Lab 1: Circuit analysis methods

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Circuit Theory and Electronics Fundamentals

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March 25, 2021

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1 Introduction

The objective of this laboratory assignment is to study a circuit from three different methods and compare the outcomes. Those are: mesh analysis, node analysis and ngspice simulation. The circuit can be seen if Figure 1. Resorting to Ohm's Law, Kirchhoff Current Law (KCL) and Kirchhoff Voltage Law (KVL) it was possible to get all the information contained in the circuit.

From KCL we know that the sum of currents converging (diverging) in a node is null

$$\sum_{i=1}^{n} I_{\text{i}} = 0$$

From KVL we deduce that the sum of Voltages in a circuit loop is null

$$\sum_{i=1}^{n} V_{\text{i}} = 0$$

In Section 2, a theoretical analysis of the circuit is presented. In Section 3, the circuit is analysed by simulation, and the results are compared to the theoretical results obtained in Section 2. The conclusions of this study are outlined in Section 4.

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Figure 1: Lab01 Circuit

R1 (kOhm)	R2 (kOhm)	R3 (kOhm)	R4 (kOhm)	R5 (kOhm)
1.01072	2.0468	3.05917	4.1965	3.07880

R6 (kOhm)	R7 (kOhm)	Va (V)	Id (mA)	Kb (mS)	Kc (kOhm)
2.08426	1.01420	5.03865	1.01597	7.14517	8.13498

Table 1: Given Data Rounded Values

2 Theoretical Analysis

In this section, the circuit shown in Figure 1 is analysed theoretically in two different ways: Mesh Analysis and Node Analysis. After obtaining circuit values, to confirm the coherence between the results from these two analysis, the voltage and current values were also calculated using Ohm's Law $R = \frac{V}{I}$ and the definition of current/voltage controled voltage/current sources: $V_c = K_c \times I_c$ and $I_b = K_b \times V_b$, repectively.

2.1 Mesh Analysis

This type of circuit analysis implies that we atribuite a current (with arbitrary direction) to each elementar mesh. Since our circuit has four elementar meshes, we assingned I_a, I_b, I_c and I_d as depicted in Figure 1. For the last three currents, the directions were specially choosen so that they had the same signal as the currents from the current generators (b and d) and the scheme given.
Since the value of I_d was known, we found 3 linaerly independent equations to find the values of the missing currents and derived the following matrix:

$$\begin{bmatrix} R_1 + R_4 + R_3 & R_3 & R_4 \\ R_4 & 0 & R_6 + R_7 + R_4 - K_c \\ K_b \times R_3 & K_b \times R_3 - 1 & 0 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix} \tag{1}$$

The first two lines of the matrix are directly derived from analysing the path of the current in the respective elementar mesh. The third line comes from finding a relation between the currents I_b and I_a, substituting voltage V_b that goes through the resistor 3:

$$I_b = K_b \times V_b \Longleftrightarrow I_b = K_b \times R_3 \times (I_a + I_b) \Leftrightarrow I_a(K_b R_3) + I_b(K_b R_3 - 1) = 0 \tag{2}$$

This system of linear equations was solved using the tool Octave and the values obtained can be consulted in Table 2.

2.2 Node Analysis

The Node Analysis was based on Kirchhoff Current Law (KCL) and Ohm Law. From node 1 to 4 and 6 to 7 KCL was applied and resulted in an equation for each node.

In order to obtain the equations of nodes 5 and 8, since between them there is a current-controlled voltage source, it isn't possible to write the node equation from KCL and we are facing a special case in this node analysis.

This case was solved with the help of Supernode analysis. This theoretical concept considers two nodes as just one, in between those nodes it must be placed a voltage source. In this particular case, we first conclude that the voltage difference between node 5 and node 8 is V_c. Then, we consider nodes 5 and 8 as just one node and apply Kirchhoff Current Law since the current that leaves node 5 is the current that arrives at 8 hence the total current in the new node is null. Now, we are able to apply the KCL to the supernode.

From the supernode analysis, we were able to find two equations

$$V5 - V8 = V_{\mathsf{c}} \tag{3}$$

in which Vc can be rewritten as

$$V_{\mathsf{c}} = Kc \times Ic = Kc \times (V7 - V6)/R6 \tag{4}$$

In addition, by applying KCL to the supernode we get

$$\frac{V5 - V6}{R4} + \frac{V5 - V2}{R3} + \frac{V5 - V4}{R5} + \frac{V8 - V7}{R7} - Id = 0 \tag{5}$$

Given that the ground was placed at node 6 the voltage in that node is null (V6 = 0 V).

The following matrix system has the information of the equations in all the nodes and its resolution leads us to the voltage value in each node (Table 2).

Gi represents 1/Ri and Vi is the voltage in node i.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0. \\ -G1 & G1+G2+G3 & -G2 & 0 & -G3 & 0 & 0 \\ 0 & -G2-Kb & G2 & 0 & Kb & 0 & 0 \\ 0 & Kb & 0 & G5 & -Kb-G5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & Kc \times G6 & -1 \\ 0 & 0 & 0 & 0 & 0 & G6+G7 & -G7 \\ 0 & -G3 & 0 & -G5 & G3+G4+G5 & -G7 & G7 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V7 \\ V8 \end{bmatrix} = \begin{bmatrix} Va \\ 0 \\ 0 \\ Id \\ 0 \\ 0 \\ -Id \end{bmatrix} \tag{6}$$

2.3 Values and Comparison

Name	Value [A or V]
Ia	1.936080e-04
Ib	-2.028901e-04
Ic	9.672069e-04
Id	1.015970e-03
V1	5.038651e+00
V2	4.842967e+00
V3	4.427687e+00
V4	8.623984e+00
V5	4.871363e+00
V6	0.000000e+00
V7	-2.015911e+00
V8	-2.996850e+00

Table 2: Table with values of current (A) and voltage (V) obtained from mesh and node analysis of the circuit using the tool Octave.

Calculating voltages using the currents from mesh analysis to compare with node analysis, we get:

$$V_b = R_3 \times (I_a + I_b) = \frac{I_b}{K_b} = -0.0283956V = V_2 - V_5(Table2) \tag{7}$$

$$V_c = K_c \times I_c = 7.868213V = V_5 - V_7(Table2) \tag{8}$$

In order to compare the voltage values to the mesh analysis, the currents were determined by the following equations

$$I_{\mathsf{a}} = \frac{V1 - V2}{R1} = 1.936 \times 10^{-4} A \quad (9)$$

$$I_{\mathsf{b}} = Kb \times (V2 - V5) = -2.0289 \times 10^{-4} A \quad (10)$$

$$I_{\mathsf{c}} = \frac{V7}{R6} = 9.6721 \times 10^{-4} A \quad (11)$$

These calculations make it possible to conclude that the two analysis are equivelent and lead to the same results.

3 Simulation Analysis

3.1 Operating Point Analysis

The Ngspice Software was used to simulate the circuit. Considering the simplicity of the circuit only Operating Point Analysis was performed. There was the need to add a virtual voltage source which would sense the controlling current I_c. For that reason, node number 7 was virtually created between node 6 and node 8. Table 3 shows the simulated operating point results for the circuit under analysis. As it can be seen, there are no differences compared to the theoretical analysis results. Take note that gb represents I_b and $id = I_d$, $v(i)$ represents the voltage in node i. Once there's an extra node, node labelling has been changed.

The nodes are numbered according to figure 2. The plus and minus signs in the resistors terminals are used to evidence the current direction choosen for each branch, i.e., it's assumed a priori the direction of the current pointing from plus to minus sign. For the other components it's used the current direction according to the arrow presented or the plus/minus signs already drawn in the figure 1.

Name	Value [A or V]

Table 3: Operating point. A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

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Figure 2: Ngspice circuit scheme representation

4 Conclusion

In this laboratory assignment the objective of analysing a simple circuit has been achieved. Static analysis has been performed both theoretically using the Octave maths tool and by circuit simulation using the Ngspice tool. The results obtained by octave in both the nodal and mesh theoretical analysis matched each other. The simulation results matched the theoretical results precisely. The reason for this perfect match is the fact that this is a straightforward circuit containing only linear components, so the theoretical and simulation models cannot differ.