PHYS2155 Methods in physics II

Assignment 3

Due date: 5:00pm, April 1, 2021

Give answers and explanations for the following questions.

1. (20 marks) (a) Evaluate the integral

$$\int_C (1 + x^2 y^2)\, \mathrm{d}s \tag{1}$$

for the curve $C : \mathbf{r}(t) = t\,\hat{\imath} + 2\sqrt{2}\, e^{t/2}\hat{\jmath} + e^t \hat{k}$, $0 \le t \le 2$.
(b) Evaluate

$$\int_C (x^4 + y^4)\, \mathrm{d}s \tag{2}$$

for the same curve.

2. (20 marks) (a) The center of a sphere of radius a is at the origin of some coordinate system. Find the area of the part of the sphere that $x \ge a - h$, where $0 \le h \le a$.
(b) Prove that the area of the part of the paraboloid $z = x^2 + y^2$ that lies under the plane $z + 2y = 8$ is

$$\int_0^{2\pi} \int_0^3 \sqrt{4r^2 - 8r \sin\theta + 5}\, r\, \mathrm{d}r\, \mathrm{d}\theta \ . \tag{3}$$

3. (20 marks) (a) A force field on the plane is given by

$$\mathbf{F} = \frac{1}{r}\,\hat{\theta} \ , \tag{4}$$

where r and θ are the polar coordinates. Calculate $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$ where C is the straight line segment $x = l$ and $-l \tan(\pi/8) \le y \le l \tan(\pi/8)$ in terms of Cartesian coordinates x and y.
(b) Hence, prove that $\mathbf{F}$ is not conservative. (No credit will be given if part (a) is not used.)

4. (20 marks) (a) Determine whether the vector field $\mathbf{F}$

$$\mathbf{F}(x, y) = \left[\frac{\ln(\ln y)}{x} + \frac{2}{3} x y^3\right] \hat{\imath} + \left(\frac{\ln x}{y \ln y} + x^2 y^2\right) \hat{\jmath} \tag{5}$$

is conservative. If so, find a potential function ϕ such that $\mathbf{F} = \nabla\phi$.
(b) Is it correct to say that the line integral of $\mathbf{F}$ along the straight line from $(x, y) = (-2, 2)$ to $(x, y) = (2, 2)$ is $\phi(2, 2) - \phi(-2, 2)$, where ϕ is the potential one found in part (a).

5. (20 marks) (a) Describe the surface S, defined by $0 \leq x \leq R$, $0 \leq y \leq x$ and $z = R$, using spherical coordinates r, θ and ϕ. (Students should express r as a function of θ and ϕ, and also specify the ranges of θ and ϕ.)
   (b) Calculate the double integral

$$\iint_S \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi \ . \tag{6}$$