Functional Dependency

Functional Dependency

A "good" database schema should not lead to *update anomalies*.

- update anomalies,
- functional dependencies,
- Armstrong Axioms,
- closures.

Update Anomalies

Redundancy in a database means storing a piece of data more than once.

Redundancy is often useful for efficiency and semantic reasons, but creates the potential for consistency problems.

A poor *redundancy control* may cause update anomalies.

Consider the example relation below (adapted from "An Introduction to Database Systems" by Desai):

STUDENTS					
Name	Course	Phone_no	Major	Prof	Grade
Jones	353	237-4539	Comp Sci	Smith	A
Ng	329	427-7390	Chemistry	Turner	B
Jones	328	237-4539	Comp Sci	Clark	B
Martin	456	388-5183	Physics	James	A
Dulles	293	371-6259	Decision Sci	Cook	C
Duke	491	823-7293	Mathematics	Lamb	B
Duke	356	823-7293	Mathematics	Bond	UN
Jones	492	237- 4539	Comp Sci	Cross	UN
Baxter	379	839-0827	English	Broes	C

Update Anomalies

Modification anomalies: e.g. Jones's phone number appears 3 times. When a phone number is changed, it must be changed in all 3 places, or the data will be inconsistent.

Insertion anomalies:

- If Jones enrolls in another course, and a different phone number is entered, again the data will be inconsistent.

- Also, if the only way that the association between course and professor is stored in this relation, we can only enter the association when someone enrolls in the course.

STUDENTS					
Name	Course	Phone_no	Major	Prof	Grade
Jones	353	237-4539	Comp Sci	Smith	A
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~~Baxter~~	~~379~~	~~839-0827~~	~~English~~	~~Broes~~	~~C~~

Update Anomalies

Deletion anomalies: If the last student in a course is deleted, the association between professor and course is lost.

Functional dependencies

A function f from S_1 to S_2 has the property

$$if\ x, y\ \in S_1 \text{ and } x = y, \text{then } f(x) = f(y)\ .$$

A generalization of keys to avoid design flaws violating the above rule.

Let X and Y be sets of attributes in R.

X (*functionally*) determines Y , $X \rightarrow Y$, iff $t_1[X] = t_2[X]$ implies $t_1[Y] = t_2[Y]$.

i.e., f (t(X)) = t [Y]

We also say $X \rightarrow Y$ is a *functional* dependency, and that Y is *functionally* dependent on X.

X is called the *left side*, Y the *right side* of the dependency.

Examples

- For every Name, there is a unique Phone_no and Major, assume Name is unique;
- For every Course, there is a unique Prof;
- For every Name and Course, there is a unique Grade.

STUDENTS					
Name	Course	Phone_no	Major	Prof	Grade
Jones	353	237-4539	Comp Sci	Smith	A
Ng	329	427-7390	Chemistry	Turner	B
Jones	328	237-4539	Comp Sci	Clark	B
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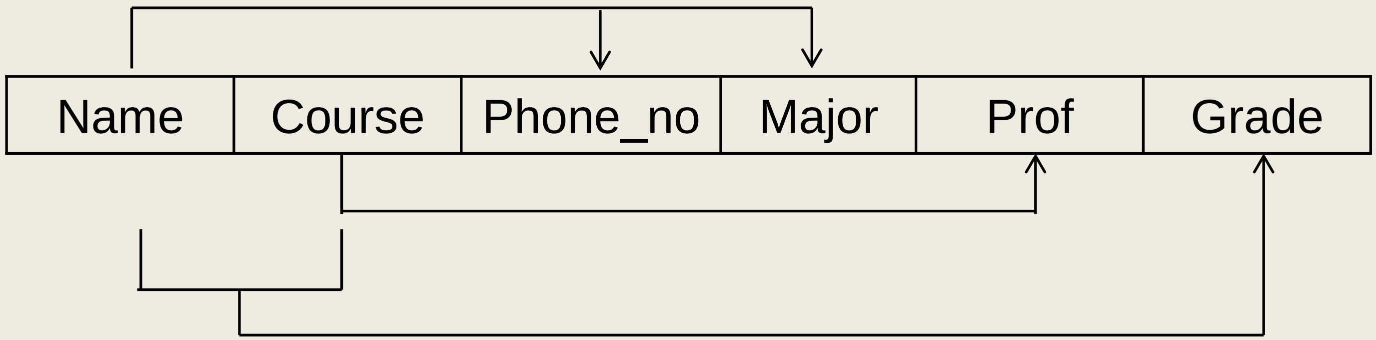
In this example:

$$\{Name\} \rightarrow \{Phone\_no\ , Major\}$$

$$\{Course\} \rightarrow \{Prof\}$$

$$\{Name\ , Course\} \rightarrow \{Grade\}$$

We can also show these in a diagram like this one:



Notice that other FD's follow from these:

$$\{Name\} \rightarrow \{Major\}$$

$$\{Course\ , Grade\} \rightarrow \{Prof\ , Grade\}$$

Functional dependencies

Let F be a set of FD's.

Definition 1: $X \rightarrow Y$ is inferred from F (or that F infers $X \rightarrow Y$), written in

$$F \models X \rightarrow Y$$

if any relation instance satisfying F must also satisfy $X \rightarrow Y$.

Impossible to list every relation to verify if $X \rightarrow Y$ is inferred from F.

A set ρ of derivation rules are required, such that:

a $X \rightarrow Y$ is inferred from F according to Definition 1 iff it can be derived using ρ.

Armstrong’s axioms (1974)

Notation: If X and Y are sets of attributes, we write XY for their union.

e.g. X = {A, B}, Y = {B, C}, XY = {A, B, C}

F1 (Reflexivity) If $X \supseteq Y$ then $X \rightarrow Y$.

F2 (Augmentation) $\{X \rightarrow Y\} \models XZ \rightarrow YZ$.

F3 (Transitivity) $\{X \rightarrow Y, Y \rightarrow Z\} \models X \rightarrow Z$.

F4 (Additivity) $\{X \rightarrow Y, X \rightarrow Z\} \models X \rightarrow YZ$.

F5 (Projectivity) $\{X \rightarrow YZ\} \models X \rightarrow Y$.

F6 (Pseudotransitivity) $\{X \rightarrow Y, YZ \rightarrow W\} \models XZ \rightarrow W$.

Example: Given F = $\{A \rightarrow B, A \rightarrow C, BC \rightarrow D\}$, derive $A \rightarrow D$:

1. $A \rightarrow B$ (given)

2. $A \rightarrow C$ (given)

3. $A \rightarrow BC$ (by F4, from 1 and 2)

4. $BC \rightarrow D$ (given)

5. $A \rightarrow D$ (by F3, from 3 and 4)

F4 (Additivity) $\{X \rightarrow Y, X \rightarrow Z\} \models X \rightarrow YZ$.

F5 (Projectivity) $\{X \rightarrow YZ\} \models X \rightarrow Y$.

F6 (Pseudotransitivity) $\{X \rightarrow Y, YZ \rightarrow W\} \models XZ \rightarrow W$.

In fact, F4, F5, and F6 can be derived from F1-F3.

Example: Prove F4 $\{X \rightarrow Y, X \rightarrow Z\} \models X \rightarrow YZ$.

1) $X \rightarrow Y$ is given.

2) $XX \rightarrow XY$ (by F2); that is, $X \rightarrow XY$

3) $X \rightarrow Z$ is given.

4) $XY \rightarrow YZ$ (by F2)

5) $X \rightarrow YZ$ (by F3, 2) and 4))

Prove: F5 $\{X \to Y Z\} \models X \to Y$

1. $X \to Y Z$ (given)

2. $YZ \to Y$ (by F1)

3. $X \to Y$ (by F3, 2 and 3)

Prove: F6 $\{X \rightarrow Y, YZ \rightarrow W\} \models XZ \rightarrow W$

1. $X \rightarrow Y$ (given)

2. $XZ \rightarrow YZ$ (F2)

3. $YZ \rightarrow W$ (given)

4. $XZ \rightarrow W$ (F3, 2 and 3)

Armstrong's axioms

We can prove that Armstrong's axioms are sound and complete:

Sound: if F derives $A \rightarrow B$ by using Armstrong's axioms, then F $\models A \rightarrow B$ by Definition 1.

Complete: if F $\models M \rightarrow N$ by Definition 1, then F derives $M \rightarrow N$ by using Armstrong's axioms.

Algorithm to Check a FD

Given F, how do we check if $X \rightarrow Y$ is in F^+?

F^+ denotes the smallest set of FD's that

- contains F, and
- is *closed* under Armstrong's axioms.

F^+ is the *closure* of F.

$F = \{ A \rightarrow B, B \rightarrow C, A \rightarrow C \}$

F^+ = {AB -> A, AB -> B, AB -> C, AC -> A, AC -> B, AC -> C, ABC -> A, ABC -> B, ABC -> C, AB -> AB, AB -> BC, AB -> AC, …….}

F^+ always has an exponential size regarding |F|.

Too expensive to compute F^+ to verify a membership.

Instead we can compute the *closure* X^+ of X under F, X^+ is the largest set of attributes functionally determined by X.

It can be proven (using additivity) that

S1: $\boxed{X^+ = \cup_{\forall X \to A \in F^+} A.}$

S2: $\boxed{X \to Y \in F^+ \text{ iff (if and only if) } Y \subseteq X^+.}$

Example:

$F = \{ A \rightarrow B, BC \rightarrow D, A \rightarrow C \}$, compute $\{A\}^+$

1st scan of F:

$X^+ := \{A\}$

$X^+ := \{A, B\}$

$X^+ := \{A, B, C\}$

2nd scan of F:

$X^+ := \{A, B, C, D\}$

3rd scan of F: no change, therefore the algorithm terminates.

$\{A\}^+ := \{A, B, C, D\}$

Algorithm to compute X^+

$X^+ := X$;

change := true;

while change do

begin

change := false;

for each FD $W \rightarrow Z$ in F do

begin

if ($W \subseteq X^+$) and ($Z \nsubseteq X^+$) then do

begin

$X^+ := X^+ \cup Z$;

change := true;

end

end

end

Algorithm to Compute a Candidate Key

Given a relational schema R and a set F of functional dependencies on R.

A key X of R must have the property that $X^+ = R$.

Algorithm to compute a candidate key

Step 1: Assign X a superkey in F.

Step 2: Iteratively remove attributes from X while retaining the property $X^+ = R$ till no reduction on X.

The remaining X is a key.

Example:

R = {A, B, C, D} and F = { A $\rightarrow$ B, BC $\rightarrow$ D, A $\rightarrow$ C }

X = {A, B, C} if the left hand side of F is a super key.

A cannot be removed because $\{BC\}^+ = \{B, C, D\} \neq R$

B can be removed because $\{AC\}^+ = \{A, B, C, D\} = R$

$\Rightarrow$ X = { A, C}

C can be further removed because $\{A\}^+ = \{A, B, C, D\}$

$\Rightarrow$ X = {A}

Algorithm to Compute All the Candidate Keys

Given a relational schema R and a set F of functional dependencies on R, the algorithm to compute all the candidate keys is as follows:

T := ∅

Main:

X := S where S is a super key which does not contain any candidate key in T

remove := true

While remove do

For each attribute A ∈ X

Compute $\{X\text{-}A\}^+$ with respect to F

If $\{X\text{-}A\}^+$ contains all attributes of R then

X := X – {A}

Else

remove := false

T :=T ∪ X

Repeat *Main* until no available S can be found. Finally, T contains all the candidate keys.

Algorithm to Compute All the Candidate Keys

Example:

R(A, B, C, D, E)

F = {A → B, BC → A, D → E}

Step 1:

Let X := {A, B, C, D}

Step 2:

Try to remove A

$\{B, C, D\}^+ = \{A, B, C, D, E\}$

Thus X := {B, C, D}

Algorithm to Compute All the Candidate Keys

Step 3:

Try to remove B, C, D

$\{C, D\}^+ = \{C, D, E\}$

$\{B, D\}^+ = \{B, D, E\}$

$\{B, C\}^+ = \{A, B, C\}$

Thus cannot be removed

So {B, C, D} is a candidate key and add to T

Step 4:

Find another superkey

Let X := {A, C, D}

Algorithm to Compute All the Candidate Keys

Step 5:

Try to remove A, C, D

$\{C, D\}^+ = \{C, D, E\}$

$\{A, D\}^+ = \{A, B, D, E\}$

$\{A, C\}^+ = \{A, B, C\}$

Thus cannot be removed

So {A, C, D} is another candidate key and add to T

Step 6:

Cannot find any other super keys,

So candidate keys are {B, C, D} and {A, C, D}

Learning Outcomes

- Armstrong’s axioms
- The algorithm to check if a FD can be derived by a given FD set
- Compute one candidate key
- Compute all candidate keys