

## Q1a Differentiate NFA vs DFA

NFA	DFA
<ul style="list-style-type: none"> <li>It stands for Nondeterministic Finite Automata.</li> <li>No need to specify how does the NFA react according to some symbol.</li> </ul>	<ul style="list-style-type: none"> <li>It stands for Deterministic Finite Automata.</li> <li>For each symbolic representation of the alphabet, there is only one state transition in DFA.</li> </ul>
<ul style="list-style-type: none"> <li>NFA can use empty string transition.</li> <li>NFA can be understood as multiple little machines computing at the same time.</li> </ul>	<ul style="list-style-type: none"> <li>DFA cannot use empty String transition.</li> <li>DFA can be understood as one machine.</li> </ul>
<ul style="list-style-type: none"> <li>NFA rejects the string in the event of all branches dying or refusing the string.</li> <li>Time needed for executing an input string is more.</li> </ul>	<ul style="list-style-type: none"> <li>DFA rejects the string in case it terminates in a state that is different from the accepting state.</li> <li>Time needed for executing an input string is less.</li> </ul>
<ul style="list-style-type: none"> <li>Not all NFA are DFA.</li> <li>NFA requires less space than DFA.</li> </ul>	<ul style="list-style-type: none"> <li>All DFA are NFA.</li> <li>DFA requires more state.</li> </ul>
<ul style="list-style-type: none"> <li>E.g.: <math>\delta: Q \times \Sigma \rightarrow 2^Q</math>, i.e. next possible state belongs to the power set of <math>Q</math>.</li> </ul>	<ul style="list-style-type: none"> <li>E.g.: <math>\delta: Q \times \Sigma \rightarrow Q</math> i.e. next possible state belongs to <math>Q</math>.</li> </ul>

Q16

ANS

STEP 1 : let m be the DFA

$$\therefore m = (Q, \Sigma, \delta, q_0, F)$$

where  $Q$  = finite set of all states $\Sigma$  = finite set of input symbols $\delta$  = transition function $q_0$  = initial state $F$  = finite set of final states

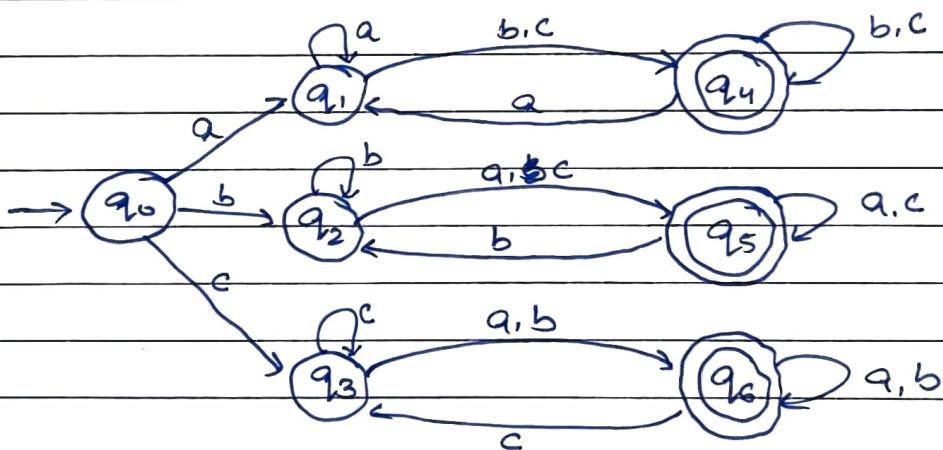
STEP 2 : LOGIC

Input alphabets are  $\{a, b, c\}$  $\therefore$  starting with  $a \Rightarrow$  end with  $b$  or  $c$ starting with  $b \Rightarrow$  end with  $a$  or  $c$ starting with  $c \Rightarrow$  end with  $a$  or  $b$ let  $q_0$  = initial state $q_1$  = string starting with 'a' and end with 'c' $q_2$  = string starting with 'b' and end with 'b' $q_3$  = string starting with 'c' and end with 'c' $q_4$  = string starting with 'a' ends with 'b' or 'c' $q_5$  = string starting with 'b' ends with 'a' or 'c' $q_6$  = string starting with 'c' ends with 'a' or 'b'

## STEP 3 : TRANSITION TABLE

$q/\Sigma$	a	b	c
$\rightarrow q_0$	$q_1$	$* q_2$	$q_3$
$q_1$	$q_1$	$* q_4$	$* q_6$
$q_2$	$* q_5$	$q_2$	$q_5$
$q_3$	$q_c$	$q_c$	$q_3$
$* q_4$	$q_1$	$* q_4$	$q_4$
$* q_5$	$* q_5$	$q_2$	$\leftarrow q_5$
$* q_6$	$q_c$	$* q_6$	$q_3$

## STEP 4 : TRANSITION DIAGRAM



## STEP 5 : SAMPLE INPUT abaa c

 $\delta(q_0, abaac)$  $\delta(q_1, baac)$  $\delta(q_4, aac)$  $\delta(q_1, ac)$  $\delta(q_6, c)$  $q_4 \rightarrow$  Final state accepted.

Q2 a) Differentiate NDPA vs DPDA.

NDPA	DPDA
• It stands for Nondeterministic Pushdown Automata	• It stands for Deterministic Pushdown Automata
• central symbol is not known in case of NDPA	• In DPDA, the central symbol is known. e.g: abcba. Here c denotes the central symbol and tells after this symbol, pop operation needs to be performed.
• There are multiple moves possible in one situation	• There is only one move allowed in one situation.
• The representation of NDPA is $w w^* \gamma$	• The representation of DPDA is $w c w^* \gamma$
• Every NDPA cannot be simulated by DPDA	• Every DPDA can be simulated by an NDPA

Q6 a church's hypothesis.

ANS

Turing machine is defined as an abstract representation of a computing device such as hardware in computers. Alan Turing proposed logical computing Machines (LCM's), i.e. Turing's expressions for Turing Machines. This was done to define algorithms properly. So, church made a mechanical method named as 'M' for manipulation of strings by using logic and mathematics.

The method M must pass the following statements:-

- Number of instructions in M must be finite
- output should be produced after performing finite number of steps.
- It should not be imaginary i.e. can be made in real life
- It should not require any complex understanding

using these statements church proposed a hypothesis called church's Turing thesis that can be stated as: "The assumption that the intuitive notion of computable functions can be identified with partial recursive functions"

In 1930, this statement was first formulated by Alonzo Church and is usually referred to as church's thesis or the church-Turing thesis. However, this hypothesis cannot be proved.

P.T.O

The recursive functions can be computable after taking the following assumptions :-

→ Each and every function must be computable

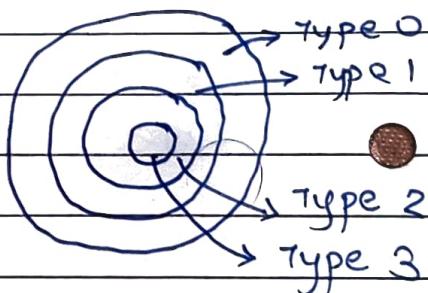
→ Let 'F' be the computable function and after performing some elementary operations to 'F', it will transform a new function 'G' then this function 'G' automatically becomes the computable function

→ If any functions that follow above two assumptions must be stated as computable function.

### b chomsky Hierarchy of languages

**ANS.** Chomsky Hierarchy represents the class of languages that are accepted by different machines. The category of language in Chomsky's Hierarchy are :-

- 1) Type 0 known as unrestricted Grammer
- 2) Type 1 known as sensitive grammer
- 3) Type 2 known as context free grammer
- 4) Type 3 known as regular grammer



This is a hierarchy, therefore every language of type 3 is also type 0, 1, 2.

## 1) TYPE 0 GRAMMER

It is known as unrestricted grammer. There is no restriction on the grammer rules. These languages can be efficiently modeled by turing machines

$$\text{e.g.: } bAa \rightarrow aa \\ S \rightarrow S$$

## 2) TYPE 1 GRAMMER

It is known as context sensitive grammer. It is used to represent context sensitive languages. It follows the following rules :-

- 1) The context sensitive grammer may have more than one symbol on the left hand side of their production rules.
- 2) The number of symbols on the LHS must not exceed the number of symbols on the RHS.
- 3) The rule of the form  $A \rightarrow \epsilon$  is not allowed unless A is start symbol. It does not occur on RHS of any rule.
- 4) The type 1 grammer should be type 0. Production is in form of  $V \rightarrow T$  where the count of symbol in  $V = T$

$$\text{e.g.: } S \rightarrow AT$$

$$T \rightarrow ny$$

$$A \rightarrow Q$$

### 3) TYPE 2 GRAMMER

It is known as context free grammar. Context free languages are the languages which can be represented by the CFG. Type 2 should be Type 1. The production rule is of the form  $A \rightarrow a$  where  $A$  is any single non-terminal and  $a$  is any combination of terminals and non-terminals.

e.g.:  $A \rightarrow ab b$   
 $A \rightarrow b$   
 $B \rightarrow a$

### 4) TYPE 3 GRAMMER

It is known as ~~singular~~ regular grammar. Regular languages are those languages which can be described using regular expressions. These languages can be modeled by NFA or DFA. Type 3 is the most restricted form of grammar. It should be in form of  $V \rightarrow \gamma^* V / T^*$

e.g.:  $A \rightarrow xy$

Q 4

$$\Sigma = \{0, 1\}$$

$$S \rightarrow 0S1 \mid 0A$$

$$A \rightarrow A0 \mid S0 \mid 0$$

ANS

let given grammar  $G = (V, T, P, S)$

$$\therefore G = (\{S, A\}, \{0\}, P, S)$$

STEP 1: Simplifying the given grammar.

a) Removing  $\epsilon$  productions / Null productions

→ There are no  $\epsilon$  productions in Grammar

b) Removing unit productions

→ There are no unit productions in grammar.

c) Removing useless variables.

→ There are no useless variable in grammar.

Hence the given grammar is already simplified.

~~QUESTION~~ converting to Chomsky Normal form (CNF)

$S \rightarrow OS_1$	$ $	$S \rightarrow C_0 S_1$	$C_0 \rightarrow O$
		$S \rightarrow C_1 S_1$	$C_1 \rightarrow I$
			$C_2 \rightarrow CO_S$
$S \rightarrow OA$	$ $	$S \rightarrow C_0 A$	
$A \rightarrow A_0$	$ $	$A \rightarrow AC_0$	
$A \rightarrow SO$	$ $	$A \rightarrow SC_0$	
$A \rightarrow O$			

So the CNF of the given grammar is

$$G = (\{S, C_0, C_1, C_2, A\}, \{O, I\}, P', S)$$

$$\begin{aligned} \therefore P' = & \left\{ \begin{array}{l} S \rightarrow C_2 A / C_0 A \\ A \rightarrow AC_0 / SC_0 / O \\ C_0 \rightarrow O \\ C_1 \rightarrow I \\ C_2 \rightarrow CO_S \end{array} \right. \end{aligned}$$

converting the Grammer into Greibach Normal Form (GNF)

STEP 2] Renaming the variables as  $A_i$

S   A  
A<sub>1</sub>   A<sub>2</sub>

$\therefore$  The productions becomes

$A_1 \rightarrow 0A_11 / 0A_2 \dots$  Invalid

$A_2 \rightarrow A_20 / A_10 / 0 \dots$  Invalid

$\therefore A_2 \rightarrow A_20 / 0A_10 / 0A_20 / 0 \longrightarrow ①$

$\therefore$  left recursion is formed in eq<sup>n</sup> ①

Using formula of removal of left recursion

$A \rightarrow A\alpha / \beta$

$\therefore A \rightarrow \beta A' / \beta$

$A' \rightarrow \alpha A' / \alpha$

$\therefore A_2 \rightarrow A_20 / 0A_10 / 0A_20 / 0$

let  $c_0 \rightarrow 0$

$c_1 \rightarrow 1$

$\therefore A_2 \rightarrow A_2 c_0 / 0A_1 c_0 / 0A_2 c_0 / 0$

$\therefore A \rightarrow A\alpha / \beta_1 / \beta_2 / \beta_3$

$\therefore A_2 \rightarrow 0A_1 c_0 A' / 0A_1 c_0 / 0A_2 c_0 A' / 0A_2 c_0 / 0A' / 0$

$\therefore A' \rightarrow c_0 A' / c_0$

NOW,  $A_1 \rightarrow 0A_11 / 0A_2$

$A_1 \rightarrow 0A_1 c_1 / 0A_2 \quad c_1 \rightarrow 1$

NOW,  $A' \rightarrow C_0 A' / C_0$   
 $A \rightarrow O A' / O$

Hence, the productions are:

$P' \Rightarrow A_1 \rightarrow O A_1 C_1 / O A_2$

$A_2 \rightarrow O A_1 C_0 A' / O A_1 C_0 / O A_2 C_0 A' / O A_2 C_0 / O A' / O$

$A' \rightarrow O A' / O$

$C_0 \rightarrow O,$

$C_1 \rightarrow I$

$\therefore G = (A, C_1, A_2, A', C_0, \{O, I\}, P', S)$

Q5 a let  $P, Q, R$  be regular expressions on  $\Sigma$ . Then if  $P$  does not contain  $\epsilon$ , the equation

$$R = Q + RP \quad \rightarrow (1)$$

has unique solution given by

$$R = QP^* \quad \rightarrow (2)$$

Proof : let us verify whether  $QP^*$  is solution of  $R = Q + RP$

on substitution of  $QP^*$  for  $R$  in (1)

$$\begin{aligned} R &= Q + RP \\ &= Q + QP^* P \\ &= Q^* (\epsilon + P^* P) \\ &= QP^* \end{aligned}$$

thus the equation (1) is satisfied by  $R = QP^*$ . We still don't know if its unique solution.

To establish uniqueness, we expand

$$\begin{aligned} R &= Q + RP \\ &= Q + (Q + RP)P \\ &= Q + QP + RP^2 \\ &= Q + QP + QP^2 + RP^3 \\ &\vdots \\ &= Q (\epsilon + P + P^2 + P^3) + RP^{i+1} \quad \rightarrow (3) \end{aligned}$$

here  $i$  is an arbitrary integer.

let us take a string  $w \in R$  | length  $|w| = k$   
substituting  $k$  for  $i$  in eqn (3)

$$R = Q (\epsilon + P + P^2 + \dots + P^k) + RP^{k+1}$$

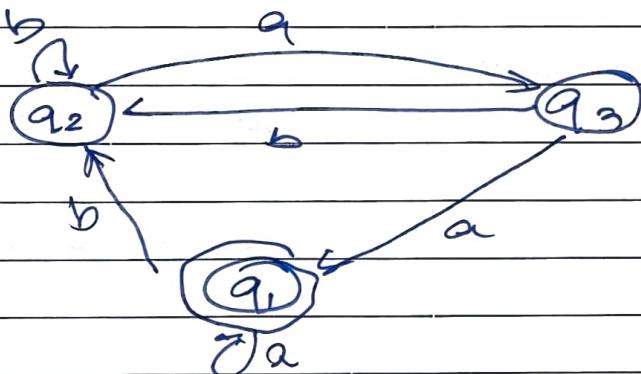
Since  $P$  does not contain  $\epsilon$ ,  $w$  is not contained in  $R P^{k+1}$  as the shortest string generated by  $R P^{k+1}$  will have a length  $q^{k+1}$

since  $w$  is contained in  $R$ , it must be contained in  $Q(\epsilon + P + P^2 \dots + P^k)$

Conversely, if  $w$  is contained in  $QP^*$  then for some integer  $k$  it must be in  $Q(\epsilon + P + P^2 \dots + P^k)$  and hence in  $R = Q + RP$

Hence verified.

Q5b



There are no dead states to eliminate.

equations from diagram :

$$q_1 = \epsilon + q_1 a + q_3 a \rightarrow 1$$

$$q_2 = q_1 b + q_2 b + q_3 b \rightarrow 2$$

$$q_3 = q_2 a \rightarrow 3$$

Substituting ③ in ②

$$q_2 = q_1 b + q_2 b + q_2 a \cdot b$$

$$q_2 = q_1 b + q_2 (b + ab)$$

Comparing with  $R = Q + RP$  we get

$$q_2 = q_1 b (b + ab)^* \rightarrow ④$$

$$q_3 = q_1 b (b + ab)^* a \rightarrow ⑤$$

Substituting ⑤ in ①

$$q_1 = q_1 a + q_2 aa + \epsilon$$

$$\begin{aligned} \therefore q_1 &= q_1 a + q_1 b (b + ab)^* aa + \epsilon \\ &= q_1 (a + b (b + ab)^* aa) + \epsilon \\ &= [a + b (b + ab)^* aa]^* \end{aligned}$$

$$\text{R.E. : } [a + b (b + ab)^* aa]^* //$$