

27/05/21

Q2

$$\text{Minimize } Z = 5n_1 + 7n_2$$

$$\text{subject to } n_1 + n_2 \leq 4$$

$$3n_1 + 8n_2 \leq 24$$

$$10n_1 + 7n_2 \leq 35$$

$$n_1, n_2 \geq 0$$

Adding  $s_1, s_2$  and  $s_3$  slack variables

$$\therefore Z = 5n_1 + 7n_2 + 0s_1 + 0s_2 + 0s_3$$

$$\begin{aligned} \text{subject to } n_1 + n_2 + s_1 &= 4 \\ 3n_1 + 8n_2 + s_2 &= 24 \\ 10n_1 + 7n_2 + s_3 &= 35 \end{aligned}$$

$$n_1, n_2, s_1, s_2, s_3 \geq 0$$

ITERATION - 1

		$C_j$	5	7	0	0	0	MIN RATIO
B	$C_B$	$n_B$	$n_1$	$n_2$	$s_1$	$s_2$	$s_3$	
$s_1$	0	4	1	1	1	0	0	4
$s_2$	0	24	3	8	0	1	0	3 $\rightarrow$
$s_3$	0	35	10	7	0	0	1	5
$Z=0$		$Z_j$	0	0	0	0	0	
		$Z_j - C_j$	-5	-7	0	0	0	

↑

$$Z_j - C_j = -7 \leq 0, \text{ column index is 2}$$

$\therefore$  Entering variable =  $n_2$

leaving variable =  $s_2$

Pivot element = 8

## ITERATION - 2

		$C_j$	5	7	0	0	0	MIN RATIO
B	$C_B$	$\pi_B$	$\pi_1$	$\pi_2$	$s_1$	$s_2$	$s_3$	
$s_1$	0	1	0.625	0	1	-1/8	0	8/5 $\rightarrow$
$\pi_2$	7	3	3/8	1	0	1/8	0	8
$s_3$	0	14	59/8	0	0	-7/8	1	12/59
$Z = 21$		$Z_j$	21/8	7	0	7/8	0	
		$Z_j - C_j$	-19/8	0	0	7/8	0	
			↑					

$$Z_j - C_j = -19/8 < 0$$

∴ Entering variable =  $\pi_1$ ,

leaving variable =  $s_1$ ,

Pivot element =  $5/8 = 0.625$

## ITERATION - 3

		$C_j$	5	7	0	0	0	MIN RATIO
B	$C_B$	$\pi_B$	$\pi_1$	$\pi_2$	$s_1$	$s_2$	$s_3$	
$\pi_1$	5	8/5	1	0	8/5	-1/5	0	
$\pi_2$	7	12/5	0	1	-3/5	1/5	0	
$s_3$	0	11/5	0	0	-59/5	3/5	1	
$Z = 124/5$		$Z_j$	5	7	19/5	2/5	0	
		$Z_j - C_j$	0	0	19/5	2/5	0	

$$\therefore Z_j - C_j \geq 0$$

Hence this is optimal solution

$$\pi_1 = \frac{8}{5}, \quad \pi_2 = \frac{12}{5}$$

$$\therefore Z = \frac{124}{5}$$

Q6

Explain Karatsuba's algorithm with an example

AN:

The Karatsuba algorithm is a fast multiplication algorithm that uses a divide and conquer approach to multiply two numbers.

Using this algorithm, multiplication of two  $n$ -digit numbers is reduced from  $O(n^2)$  to  $O(n^{\log 3}) = O(n^{1.585})$

~~RE~~ The key idea is to reduce the four sub-problems in multiplication to three unique problems. Thus, on calculating the three unique sub-problems, the original 4 sub-problems are solved using addition or subtraction.

Basically Karatsuba stated that if we have to multiply two  $n$ -digit numbers  $x$  and  $y$ , this can be done with the following operations, assuming that  $B$  is the base of  $m$  and  $m < n$ .

First both numbers  $x$  and  $y$  can be represented as  $x_1, x_2$  and  $y_1, y_2$  with the following formula:

$$x = x_1 * B^m + x_2$$

$$y = y_1 * B^m + y_2$$

~~RE~~

$$\begin{aligned} xy &= (x_1 * B^m + x_2)(y_1 * B^m + y_2) \\ &= x_1 y_1 B^{2m} + x_1 y_2 B^m + x_2 y_1 B^m \end{aligned}$$

There are 4 sub problems:  $x_1 y_1, x_1 y_2, x_2 y_1, x_2 y_2$   
we can reduce this to 3 sub-problems

$$\text{let } a = n_1 y_1$$

$$b = n_1 y_2 + n_2 y_1,$$

$$c = n_2 y_2$$

$$\therefore xy = a * B^{2m} + bB^m + c \\ = AB^{2m} + bB^m + c$$

Karatsuba's formula to calculate  $b$ :

$$b = (n_1 + n_2)(y_1 + y_2) - a - c$$

EXAMPLE :

Consider the following multiplication :  $47 \times 78$

$$n = 47$$

$$n = 4 \times 10 + 7$$

$$n_1 = 4$$

$$n_2 = 7$$

$$y = 78$$

$$y = 7 \times 10 + 8$$

$$y_1 = 7$$

$$y_2 = 8$$

$$a = n_1 y_1 = 4 \times 7 = 28$$

$$c = n_2 y_2 = 7 \times 8 = 56$$

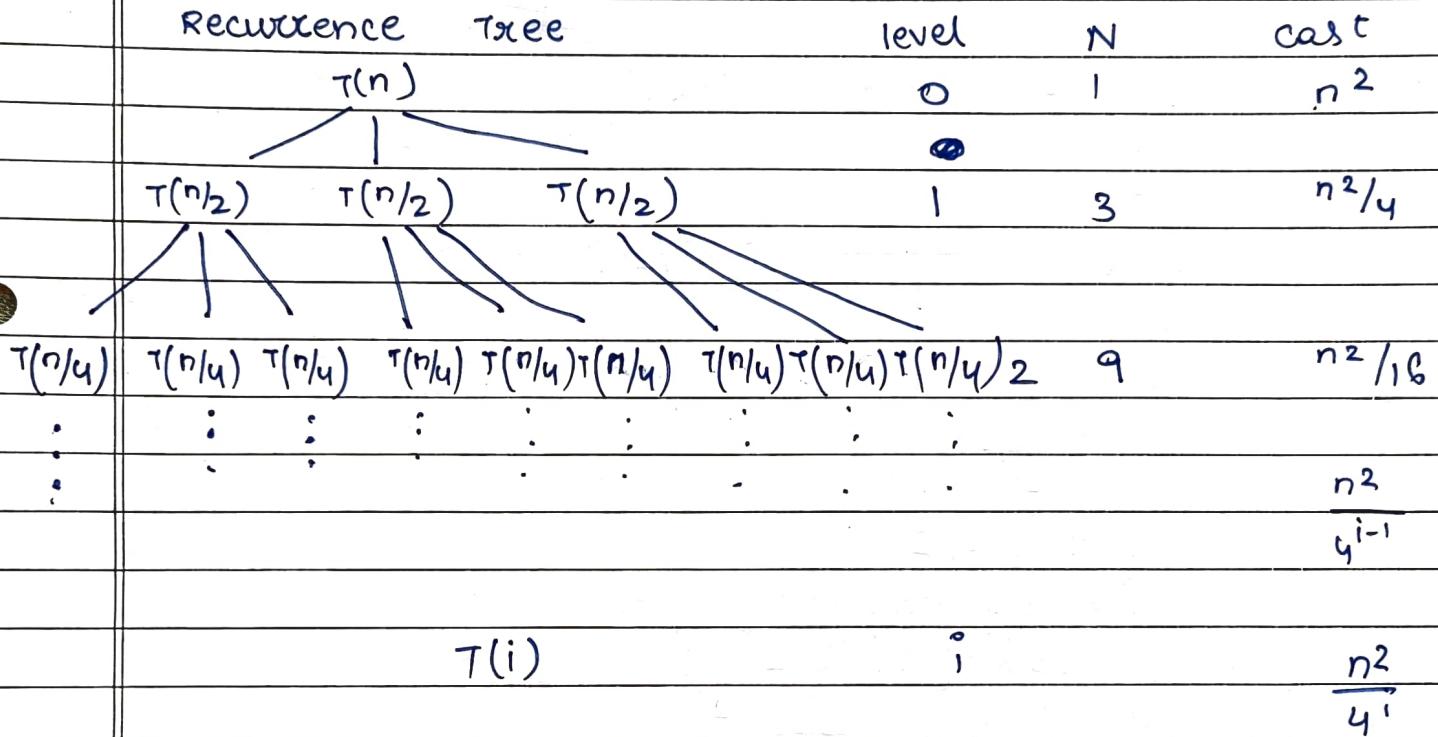
$$b = (n_1 + n_2)(y_1 + y_2) - a - c \\ = 11 \times 15 - 28 - 56 .$$

$11 \times 15$  can be then multiplied using Karatsuba's algorithm recursively.

$$T(n) = 3 \times T(n/2) + O(n)$$

Q1

Using Recursion tree method find time complexity  
 $T(n) = 3T(n/2) + n^2$



LEVEL	NO OF TERMS	TOTAL COST
0	$1 = 3^0$	$n^2$
1	$3 = 3^1$	$3 \cdot n^2/4 = 3n^2/4$
2	$9 = 3^2$	$9 \cdot n^2/16 = 9n^2/16$
3	$27 = 3^3$	$27 \cdot n^2/64 = 27n^2/64$
$\vdots$	$\vdots$	$\vdots$
$i$	$3^i$	$3^i T(1)$

Now,

$$\text{level } 0 \longrightarrow T(n) = T(n/2^0)$$

$$\text{level } 1 \longrightarrow T(n/2) = T(n/2^1)$$

$$\text{level } 2 \longrightarrow T(n/4) = T(n/2^2)$$

⋮

$$\text{level } i \longrightarrow T(n/2^i) = T(1)$$

$$\therefore \frac{n}{2^i} = 1$$

$$\therefore n = 2^i$$

Taking log on both sides

$$\log n = i \log 2$$

$$\boxed{\therefore i = \log_2 n}$$

Now,

$$T(n) = n^2 + \frac{3n^3}{4} + \frac{9n^2}{16} + \frac{27n^2}{64} + \dots + 3^i T(1)$$

$$= n^2 \left[ 1 + \frac{3}{4} + \frac{9}{16} + \frac{27}{64} + \dots + \left(\frac{3}{4}\right)^{i-1} \right] + 3^i T(1)$$

$$= n^2 \left[ 1 - \left( \frac{1 - (3/4)^i}{1 - 3/4} \right) \right] + 3^i T(1)$$

$$= n^2 \left[ 1 - \frac{(3/4)^{\log_2 n}}{1/4} \right] + 3^{\log_2 n} \cdot 1$$

$$= 4n^2 \left[ 1 - \left( \frac{3}{4} \right)^{\log_2 n} \right] + 3^{\log_2 n}$$

$$= 4n^2 \left[ 1 - n^{\log_2 \frac{3}{4}} \right] + n^{\log_2 3}$$

$$= 4n^2 (1 - n^{\log_2 3 - 2})$$

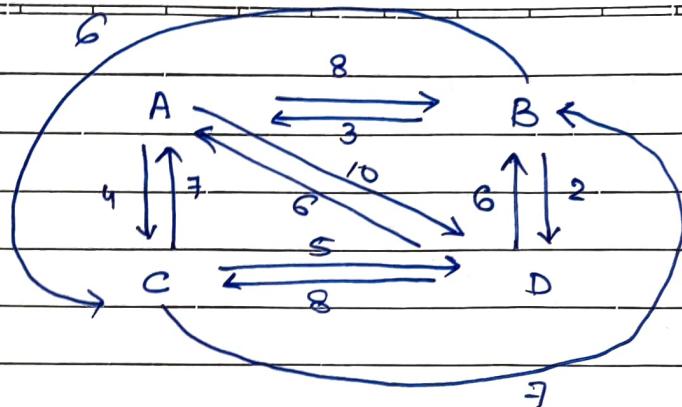
$$= 4n^2 - 4n^{\log_2 3} + n^{\log_2 3}$$

$$= 4n^2 - 3n^{\log_2 3}$$

$$\approx 4n^2 \quad \because n >> 10^3$$

$T(n) = O(n^2)$

Q3



$$D^0 = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 8 & 4 & 10 \\ 3 & 0 & 6 & 2 \\ 7 & 6 & 0 & 5 \\ 6 & 6 & 8 & 0 \end{bmatrix} \end{matrix} \quad \pi^0 = \begin{bmatrix} -A & A & A \\ B & -B & B \\ C & C & -C \\ D & D & D \end{bmatrix}$$

$$D^K[i,j] = \min \{ D^{K-1}[i,j], D^{K-1}[i,K] + D^{K-1}[K,j] \}$$

ITERATION 1 ( $K=1$ )

$$\begin{aligned} D'[1,2] &= \min \{ D^0(1,2), D^0(1,1) + D^0(1,2) \} \\ &= \min \{ 8, 0 + 8 \} \\ &= 8 \end{aligned}$$

$$\begin{aligned} D'[1,3] &= \min \{ D^0(1,3), D^0(1,1) + D^0(1,3) \} \\ &= \min \{ 4, 0 + 4 \} \\ &= 4 \end{aligned}$$

$$\begin{aligned} D'[1,4] &= \min \{ D^0(1,4), D^0(1,1) + D^0(1,4) \} \\ &= \min \{ 10, 0 + 10 \} \\ &= 10 \end{aligned}$$

$$\begin{aligned} D'[2,1] &= \min \{ D^0(2,1), D^0(2,1) + D^0(1,1) \} \\ &= \min \{ 3, 3 + 0 \} \\ &= 3 \end{aligned}$$

$$D'[2,1] = \min \{ D^o[2,1], D^o[2,1] + D^o[4,3] \}$$

$$\begin{aligned} D'[2,3] &= \min \{ D^o[2,3], D^o[2,3] + D^o[1,3] \} \\ &= \min \{ 6, 3+4 \} \\ &= 5 \end{aligned}$$

$$\begin{aligned} D'[2,4] &= \min \{ D^o[2,4], D^o[2,1] + D^o[1,4] \} \\ &= \min \{ 2, 3+10 \} \\ &= 2 \end{aligned}$$

$$\begin{aligned} D'[3,1] &= \min \{ D^o[3,1], D^o[3,1] + D^o[1,1] \} \\ &= \min \{ 7, 7 \} \\ &= 7 \end{aligned}$$

$$\begin{aligned} D'[3,2] &= \min \{ D^o[3,2], D^o[3,1] + D^o[1,2] \} \\ &= \min \{ 7, 7+8 \} \\ &= 7 \end{aligned}$$

$$\begin{aligned} D'[3,4] &= \min \{ D^o[3,4] + D^o[3,1] + D^o[1,4] \} \\ &= \min \{ 5, 7+10 \} \\ &= 5 \end{aligned}$$

$$\begin{aligned} D'[4,1] &= \min \{ D^o[4,1], D^o[4,1] + D^o[1,1] \} \\ &= \min \{ 6, 6 \} \\ &= 6 \end{aligned}$$

$$\begin{aligned} D'[4,2] &= \min \{ D^o[4,2], D^o[4,2] + D^o[1,2] \} \\ &= \min \{ 6, 6+8 \} \\ &= 6 \end{aligned}$$

$$\begin{aligned} D'[4,3] &= \min \{ D^o[4,3], D^o[4,1] + D^o[1,3] \} \\ &= \min \{ 8, 6+4 \} \\ &= 8 \end{aligned}$$

$$D^1 = A \begin{bmatrix} A & B & C & D \\ 0 & 8 & 4 & 10 \\ 3 & 0 & 6 & 2 \\ 7 & 7 & 0 & 5 \\ 6 & 6 & 8 & 0 \end{bmatrix} \quad \pi^1 = \begin{bmatrix} 0 - AAA \\ B - BBB \\ C C - C \\ DDD - \end{bmatrix}$$

## ITERATION - 2

$$D^2[2,1] = 3$$

$$D^2[2,3] = 6$$

$$D^2[2,4] = 2$$

$$D^2[1,2] = 8$$

$$D^2[1,3] = 4$$

$$D^2[1,4] = 10$$

$$D^2[3,1] = 7$$

$$D^2[3,2] = 7$$

$$D^2[3,4] = 5$$

$$D^2[4,1] = 6$$

$$D^2[4,2] = 6$$

$$D^2[4,3] = 8$$

$$\pi^2 = \begin{bmatrix} - AAA \\ B - BBB \\ C C - C \\ DDD - \end{bmatrix}$$

$$D^2 = A \begin{bmatrix} A & B & C & D \\ 0 & 8 & 4 & 10 \\ 3 & 0 & 6 & 2 \\ 7 & 7 & 0 & 5 \\ 6 & 6 & 8 & 0 \end{bmatrix}$$

ITERATION 3 ( $k=3$ )

$$D^3[2,1] = \min(3, 6+7) = 3$$

$$D^3[2,3] = \min(6, 6+0) = 6$$

$$D^3[2,4] = \min(2, 6+5) = 2$$

$$D^3[1,2] = \min(8, 4+7) = 8$$

$$D^3[1,3] = \min(4, 4+0) = 4$$

$$D^3[1,4] = \min(10, 4+5) = 9$$

$$D^3[4,1] = 6$$

$$D^3[4,2] = 6$$

$$D^3[4,3] = 8$$

$$\pi^3 = \begin{bmatrix} - & A & A & A \\ B & - & B & B \\ C & C & - & C \\ D & D & D & - \end{bmatrix}$$

$$D^3 = A \begin{bmatrix} A & B & C & D \\ 0 & 8 & 4 & 9 \\ 3 & 0 & 6 & 2 \\ 7 & 7 & 0 & 5 \\ 6 & 6 & 8 & 0 \end{bmatrix}$$

ITERATION -4 ( $k=4$ )

$$D^4[1,2] = 8$$

$$D^4[1,3] = 4$$

$$D^4[1,4] = 9$$

$$D^4[2,1] = 3$$

$$D^4[2,3] = 6$$

$$D^4[2,4] = 2$$

$$D^4[3,1] = \rightarrow$$

$$D^4[3,2] = \rightarrow$$

$$D^4[3,4] = 5$$

$$D^4 = A \begin{bmatrix} A & B & C & D \\ 0 & 8 & 4 & 9 \\ 3 & 0 & 6 & 2 \\ 7 & 7 & 0 & 5 \\ 6 & 6 & 8 & 0 \end{bmatrix}$$

$$\pi^4 = \begin{bmatrix} - & A & A & A \\ B & - & B & B \\ C & C & - & C \\ D & D & D & - \end{bmatrix}$$

∴ All pair shortest path is

$$D^4 = \begin{bmatrix} 0 & 8 & 4 & 9 \\ 3 & 0 & 6 & 2 \\ 7 & 7 & 0 & 5 \\ 6 & 6 & 8 & 0 \end{bmatrix}$$

Q5 write an algorithm for sum of subset problem.

Solve the following using sum of subset

$$M = 30, W = \{5, 8, 10, 12, 15, 17\}$$

Ans.

ALGORITHM :

Sum\_of\_Subset ( $s, k, x$ )

$$n[k] = 1$$

$$\text{if } s + w[k] == m$$

print  $x[1:k]$

$$\text{else if } s + w[k] + w[k+1] \leq m$$

sum\_of\_subset ( $s + w[k], k+1, x - w[k]$ )

$$\text{if } s + x - w[k] \geq m \text{ & } s + w[k+1] \leq m$$

$$n[k] = 0$$

Sum\_of\_subset ( $s, k+1, x - w[k]$ )

Worst case Time complexity =  $2^n$  since the number  
of subsets for a given set is  $2^n$

$$M = 30$$

$$W = \{5, 8, 10, 12, 15, 17\}$$

∴ Possible subsets are : [12, 10, 8]

[15, 10, 5]

[17, 8, 5]

$\lambda$        $\pi$

Q4	A	83	B
	B	0	-
	C	82	+B
	D	84	+B
	E	87	+B
	F	8167	+BO

source node B == B

$$B = \{A, C, E, D\}$$

$$\lambda[v] = d[v] + c(u, v)$$

$$\begin{aligned}\lambda[A] &= 0 + 3 \\ &= 3\end{aligned}$$

$$\begin{aligned}\lambda[C] &= 0 + 7 \\ &= 7\end{aligned}$$

$$\begin{aligned}\lambda[D] &= 0 + 4 \\ &= 4\end{aligned}$$

$$\begin{aligned}\lambda[E] &= 0 + 7 \\ &= 7.\end{aligned}$$

next node = C    C - [A, B, D, E, =]

$$\begin{aligned}d[A] &= 2 + 5 \\ &= 7 > 3\end{aligned}$$

$$d[A] = 3 \text{ (min)}$$

$$\begin{aligned}d[D] &= 2 + 3 \\ &= 5 > 4\end{aligned}$$

$$\therefore d[D] = 4$$

$$d[E] = 2 + 6 \\ = 8 > 7$$

$$d[E] = 7$$

$$d[F] = 2 + 8 \\ = 10$$

Nächst vertem = A  
 $A - [B, C, D]$

$$d[D] = 3 + 9 \\ = 12 > 4 \\ \therefore d[D] = 4$$

Nächst vertem = D  
 $D - [A, B, C, E, F]$

$$d(E) = 4 + 3 \\ = 7$$

$$d[F] = 4 + 3 \\ = 7$$

Nächst vertem - E

$$E - [C, D, F]$$

$$d[F] = 9 + 5 \\ = 12 > 7$$

$$\therefore d[F] = 7$$

	$d$	$\pi$
A	3	B
B	0	-
C	2	B
D	4	B
E	7	B
F	7	D

