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END SEM EXAMS
AOA

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SE COMS A-3

Q2 Maximize $Z = 5x_1 + 7x_2$

subject to $x_1 + x_2 <= 4$

$3x_1 + 8x_2 <= 24$

$10x_1 + 7x_2 <= 35$

$x_1, x_2 >= 0$

Adding s_1, s_2 and s_3 slack variables

$\therefore \quad Z = 5x_1 + 7x_2 + 0s_1 + 0s_2 + 0s_3$

subject to

$x_1 + x_2 + s_1 = 4$

$3x_1 + 8x_2 + s_2 = 24$

$10x_1 + 7x_2 + s_3 = 25$

$x_1, x_2, s_1, s_2, s_3 \geq 0$

ITERATION - 1

		C_j	5	7	0	0	0	MIN RATIO
B	C_B	x_B	x_1	x_2	s_1	s_2	s_3	
s_1	0	4	1	1	1	0	0	4
s_2	0	24	3	8	0	1	0	3 →
s_3	0	35	10	7	0	0	1	5
Z=0		Z_j	0	0	0	0	0	
		$Z_j - C_j$	-5	-7 ↑	0	0	0	

$Z_j - C_j = -7 < 0$, column index is 2

$\therefore$ Entering variable $= x_2$

leaving variable $= s_2$

Pivot element $= 8$

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ITERATION - 2

		C_j	5	7	0	0	0	MIN RATIO
B	C_B	x_B	x_1	x_2	S_1	S_2	S_3	
S_1	0	1	$\boxed{0.625}$	0	1	$-1/8$	0	$8/5 \longrightarrow$
x_2	7	3	$3/8$	1	0	$1/8$	0	8
S_3	0	14	$59/8$	0	0	$-7/8$	1	$112/59$
$Z=21$		Z_j	$21/8$	7	0	$7/8$	0	
		$Z_j - C_j$	$-19/8$	0	0	$7/8$	0	
			$\uparrow$					

$Z_j - C_j = -19/8 < 0$

$\therefore$ Entering variable $= x_1$

leaving variable $= S_1$

Pivot element $= 5/8 = 0.625$

ITERATION - 3

		C_j	5	7	0	0	0	MIN RATIO
B	C_B	x_B	x_1	x_2	S_1	S_2	S_3	
x_1	5	$8/5$	1	0	$8/5$	$-1/5$	0	
x_2	7	$12/5$	0	1	$-3/5$	$1/5$	0	
S_3	0	$11/5$	0	0	$-59/5$	$3/5$	1	
$Z=124/5$		Z_j	5	7	$19/5$	$2/5$	0	
		$Z_j - C_j$	0	0	$19/5$	$2/5$	0	

$\because Z_j - C_j \geq 0$

Hence this is optimal solution

$x_1 = \frac{8}{5}$, $x_2 = \frac{12}{5}$

$$\boxed{\therefore\ Z = \frac{124}{5}}$$

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Q6 Explain Karatsuba's algorithm with an example

ANS The Karatsuba algorithm is a fast multiplication algorithm that uses a divide and conquer approach to multiply two numbers.

Using this algorithm, multiplication of two n-digit numbers is reduced from $O(n^2)$ to $O(n^{\log 3}) = O(n^{1.585})$

The key idea is to reduce the four sub-problems in multiplication to three unique problems. Thus, on calculating the three unique sub-problems, the original 4 sub-problems are solved using addition or subtraction.

Basically Karatsuba stated that if we have to multiply two n-digit numbers x and y, this can be done with the following operations, assuming that B is the base of m and m<n

First both numbers x and y can be represented as x_1, x_2 and y_1, y_2 with the following formula:

$$x = x_1 * B^m + x_2$$
$$y = y_1 * B^m + y_2$$

$$\therefore xy = (x_1 * B^m + x_2)(y_1 * B^m + y_2)$$
$$= x_1 y_1 B^{2m} + x_1 y_2 B^m + x_2 y_1 B^m$$

There are 4 sub problems : $x_1 y_1$, $x_1 y_2$, $x_2 y_1$, $x_2 y_2$

we can reduce this to 3 sub-problems

let $a = x_1 y_1$
$b = x_1 y_2 + x_2 y_1$
$c = x_2 y_2$

$$\therefore xy = a * B^{2m} + bB^{m} + c$$
$$= aB^{2m} + bB^{m} + c$$

Karatsuba's formula to calculate b:
$$b = (x_1 + x_2)(y_1 + y_2) - a - c$$

EXAMPLE:

Consider the following multiplication : 47×78

$x = 47$
$x = 4 \times 10 + 7$
$x_1 = 4$
$x_2 = 7$

$y = 78$
$y = 7 \times 10 + 8$
$y_1 = 7$
$y_2 = 8$

$a = x_1 y_1 = 4 \times 7 = 28$
$c = x_2 y_2 = 7 \times 8 = 56$
$b = (x_1 + x_2)(y_1 + y_2) - a - c$
$= 11 \times 15 - 28 - 56$.

11×15 can be then multiplied using Karatsuba's algorithm recursively.

$$T(n) = 3 \times T(n/2) + O(n)$$

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Q1 Using Recursion Tree Method find time complexity

$T(n) = 3T(n/2) + n^2$

Recurrence Tree	level	N	cost
$T(n)$	0	1	n^2
$T(n/2)$ $T(n/2)$ $T(n/2)$	1	3	$n^2/4$
$T(n/4)$ $T(n/4)$ $T(n/4)$ $T(n/4)$ $T(n/4)$ $T(n/4)$ $T(n/4)$ $T(n/4)$ $T(n/4)$	2	9	$n^2/16$
⋮			$\frac{n^2}{4^{i-1}}$
$T(i)$	i		$\frac{n^2}{4^i}$

LEVEL	NO OF TERMS	TOTAL COST
0	$1 = 3^0$	n^2
1	$3 = 3^1$	$3\,n^2/4 = 3n^2/4$
2	$9 = 3^2$	$9\,n^2/16 = 9n^2/16$
3	$27 = 3^3$	$27\,n^2/64 = 27n^2/64$
⋮	⋮	⋮
i	3^i	$3^i\,T(1)$

Now,

level 0 $\longrightarrow$ $T(n) = T(n/2^0)$

level 1 $\longrightarrow$ $T(n/2) = T(n/2^1)$

level 2 $\longrightarrow$ $T(n/4) = T(n/2^2)$

⋮

level i $\longrightarrow$ $T(n/2^i) = T(1)$

$$\therefore \frac{n}{2^i} = 1$$

$$\therefore n = 2^i$$

Taking log on both sides

$$\log n = i \log 2$$

$$\boxed{\therefore i = \log_2 n}$$

Now,

$$T(n) = n^2 + \frac{3n^2}{4} + \frac{9n^2}{16} + \frac{27n^2}{64} + \dots + 3^i T(1)$$

$$= n^2 \left[1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \dots + \left(\frac{3}{4}\right)^{i-1} \right] + 3^i T(1)$$

$$= n^2 \left[1 - \left(\frac{1^i - (3/4)^i}{1 - 3/4} \right) \right] + 3^i T(1)$$

$$= n^2 \left[\frac{1 - (3/4)^{\log_2 n}}{1/4} \right] + 3^{\log_2 n} \cdot 1$$

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$$= 4n^2\left[1-\left(\frac{3}{4}\right)^{\log_2 n}\right] + 3^{\log_2 n}$$

$$= 4n^2\left[1 - n^{\log_2 \frac{3}{4}}\right] + n^{\log_2 3}$$

$$= 4n^2\left(1 - n^{\log_2 3 - 2}\right)$$

$$= 4n^2 - 4n^{\log_2 3} + n^{\log_2 3}$$

$$= 4n^2 - 3n^{\log_2 3}$$

$$\approx 4n^2 \qquad \because n \gg 10^3$$

$$\boxed{T(n) = O(n^2)}$$

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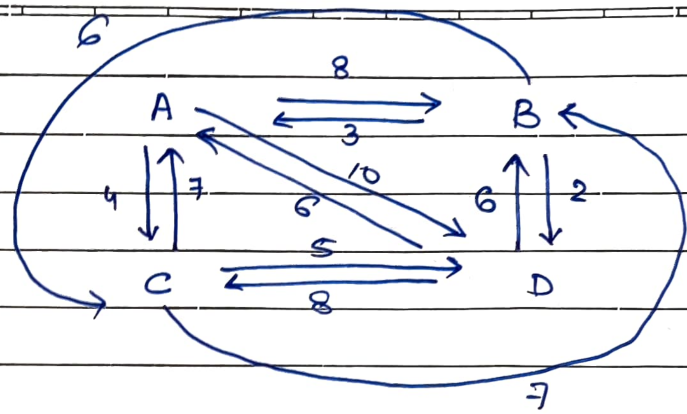
Q3



$$D^0 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 8 & 4 & 10 \\ B & 3 & 0 & 6 & 2 \\ C & 7 & 7 & 0 & 5 \\ D & 6 & 6 & 8 & 0 \end{array} \qquad \Pi^0 = \begin{bmatrix} - & A & A & A \\ B & - & B & B \\ C & C & - & C \\ D & D & D & - \end{bmatrix}$$

$D^K[i,j] = \min\{D^{K-1}[i,j],\ D^{K-1}[i,K] + D^{K-1}[K,j]\}$

ITERATION 1 $(K=1)$

$D'[1,2] = \min\{D^0(1,2), D^0[1,1] + D^0[1,2]\}$
$= \min(8, 0+8)$
$= 8$

$D'[1,3] = \min[D^0[1,3], D^0[1,1] + D^0[1,3]\}$
$= \min\{4, 0+4\}$
$= 4$

$D'[1,4] = \min\{D^0[1,4], D^0[1,1] + D^0[1,4]\}$
$= \min\{10, 0+10\}$
$= 10$

$D'[2,1] = \min\{D^0[2,1], D^0[2,1] + D^0[1,1]\}$
$= \min\{3, 3+0\}$
$= 3$

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~~$D'[2,1] = \min\{D^0[2,1], D^0[2,1] + D^0[4,3]\}$~~

$$D'[2,3] = \min\{D^0[2,3], D^0[2,1] + D^0[1,3]\}$$
$$= \min(6, 3+4)$$
$$= 6$$

$$D'[2,4] = \min\{D^0(2,4], D^0[2,1] + D^0[1,4]\}$$
$$= \min\{2, 3+10\}$$
$$= 2$$

$$D'[3,1] = \min\{D^0[3,1], D^0[3,1] + D^0[1,1]\}$$
$$= \min(7,7)$$
$$= 7$$

$$D'[3,2] = \min\{D^0[3,2], D^0[3,1] + D^0(1,2]\}$$
$$= \min\{7, 7+8\}$$
$$= 7$$

$$D'[3,4] = \min\{D^0(3,4] + D^0[3,1] + D^0[1,4]\}$$
$$= \min\{5, 7+10\}$$
$$= 5$$

$$D'[4,1] = \min\{D^0[4,1], D^0[4,1] + D^0(1,1]\}$$
$$= \min\{6, 6\}$$
$$= 6$$

$$D'(4,2] = \min\{D^0(4,2], D^0[4,1] + D^0[1,2]\}$$
$$= \min\{6, 6+8\}$$
$$= 6$$

$$D'[4,3] = \min\{D^0[4,3], D^0(4,1] + D^0(1,3]\}$$
$$= \min\{8, 6+4\}$$
$$= 8$$

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$$D^1 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 8 & 4 & 10 \\ B & 3 & 0 & 6 & 2 \\ C & 7 & 7 & 0 & 5 \\ D & 6 & 6 & 8 & 0 \end{array} \qquad \Pi^1 = \begin{bmatrix} B & - & A & A & A \\ & B & - & B & B \\ & C & C & - & C \\ & D & D & D & - \end{bmatrix}$$

ITERATION - 2

$D^2[2,1] = 3$
$D^2[2,3] = 6$
$D^2[2,4] = 2$
$D^2[1,2] = 8$
$D^2[1,3] = 4$
$D^2[1,4] = 10$
$D^2[3,1] = 7$
$D^2[3,2] = 7$
$D^2[3,4] = 5$
$D^2[4,1] = 6$
$D^2[4,2] = 6$
$D^2[4,3] = 8$

$$\Pi^2 = \begin{bmatrix} - & A & A & A \\ B & - & B & B \\ C & C & - & C \\ D & D & D & - \end{bmatrix}$$

$$D^2 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 8 & 4 & 10 \\ B & 3 & 0 & 6 & 2 \\ C & 7 & 7 & 0 & 5 \\ D & 6 & 6 & 8 & 0 \end{array}$$

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ITERATION 3 $(K = 3)$

$D^3[2,1] = \min(3, 6+7) = 3$

$D^3[2,3] = \min(6, 6+0) = 6$

$D^3[2,4] = \min(2, 6+5) = 2$

$D^3[1,2] = \min(8, 4+7) = 8$

$D^3[1,3] = \min(4, 4+0) = 4$

$D^3[1,4] = \min(10, 4+5) = 9$

$D^3[4,1] = 6$

$D^3[4,2] = 6$

$D^3[4,3] = 8$

$$\Pi^3 = \begin{bmatrix} - & A & A & A \\ B & - & B & B \\ C & C & - & C \\ D & D & D & - \end{bmatrix}$$

$$D^3 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 8 & 4 & 9 \\ B & 3 & 0 & 6 & 2 \\ C & 7 & 7 & 0 & 5 \\ D & 6 & 6 & 8 & 0 \end{array}$$

ITERATION - 4 $(K = 4)$

$D^4[1,2] = 8$

$D^4[1,3] = 4$

$D^4[1,4] = 9$

$D^4[2,1] = 3$

$D^4[2,3] = 6$

$D^4[2,4] = 2$

$D^4[3,1] = 7$

$D^4[3,2] = 7$

$D^4[3,4] = 5$

$$D^4 = \begin{array}{c|cccc} & A & B & C & D \\ \hline A & 0 & 8 & 4 & 9 \\ B & 3 & 0 & 6 & 2 \\ C & 7 & 7 & 0 & 5 \\ D & 6 & 6 & 8 & 0 \end{array}$$

$$\Pi^4 = \begin{bmatrix} - & A & A & A \\ B & - & B & B \\ C & C & - & C \\ D & D & D & - \end{bmatrix}$$

∴ All pair shortest path is

$$D^4 = \begin{bmatrix} 0 & 8 & 4 & 9 \\ 3 & 0 & 6 & 2 \\ 7 & 7 & 0 & 5 \\ 6 & 6 & 8 & 0 \end{bmatrix}$$

Q5 Write an algorithm for sum of subset problem. Solve the following using sum of subset
$M = 30$, $w = \{5, 8, 10, 12, 15, 17\}$

ANS. ALGORITHM :

```
Sum_of_Subset(s,k,r)
x[k] = 1
if s + w[k] == m
        print x[1:k]
else if  s + w[k] + w[k+1] <= m
        sum_of_Subset(s + w[k], k+1, r - w[k])
if  s + r - w[k] >= m  &&  s + w[k+1] <= m
        x[k] = 0
        Sum_of_subset(s, k+1, r - w[k])
```

Worst case Time complexity $= 2^n$ since the number of subsets for a given set is 2^n

$M = 30$
$w = \{5, 8, 10, 12, 15, 17\}$

∴ Possible subsets are : [12, 10, 8]
[15, 10, 5]
[17, 8, 5]

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Q4

	l	π
A	~~8~~ 3	B
B	0	-
C	~~8~~ 2	+ B
D	~~8~~ 4	+ B
E	~~8~~ 7	+ B
F	~~8~~ ~~16~~ 7	+ ~~C~~ 0

souce node B == B

B = (A, C, E D)

$l[v] = d[v] + c(u, v)$

$l[A] = 0 + 3$

$= 3$

$l[C] = 0 + 7$

$= 7$

$l[D] = 0 + 4$

$= 4$

$l[E] = 0 + 7$

$= 7.$

next node = C   C - [A, B, D, E, =]

$d[A] = 2 + 5$

$= 7 > 3$