23-10-2020

DISCRETE STRUCTURES

TUTORIAL - 3

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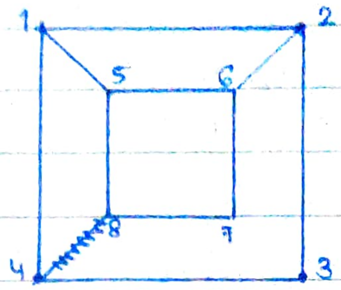
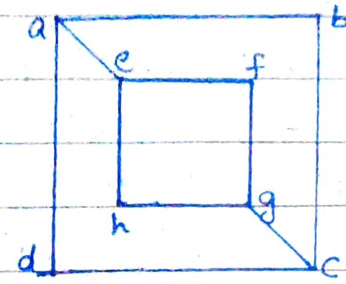
SE A3

Q1. Determine whether the following graphs are isomorphic.

A.



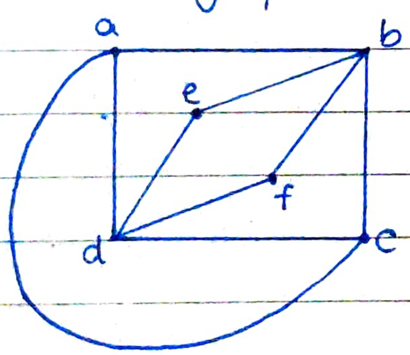
G_1



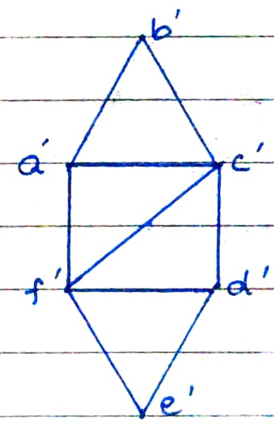
G_2

There are 8 vertices in G_1 and 8 vertices in G_2.
Also, there are 4 vertices of degree 3 in G_1 and 4 vertices of degree 3 in G_2.
There are 4 vertices of degree 2 in G_1 and 4 vertices of degree 2 in G_2.
But if $5 \rightarrow e$ and $6 \rightarrow g$ then, although 5 and 6 are adjacent, e and g are not adjacent. Thus adjacency is not preserved.
Hence the graphs are not isomorphic.

B



G_1



G_2

Here, we note that both the graphs have
1. The same number of vertices (6)
2. The same number of edges (9)

In g, there are 2 vertices e and f of degree 2 ; 2 vertices c and a of degree 3 ; two vertices b and d of degree 4.

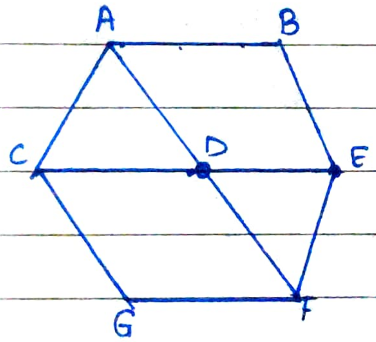
In g' also, there are 2 vertices b' and e' of degree 2 ; 2 vertices a' and d' of degree 3 and 2 vertices c' and f' of degree 4.

The property of adjacency, however, it is not observed. In G, one vertex e (and also f) of degree 2 is adjacent to two vertices d and b of degree 4.

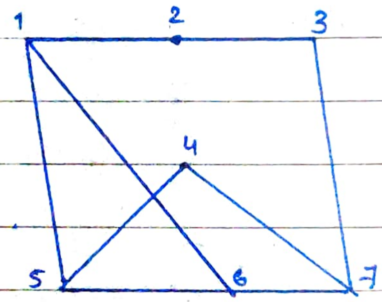
The property of adjacency, however, is not observed. In G, one vertex e (and also f) of degree 2 is adjacent to two vertices d and b of degree 4. In G', one vertex b' (and also e') of degree 2 is adjacent to the vertex c' with degree 4 but to a' with degree 3.

Hence the graphs are not isomorphic.

C



G



G'

Here G and G' have

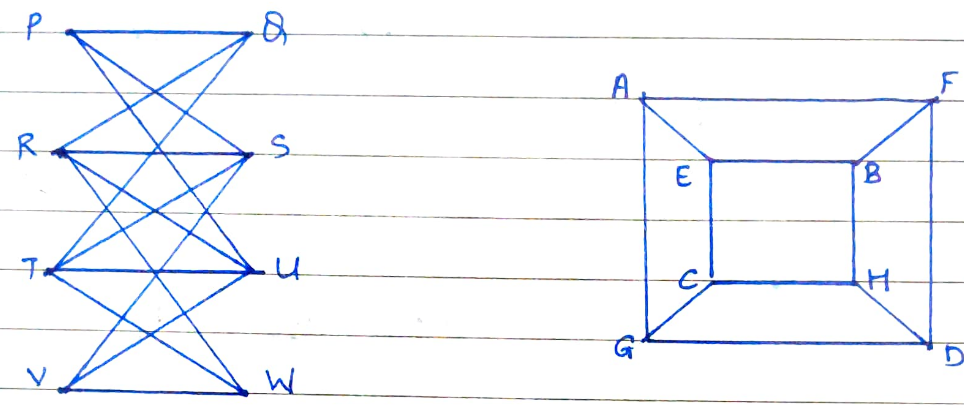
(i) The same number of vertices (7)

(ii) The same number of edges (9)

(iii) Three vertices of degree two [G has B, C and G ; G' has 2,3 and 4] four vertices of degree three [G has A, D, E, F, G' has 1, 5, 6, 7]

Also, adjaceny is preserved.
we can define one-to-one correspondence as follows:
A → 1, B → 2, C → 3, D → 6, E → 5, F → 7, G → 4
Hence the graphs are isomorphic.

(D)



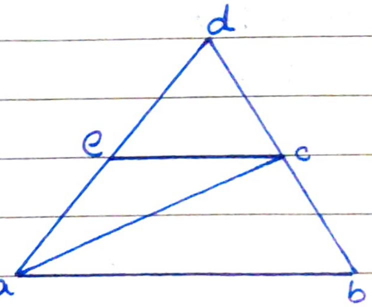
Both graphs have 8 vertices and 12 edges. In each graph, each vertex is of degree three. Also, adjacency is preserved. Since all four conditions are satisfied, the graphs are isomorphic. we can also obtain the second figure from the first by interchanging the vertices R and S. Also, interchanging the vertices T, U and putting them down, so that the vertices V and W are inside the square.

The required correspondence is :

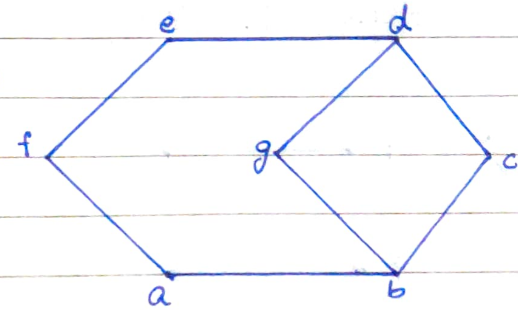
A → P , F → Q ,
E → S , B → R ,
C → V , H → W ,
G → U , D → T ,

Q.2 Determine whether the graphs below have a Hamiltonian circuit, Eulerian circuit. If so, find them.

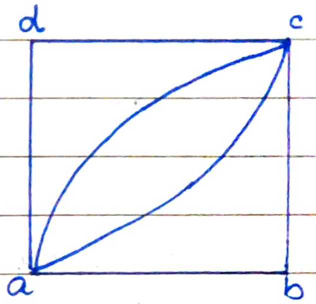
A



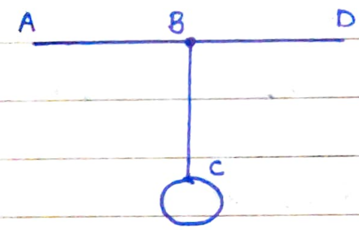
B.



C.



D.



SOLUTION:

A) In this graph there are three groups of vertices. In the first group, the vertices a and b are of degree 2, in the second group, the vertices e and a are of degree 3. and in the third group, the vertex c has a degree 4.

Although the vertices d, b and c are of even degree, the remaining 2 vertices e and a are of odd degree 3. Hence the graph is not Eulerian. Therefore, the graph doesn't have a Eulerian circuit.

In the graph, the number of vertices $n = 5$. The sum of the degree of each pair of vertices is 5 or greater than 5 [equal to $(n-1)$]

Hence there exists a Hamiltonian circuit.

$\therefore$ T_1 : e, d, c, b, a, e is a Hamiltonian circuit.

B) In the graph, there are two groups of vertices. In the first group, the vertices a, c, g, e and f are of degree 2 and in second group the vertices b and d are of degree 3. Although the vertices a, c, g, e, f are of even degree, the remaining 2 vertices b and d are of odd degree 3. Hence the graph is not Eulerian. Therefore the graph doesn't have a eulerian circuit.

In the graph, the number of vertices $n = 7$, the sum of the degree of each pair of vertices is less than $(n-1)$. Therefore, according to the Hamiltonian ~~circuit~~ theorem, for the given graph, Hamiltonian circuit doesn't exist.

c) In this graph there are two groups of vertices. In the first group, the vertices b and d have degree 2 and the second group of vertices c, a are of degree 4. All the vertices are of even degree. Hence according to Euler's 1st theorem, the eulerian circuit for this graph exists.

π : a, d, c, a, c, b, a is the Eulerian circuit.

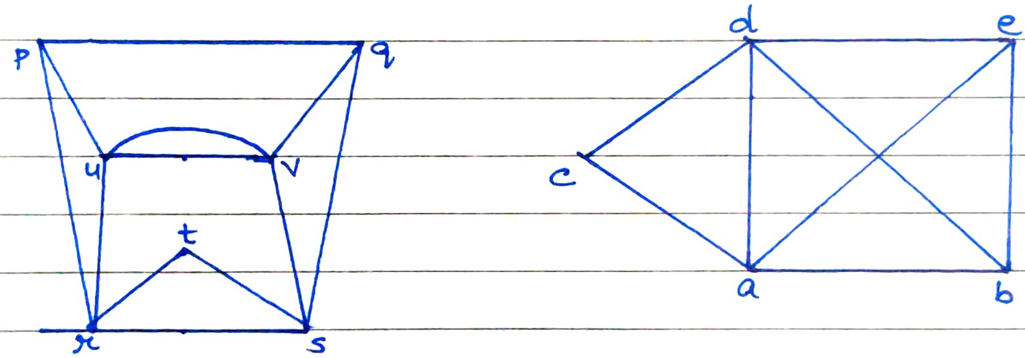
In the graph, the number of vertices $n = 4$, the sum of the degree of each pair of vertices is greater than $(n-1)$ i.e 3. Therefore according to Hamiltonian theorem 1, there exists a Hamiltonian circuit.

π : a, d, c, b, a is the Hamiltonian circuit.

D) The graph contains vertices with odd degree. Hence theorem 1 is not satisfied and hence it is not a Eulerian graph.
Since the graph is not Eulerian, it doesn't contain a Eulerian circuit.

There is no Hamiltonian circuit which contains all the vertices exactly once.
∴ There is no Hamiltonian circuit for the given graph.

Q3. Find out the Eulerian path, Hamiltonian path, if they exist in the following graphs.

A) 

Ans A. In this graph, there are three groups of vertices. The first group of vertices p, q have degree 3, second group of vertex t having degree 2 and third group of vertices u, v, r, s are of degree 4. There are two odd degree vertices p and q. According to Euler's theorem 2, there exists a Eulerian path if there are exactly two odd degree vertices.

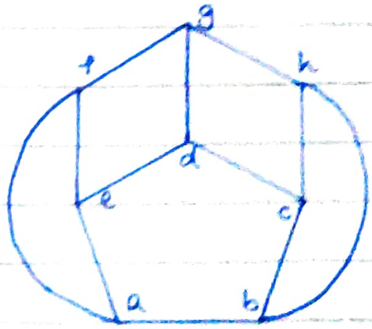
π : p, u, v, q, s, v, u, r, t, s is the Eulerian path.
Hamiltonian path is a path that contains each vertex exactly once. Here, the Hamiltonian path is
π : p, u, v, q, s, t, r contains each vertex exactly once. Hence its a Hamiltonian path.

B) In this graph there are three groups of vertices. The first group of vertices d, a are of degree 4, second group of vertices e, b are of degree 3 and the third group of vertex c of degree 2. There are two odd vertices b, e. According to Euler's theorem 2, there exists a Eulerian path if there are only two odd degree vertices.
∴ The Euler path is; π : e, d, b, a, d, c, a.

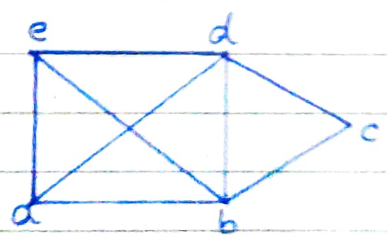
Hamiltonian path is a path that contains each vertex exactly once. In the graph, the Hamiltonian path is π : a, b, e, d, c as it contains each vertex only once.

Q.4. Discuss whether the following graph has Hamiltonian path, Hamiltonian circuit.

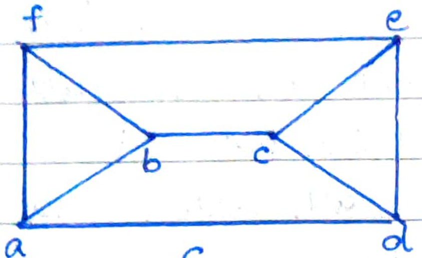
(A)



(B)

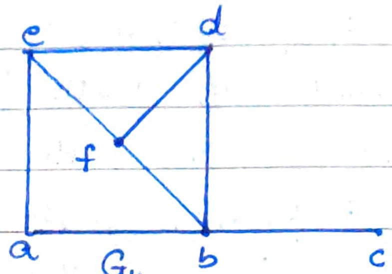


Ⓒ



G_3

Ⓓ



G_4

SOLUTION:

ANS A) G_1 is a simple graph with $n=8$ vertices. The degree of each vertex is $3 \times (n/2) = 4$.
Hence, by theorem 2, there is no Hamiltonian circuit. and hence no hamiltonian (according to theorem 2).
But there are Hamiltonian paths. One of them is :
π : a, b, c, d, e, f, g, h. However, we can clearly see the Hamiltonian circuit $\boxed{\text{abedghcba}}$ hence it is a hamiltonian graph.

ANS B) The graph G2 has 6 vertices a, b, c, d, e, f with degree 3, 4, 2, 4, 3, 4. The sum of the degrees of any two vertices is greater than or equal to $(n-1) = 5$. Hence by theorem 1; there is a Hamiltonian circuit and hence it is a Hamiltonian graph.

π = a, f, b, c, d, e, a

Hamiltonian path is :-

π : a, f, b, c, d, e

ANS c) The graph G_3 has 6 vertices a, b, c, d, e, f with 2, 3, 3, 2, 2, 2 degrees respectively.
Since the degree 2 of a, f, d, e is less than $6/2 = 3$, the graph does not have a Hamiltonian circuit.
∴ The graph is not Hamiltonian. However, there is a Hamiltonian path.

π : a, f, b, c, d, e.

ANS D) The graph G_4 has 6 vertices a, b, c, d, e, f with 2, 4, 1, 3, 3, 3 degrees respectively. The sum of the degrees of a and c is $2+1=3$ is less than $(n-1)=5$. Hence there is no Hamiltonian circuit.
The Hamiltonian path is :

π : a, e, d, f, b, c //

* **THEOREM 1:** If G is a simple connected graph with n vertices and if the sum of the degrees of each pair of vertices is greater than or equal to $(n-1)$, then there exists a Hamiltonian path is G.

* **THEOREM 2:** In a simply connected graph G (with no loops) with n vertices, if the degree of each vertex is greater than or equal to $n/2$, then G will contain a Hamiltonian circuit.