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Question: 120 Dynamics of Structures 4.2 Compute the vertical motion o...

120 Dynamics of Structures

4.2 Compute the vertical motion of the car shown schematically in Figure 4.32 when it is crossing a bridge at a velocity of 60 km/h. The spring stiffness was evaluated by a test during which the measured vertical displacement of the car was 2.5 mm when its weight was increased by 500 N. The bridge profile is idealized by a sinusoidal curve with a wave length of 12 m and a half-amplitude of 30 mm. Consider a damping ratio of 50% of critical.

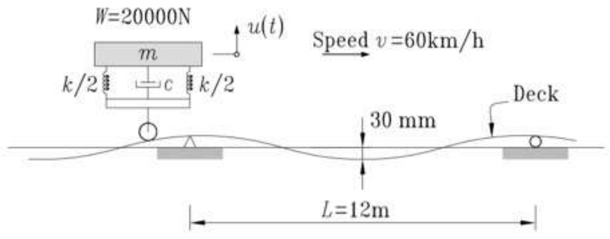

Figure 4.32. *Problem 4.2*

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Expert Answer

Anonymous answered this
353 answers

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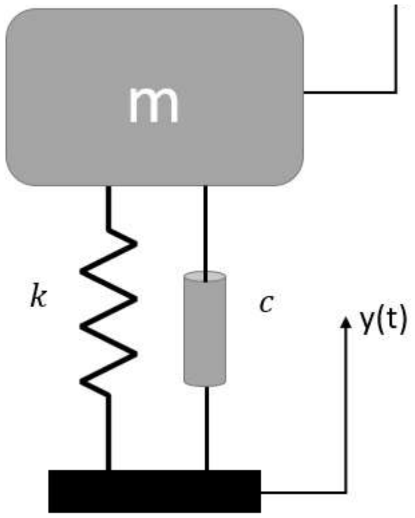
Given;

$$v = 60kmph$$
$$\Delta F = 500N$$
$$\Delta x = 2.5mm$$
$$W = 20000N$$
$$\lambda = 12m$$
$$A = 30mm$$

Solution



$$m = \frac{W}{9.81} = \frac{20000}{9.81} = 2038.736kg$$

$$k = \frac{\Delta F}{\Delta x} = \frac{500}{0.0025} = 2 \times 10^5 N/m$$

$$c = \zeta c_c = 0.5(2\sqrt{km}) = 20192.751 Ns/m$$

Road profile:

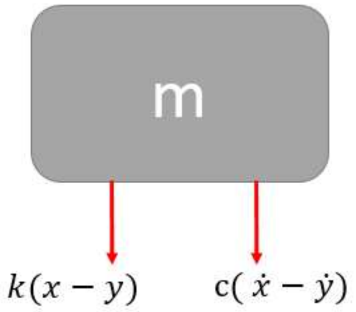
Vehicle velocity = $60kmph = 16.67m/s$

Frequency of base excitation, $\omega = \frac{2\pi v}{\lambda} = \frac{2\pi(16.67)}{12} = 8.7284\ rad/s$

Amplitude of base excitation= 0.06 m.

Therefore, $y(t) = 0.03\sin(8.7284t)$

Free body diagram



Using Newton's law of motion,

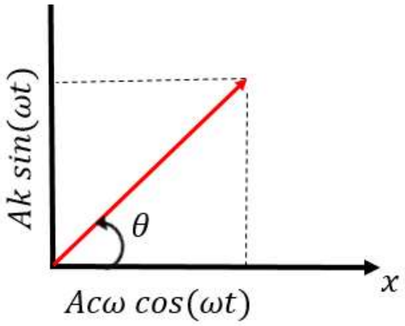
$$-k(x - y) - c(\dot{x} - \dot{y}) = m\ddot{x}$$

$$m\ddot{x} + c\dot{x} + kx = ky + c\dot{y}$$

Substituting y(t), we get,

$$m\ddot{x} + c\dot{x} + kx = Ak\sin(\omega t) + Ac\omega\cos(\omega t)$$

Using properties of vectors,



$$m\ddot{x} + c\dot{x} + kx = A\sqrt{k^2 + (c\omega)^2}\sin(\omega t + \theta)$$

$$\theta = tan^{-1}\left(\frac{k}{c\omega}\right)$$

$$m\ddot{x} + c\dot{x} + kx = (0.3)\sqrt{(2\times 10^5)^2 + 176250.41^2}\sin(8.7284t + \theta)$$

$$m\ddot{x} + c\dot{x} + kx = (52997.534)\sin(8.7284t + \theta)$$

Let $F_o = 7997.3612N$

Therefore this equation will be of the form,

$$m\ddot{x} + c\dot{x} + kx = F\ldots(1)$$

To solve this differential equation let us assume,

$$x = Xe^{i\omega t}$$

Therefore,

$$\dot{x} = i\omega Xe^{i\omega t}$$
$$\ddot{x} = i^2\omega^2Xe^{i\omega t} = -\omega^2Xe^{i\omega t}$$
$$F = F_oe^{i\omega t}$$

Substituting in the equation we get,

$$\left(-m\omega^2 + i\omega c + k\right)Xe^{i\omega t} = F_oe^{i\omega t}$$

$$X = \frac{F_o}{(k - m\omega^2) + i(\omega c)}$$

$$|X| = \frac{F_o}{\sqrt{(k - m\omega^2)^2 + (\omega c)^2}}$$

$$|X| = \frac{7997.3612}{\sqrt{(2\times 10^5 - 2038.736\times 8.7284^2)^2 + (8.7284\times 20192.751)^2}}$$

$$|X| = 0.9716m$$

Therefore,

$$x(t) = (0.04398)e^{i(8.7284)t}\quad m$$

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