1 LAPLACE TRANSFORMS

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = \bar{f}(s)\text{; } s = \text{parameter (sufficiently large)}$$

Laplace transform table :

	f (t)	$L f(t) = \bar{f}(s)$
1.	1	$\frac{1}{s}$
2.	e^{at}	$\frac{1}{s-a}$
3.	e^{-at}	$\frac{1}{s+a}$
4.	sin at	$\frac{a}{s^2+a^2}$
5.	cost at	$\frac{s}{s^2+a^2}$
6.	sinh at	$\frac{a}{s^2-a^2}$
7.	cosh at	$\frac{s}{s^2-a^2}$
8.	t^n	$\lceil n+1 / s^{n+1}$

Theorem : 1) Laplace transform is an linear transform.

Statement : If α and β are constants then $L\{\alpha f(t) + \beta g(t)\} = \alpha L\{f(t)\} + \beta L\{g(t)\}$

Problems ;

Type - I (Based on definiction) :

1. Obtain Laplace transform of : **Dec. 99**
   i) $\sin^3 t$
   ii) $\cos^4 t$
   iii) cos t. cos 2t. cos 3 t **May. 2000**
   iv) $\sin^4 t$
   v) $\cos(wt + \alpha)$; α = constant. **Dec. 99**

2. Prove that :$L\{\sin^5 t\} = \left[\frac{5!}{(s^2+1)(s^2+9)(s^2+25)}\right]$ **April. 93**

3. Find Laplace of f (t) defined as :

$$f(t) = \begin{cases} t/k & ; 0 < t < k \\ 1 & ; t > k \end{cases}$$

Ans : $\frac{1 - e^{-ks}}{ks^2}$

4. Find Laplace transform of : $f(t) = \begin{cases} \cos t \ ; & 0 < t < 2\pi \\ 0 & ; t < 2\pi \end{cases}$

Ans : $\frac{s}{s^2 + 1} \ [1 - e^{-2\pi s}]$. April. 93

5. Find Laplace of $f(t) = \begin{cases} 1 & ; 0 \le t \le 1 \\ e^t & ; 1 \le t \le 4 \\ 0 & ; t > 4 \end{cases}$

Ans : $\frac{e^{-s}}{s} + \frac{1}{s} - \frac{e^{-4(s-1)}}{s-1} + \frac{e^{-(s-1)}}{s-1}$ May. 95

6. Find the Laplace transform of $f(t) = \begin{cases} \sin 2t \ ; 0 < t < \pi \\ 0 \qquad ; t \ge \pi \end{cases}$

Ans : $\frac{2}{s^2 + 4} \ [1 - e^{-\pi s}]$. Nov. 90

Type - II : (First Shifting Theorem)

Statement : If $L\{f(t)\} = \bar{f}(s)$ then $L\{e^{-at} f(t)\} = \bar{f}(s+a)$ and $L\{e^{at} f(t)\} = \bar{f}(s-a)$.

7. Obtain Laplace transform of :

i) $L\{t^5 \cosh 4t\}$

Ans : $60\left[\frac{1}{(s-4)^6} + \frac{1}{(s+4)^6}\right]$

ii) $L\{e^{-2t} \cos 2t . \sinh t\}$ Nov. 90

Ans $\frac{1}{2}\left[\frac{s+1}{(s+1)^2+4} - \frac{s+3}{(s+3)^2+4}\right]$

iii) $L\{e^{4t} \ t^{5/2}\}$

Ans : $\frac{\overline{|7/2}}{(s-4)^{7/2}}$

iv) $L\{t+2)^2 \ e^t\}$.

Ans : $\frac{2}{(s-1)^3} - \frac{4}{(s-1)^2} + \frac{4}{(s-1)}$ April 90

v) $L\left\{\left(\frac{\cos t + \sin t}{e^t}\right)^2\right\}$

Ans : $\frac{1}{s+2} + \frac{2}{(s+2)^2+4}$ April 92

vi) $L\{\sin 2t.\ \sin 4t.\ \sinh t.\}$

Ans : $\frac{1}{4}\cdot\left[\frac{(s-1)}{(s-1)^2+4}-\frac{(s-1)}{(s-1)^2+36}-\frac{(s+1)}{(s+1)^2+4}+\frac{(s+1)}{(s+1)^2+36}\right]$ May. 95

vii) $L\{\sinh at.\ \cos t\ at\}$

Ans : $\frac{1}{2}\left[\frac{(s-a)}{(s-a)^2+a^2}-\frac{(s+a)}{(s+a)^2+a^2}\right]$ April. 90

viii) $L\{\cosh at.\ \cos t\ at\}$

Ans : $\frac{1}{2}\left[\frac{(s-a)}{(s-a)^2+a^2}+\frac{(s+a)}{(s+a)^2+a^2}\right]$ April. 91

ix) $L\left\{\left(\frac{\cos 2t\ \sin t}{e^t}\right)\right\}$

Ans : $\frac{1}{2}\left[\frac{3}{(s+1)^2+9}-\frac{1}{(s+1)^2+1}\right]$ Dec. 93

x) $L\{(1+t\,e^{-t})^3\}$

Ans : $\frac{1}{s}+\frac{3}{(s+1)^2}+\frac{6}{(s+2)^3}+\frac{6}{(s+3)^3}$ May. 97

xi) $L\{\sin 2t.\ \cos t.\ \cosh 2t.\}$

Ans : $\frac{1}{4}\left[\frac{3}{(s-2)^2+9}\quad\frac{3}{(s+2)^2+9}+\frac{1}{(s-2)^2+1}+\frac{1}{(s+2)^2+1}\right]$ May. 94

xii) Prove that : $L\left\{\sinh\frac{t}{2}\sin\frac{\sqrt{3}}{2}t\right\}=\frac{\sqrt{3}}{2}\left(\frac{2}{s^4+s^2+1}\right)$ April. 93

xiii) $L\{\cosh^2 4t.\}$

Ans : $\left[\frac{1}{s-8}+\frac{1}{s+8}+\frac{2}{s}\right]\frac{1}{4}$.

xiv) $L\{t\,e^{3t}\ \cosh 4t.\}$ Nov. 98

Ans : $\frac{1}{2}\left[\frac{1}{(s-7)^2}+\frac{1}{(s+1)^2}\right]$

xv) $L\{t^2 \sinh 4t.\}$

Ans : $\dfrac{1}{(s-4)^3} - \dfrac{1}{(s+4)^3}$ May. 95

xvi) $L\{e^{-t} \sin^2 t.\}$

Ans : $\dfrac{2}{(s+1)(s^2+2s+5)}$

Type - III (Change of Scale property)

Statement : If $L\{f(t)\} = \bar{f}(s)$ then

$$L\{f(at)\} = \frac{1}{a}\,\bar{f}\left(\frac{s}{a}\right)$$

8. Obtain Laplace transform of the following :

i) If $L\{J_0(t)\} = \dfrac{1}{\sqrt{1+s^2}}$; find $L\{J_0(3t)\}$. **Ans :** $\dfrac{1}{\sqrt{s^2+9}}$

ii) If $L\{f(t)\} = \dfrac{8+12s-2s^2}{(s^2+4^2)}$; find $L\,f(2t)$. **Ans :** $\dfrac{4(16+12s-s^2)}{(s^2+16)^2}$

iii) If $L\{(\cos t)\} = \dfrac{s}{s^2+1}$; find $L\{\cos 4t\}$ **Ans :** $\dfrac{s}{(s^2+16)}$

Type : IV (Effect of Multiplication by t)

Statement : If $L\{f(t)\} = \bar{f}(s)$ then $L\{t^n f(t)\} = (-1)^n \dfrac{d^n}{ds^n}\{\bar{f}(s)\}$. May 2000

9. Obtain Laplace transform of the following :

i) $L\{t \sin^2 t)\}$

Ans : $\dfrac{1}{2}\left[\dfrac{1}{s^2} + \dfrac{4-s^2}{(s^2+4)^2}\right]$ May. 97

ii) $L\{t\sqrt{1+\sin t}\}$

Ans : $4\left[\dfrac{4s^2+4s-1}{(4s^2+1)^2}\right]$ Dec. 93

iii) $L\{t \sin 2t \cosh t\}$

Ans : $\dfrac{2(s-1)}{[(s-1)^2+4]^2} + \dfrac{2(s+1)}{[(s+1)^2+4]^2}$ Dec. 94

iv) $L\{t \sin^3 t\}$

Ans : $\dfrac{3s}{2(s^2-1)^2} - \dfrac{3s}{2(s^2+9)^2}$ May. 94

v) $L\{t e^{at} \sin a t\}$

Ans : $\dfrac{2a(s-a)}{[(s-a)^2+a^2]^2}$ **April.90**

vi) $L\{t(3\sin 2t - 2\cos 2t\}$

Ans : $-\left[\dfrac{2s^2-12s-8}{(s^2+4)^2}\right]$ **Dec. 91**

vii) $L\{t\cos^2 t.\}$

Ans : $\dfrac{1}{2}\left[\dfrac{1}{s^2}+\dfrac{s^2-4}{(s^2+4)^2}\right]$ **Dec. 93**

viii) $L\{t^3 e^{-3t}\}$

Ans : $\dfrac{1}{(s+3)^4}$

ix) $L\{t^2 \cos a t.\}$

Ans : $\dfrac{2s(s^2-3a^2)}{(s^2-a^2)^3}$

Type V (Effect of division by t)

Statement : If $L\{f(t)\}=\bar{f}(s)$ then $L\left\{\dfrac{f(t)}{t}\right\}=\int_s^{\infty}\bar{f}(s)\,ds$, if $\lim\limits_{t\to 0^+}\dfrac{f(t)}{t}$ exists. **Dec. 98**

10. Obtain Laplace transform of the following :

i) Find $L\left\{\dfrac{\sin t}{t}\right\}$ and hence show that $\int_0^{\infty}\dfrac{\sin t}{t}\,dt=\dfrac{\pi}{2}$ **Ans :** $\cot^{-1} s$. **May. 94**

ii) $L\left\{\dfrac{\cos a t - \cos b t}{t}\right\}$ **Ans :** $\dfrac{1}{2}\log\left(\dfrac{s^2+b^2}{s^2+a^2}\right)$ **May. 96**

iii) $L\left\{\dfrac{\cos 6t - \cos 4t}{t}\right\}$ **Ans :** $\dfrac{1}{2}\log\left(\dfrac{s^2+16}{s^2+36}\right)$ **Nov. 91**

iv) $L\left\{\dfrac{e^{-at}-e^{-bt}}{t}\right\}$ **Ans :** $\log\left(\dfrac{s+b}{s+a}\right)$ **Nov. 92**

v) $L\left\{\dfrac{1-e^{t}}{t}\right\}$ **Ans :** $\log\left(\dfrac{s-1}{s}\right)$ **April. 91**

vi) $L\left\{\dfrac{\sinh t}{t}\right\}$ **Ans :** $\dfrac{1}{2}\log\left(\dfrac{s+1}{s-1}\right)$

vii) $L\left\{\frac{\sin^2 2t}{t}\right\}$ Ans : $\frac{1}{4}\log\left(\frac{s^2+16}{s^2}\right)$ Dec. 93

viii) $L\{e^{-2t}t^{-1}\sin 3t\}$ Ans : $\cot^{-1}\left(\frac{s+2}{3}\right)$

ix) $L\left\{\frac{1-\cos t}{t^2}\right\}$ Ans : $\frac{\pi}{2}+\tan^{-1}s-\frac{s}{2}\log\left(\frac{s^2+1}{s^2}\right)$ Dec. 93

x) $L\left\{\frac{\sin 2t}{t}\right\}$ Ans : $\cot^{-1}(s/2)$

xi) $L\left\{\frac{e^{-2t}\sin 2t\cosh t}{t}\right\}$ Ans : $\frac{1}{2}\left\{\cot^{-1}\left(\frac{s+1}{2}\right)+\cot^{-1}\left(\frac{s+3}{2}\right)\right\}$ May. 2000

xii) $L\{t^{-1}\cosh 2t\sin 2t\}$ Ans : $\frac{1}{2}\left\{\cot^{-1}\left(\frac{s-2}{2}\right)+\cot^{-1}\left(\frac{s+2}{2}\right)\right\}$ Dec. 99

Type VI (Laplace transform of derivative)

Statement : If $L\{f(t)\}=\bar{f}(s)$ then

$$L\left\{\frac{d}{dt}f(t)\right\}=s\bar{f}(s)-f(0)$$

$$L\left\{\frac{d^2}{dt^2}f(t)\right\}=s^2\bar{f}(s)-sf(0)-f'(0)$$

$$L\left\{\frac{d^3}{dt^3}f(t)\right\}=s^3\bar{f}(s)-s^2f(0)-sf'(0)-f''(0)$$ and so on

11.

i) Find $L\left\{\frac{d}{dt}\left(\frac{\sin t}{t}\right)\right\}$ Ans : $s\cot^{-1}s-1$

ii) Find Laplace transform of $\frac{d^2y}{dt^2}-3\frac{dy}{dt}+5y$; $y(0)=2$ and $y'(0)=4$.

Ans : $(s^2-3s+5)\bar{y}(s)-2s+10$

Type VII (Laplace transform of integral)

Statement : If $L\{f(t)\}=\bar{f}(s)$ then $L\left[\int_0^t f(u)\,du\right]=\frac{1}{s}\bar{f}(s)$

12. Find Laplace transform of the following : April. 91

i) $L\left\{\int_0^t e^{-2u}u^3\,du\right\}$ Ans : $\frac{6}{s(s+2)^4}$

ii) $L\left\{\int_0^t \frac{\sin u}{u}\, d u\right\}$ **Ans :** $\frac{\cot^{-1} s}{s}$ **May. 97**

iii) $L\left\{\int_0^t u \cos^2 u\, d u\right\}$ **Ans :** $\frac{1}{2s^3} - \frac{1}{2s}\,\frac{(4 - s^2)}{(s^2+4)^2}$ **Dec. 94**

iv) $L\left\{\int_0^t e^{-t}\, \frac{\sin u}{u}\, d t\right\}$ **Ans :** $\frac{1}{s}\cot^{-1}(s+1)$

v) $L\left\{t \int_0^t \frac{\sin u}{u}\, d u\right\}$ **Ans :** $\frac{1}{s(s^2+1)} + \frac{1}{s^2}\cot^{-1} s$

vi) $L\left\{\int_0^t u \cosh u\, d u\right\}$ **Ans :** $\frac{1}{s}\;\frac{s^2+1}{(s^2-1)^2}$

vii) $L\left\{\cosh t \int_0^t e^t \cosh t\, d t\right\}$ **Ans :** $\frac{1}{2}\left[\frac{s-2}{(s-1)^2\,(s-3)}\right]+\frac{1}{2}\left[\frac{s}{(s+1)^2\,(s-1)}\right]$

viii) $L\left\{e^{-2t} \int_0^t t \sin 3t\, d t\right\}$ **Ans :** L $\frac{6}{(s^2+4s+13)^2}$

ix) $L\left\{\int_0^t t e^{-3t} \sin 4t\, d t\right\}$ **Ans :** $\frac{8(s+3)}{s(s^2+6s+25)^2}$

x) $L\left\{\int_0^t x \cosh x\, d x\right\}$ **Ans :** $\frac{s^2+1}{s(s^2-1)^2}$

Type VIII (Evaluation of Integrals using Laplace transform)

13. By using Laplace Transform Prove that :

i) $\int_0^\infty e^{-3t} \sin 2t\, d t = \frac{2}{13}$

ii) $\int_0^\infty e^{-3t}\, t \sin^2 t\, d t = \frac{62}{1521}$ **Nov. 92**

iii) $\int_0^\infty \frac{\sin 2t + \sin 3t}{t e^t}\, d t = \frac{3\pi}{4}$ **Dec. 94**

iv) $\int_0^{\infty} e^{-2t} \sin^3 t \, dt = \frac{6}{65}$ Nov. 95

v) $\int_0^{\infty} \frac{e^{-\sqrt{2}t} \sinh t \sin t}{t} \, dt = \frac{\pi}{8}$ April 93, May. 97.

vi) $\int_0^{\infty} \frac{\cos 6t - \cos 4t}{t} \log = (2/3).$ April. 91

vii) $\int_0^{\infty} e^{-3t} \cos^2 2t \, dt = \frac{17}{75}$ Dec. 93

viii) $\int_0^{\infty} \frac{t \sin 3t}{e^{2t}} \, dt = \frac{12}{169}$ Nov. 90, April. 92

ix) $\int_0^{\infty} t^3 e^{-t} \sin t \, dt = 0$

x) $\int_0^{\infty} e^{-2t} \frac{\sinh t \sin t}{t} \, dt = \frac{1}{2} \tan^{-1}\left(\frac{1}{2}\right)$

xi) $\int_0^{\infty} e^{-t} \frac{\sin^2 t}{t} \, dt = \frac{1}{4} \log 5$ Dec. 99

Type IX (Typical problems)

1. Prove that $L\{\sin\sqrt{t}\} = \frac{\sqrt{\pi}}{2s^{3/2}} e^{-1/4s}$ May. 96

2. Prove that $L\left\{\frac{\cos\sqrt{t}}{\sqrt{t}}\right\} = \sqrt{\frac{\pi}{s}} \, e^{-\frac{1}{4s}}$ May. 99

3. Show that $L\{\text{erf}\sqrt{t}\} = \frac{1}{s\sqrt{s+1}}$ and find $L\{\text{erf } 2\sqrt{t}\}$

Ans : $\frac{2}{s\sqrt{(s+4)}}$

4. Given that : $L\left\{2\sqrt{\frac{t}{\pi}}\right\} = \frac{1}{s^{3/2}}$ then show that $L\left\{\frac{1}{\sqrt{\pi t}}\right\} = \frac{1}{\sqrt{s}}$

5. Given that : $L\{J_0(t)\} = \frac{1}{\sqrt{s^2+1}}$ then show that $\int_0^{\infty} t e^{-3t} J_0(4t) \, dt = 3/125$ May. 2000

6. Prove that : $L\left\{ \operatorname{erf}_c (\sqrt{t}) \right\} = \dfrac{1}{\sqrt{s+1}\ (\sqrt{s+1}+1)}$

7. Show that : $L\left\{ \operatorname{erf}\ (t) \right\} = e^{s^2/4}\ \operatorname{erf}_c \left(\dfrac{s}{2}\right) \dfrac{1}{s}$

8. Show that : $L\left\{ e^{-2t} \operatorname{erf} \sqrt{t} \right\} = \dfrac{1}{(s+2)\sqrt{s+3}}$ Dec. 99,May. 2000

Type X (Second Shifting Property)

Statement : If $L\ \left\{ f(t) \right\} = \bar{f}(s)$ and $g(t) = 0 \quad ; t < a$

$= f(t-a)\ ;\ t > o.$ May. 99

then $L\left\{ g(t) \right\} = e^{-as} \bar{f}(s) = e^{-as} L\left\{ f(t) \right\}$

1. Find $Lf(t)$: if

$$f(t) = (t-1)^4 ; t < 1$$
$$= 0 \quad ; t < 1$$

Ans : $e^{-s} \dfrac{24}{s^5}$

INVERSE LAPLACE TRANSFORM

Inverse Laplace transform table :

	$\bar{f}(s)$	$L^{-1}\bar{f}(s) = f(t)$
1.	$\frac{1}{s-a}$	e^{at}
2.	$\frac{1}{s+a}$	e^{-at}
3.	$\frac{1}{s}$	1
4.	$\frac{1}{s^n}$	$\frac{t^{n-1}}{\lceil n}$
5.	$\frac{1}{s^2+a^2}$	$\frac{1}{a}\sin at$
6.	$\frac{s}{s^2+a^2}$	$\cos at$
7.	$\frac{1}{s^2-a^2}$	$\frac{1}{a}\sinh at$
8.	$\frac{s}{s^2-a^2}$	$\cosh at$

Problems :

Type - I (From Table)

1. Find

i) $L^{-1}\left\{\dfrac{1}{s^2+9}\right\}$ Ans : $\dfrac{1}{3}\sin 3t$

ii) $L^{-1}\left\{\dfrac{5}{3s-1}\right\}$ Ans : $\dfrac{5}{3}e^{t/3}$

iii) $L^{-1}\left\{\dfrac{7s}{s^2+4}\right\}$ Ans : $7\cos 2t$

iv) $L^{-1}\left\{\dfrac{s^2-3s+4}{s^3}\right\}$ Ans : $2t^2-3t+1$

v) $L^{-1}\left\{\dfrac{3s+4\sqrt{3}}{s^2+7}\right\}$ Ans : $3\cos\sqrt{7}\,t+\dfrac{4\sqrt{3}}{\sqrt{7}}\sin\sqrt{7}\,t.$

vi) $L^{-1}\left\{\dfrac{4s+15}{16s^2-25}\right\}$ Ans : $\dfrac{1}{4}\left[\cosh\dfrac{5t}{4}+3\sinh\dfrac{5t}{4}\right]$

Type II :

Theorem : If $L^{-1}\{\bar{f}(s)\}=f(t)$ then $L^{-1}\{\bar{f}(s-a)\}=e^{at}L^{-1}\{\bar{f}(s)\}$ and $L^{-1}\{\bar{f}(s+a)\}=e^{-at}L^{-1}\{\bar{f}(s)\}$.

2. Determine :

i) $L^{-1}\left\{\dfrac{2s+2}{s^2+2s+10}\right\}$ Ans : $2e^{-t}\cos 3t.$

ii) $L^{-1}\left\{\dfrac{s}{(s-3)^5}\right\}$ Ans : $e^{3t}.\,t^3\left(\dfrac{1}{6}+\dfrac{t}{8}\right)$

iii) $L^{-1}\left\{\dfrac{3}{s^2-4s+13}\right\}$ Ans : $e^{2t}\sin 3t$

iv) $L^{-1}\left\{\dfrac{2}{(s-3)^5}\right\}$ Ans : $\dfrac{e^{3t}t^4}{12}$

v) $L^{-1}\left\{\dfrac{2s+3}{s^2+2s+2}\right\}$ Ans : $2e^{-t}\cos t+e^{-t}\sin t.$ May. 94

vi) $L^{-1}\left\{\dfrac{3s+1}{(s+1)^4}\right\}$ Ans : $\dfrac{3}{2}e^{-t}t^2-e^{-t}\dfrac{t^3}{3}$

vii) $L^{-1}\left\{\dfrac{s+7}{s^2+2s+2}\right\}$ Ans : $e^{-t}(\cos t+6\sin t)$

Type III :

Theorem: If $L^{-1}\{\bar{f}(s)\}=f(t)$ then $L^{-1}\left(\frac{d}{ds}\bar{f}(s)\right)=-t\,f(t)$.

3. Determine :

i) $L^{-1}\left\{\log\left(\frac{s+a}{s+b}\right)\right\}$ **Ans :** $\frac{e^{-bt}-e^{-at}}{t}$ **May. 96**

ii) $L^{-1}\left\{\log\left(\frac{s^2+b^2}{s^2+a^2}\right)\right\}$ **Ans :** $2\left[\frac{\cos at-\cos bt}{t}\right]$ **Nov. 92**

iii) Prove that : $L^{-1}\left\{\frac{1}{2}\log\left(\frac{r^2+b^2}{r^2+a^2}\right)\right\} = \frac{\cos at-\cos bt}{t}$

iv) $L^{-1}\left\{\log\left(\frac{2s+1}{2s+3}\right)\right\}$ **Ans :** $\frac{e^{-\frac{3}{2}t}-e^{-\frac{1}{2}t}}{t}$ **May. 95**

v) $L^{-1}\left\{\log\left(1+\frac{1}{s^2}\right)\right\}$ **Ans :** $\frac{2(1-\cos t)}{t}$

vi) $L^{-1}\left\{\log\sqrt{\frac{s^2+a^2}{s^2}}\right\}$ **Ans :** $\frac{(1-\cos at)}{t}$

vii) $L^{-1}\left\{\log\sqrt{\frac{s-1}{s+1}}\right\}$ **Ans :** $-\frac{1}{t}\sinh t.$

viii) $L^{-1}\left\{\log\left(\frac{s-m}{s-n}\right)\right\}$ **Ans :** $\frac{e^{nt}-e^{mt}}{t}$ **Dec. 93**

4. Determine :

i) $L^{-1}\{\cot^{-1}(s)\}$ **Ans :** $\frac{\sin t}{t}$

ii) $L^{-1}\left\{\tan^{-1}\left(\frac{a}{s}\right)\right\}$ **Ans :** $\frac{\sin at}{t}$

iii) $L^{-1}\{\tan^{-1}(s+2)\}$ **Ans :** $-e^{-2t}\frac{\sin t}{t}$

Type IV :

Theorem : If $L^{-1}\{f(s)\} = \bar{f}(t)$ then $L^{-1}\left\{\int_{s}^{\infty} \bar{f}(s)\, ds\right\} = \frac{f(t)}{t}$

5. Determine :

i) $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ **Ans :** $\frac{t \sin a t}{2a}$

ii) $L^{-1}\left\{\frac{2s+1}{(s^2+s+1)^2}\right\}$ **Ans :** $\frac{2}{\sqrt{3}}\, t e^{-t/2} \sin\left(\frac{\sqrt{3}}{2} t\right)$

Type : V

Theorem : If $L^{-1}\{\bar{f}(s)\} = f(t)$ then $L^{-1}\left\{\frac{\bar{f}(s)}{s}\right\} = \int_0^t f(t)\, dt.$

6 Find :

i) $L^{-1}\left\{\frac{1}{s} \log\left(\frac{s^2+a^2}{s^2+b^2}\right)\right\}$ **Ans :** $\int_0^t \frac{2(\cos bt - \cos at)}{t}\, dt.$

ii) Show that : $L^{-1}\left\{\frac{1}{s} \log\left(1 + \frac{1}{s^2}\right)\right\} = \int_0^t \frac{2(1-\cos x)}{x}\, dx$ **May. 2000**

Type : VI :

Convolution Theorem :

If $L^{-1}\{\bar{f}(s)\} = f(t)$ and $L^{-1}\{\bar{g}(s)\} = g(t)$ then :

$$L^{-1}\{\bar{f}(s).\bar{g}\{s)\} = \int_0^t f(u)\, g(t-u)\, du = \int_0^t f(t-u)\, g(u)\, du.$$

7. Apply convolution theorem to evaluate the following :

i) $L^{-1}\left\{\frac{s}{(s^2+a^2)^2}\right\}$ **Ans :** $\frac{t}{2a} \sin at.$

ii) $L^{-1}\left\{\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right\}$ **Ans :** $\frac{a \sin at - b \sin bt}{a^2 - b^2}$ **Dec. 98**

iii) $L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\}$ **Ans :** $\frac{1}{2a}(\sin at + at \cos at)$

iv) $L^{-1}\left\{\dfrac{s^2}{s^2(s^2+1^2)^2}\right\}$ Ans : $te^{-t}+2e^{-2t}+t-2$

v) $L^{-1}\left\{\dfrac{2}{(s^2+a^2)^2}\right\}$ Ans : $\dfrac{1}{a^3}(\sin at - at\cos at)$ Nov. 91

vi) $L^{-1}\left\{\dfrac{1}{(s-2)(s+2)^2}\right\}$ Ans : $\dfrac{1}{16}(e^{2t}-e^{-2t}-4te^{-2t})$. May. 94

vii) $L^{-1}\left\{\dfrac{s}{(s^2+4)(s^2+1)}\right\}$ Ans : $\dfrac{1}{3}\left[\cos t-\cos 2t\right]$ Dec. 94

viii) $L^{-1}\left\{\dfrac{1}{(s-4)^4\,(s+3)}\right\}$

Ans : $\dfrac{e^{4t}}{6\,.\,7^4}\left[7^3t^3-7^2\,.\,3t^2+42t+6e^{-7t}-6\right]$ Dec. 93, May. 98, Dec. 99

Type : VII :

Using partial fraction :

8. Determine :

i) $L^{-1}\left\{\dfrac{2s^2-6s+5}{s^3-6s^2+11s-6}\right\}$ Ans : $\dfrac{1}{2}e^{t}-e^{2t}+\dfrac{5}{2}e^{3t}$

ii) $L^{-1}\left\{\dfrac{9s^2+4s-10}{s(s-1)(s+2)}\right\}$ Ans : $5+e^{t}+3e^{-2t}$

iii) $L^{-1}\left\{\dfrac{3s^3+s^2+12s+2}{(s-3)(s+1)^3}\right\}$ Ans : $2e^{3t}+e^{-t}-4te^{-t}+\dfrac{3}{2}t^2e^{-t}$.

iv) $L^{-1}\left\{\dfrac{7s+13}{s(s^2+4s+13)}\right\}$ Ans : $1-e^{-2t}\cos 3t+\dfrac{5}{3}e^{-2t}\sin 3t$

v) $L^{-1}\left\{\dfrac{s^2+2s+3}{(s^2+2s+2)(s^2+2s+5)}\right\}$ Ans : $\dfrac{e^{-t}}{3}(\sin t+\sin 2t)$ May. 2000

vi) $L^{-1}\left\{\dfrac{s}{(s^2+1)(s^2+4)}\right\}$ Ans : $\dfrac{1}{3}(\cos t-\cos 2t)$

vii) $L^{-1}\left\{\frac{1}{s^3+a^3}\right\}$

Ans : $\frac{e^{-at}}{3a^2} - \frac{e^{at/2}}{3a^2}\cos\left(\frac{\sqrt{3}}{2}at\right) + \frac{e^{at/2}}{\sqrt{3}a^2}\sin\left(\frac{\sqrt{3}}{2}at\right)$

viii) $L^{-1}\left\{\frac{s}{s^4+s^2+1}\right\}$. **Ans :** $\frac{2}{\sqrt{3}}\sin h\left(\frac{t}{2}\right)\ \sin\left(\frac{\sqrt{3}}{2}t\right)$

ix) $L^{-1}\left\{\frac{1}{s^4+s^2+1}\right\}$ **Ans :** $-\cos\left(\frac{\sqrt{3}}{2}t\right)\sinh\left(\frac{t}{2}\right) + \frac{1}{\sqrt{3}}\sin\left(\frac{\sqrt{3}}{2}t\right)\cosh\left(\frac{t}{2}\right)$

Type : VIII :

Second Shifting Theorem :

if $L^{-1}\{\bar{f}(s)\} = f(t)$ then

$L^{-1}\{e^{-as}\bar{f}(s)\} = 0 \qquad ; t < a$

$\qquad = f(t-a) \qquad ; t \geq a$

9. Determine $L^{-1}\left\{\frac{e^{-3s}}{(s-2)^4}\right\}$

Ans : $\frac{1}{6}\frac{(t-3)^3}{6}\ e^{2(t-3)}$; $t > 3$. Otherwise 0 when $t < 3$

10. Find $L^{-1}\left\{\frac{e^{-3s}}{(s+4)^3}\right\}$ **May. 2000**

Inverse Laplace Transform Theorem Table :

	$\bar{f}(s)$	$L^{-1}\bar{f}(s) = f(t)$
1.	$\bar{f}(s-a)$	$e^{at} f(t)$
2.	$\bar{f}(s+a)$	$e^{-at} f(t)$
3.	$e^{-as}\bar{f}(s)$	$= f(t-a)\ ;\ t \geq a$ $= 0\ \ \ \ ;\ t < a$
4.	$\bar{f}(s/a)$	$a f(at)$
5.	$(-1)^n \dfrac{d^n}{ds^n} \bar{f}(s)$	$t^n f(t)$
6.	$\dfrac{1}{s}\bar{f}(s)$	$\int_0^t f(t)\, dt$
7.	$\int_0^\infty \bar{f}(s)\, ds$	$\dfrac{1}{t} f(t)$
8.	$\bar{f}(s)\,\bar{g}(s)$	$\int_0^t f(u)\, g(t-u)\, du$

Miscellaneous Problems :

10. Find Inverse Laplace transform of the following :

i) $L^{-1}\left\{\dfrac{s}{(s+1)(s^2+4)}\right\}$

Ans : $\dfrac{-1}{5}\, e^{-t} + \dfrac{1}{5}\cos 2t + \dfrac{2}{5}\sin 2t$ **April. 90**

ii) $L^{-1}\left\{\dfrac{s^4}{(s^2+a^2)\,(s^2+b^2)\, s^2+c^2)}\right\}$

Ans : $\dfrac{a^3 \sin at}{(b^2-a^2)\,(c^2-a^2)} + \dfrac{b^3 \sin bt}{(c^2-b^2)\,(a^2-b^2)} + \dfrac{c^3 \sin ct}{(a^2-c^2)\,(b^2-c^2)}$

iii) $L^{-1}\left\{\dfrac{4s+12}{s^2+8s+12}\right\}$ **Ans :** $3\,e^{-6t} + e^{-2t}$ **Nov. 91, Dec. 99**

iv) $L^{-1}\left\{\dfrac{3s+7}{s^2-2s-3}\right\}$ **Ans :** $4\,e^{3t} - e^{-t}$ **April. 91**

v) $L^{-1}\left\{\dfrac{s+2}{s(s+1)\,(s+3)}\right\}$ **Ans :** $\dfrac{2}{3} - \dfrac{1}{2}\, e^{-t} - \dfrac{1}{6}\, e^{-3t}$ **April. 91**

vi) $L^{-1}\left\{\dfrac{2s^2-1}{(s^2+1)\,(s^2+4)}\right\}$ **Ans :** $-\sin t + \dfrac{3}{2}\sin 2t$ **April. 90**

vii) $L^{-1}\left\{\dfrac{5s-2}{3s^2+4s+8}\right\}$

Ans : $\dfrac{5}{3}\, e^{-\frac{2t}{3}} \cos \dfrac{\sqrt{20}}{3}\, t - \dfrac{8}{3\sqrt{5}}\, e^{-\frac{2t}{3}} \sin \dfrac{\sqrt{20}}{3}\, t$ **Nov. 90**

viii) $L^{-1}\left\{\dfrac{2}{(s+1)^2(s^2+4)}\right\}$ **Nov. 91**

Ans: $\dfrac{1}{25}\left[4e^{-t} + 10\,t\,e^{-t} - 4\cos 2t - 3\sin 2t\right]$

ix) $L^{-1}\left\{\frac{1}{s^3(1-s)}\right\}$ Ans : $\frac{t^2}{2}+t+1-e^t$ April. 93

x) $L^{-1}\left\{\frac{1}{s^3(s^2+1)}\right\}$ Ans : $\frac{t^2}{2}-1+\cos t$ May. 94

xi) $L^{-1}\left\{\frac{1}{s(s+1)^3}\right\}$ Ans : $1-e^{-t}\left\{1+t+\frac{t^2}{2}\right\}$

xii) $L^{-1}\left\{\frac{s}{(s+2)^2(s+1)}\right\}$ Ans : $e^{-2t}+2e^{-2t}t-e^{-t}$ April.91

xiii) $L^{-1}\left\{\frac{s+2}{s^2-4s+7}\right\}$ Ans : $e^{2t}\cos(\sqrt{3}t)+\frac{4}{\sqrt{3}}e^{2t}\sin\sqrt{3}t.$ April.93

xiv) $L^{-1}\left\{\frac{s}{s^4+4a^4}\right\}$ Ans : $\frac{1}{2a^2}$ sin at sinh at

xv) $L^{-1}\left\{\frac{1}{s^4+4a^4}\right\}$ Ans : $\frac{1}{4a^3}$ sin at cosh at $+\frac{1}{4a^3}$ cos at sinh at.

xvi) $L^{-1}\left\{\frac{1}{(s^2+4s++13)^2}\right\}$ Ans : $\frac{1}{54}e^{-2t}[\sin 3t-3t\cos 3t]$. April.92

xvii) $L^{-1}\left\{\tan^{-1}\frac{2}{s^2}\right\}$ Ans : $\frac{2\sin t\sinh t}{t}$ Dec.94,97, 98

xviii) $L^{-1}\left\{\tan^{-1}(s^2+2)\right\}$ Ans : April.90

xix) $L^{-1}\left\{\frac{s^2}{(s^2-a^2)^2}\right\}$ Ans : $\frac{1}{2a}$(sinh at + at cosh at)

XX) $L^{-1}\left\{\frac{(s+2)^2}{(s^2+4s+8)^2}\right\}$ Ans : $\frac{e^{-2t}}{4}[2t\cos 2t+\sin 2t]$

xxi) $L^{-1}\left\{\frac{1}{\sqrt{1+s^2}}\right\}$ Ans : $1-\frac{t^2}{4}+\frac{t^4}{64}$ .....

xxii) $L^{-1}\left\{\frac{e^{4-3s}}{(s+4)^{5/2}}\right\}$ Ans : $\left\{e^4\frac{4}{\sqrt{3\pi}}(t-3)^{3/2}e^{-4(t-3)}\right\}$: t > 2 otherwise 0.

May.97

xxiii) $L^{-1}\left\{\frac{1}{2}\log\left(1-\frac{a^2}{s^2}\right)\right\}$ Ans : $\frac{1-\cosh at}{t}$ Dec. 99

xxiv) $L^{-1}\left\{\frac{s+29}{(s+4)(s^2+9)}\right\}$ Ans : $e^{-4t} - \cos 3t + \frac{5}{3}\sin 3t$

May. 99

Linear Differential Equations With Constant Coefficients Using Laplace Transform Method

Note : $Ly(t) = \bar{y}(s)$ and $L^{-1}\{\bar{y}(s)\} = y(t)$.

$$L\left\{\frac{dy}{dt}\right\} = s\,\bar{y}(s) - y(o)$$

$$L\left\{\frac{d^2y}{dt^2}\right\} = s^2\bar{y}(s) - sy(0) - y'(0)$$

$$L\left\{\frac{d^3y}{dt^3}\right\} = s^3\bar{y}(s) - s^2y(0) - sy'(0) - y''(0)$$

$$L\left\{\frac{d^ny}{dt^n}\right\} = s^n\bar{y}(s) - s^{n-1}y(0) - s^{n-2}y'(0) - \dots - y^{n-1}(0).$$

Problems :

Using Laplace transform solve the following differential equations :

1) $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t}\sin t$ with $y(0) = 0$, $y'(0) = 0$

Ans : $\frac{1}{3}e^{-t}\left[\sin t - \frac{\sin 2t}{2}\right]$. April 91, Nov. 95

2) $\frac{dy}{dt} + 2y + \int_0^t y\,dt = \sin t$ when $y(0) = 1$

Ans : $y = e^{-t} - \frac{3}{2}te^{-t} + \frac{1}{2}\sin t$

3) $\frac{d^2y}{dt^2} + 9y = 18t$ where $y(0) = 0$ and $y(\pi/2) = 0$.

Ans : $y = 2t + \pi\sin 3t$.

4) $\frac{d^2y}{dt^2} + \frac{dy}{dt} = t^2 + 2t$; given $y = 4$ and $\frac{dy}{dt} = -2$ at $t = 0$.

Ans : $y = \frac{t^3}{3} + 2e^{-t} + 2.$

5) $D^2 + 4D + 8y = 1$ given $y(0) = 0$, $y'(0) = 1$, $D = \frac{d}{dt}$

Ans : $y = \frac{1}{8}[1 - e^{-2t}\cos 2t + 3e^{-2t}\sin 2t].$

6) $\frac{d^2y}{dt^2} + 9y = \cos 2t$, $y(0) = 0$, $y(\pi/2) = -1.$

Ans : $y = \frac{4}{5}\sin 3t - \frac{1}{5}\cos 3t + \frac{1}{5}\cos 2t$

7) $(D^2 - D - 2)X = 20\sin 2t$ given $x = -1$, $DX = 2$ at $t = 0$, $D = \frac{d}{dt}$.

Ans : $x(t) = 2e^{2t} - 4e^{-t} + \cos 2t - 3\sin 2t.$

8) $\frac{d^3y}{dt^3} - 3\frac{d^2y}{dt^2} + 3\frac{dy}{dt} - y = t^2e^t$ when $y(0) = 0$, $y'(0) = 0$, $y''(0) = -2$

Ans : $y = -e^t t^2 + \frac{e^t t^5}{60}$ Nov. 90, Dec. 93

9) $(D^3 - 2D^2 + 5D)y = 0$ with $y(0) = 0$, $y'(0) = 0$, $y''(0) = 1$, $D = \frac{d}{dt}$

Ans : $y = \frac{1}{5} - \frac{1}{5}e^t\cos 2t + \frac{1}{10}e^t\sin 2t.$

10) $(D + 1)^2 y = \sin t$ with $y = \frac{dy}{dt} = 1$ at $t = 0$

Ans : $y = \frac{5}{2}te^{-t} + \frac{3}{2}e^{-t} - \frac{1}{2}\cos t.$

11) $\frac{d^2x}{dt^2} + X = 6\cos 2t$ with $X = 3$, $\frac{dx}{dt} = 1$ at $t = 0.$

Ans : $x = 5\cos t + \sin t - 2\cos 2t.$

12) $\frac{d^2y}{dt^2} - \frac{dy}{dt} - 6y = 2$ with $y(0) = 1$, $y'(0) = 0.$

Ans : $y = \frac{-1}{3} + \frac{8}{15}e^{3t} + \frac{4}{5}e^{-2t}$

13) $\frac{dx}{dt} + X = \sin\omega t$; $X(0) = 2.$

Ans :

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14) $\frac{d^2y}{dt^2} + \frac{dy}{dt} = t$ with $y(0) = 1$, $y'(0) = 0$.

Ans : $y(t) = 2 - t + t^2/2 - e^{-t}$.

15) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + y = 1$ with $y(0) = 1$, $y'(0) = 1$.

Ans :

16) $\frac{d^2y}{dt^2} + \frac{dy}{dt} - 2y = t$ with $y(0) = 1$, $y'(0) = 0$.

Ans :

17) $x''(t) + 4x(t) = 9t$ with $x(0) = 0$, $x'(0) = 7$.

Ans :

$x(t) = \frac{9}{4}t + \frac{19}{8}\sin 2t$.

18) $(D^2 - 2D + 1)x = e^t$ with $x(t) = 2$, $x'(t) = -1$ at $t = 0$. $D = \frac{d}{dt}$

Ans : $x = \frac{e^t t^2}{2} + 2e^t - 3te^t$ **Dec. 97, May. 2000**

19) $(D^2 + 4D + 8)y = 1$ Given $y = 0$ & $\frac{dy}{dt} = 0$ at $t = 0$.

Ans : $y = \frac{1}{8}(1 - e^{-2t}\cos 2t - e^{-2t}\sin 2t)$ **May. 99**

Laplace transform of Special Function :

Type I : (Periodic function)

Statement : If f (t) is periodic function with period 'T' then show that :

$$L\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\, dt.$$

Dec. 94, May. 96

Problems :

1) Find L. T. of $f(t) = \begin{cases} k ; 0 < t < T/2 \\ -k ; T/2 < t < T \end{cases}$ and $f(t+T) = f(t)$.

Ans : $\frac{k}{s} \tanh\left(\frac{sT}{4}\right)$

2) Find L. T. of $f(t) = \begin{cases} a \sin \omega t ; 0 < t < \pi/\omega \\ 0 \quad ; \pi/\omega < t < 2\pi/\omega \end{cases}$ and $f\left(t + \frac{2\pi}{\omega}\right) = f(t)$

Ans : $\frac{aw}{s^2 + w^2} \left(1 - e^{\frac{-\pi s}{\omega}}\right)^{-1}$

3) Find L. T. of $f(t) = \begin{cases} t \quad , 0 < t < \pi \\ \pi - t , \pi < t < 2\pi \end{cases}$ and $f(t) = f(t + 2\pi)$

Ans : $\frac{1-(1+\pi s)e^{-\pi s}}{s^2(1+e)^{-\pi s}}$

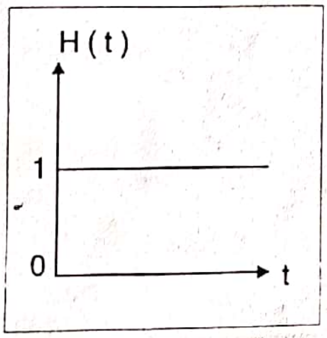
4) Find L. T. of $f(t) = \begin{cases} 1 & 0 < t < a \\ -1 & 0 < t < 2a \end{cases}$ and $f(t) = f(t + 2a)$

Ans : $\frac{1}{s} \tan h\left(\frac{as}{2}\right)$

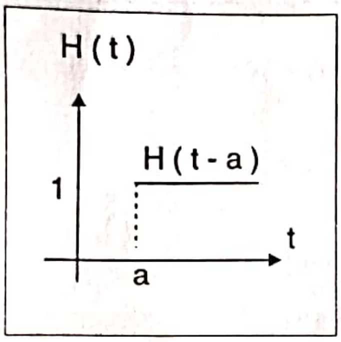
May. 99

Type - II : (Heaviside Unit Step Function

It is denoted by H (t) or u (t).



$$H(t) = \begin{cases} 0 ; t < 0 \\ 1 ; t \geq 0 \end{cases}$$



$$H(t-a) = \begin{cases} 0 \,; t < a \\ 1 \,; t \geq a \end{cases}$$

Theoreons :

1) $L\left\{H(t)\right\} = \dfrac{1}{s}.$ $\quad L^{-1}\left\{\dfrac{1}{s}\right\} = H(t).$

2) $L\left\{H(t-a)\right\} = \dfrac{1}{s}e^{-as}$ $\quad L^{-1}\left\{\dfrac{1}{s}e^{-as}\right\} = H(t-a).$ **May. 2000**

3) $L\{f(t-a)H(t-a)\} = e^{-as}\bar{f}(s) = e^{-as}L\{f(t)\}$

$L^{-1}\{e^{-as}\bar{f}(s)\} = \bar{f}(t-a)H(t-a).$ **Nov. 96**

4) Put $a = 0$ in 3.

$L\{f(t)H(t)\} = \bar{f}(s)$ and $L^{-1}\{\bar{f}(s)\} = f(t)H(t).$

5) $L\{f(t)H(t-a)\} = e^{-as}L\{f(t+a)\}$

6) If $L\{f(t)\} = \bar{f}(s)$ and $g(t) = f(t-a);\ t > a$

$= 0 \qquad ;\ t < a.$

then $L\{g(t)\} = e^{-as}\bar{f}(s).$

Problems :

1) i) Find $L\left\{\sin t\, H\left(t - \dfrac{\pi}{2}\right) - H\left(t - \dfrac{3\pi}{2}\right)\right\}$

Ans : $e^{-\pi s/2}\dfrac{s}{s^2+1} - \dfrac{1}{s}e^{-3\pi s/2}$ **May. 96**

ii) Find $L\{t^2 U(t-2)\}$ where $U(t-2)$ is heaviside unit step function.

Ans : $e^{-2s}\left[\dfrac{2}{s^3} + \dfrac{4}{s^2} + \dfrac{4}{s}\right]$ **May. 97**

iii) Find $L\left\{t^2 H(t-4)\right\}$

Ans : $e^{-4s}\left[\frac{2}{s^3}+\frac{8}{s^2}+\frac{16}{s}\right]$

iv) Find $L\{(1+2t-3t^2+4t^3)H(t-2)\}$

Ans : $e^{-2s}\left[\frac{25}{s}+\frac{38}{s^2}+\frac{42}{s^3}+\frac{24}{s^4}\right]$ Dec. 98.

v) Find $L\{e^{-t}\sin t\, H(t-\pi)\}$

Ans : $e^{-\pi(s+1)}\frac{1}{[(s+1)^2+1]}$

vi) $L\{t^4 H(t-2)\}$

ans : $e^{-2s}\left[\frac{24}{s^5}+\frac{48}{s^4}+\frac{48}{s^3}+\frac{32}{s^2}+\frac{16}{s}\right]$

2. Find Inverse Laplace transform of the following

i) $L^{-1}\left\{\frac{e^{-2s}}{s^2+8s+25}\right\}$

Ans : $\frac{1}{3}\ e^{8-4t}\sin(3t-6)\,H(t-2)$

ii) $L^{-1}\left\{\frac{s\,e^{-\pi s}}{s^2+3s+2}\right\}$

Ans : $\left[2e^{2\pi-2t}-e^{\pi-t}\right]H(t-\pi)$

iii) $L^{-1}\left\{\frac{s\,e^{-2s}}{s^2-4s+29}\right\}$

Ans : $e^{2t-4}\left[\cos(5t-10)+\frac{2}{5}\sin(5t-10)\right]H(t-2)$

iv) $L^{-1}\left\{\frac{(1-\sqrt{s})^2 e^{-3s}}{s^4}\right\}$

Ans : $\left[\frac{(t-3)^3}{6}-\frac{16}{15\sqrt{\pi}}(t-3)^{5/2}+\frac{1}{2}(t-3)^2\right]H(t-3)$

v) $L^{-1}\left\{\frac{8\,e^{-3s}}{s^2+4}\right\}$

Ans : $4\sin(2t-6)\,H(t-3)$ Dec. 98.

Type III (Dirac Delta Function or Unit Impulse Function)

Definition : If $f(t)\begin{cases} =0 & ; o<t<a \\ =1/t & ; a \leq t \leq a+\varepsilon \\ =0 & ; t>a+\varepsilon \end{cases}$

Then $\delta(t-a) = \lim\limits_{\varepsilon \to 0} f(t)$.

Theorem : 1) $L\{\delta(t-a)\} = e^{-as}$ **May. 98**

Corollary : If a = 0 then $L[\delta(t)] = 1 \therefore L^{-1}\{1\} = \delta(t)$.

2) $L\{f(t)\,\delta(t-a)\} = e^{-as} f(a)$.

3) $\int\limits_0^\infty f(t)\,\delta(t-a)\,dt = f(a)$

Problems :

1) Find $L\{\sin 2t\,\delta(t-2)\}$

ans : $e^{-2s} \sin 4$.

2) Find $L\left\{\sin 2t\,\delta\left(t-\frac{\pi}{4}\right)\right\}$

Ans : $e^{-\frac{\pi s}{4}}$

3) Find $L\{t\,H(t-4) + t^2\,\delta(t-4)\}$

Ans : $e^{-4s}\left\{\frac{1}{s^2} + \frac{4}{s} + 16\right\}$

4) $L[t^2 H(t-2) - \cosh t\,\delta(t-4)]$

Ans : $e^{-2s}\left\{\frac{2}{s^3} + \frac{4}{s^2} + \frac{4}{s}\right\} - e^{-4s}\cosh 4$.

5) Evaluate $\int\limits_0^\infty t^2 e^{-t} \sin t\,\delta(t-2)\,dt$.

Ans : $4e^{-2} \sin 2$.

6) Evaluate $\int\limits_0^\infty \cos 2t\,\delta\left(t-\frac{\pi}{4}\right) dt$.

Ans : 0