since 70% is at the boundary of Medium-High)
  - **High**: 0.8 (since 70% is close to the beginning of the High range)

2. Fuzzy Rules

These are "IF-THEN" rules that relate inputs to outputs.
Examples of fuzzy rules for controlling the speed of the fan:

1. **IF** Temperature is Hot **AND** Humidity is High, **THEN** Fan Speed is High.
2. **IF** Temperature is Warm **AND** Humidity is Medium, **THEN** Fan Speed is Medium.
3. **IF** Temperature is Cold **OR** Humidity is Low, **THEN** Fan Speed is Low.

3. Inference (Applying Rules)

Now, apply the inputs to the fuzzy rules.

- Temperature = 30°C (Warm = 0.2, Hot = 0.8)
- Humidity = 70% (Medium = 0.2, High = 0.8)

Apply the rules:

1. **IF** Temperature is Hot (0.8) **AND** Humidity is High (0.8), **THEN** Fan Speed is High.

$$\text{Fuzzy Output} = \min(0.8, 0.8) = 0.8$$

2. **IF** Temperature is Warm (0.2) **AND** Humidity is Medium (0.2), **THEN** Fan Speed is Medium.

$$\text{Fuzzy Output} = \min(0.2, 0.2) = 0.2$$

3. **IF** Temperature is Cold **OR** Humidity is Low, **THEN** Fan Speed is Low.
   - Temperature is not cold (0)
   - Humidity is not low (0)
     So this rule does not apply (contributes nothing).

4. Defuzzification

Convert the fuzzy output back to a crisp value.
Suppose the membership functions for **Fan Speed** are:

- **Low** (0 to 30 RPM)
- **Medium** (20 to 50 RPM)
- **High** (40 to 100 RPM)

Use the **Centroid Method** to calculate the crisp output.

1. The fuzzy output for **High** speed is 0.8.
2. The fuzzy output for **Medium** speed is 0.2.

Use the centroid method to calculate the final speed:

$$\text{Speed} = \frac{(0.8 \cdot 70) + (0.2 \cdot 35)}{0.8 + 0.2} = \frac{56 + 7}{1} = 63 \text{ RPM}$$

Output: The fan speed should be set to **63 RPM**.

How to Fuzzify a Scenario?

To fuzzify a scenario, follow these steps:

1. **Define Input and Output Variables**:
   - Input 1: Temperature (Crisp input)
   - Input 2: Humidity (Crisp input)
2. **Define Membership Functions**:
   - Partition the range of possible values for temperature (e.g., Cold, Warm, Hot) and humidity (e.g., Low, Medium, High).
   - Use triangular, trapezoidal, or Gaussian membership functions to represent these ranges.
3. **Assign Membership Values**:
   - Calculate the degree of membership for a given crisp input using the membership functions.
   - Example: Temperature = 30°C could be partially **Warm** (0.2) and partially **Hot** (0.8).

Real-Life Applications of Fuzzy Systems

1. **Air Conditioning System**: Adjusts cooling power based on temperature and humidity.
2. **Washing Machines**: Automatically adjusts the washing time based on the dirtiness of the clothes.
3. **Traffic Control**: Controls traffic lights by assessing the traffic density.
4. **Medical Diagnosis**: Diagnoses diseases based on symptoms with degrees of certainty.
5. **Stock Market Predictions**: Assesses the likelihood of price increase or decrease based on various factors.

Advantages of Fuzzy Systems

- Handles **uncertain, vague, or imprecise information**.
- **Models human-like reasoning**.
- Used in systems where exact mathematical models are hard to derive.

Scenario: Smart Air Conditioner (AC) System

A smart air conditioner uses **fuzzy logic** to maintain a comfortable room temperature based on:

1. **Current Room Temperature** (T) relative to a desired setpoint (T_{set}).
2. **Fan Speed** (F) to control cooling intensity.

The AC system uses **fuzzy inference rules** to decide the appropriate fan speed.

Problem:

The system evaluates two inputs:

- **Temperature Difference** ($T_{\text{diff}} = T - T_{\text{set}}$) with fuzzy sets:
  - **Low**, **Medium**, **High**.
- **Room Humidity** (H) with fuzzy sets:
  - **Low**, **Normal**, **High**.

The output is the **Fan Speed** (F) with fuzzy sets:

- **Slow**, **Moderate**, **Fast**.

The goal is to determine the fan speed based on fuzzy rules.

Solution Using Fuzzy Inference

Step 1: Define Fuzzy Membership Functions

1. **Temperature Difference** (T_{diff}):
   - Low: $0 \leq T_{\text{diff}} \leq 2$
   - Medium: $1 \leq T_{\text{diff}} \leq 5$
   - High: $4 \leq T_{\text{diff}} \leq 8$
2. **Humidity** (H):
   - Low: $0 \leq H \leq 30\%$
   - Normal: $20\% \leq H \leq 60\%$
   - High: $50\% \leq H \leq 100\%$
3. **Fan Speed** (F):
   - Slow: $0 \leq F \leq 3$
   - Moderate: $2 \leq F \leq 6$
   - Fast: $5 \leq F \leq 10$

Step 2: Fuzzy Rules

Define fuzzy rules based on expert knowledge:

1. **Rule 1**: If T_{diff} is **Low** AND H is **Low**, THEN F is **Slow**.
2. **Rule 2**: If T_{diff} is **Medium** AND H is **Normal**, THEN F is **Moderate**.
3. **Rule 3**: If T_{diff} is **High** OR H is **High**, THEN F is **Fast**.

Step 3: Apply Fuzzification

For example:

- Current room temperature $T = 26^\circ$, setpoint $T_{\text{set}} = 22^\circ$:
  - $T_{\text{diff}} = 4^\circ$ → Membership in fuzzy sets:
    - Low: 0, Medium: 0.5, High: 0.8.
- Current room humidity $H = 50\%$:
  - Membership in fuzzy sets:
    - Low: 0, Normal: 0.6, High: 0.4.

Step 4: Apply Fuzzy Rules

Use fuzzy inference to evaluate each rule:

- **Rule 1**: $\min(0, 0) = 0$ → No activation.
- **Rule 2**: $\min(0.5, 0.6) = 0.5$ → Partial activation for **Moderate**.
- **Rule 3**: $\max(0.8, 0.4) = 0.8$ → Strong activation for **Fast**.

Step 5: Defuzzification

Combine results to determine a crisp output for F:

- Use methods like the **centroid** or **weighted average** to calculate the final fan speed.

For this example:

$$F = (0.5 \cdot \text{Moderate}) + (0.8 \cdot \text{Fast})$$

Suppose this yields a fan speed of $F = 7$ (Fast).

Final Output:

The smart AC system sets the **fan speed to Fast** based on fuzzy rules.

Summary

A **fuzzy system** models decision-making with uncertain information. It consists of **fuzzification**, **fuzzy rules**, **inference**, and **defuzzification**.

- **Fuzzification**: Convert crisp inputs to fuzzy values.
- **Fuzzy Rules**: Apply "IF-THEN" logic rules.
- **Inference**: Determine fuzzy output using logical operators.
- **Defuzzification**: Convert fuzzy output back to a crisp value.

Example: Control the speed of a fan using fuzzy inputs (temperature and humidity) and fuzzy rules.