Tercera Práctica Calificada

Solucionario

Resolver las siguientes ecuaciones diferenciales:

1. $$y''' - 2y' + 4y = x^2 + \operatorname{sen}x + e^x\cos x \quad \ldots(\mathrm{I})$$

Solución: $$(D^3-2D+4)y = (D+2)(D^2-2D+2)y = x^2+\operatorname{sen}x+e^x\cos x \ldots(\mathrm{II})$$

a) y_H: $(r+2)(r^2-2r+2)=0 \Rightarrow r+2=0 \;,\; (r-1)^2+1=0$

$\Rightarrow r_1=-2 \;,\; r_{2,3}=1\pm i(1)$

$$\Rightarrow \quad y_H = c_1e^{-2x}+(c_2\cos x + c_3 \operatorname{sen}x)e^x \quad \ldots(1)$$

b) y_p: De (II): $$y_p = \frac{1}{4-2D+D^3}{}^{*}x^2 + \frac{1}{\overset{D^2=-1^2}{D^2}.D-2D+4}\operatorname{sen}x + \frac{1}{((D-1)^2+1)(D+2)}\overset{D:D+1}{e^x\cos x}$$

$$\Rightarrow y_p = \frac14\Big(1+\frac12 D+\frac14 D^2 - \underbrace{\frac18 D^3 \ldots}_{=0}\Big)x^2 + \frac{1}{(-1).D-2D+4}\operatorname{sen}x + e^x.\frac{1}{(D^2+1)(D+3)}\cos x$$

$$\Rightarrow y_p = \frac14\Big(x^2+\frac14 \overbrace{Dx^2}^{2x} + \frac14 \overbrace{D^2x^2}^{2} + 0\Big) - \frac{1}{3D-4}\operatorname{sen}x + e^x.\frac{1}{D^2+1}\Big[(D-3).\frac{1}{\underset{D^2=-1^2}{D^2-3^2}}\cos x\Big]$$

$$\Rightarrow y_p = \frac14\Big(x^2+\frac14(2x)+\frac14(2)\Big) - (3D+4).\frac{1}{9\overset{D^2=-1^2}{D^2}-16}\operatorname{sen}x + e^x.\frac{1}{D^2+1}\Big[(D-3).\frac{1}{\underbrace{-1^2-3^2}_{-10}}\cos x\Big]$$

$$\Rightarrow y_p = \frac14\Big(x^2+\frac12 x+\frac12\Big) - (3D+4).\frac{1}{\underbrace{9(-1)-16}_{-25}}\operatorname{sen}x - \frac{1}{10}e^x.\frac{1}{D^2+1}(D-3)\cos x$$

$$\Rightarrow y_p = \frac{2x^2+x+1}{8} + \frac{1}{25}\big(3\overbrace{D\operatorname{sen}x}^{\cos x}+4\operatorname{sen}x\big) - \frac1{10}e^x.\frac{1}{D^2+1}\big(\overbrace{D\cos x}^{-\operatorname{sen}x}-3\cos x\big)$$

$$\Rightarrow y_p = \frac{2x^2+x+1}{8}+\frac1{25}(3\cos x+4\operatorname{sen}x)+\frac1{10}e^x\Big[\frac{1}{D^2+1}\operatorname{sen}x + 3\frac{1}{D^2+1}\cos x\Big]$$

$$\Rightarrow y_p = \frac{2x^2+x+1}{8}+\frac{3\cos x+4\operatorname{sen}x}{25}+\frac{e^x}{10}\Big(-\frac{x\cos x}{2(1)}+3.\frac{x\operatorname{sen}x}{2(1)}\Big)$$

$$\Rightarrow \quad y_p = \frac{2x^2+x+1}{8}+\frac{3\cos x+4\operatorname{sen}x}{25}+\frac{xe^x}{20}(3\operatorname{sen}x-\cos x)$$

* $1 \div (D^3-2D+4) \Rightarrow$

$$\begin{array}{l|l}
1 & 4-2D+D^3 \\ \hline
-1+\frac12 D \quad -\frac14 D^3 & \frac14+\frac18 D+\frac1{16}D^2-\frac1{32}D^3- \\
\quad -\frac12 D+\frac14 D^2 \quad -\frac18 D^4 & \\
\quad\quad -\frac14 D^2+\frac18 D^3 \quad -\frac1{16}D^5 & \\
\quad\quad\quad -\frac18 D^3-\frac18 D^4-\frac1{16}D^5 & \\
\quad\quad\quad \frac18 D^3-\frac1{16}D^4+\frac1{32}D^5 & \\
\end{array}$$

1. b)

$$D^4(D^2-1)y = x^3 + 4e^{-x} \quad \ldots(\mathrm{I})$$

i) y_H:

$$r^4(r+1)(r-1)=0 \Rightarrow r_{1,2,3,4}=0,\ r_5=-1,\ r_6=1$$

$$\Rightarrow \quad y_H = c_1x^3+c_2x^2+c_3x+c_4 + c_5e^{-x}+c_6e^{x} \quad \ldots(1)$$

ii) y_p: De (I), $y_p = \dfrac{1}{D^4}\cdot\dfrac{1}{D^2-1}x^3 + 4\cdot\dfrac{1}{(D+1).\underbrace{D^4(D-1)}_{P(D)}}e^{-x}$

$$\Rightarrow \quad y_p = \frac{1}{D^4}\left[-(1+D^2+D^4+D^6+\ldots)x^3 + e^{-x}\cdot\frac{x}{1!(-1)^4(-2)}\right.$$

$$\Rightarrow \quad y_p = -\frac{1}{D^4}\left(x^3 + \underbrace{D^2x^3}_{6x} + \underbrace{D^4x^3 + D^6x^3+\ldots}_{=0}\right) - \frac{xe^{-x}}{2}$$

$$\Rightarrow \quad y_p = -\iiiint(x^3+6x)\,dx\,dx\,dx\,dx - \frac{xe^{-x}}{2}$$

$$y_p = -\frac{x^7}{4.5.6.7} - \frac{xe^{-x}}{2}$$

1. c)

$$y'' + y' = \sec x\tan x \quad \ldots(\mathrm{I})$$

<u>Solución:</u>

$$(D^2+D)y = D(D+1)y = \sec x\tan x \quad \ldots(\mathrm{II})$$

$$r_1=0,\ r_2=-1$$

a) y_H:

$$y_H = c_1e^{0x}+c_2e^{-x} = c_1+c_2e^{-x} \quad \ldots(1)$$

$$\Rightarrow \quad y_p = c_1(x).1 + c_2(x).e^{-x} \quad \ldots(\alpha)$$

b) y_p:

donde: $W[y_1,y_2] = \begin{vmatrix} 1 & e^{-x} \\ 0 & -e^{-x} \end{vmatrix} \Rightarrow W[y_1,y_2] = -e^{-x}$

b₁) $c_1(x) = -\displaystyle\int\frac{y_2\,Q(x)}{a_0(x)W[y_1,y_2]}dx = -\int\frac{e^{-x}.\sec x\tan x\,dx}{1.(-e^{-x})} = \int\sec x\tan x\,dx$

$$\Rightarrow \quad c_1(x) = \sec x$$

b₂) $c_2(x) = \displaystyle\int\frac{y_1\,Q(x)}{a_0(x)W[y_1,y_2]}dx = \int\frac{1.\sec x\tan x}{1.(-e^{-x})}dx = \int\underbrace{e^{x}}_{u}.\underbrace{\sec x\tan x\,dx}_{dv}$

$du = e^{x}dx,\ v = \sec x$

$$\Rightarrow \quad c_2(x) = e^{x}\sec x - \int e^{x}\sec x\,dx$$

$\Rightarrow$ En (α): $y_p = \sec x + \left(e^{x}\sec x - \int e^{x}\sec x\,dx\right)e^{-x}$

$$y_p = 2\sec x - e^{-x}\int e^{x}\sec x\,dx$$

2. Resolver la ecuación: $\underbrace{x^4}_{a_0(x)}y'' + \underbrace{x^3}_{a_1(x)}y' - 4x^2y = 1 \quad \ldots(I)$

$y_1 = x^2$

Solución: La ecuación diferencial homogénea asociada la ecuación propuesta es: $x^4y'' + x^3y' - 4x^2y = 0 \quad \ldots(II)$

siendo una de sus soluciones $y_1 = x^2 \ldots(1) \Rightarrow y_2 = u(x)x^2 \ldots(2)$, también será otra solución, donde:

$$u = u(x) = \int \frac{e^{-\int \frac{a_1(x)}{a_0(x)}dx}}{y_1^2}dx = \int \frac{e^{-\int \frac{x^3}{x^4}dx}}{(x^2)^2}dx = \int \frac{e^{-\int \frac{dx}{x}}}{x^4}dx = \int \frac{e^{-\ln x}}{x^4}dx$$

$$\Rightarrow u = u(x) = \int \frac{x^{-1}}{x^4}dx = \int x^{-5}dx \Rightarrow u = -\frac{1}{4x^4}$$

$$\Rightarrow \text{en } (2): \quad y_2 = \frac{1}{x^2} \quad \ldots(3)$$

$$\text{De (1) y (3)}, \quad y_H = c_1x^2 + c_2 \cdot \frac{1}{x^2} \quad \ldots(\alpha)$$

$$\Rightarrow y_p: \quad y_p = c_1(x)x^2 + c_2 \cdot \frac{1}{x^2} \quad \ldots(\beta)$$

donde: $W[y_1, y_2] = \begin{vmatrix} x^2 & \frac{1}{x^2} \\ 2x & -\frac{2}{x^3} \end{vmatrix} = x^2\left(-\frac{2}{x^3}\right) - 2x\left(\frac{1}{x^2}\right) = -\frac{2}{x} - \frac{2}{x}$

$\Rightarrow W[y_1, y_2] = -\frac{4}{x}$

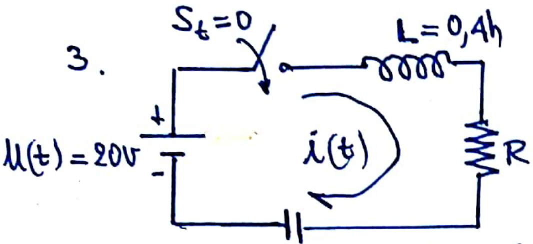
a) $$c_1(x) = -\int \frac{y_2\,Q(x)}{a_0(x)W[y_1,y_2]}dx = -\int \frac{\frac{1}{x^2}\cdot 1}{x^4 \cdot \left(-\frac{4}{x}\right)}dx = \frac{1}{4}\int x^{-5}dx = -\frac{x^{-4}}{16}$$

b) $$c_2(x) = \int \frac{y_1\,Q(x)}{a_0(x)W[y_1,y_2]}dx = \int \frac{x^2 \cdot 1}{x^4 \cdot \left(-\frac{4}{x}\right)}dx = -\frac{1}{4}\int \frac{dx}{x} \Rightarrow c_2(x) = -\frac{1}{4}\ln x$$

$c_1(x)$ y $c_2(x)$ en (β): $$y_p = \left(-\frac{x^{-4}}{16}\right)x^2 + \left(-\frac{1}{4}\ln x\right)\cdot\frac{1}{x^2}$$

$$y_p = -\frac{x^{-2}}{16} - \frac{\ln x}{4x^2} = -\frac{1 + 4\ln x}{16x^2}$$

3.



$\mathbb{C} = 0,001 f \Rightarrow \frac{1}{\mathbb{C}} = 10^3$
$= 10^{-3} f$

$$V_L + V_C = u(t)$$

$$L Q''(t) + \frac{1}{C} Q(t) = 20V$$

$$0,4 Q''(t) + 10^3 Q(t) = 20$$

$$\rightarrow \quad Q''(t) + 2500 Q(t) = 50 \quad \ldots (I)$$

$$(D^2 + 2500) Q(t) = 50 \quad \ldots (II)$$

a) $Q_H(t)$: $\quad r^2 + 2500 = 0 \Rightarrow r = \pm i50$

$$\Longrightarrow \quad Q_H(t) = C_1 \cos 50t + C_2 \operatorname{sen} 50t \quad \ldots (1)$$

b) $Q_p(t)$: De (II), $\quad Q_p(t) = 50 \cdot \frac{1}{D^2+2500} e^{0t} = 50 \cdot \frac{1}{0^2+2500} (1)$

$$Q_p(t) = \frac{1}{50} \quad \ldots (2)$$

$\Rightarrow$ De (1) y (2): $\quad Q(t) = C_1 \cos 50t + C_2 \operatorname{sen} 50t + \frac{1}{50} \quad \ldots (\alpha)$

$\rightarrow D_t$: $\quad i(t) = Q'(t) = -50 C_1 \operatorname{sen} 50t + 50 C_2 \cos 50t \quad \ldots (\beta)$

si $t=0$

i) En (α): $\quad Q(0) = C_1(1) + C_2(0) + \frac{1}{50} = 0 \Rightarrow C_1 = -\frac{1}{50}$

ii) En (β): $\quad i(0) = -50 \cancel{C_1}^{0}(0) + 50 C_2(1) = 0 \Rightarrow C_2 = 0$

$\rightarrow$ En (α) y (β): $\quad Q(t) = -\frac{1}{50} \cos 50t + \frac{1}{50}$

$$i(t) = \operatorname{sen} 50t$$