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Human Evolutionary Optimization Algorithm

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A R T I C L E I N F O

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A B S T R A C T

This paper introduces the Human Evolutionary Optimization Algorithm (HEOA), a metaheuristic algorithm inspired by human evolution. HEOA divides the global search process into two distinct phases: human exploration and human development. Logistic Chaos Mapping is employed for initialization. In the human exploration phase, an initial global search is conducted, followed by the human development phase, in which the population is categorized into leaders, explorers, followers, and losers, each utilizing distinct search strategies. The convergence speed and search accuracy of HEOA are evaluated using 23 well-established test functions. Furthermore, the algorithm's applicability in engineering optimization is assessed with four engineering problems. A comparative analysis with ten other algorithms highlights HEOA's effectiveness, as evidenced by various performance metrics and statistical measures. Consistently, the results demonstrate that HEOA surpasses most current state-of-the-art algorithms in approximating optimal solutions for complex global optimization problems. The MATLAB code for HEOA is available at https://github.com/junbolian/HEOA.git.

1. Introduction and background

Optimization is the process of finding optimal values for specific system parameters to meet design requirements at the lowest cost (Hajipour et al., 2015). In various real-world applications, particularly within fields like artificial intelligence and machine learning, optimization problems often exhibit discrete, unconstrained, or discontinuous characteristics (Hajipour et al., 2014). Traditional mathematical programming methods, reliant on gradients and sensitive to initial conditions (Agushaka et al., 2015), can fall short in solving such complex problems (Wu et al., 2016). Consequently, optimization challenges span across a multitude of scientific domains.

Over recent decades, significant efforts have led to the development of numerous optimization algorithms, aimed at improving system performance and reducing computational costs. These approaches typically fall into two categories: mathematical methods and metaheuristic algorithms. While mathematical methods are effective when gradients are available, they can struggle with global optimization problems (Ezugwu et al., 2021). This limitation has spurred the growth of nature-inspired algorithms (Ezugwu et al., 2021), despite facing some criticism (Zapata et al., 2020). Nevertheless, these algorithms have demonstrated remarkable success in addressing complex optimization challenges in benchmark datasets like CEC 2020 (Liang et al., 2013) and real-world engineering applications (Qin et al., 2008).

Metaheuristic algorithms (MAs) are global optimization techniques rooted in simulations and nature-inspired principles (Nabil et al., 2016; Wang et al., 2018). Extensively studied for several decades, MAs have proven their efficacy in solving diverse optimization problems across various domains, including the traveling salesman problem (Johnson et al., 1990), optimal control (Michalewicz et al., 1990), and medical image processing (Oyelade et al., 2021), among others (Nadimi-Shahraki et al., 2021; Zheng et al., 2022).

These algorithms draw inspiration from the incredible problem-solving abilities observed in nature, such as the cooperative behaviors of creatures like fish, birds, and ants. This field of research, known as swarm intelligence (SI) (Beni et al., 1993), has led to the development of numerous swarm-based metaheuristic algorithms, including particle swarm optimization (PSO) (Eberhart et al., 1995), ant colony optimization (ACO) (Colorni et al., 1991), artificial bee colony (ABC) algorithm (Karaboga et al., 2007), firefly algorithm (FA) (Yang et al., 2009a), cuckoo search (CS) algorithm (Yang et al., 2009b), bat algorithm (BA) (Yang et al., 2010), fruit fly optimization algorithm (FOA) (Pan et al., 2012), krill herd (KH) algorithm (Gandomi et al., 2012), grey wolf optimizer (GWO) (Saremi et al., 2015), dragonfly algorithm (DA) (Mirjalili, 2016b), whale optimization algorithm (WOA) (Mirjalili et al., 2016d), moth search (MS) algorithm (Wang et al., 2018), and many

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Table 1
A summary of popular MH algorithms.

*Type	Algorithm	Inspired by
SI	Particle Swarm Optimization (PSO)	The natural behaviors of swarm particles (Eberhart et al., 1995)
	Ant Colony Optimization (ACO)	Ants deposit pheromone on the ground (Colorini et al., 1991)
	Artificial Bee Colony (ABC)	The behaviors of the honey bees colony (Karaboga et al., 2007)
	Firefly Algorithm (FA)	Flashing light of fireflies in oceans (Yang et al., 2009a)
	Cuckoo Search (CS)	The behavior of cuckoo breeding parasitism (Yang et al., 2009b)
	Whale Optimization Algorithm (WOA)	The behavior of humpback whales (Mirjalili et al., 2016d)
	Dwarf mongoose optimization algorithm (DMOA)	the behavior of the dwarf mongoose (Agushaka et al., 2022)
	Salp Swarm Algorithm (SSA)	The behavior of salps navigating in oceans (Mirjalili et al., 2015)
	Moth Flame Optimization (MFO)	The moths navigation method in nature (Mirjalili et al., 2008a)
	Marine Predators Algorithm (MPA)	predators foraging strategy in oceans (Faramarzi et al., 2020b)
	Lion Optimization Algorithm (LOA)	Lifestyle and cooperation of lions (Yazdani et al., 2016)
	Grasshopper Optimization Algorithm (GOA)	The behavior of grasshopper swarms (Saremi et al., 2017)
	Squirrel Search Algorithm (SSA)	The behavior of southern flying squirrels (Jain et al., 2019)
	sparrow search algorithm (SSA)	The behavior of sparrows (Xue et al., 2020)
	Golden Eagle Optimizer (GEO)	The behavior of golden eagles in tuning speed (Mohammadi-Balani et al., 2021)
EA	Genetic Algorithm (GA)	Darwinian theory of evolution (John et al., 1992)
	Differential evolution (DE)	the natural phenomenon of evolution (Storn et al., 1997)
	Biogeography-Based Optimizer (BBO)	Biogeography related to species migration (Simon et al., 2008)
	Invasive Tumor Growth (ITG)	Kidney process. (Tang et al., 2015)
	Tree Growth Algorithm (TGA)	Competition of trees for acquiring foods and light (Cheraghalipour et al., 2018)
	Arithmetic Optimization Algorithm (AOA)	The behavior of the main arithmetic operators (Abualigah et al., 2021)
PhA	Big Bang–Big Crunch (BBBC)	The evolution of the universe (Erol et al., 2006)
	Gravitational Search Algorithm (GSA)	The law of gravity and mass interactions (Rashedi et al., 2009)
	Multi-verse Optimizer (MVO)	multi-verses theory (Mirjalili et al., 2016c)
	Central Force Optimization (CFO)	The metaphor of gravitational kinematics (Formato et al., 2007)
	Henry Gas Solubility Optimization (HGSO)	The behavior of Henry's law (Hashim et al., 2019)
	Thermal Exchange Optimization (TEO)	Newton's law of cooling (Kaveh et al., 2017)
	Electromagnetic Field Optimization (EFO)	The behavior of electromagnets (Abedinpourshotorban et al., 2016)
Human based	Teaching based learning algorithm (TBLA)	The influence of a teacher on the output of learners (Rao et al., 2012)
	Collective Decision Optimization (CSO)	Human decision-making characteristics (Zhang et al., 2017)
	Socio Evolution & Learning Optimization Algorithm (SELOA)	Social learning behavior of humans (Kumar et al., 2018)
Others	Sine Cosine Algorithm (SCA)	Sine and cosine functions (Mirjalili et al., 2016a)
	Volleyball Premier League Algorithm (VPLA)	Competitions of volleyball teams (Moghdani et al., 2018)

others. The success of nature-inspired algorithms lies in their ability to emulate the best aspects of nature (Agushaka et al., 2022). These algorithms can generally be classified into four categories: swarm intelligence (SI) algorithms, Evolutionary Algorithms (EA), Physics-based algorithms (PhA), and Human-based algorithms. SI algorithms, inspired by social behaviors of swarms and animals, have shown promising results (Eberhart et al., 1995; Colorni et al., 1991; Karaboga et al., 2007; Yang et al., 2009a; Yang et al., 2009b). EA algorithms simulate biological evolutionary behaviors, including mutation, crossover, and selection, to guide the search process (John et al., 1992; Storn et al., 1997; Simon et al., 2008; Tang et al., 2015; Cheraghalipour et al., 2018; Abualigah et al., 2021). PhA algorithms draw inspiration from physical laws to guide optimization, as demonstrated in (Erol et al., 2006; Rashedi et al., 2009; Mirjalili et al., 2016c; Formato et al., 2007; Hashim et al., 2019; Kaveh et al., 2017; Abedinpourshotorban et al., 2016). Human-based methods leverage human behaviors as inspiration, as exemplified in (Rao et al., 2012; Zhang et al., 2017; Kumar et al., 2018). A short review of some of the well-known nature-inspired metaheuristic algorithms is presented in Table 1.

These MH algorithms share a common characteristic—a search process comprising two distinct phases: diversification (exploration) and intensification (exploitation) (Abualigah et al., 2020; Salcedo-Sanz et al., 2016). During diversification, the MH algorithm employs random operators to explore various regions within the search space. Conversely, the intensification phase concentrates on identifying the optimal solution within the search space. An effective MH optimization algorithm must carefully balance exploration and exploitation to avoid getting trapped in local optima (Abualigah et al., 2021).

It is widely accepted that no single algorithm can universally provide optimal solutions for all problems. Hence, there is a continuous drive to develop novel high-performance algorithms tailored to specific problem types. Adhering to the principle of the no-free lunch theory, researchers continually devise metaheuristic algorithms that offer improved solutions for complex and large-scale optimization problems, with a focus on effective exploration and exploitation (Jerebic et al., 2021). Over the past decade, there has been an exponential growth in the number of nature-inspired optimization algorithms, emphasizing novelty and robust optimization capabilities.

Despite the abundance of nature-inspired algorithms, the potential for optimizing the Human Evolutionary Optimization Algorithm (HEOA), inspired by the process of human evolution, remains largely untapped. HEOA divides the global search process into two distinct phases: an exploration phase and a development phase. Logistic Chaos Mapping is employed during the human exploration phase to initiate an initial global search. Subsequently, in the human development phase, the population is categorized into leaders, explorers, followers, and losers, each utilizing distinct search strategies.

In this paper, we assess the performance of HEOA by applying it to benchmark test functions and four distinct engineering optimization problems. Comparative analyses, based on five performance metrics, are conducted against ten other optimization methods. Our results consistently demonstrate that HEOA excels, consistently outperforming alternative methods and showcasing its highly effective search capabilities.

The HEOA possesses several unique properties that enable it to theoretically discover globally optimal solutions for various optimization problems. These properties can be summarized as follows:

- Logistic Chaos Mapping during initialization: HEOA draws inspiration from the chaotic nature of the universe and utilizes Logistic Chaos Mapping to enhance the quality of the initial solution. This initialization process not only improves the starting point but also increases the diversity within the population.
- Jump-shift strategy inspired by image compression: To ensure randomness and dispersion during the search phase, HEOA incorporates a jump-shift strategy inspired by image compression

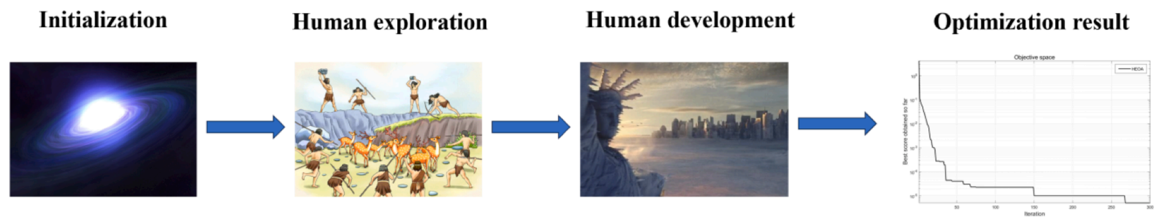

Fig. 1. Process of human evolutionary optimization.

techniques. This strategy helps to explore the search space more effectively and avoid getting stuck in local optima.

- Levy search strategy for enhanced global search capability: HEOA employs the Levy search strategy to improve its global search capability. The Levy flight behavior allows for efficient exploration of the search space, increasing the algorithm's ability to locate globally optimal solutions.
- Categorization of populations in the developmental stage: In the developmental stage of the HEOA algorithm, populations are classified into four distinct classes based on their fitness values. Each class employs different search strategies to ensure diversity in search methods. This categorization facilitates a more comprehensive exploration of the search space.
- Sampling adaptive weights for improved search accuracy and efficiency: HEOA utilizes the method of sampling adaptive weights to control the search step size. This technique enhances the algorithm's search accuracy and efficiency, enabling it to converge towards optimal solutions more effectively.

The proposed human evolutionary optimization algorithm combines the historical and social characteristics of human evolution and development processes, offering a promising approach to solving optimization problems. Its superior performance, compared to existing methods, highlights its potential for discovering globally optimal solutions across various domains.

The structure of this paper is as follows: Section 2 provides a detailed

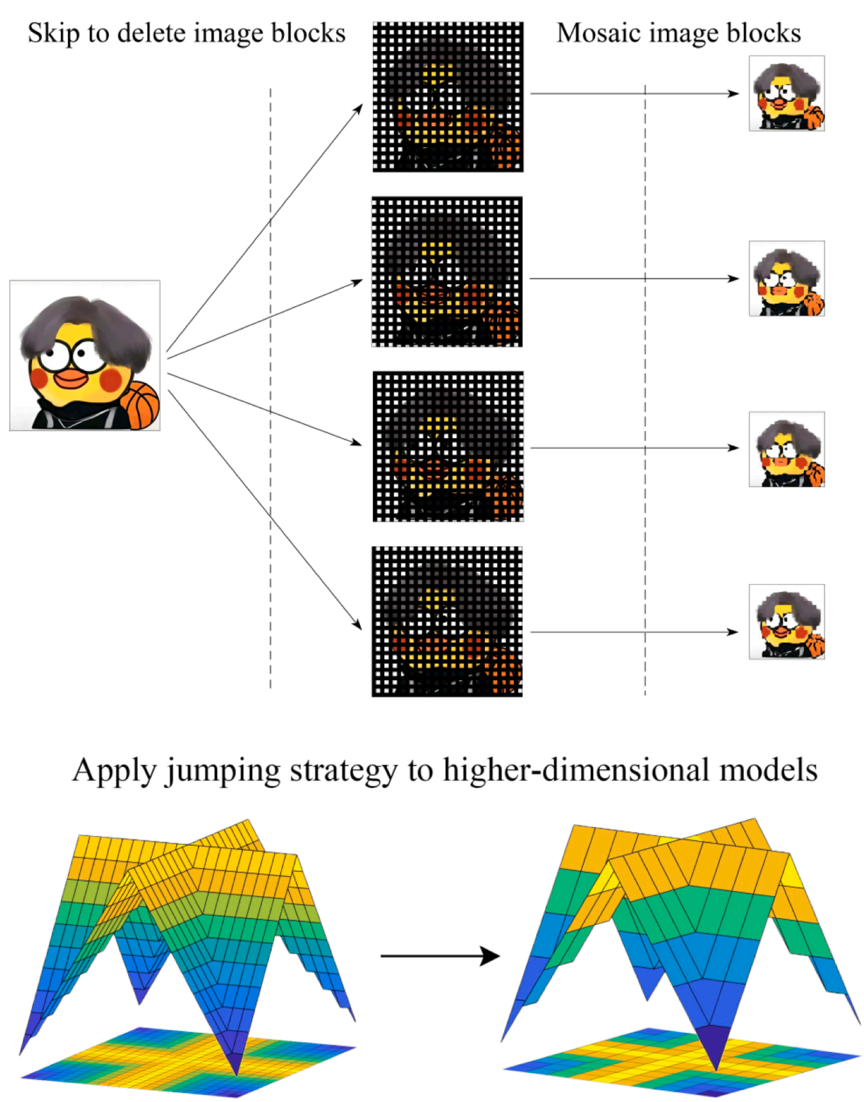

Fig. 2. Jumping strategy: (b) Apply to higher-dimensional models.

```
Initialization phase:
  Start
  Logistic chaotic mapping initialization
  Compute initial fitness values
  End
Start search:
  For i = 1:Max_iter do
    BestF = fitness(1)
    WorstF = fitness(end)
    R = rand(1)
    For j = 1:size(X, 1) do
      Human exploration stage:
      If i <= (1 / 4) * Max_iter Then
        X_new(j, :) = GBestX * (1 - i / Max_iter) + (mean(X(j, :)) - GBestX) * floor(rand() /
f_jump) * f_jump + b.* Levy(dim)
      Human development stage:
      Else
        Leaders:
        For j = 1:LNNumber do
          If R < A Then
            X_new(j, :) = w * X(j, :) * exp((-i * randn(1)) / (rand(1) * Max_iter))
          Else
            X_new(j, :) = w * X(j, :) + randn() * ones(1, dim)
          End
      End
        Explorers:
        For j = LNNumber + 1:LNNumber + ENNumber do
          X_new(j, :) = randn() .* exp((X(end, :) - X(j, :)) / j^2)
        End
        Followers:
        For j = LNNumber + ENNumber + 1:LNNumber + ENNumber + FNNumber do
          X_new(j, :) = X(j, :) + w * rand(1, dim) .* (X(1, :) - X(j, :))
        End
        Losers:
        For j = LNNumber + ENNumber + FNNumber + 1:N do
          X_new(j, :) = GBestX + (GBestX - X(j, :)) * randn(1)
        End
    End
    Update and boundary control
  End
  Return Best solution
```

Fig. 3. Pseudo-code of the HEOA.

explanation of the implementation of the proposed HEOA. In Section 3, we present the experimental setup, results, and engage in a comprehensive discussion. Finally, Section 4 summarizes the findings of this study and outlines potential directions for future research.

2. The human evolutionary optimization algorithm (HEOA)

The general framework of the human evolutionary optimization algorithm (HEOA) is presented, and the optimization processes are formulated.

2.1. Inspiration and model of HEOA

The inspiration behind the development of HEOA stems from several key factors. A significant source of inspiration is the remarkable adaptability of human evolution and its ability to find optimal solutions within complex environments. Human evolution has served as a driving force for human survival and progress, highlighting the effectiveness of natural selection and adaptation. This exceptional capacity to navigate and thrive in challenging circumstances has spurred the creation of HEOA as a metaheuristic algorithm, engineered to efficiently seek

Table 2
Unimodal benchmark functions.

Function	Dim	Range	Shift position	f_{min}
$F_1(x) = \sum_{i=1}^{n} x_i^2$	30	$[-100, 100]$	$[-30, -30, \cdots, -30]$	0
$F_2(x) = \sum_{i=1}^{n} \lvert x_i \rvert + \prod_{i=1}^{n} \lvert x_i \rvert$	30	$[-10, 10]$	$[-3, -3, \cdots, -3]$	0
$F_3(x) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j\right)^2$	30	$[-100, 100]$	$[-30, -30, \cdots, -30]$	0
$F_4(x) = \sum_{i=1}^{n} \left(\sum_{j=1}^{i} x_j\right)^2$	30	$[-100, 100]$	$[-30, -30, \cdots, -30]$	0
$F_5(x) = \sum_{i=1}^{n-1} \left[100\left(x_{i+1} - x_i^2\right)^2 + (x_i - 1)^2\right]$	30	$[-30, 30]$	$[-15, -15, \cdots, -15]$	0
$F_6(x) = \sum_{i=1}^{n} (\lvert x_i + 0.5 \rvert)^2$	30	$[-100, 100]$	$[-750, -750, \cdots, -750]$	0
$F_7(x) = \sum_{i=1}^{n} ix^4 + random[0, 1)$	30	$[-1.28, 1.28]$	$[-0.25, -0.25, \cdots, -0.25]$	0

Table 3
Multimodal benchmark functions.

Function	Dim	Range	Shift position	f_{min}
$F_8(x) = \sum_{i=1}^{n} -x_i sin\left(\sqrt{\lvert x_i \rvert}\right)$	30	$[-500, 500]$	$[-300, \cdots, -300]$	−12569.5
$F_9(x) = \sum_{i=1}^{n} \left[x_i^2 - 10\cos(2\pi x_i) + 10\right]$	30	$[-5.12, 5.12]$	$[-2, -2, \cdots, -2]$	0
$F_{10}(x) = -20exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	30	$[-32, 32]$		0
$F_{11}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_i^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{(i)}}\right) + 1$	30	$[-600, 600]$	$[-400, \cdots, -400]$	0
$F_{12}(x) = \frac{\pi}{n}\left\{10sin(\pi y_i) + \sum_{i=1}^{n-1}(y_i - 1)^2[1 + 10sin^2(\pi y_{i+1})\right] + (y_n - 1)^2\Big\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$, $y_i = 1 + \frac{x_i + 1}{4}$, $u(x_i, a, k, m) = \begin{cases} k(x_i - a)^m x_i > a \\ 0 - a < x_i < a \\ k(-x_i - a)^m x_i < -a \end{cases}$	30	$[-50, 50]$	$[-30, -30, \cdots, -30]$	0
$F_{13}(x) = 0.1\Big\{sin^2(3\pi x_1) + \sum_{i=1}^{n1}(x_i - 1)^2[1 + sin^2(3\pi x_i + 1)] + (x_n - 1)^2\left[1 + sin^2(2\pi x_n)\right]\Big\} + \sum_{i=1}^{n} u(x_i, 5, 100, 4)$	30	$[-50, 50]$	$[-100, \cdots, -100]$	0

Table 4
Fixed-dimension multimodal benchmark functions.

Function	Dim	Range	Shift position	f_{min}
$F_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25}\frac{1}{j + \sum_{i=1}^{2}(x_i - a_{ij})^6}\right)^{-1}$	2	$[-65, 65]$	$[-2, -2, \cdots, -2]$	1
$F_{15}(x) = \sum_{i=1}^{11}\left[a_i - \frac{x_1\left(b_i^2 + b_i x_2\right)}{b_i^2 + b_i x_3 + x_4}\right]^2$	4	$[-5, 5]$	$[-2, -2, \cdots, -2]$	0.0003075
$F_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1x_2 - 4x_2^2 + 4x_2^4$	2	$[-5, 5]$	$[-2, -2, \cdots, -2]$	−1.0316285
$F_{17}(x) = \left(x_2 - \frac{5.1}{4\pi^2}x_1^2 + \frac{5}{\pi}x_1 - 6\right) + 10\left(1 - \frac{1}{8\pi}\right)\cos x_1 + 10$	2	$[-5, 5]$	$[-2, -2, \cdots, -2]$	0.398
$F_{18}(x) = \left[1 + (x_1 + x_2 + 1)^2\left(19 - 14x_1 + 3x_1^2 - 14x_2 + 16x_1x_2 + 3x_2^2\right)\right] \times \left[30 + (2x_1 - 3x_2)^2 \times \left(18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2\right)\right]$	2	$[-2, 2]$	$[-2, -2, \cdots, -2]$	3
$F_{19}(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{3} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	3	$[1, 3]$	$[-2, -2, \cdots, -2]$	−3.86
$F_{20}(x) = -\sum_{i=1}^{4} c_i exp\left(-\sum_{j=1}^{6} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	6	$[0, 1]$	$[-2, -2, \cdots, -2]$	−3.32
$F_{21}(x) = -\sum_{i=1}^{5}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	$[0, 10]$	$[-2, -2, \cdots, -2]$	−10.1532
$F_{22}(x) = -\sum_{i=1}^{7}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	$[0, 10]$	$[-2, -2, \cdots, -2]$	−10.4028
$F_{23}(x) = -\sum_{i=1}^{1}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	$[0, 10]$	$[-2, -2, \cdots, -2]$	−10.536

Table 5
Results of unimodal benchmark functions (different population).

Function	Item	N = 10	N = 20	N = 30	N = 40	N = 50
F1	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Median	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Worst	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F2	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Median	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Mean	6.99E-265	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Worst	1.40E-263	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F3	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Median	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Mean	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Worst	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F4	Best	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Median	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Mean	1.10E-238	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	Worst	2.20E-237	0.00E+00	0.00E+00	0.00E+00	0.00E+00
	STD	0.00E+00	0.00E+00	0.00E+00	0.00E+00	0.00E+00
F5	Best	1.74E-08	1.39E-10	3.49E-10	1.70E-11	5.16E-11
	Median	1.10E-05	1.70E-07	8.69E-08	3.18E-09	1.17E-09
	Mean	7.83E-05	1.89E-06	2.07E-07	9.46E-08	7.24E-08
	Worst	7.96E-04	1.21E-05	1.22E-06	7.29E-07	9.75E-07
	STD	3.60E-08	1.16E-11	1.14E-13	4.44E-14	5.06E-14
F6	Best	6.64E-11	1.96E-13	9.20E-13	4.36E-13	1.25E-14
	Median	1.02E-09	3.60E-10	9.77E-11	2.05E-11	6.58E-12
	Mean	1.92E-08	3.69E-09	4.53E-10	1.95E-10	1.85E-10
	Worst	1.28E-07	3.30E-08	4.17E-09	1.85E-09	2.00E-09
	STD	1.80E-15	6.58E-17	9.72E-19	1.87E-19	2.27E-19
F7	Best	1.26E-06	9.02E-07	1.56E-07	1.13E-07	1.12E-07
	Median	8.35E-05	2.43E-05	8.82E-06	5.15E-06	3.53E-06
	Mean	9.08E-05	2.62E-05	1.32E-05	8.12E-06	4.44E-06
	Worst	3.02E-04	7.67E-05	5.31E-05	2.51E-05	2.03E-05
	STD	5.02E-09	4.78E-10	2.1E-10	6.28E-11	2.27E-11

optimal solutions in intricate optimization landscapes.

The Chaotic Universe Theory (Barrow et al., 1977) presents a distinctive perspective on the origin and evolution of the universe. It posits that the universe lacks a specific initial state or a single creation event, evolving gradually through chaotic processes and self-organizational phenomena. This theory serves as the inspiration for Logistic Chaos Mapping (Kanso et al., 2009), a method integrated into HEOA as an initialization technique. By infusing chaos theory into the algorithm's design, HEOA endeavors to introduce a dynamic and exploratory element into the optimization process.

The human evolutionary process can be bifurcated into two distinct stages: human exploration and human development. While this division simplifies the intricate nature of human evolution, it offers a scientific foundation for comprehending the progression of human societies. In the phase of human exploration, early humans confronted new environments and resources, relying on trial and error and adaptation to devise survival strategies. They gradually amassed knowledge and skills through experimentation and feedback. This phase of exploration and adaptation aligns with the initial global search conducted by HEOA, mirroring the exploratory nature of human evolution.

The stage of human development signifies the gradual formation of human societies, the emergence of diverse cultures, technologies, and forms of social organization. Human development is rooted in observation, practice, and reflection on the environment and society. Through the accumulation and transmission of truths, humans constructed a system of knowledge about the world and themselves. HEOA categorizes the human community into four distinct roles: leaders, seekers, followers, and losers. Leaders seek superior areas of human development based on existing knowledge, while seekers venture into uncharted territories. Followers heed the discoveries of leaders and accompany them on their journey. Losers, those who fail to adapt, are eliminated by society, and the population is replenished in regions conducive to human development. Each role adopts distinct search strategies to explore the global optimal solution.

Two additional inspirations incorporated into HEOA are the Levy flight strategy and jumping strategy. The Levy flight algorithm (Barthelemy et al., 2008) is a widely used method in intelligent optimization algorithms (Abualigah et al., 2021; Abualigah et al., 2021; Haklı & Uğuz, 2014; Jain et al., 2019; Kaidi et al., 2022), known for improving algorithm performance through the Levy flight mechanism. This strategy enhances exploration capabilities by introducing stochastic movements. The jumping strategy in HEOA draws inspiration from image compression techniques, ensuring decentralization of the pre-search position. It employs a compression-like jumping strategy to facilitate exploration and diversification.

These various inspirations collectively contribute to the design and functionality of the Human Evolutionary Optimization Algorithm, enabling it to leverage the principles of human evolution and effectively address complex optimization problems. The process of human evolutionary optimization is shown in Fig. 1.

2.2. Population initialization

To simulate the chaotic stage at the beginning of human evolution, the Human Evolutionary Optimization Algorithm initializes the population using the Logistic Chaos Mapping. The initialization formula for Logistic Chaos Mapping, considering a population size of N, maximum iterations of Max_{iter}, and search space boundaries of lb (lower bound) and ub (upper bound), can be expressed as:

$$x_i = \alpha \cdot x_{i-1} \cdot (1 - x_{i-1}), 0 \le x_0 \le 1, i = 1, 2, ..., N, \alpha = 4 \quad (1)$$

where x_i represents the i^{th} iteration value and x_{i-1} represents the previous iteration value. Map the chaotic value, x_i, to the search space:

$$X_i^0 = lb + (ub - lb) \cdot x_i \quad (2)$$

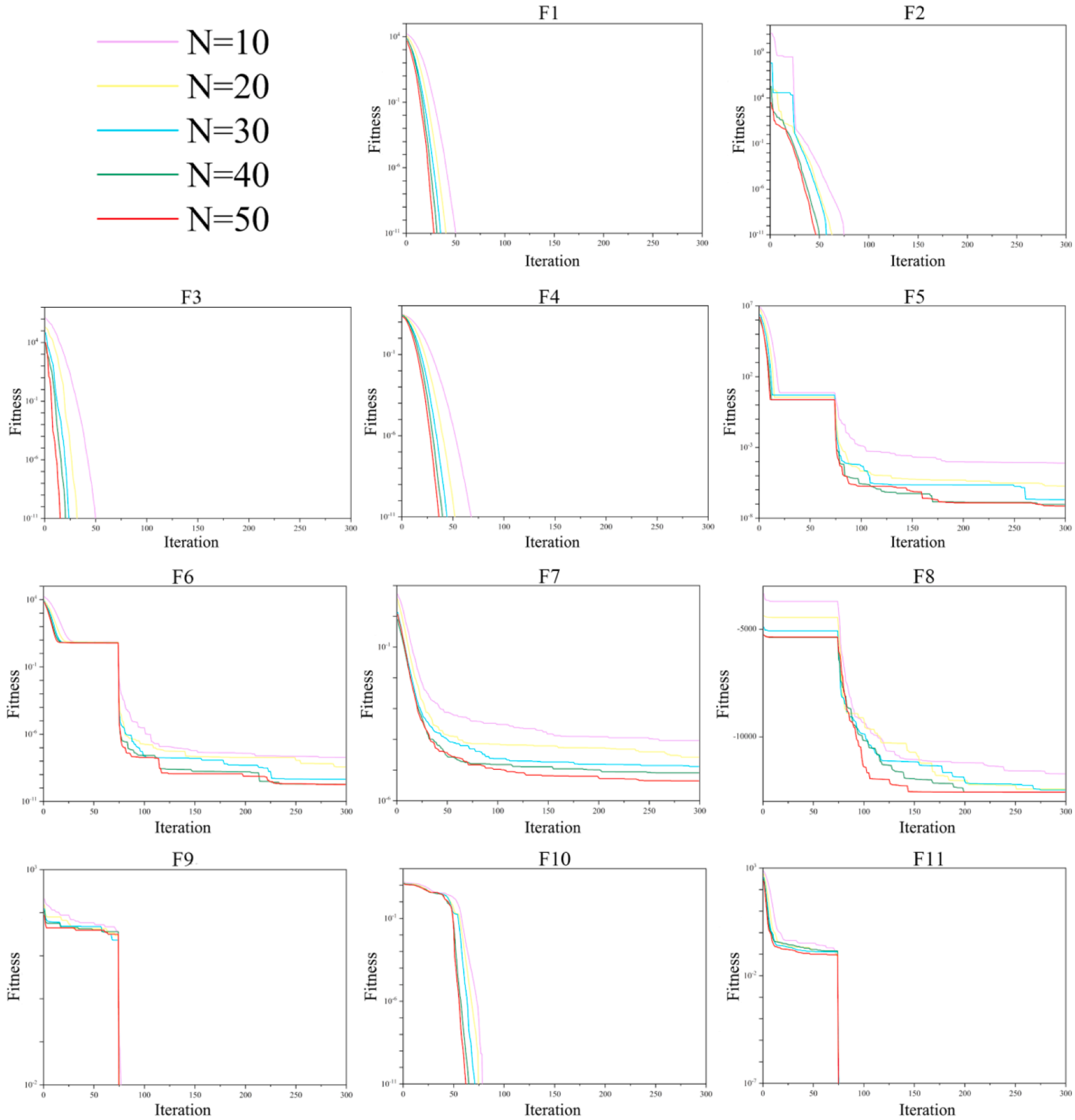

Fig. 4. Comparison of convergence rates for different population sizes.

2.3. Human exploration stage

After initializing the population, the next step is to calculate the fitness of each solution. In the conducted experiments, the exploration phase was defined as the initial 1/4 of the maximum number of iterations. In the stages of human development, when faced with unexplored territories and limited knowledge, individuals tend to adopt a uniform strategy for searching. This can be mathematically represented by the following expression, denoted as Eq. (3):

$$X_i^{t+1} = \beta\cdot(1-\frac{t}{Max_{iter}})\cdot(X_i^t - X_{best})\cdot Levy(dim)+ X_{best}\cdot(1-\frac{t}{Max_{iter}}) + (X_{mean}^t - X_{best})\cdot floor(\frac{rand}{f_{jump}})\cdot f_{jump} \quad (3)$$

In Eq. (3), the adaptive function β is defined, where t represents the current number of iterations. *dim* signifies the dimensionality of the problem or the number of variables involved. X_i^t denotes the current position, while X_i^{t+1} signifies the position of the subsequent update. X_{best} corresponds to the best position explored thus far, and X_{mean}^t represents the average position within the current population. The term *floor* refers to the operation of rounding downwards.*Levy* denotes the *Levy* distribution, f_{jump} is the jump coefficient, and *rand* is a random number in the range of [0, 1].

The relevant parameters and functions can be further explained as follows:

Average position X_{mean}^t: The average position of the current population, denoted as X_{mean}^t, is calculated using the expression shown in the Eq. (4).

$$X_{mean}^t = \frac{1}{N}\sum_{k=1}^{N} X_k^t \quad (4)$$

Adaptive function β : The adaptive function, β, is responsible for adjusting the parameters based on the number of iterations and the current position. This function accounts for the increasing difficulty of human exploration of knowledge and the characteristics of swarming. The expression for the adaptive function, β, is shown in the Eq. (5).

$$\beta = 0.2\left(1-\frac{t}{Max_{iter}}\right)\cdot(X_i^t - X_{mean}^t) \quad (5)$$

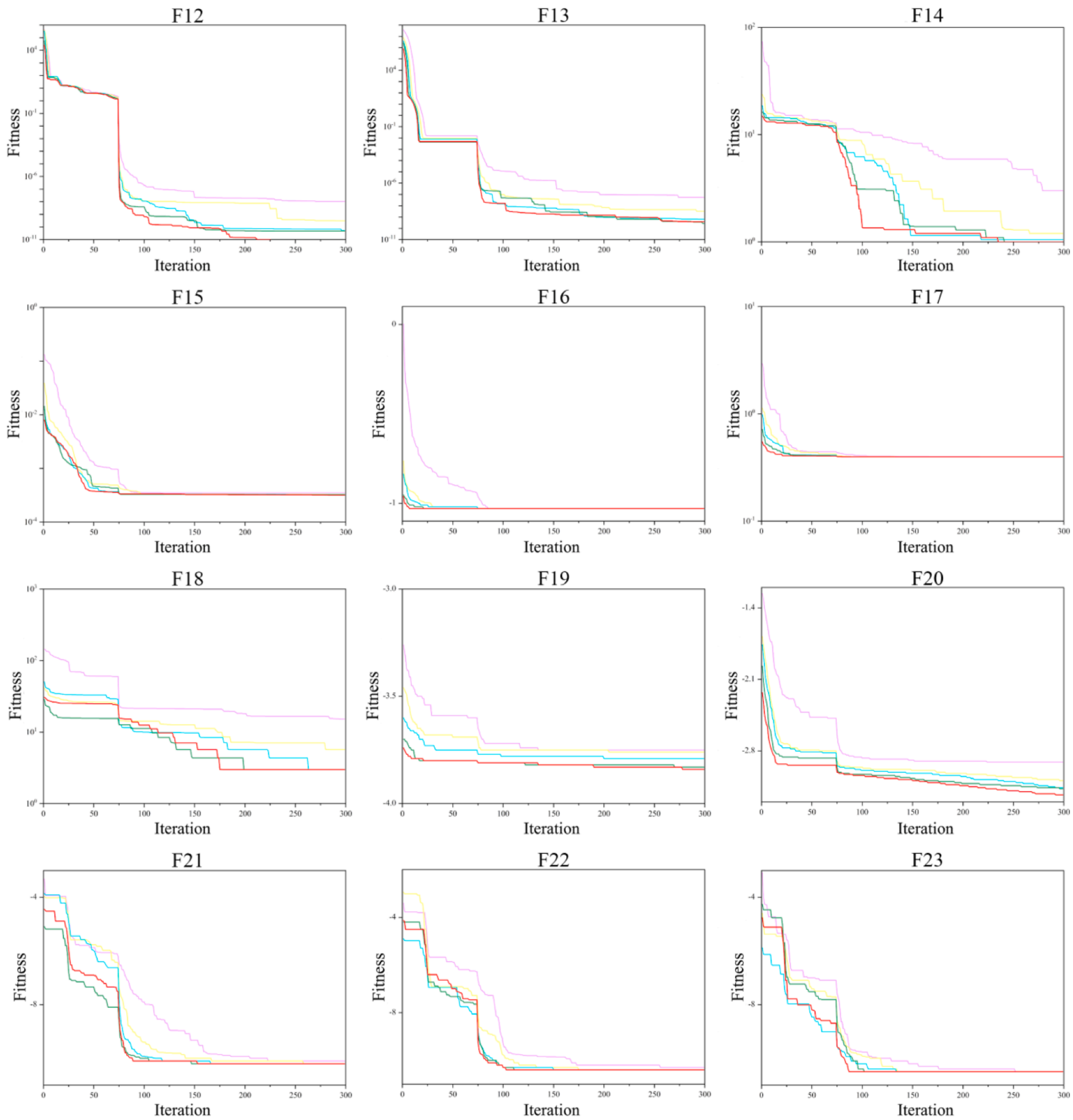

Fig. 4. (*continued*).

Levy **distribution**: To simulate the convoluted nature of knowledge acquisition in the human exploration stage and the characteristic of spiral development, the *Levy* distribution is utilized. In this context, the expression for the Levy distribution is shown in the Eq. (6), where γ is assigned the value of 1.5.

$$\begin{cases} Levy(D) = \dfrac{\mu\cdot\sigma}{|v|^{\frac{1}{\gamma}}} \\ \mu \sim N(0, D) \\ v \sim N(0, D) \\ \sigma = \left(\dfrac{\Gamma(1+\gamma)\cdot\sin\left(\frac{\pi\gamma}{2}\right)}{\Gamma\left(\frac{1+\gamma}{2}\right)\cdot\gamma\cdot 2^{\frac{1+\gamma}{2}}}\right)^{\gamma+1} \end{cases} \tag{6}$$

Jumping strategy: The human exploration phase incorporates a jumping strategy inspired by image cropping and reorganization techniques, aiming to enhance the dispersion of search locations. In Fig. 2(a), an image is divided into small 36×36 pieces and then rearranged according to specific rules to maintain the overall shape. This technique leverages the fact that the human eye perceives an image through a combination of overall and local features, making it relatively insensitive to the loss of specific local information. Extending this concept to high-dimensional data, as illustrated in Fig. 2(b), this approach preserves the essential characteristics of the metadata while distributing the search across multiple regions, thus improving search efficiency. The jump coefficient, denoted as f_{jump} quantifies the extent of jumping and is expressed as follows:

$$f_{jump} = \frac{(lb(1) - ub(1))}{\delta}, \delta \in [100, 2000] \tag{7}$$

2.4. Human development stage

In the stages of human development, the HEOA categorizes human society into four distinct roles: leaders, explorers, followers, and losers. Each role adopts a unique search strategy, and collectively they collaborate to explore the global optimal solution. The specific search strategies for each role are outlined below:

Leaders: Leaders, possessing a wealth of knowledge, are typically situated in optimal regions. In the conducted experiments, individuals within the top 40 % of pre-adaptation are designated as leaders. They embark on a quest to discover superior areas for human development,

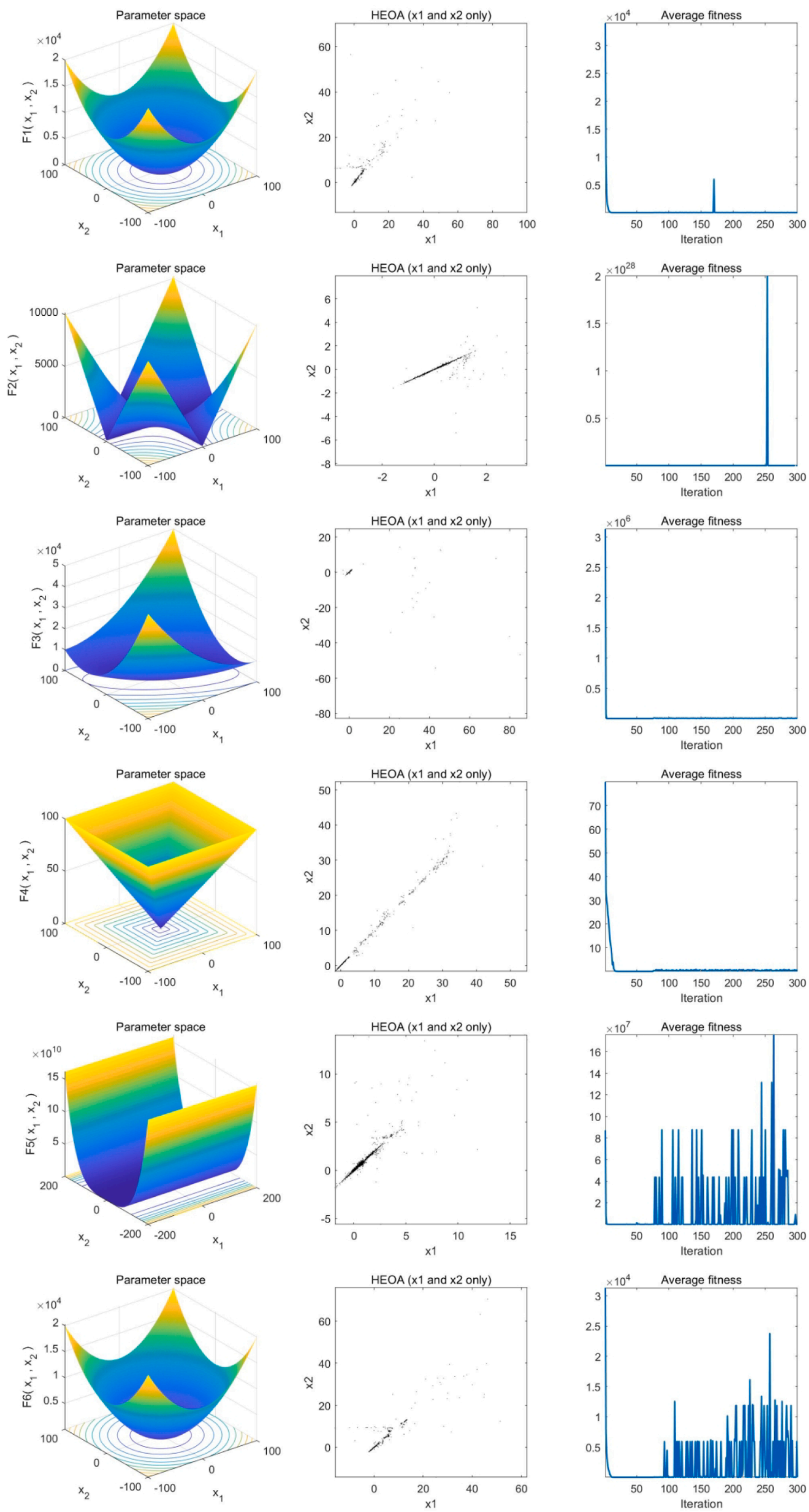

Fig. 5. Classical function images and HEOA search results.

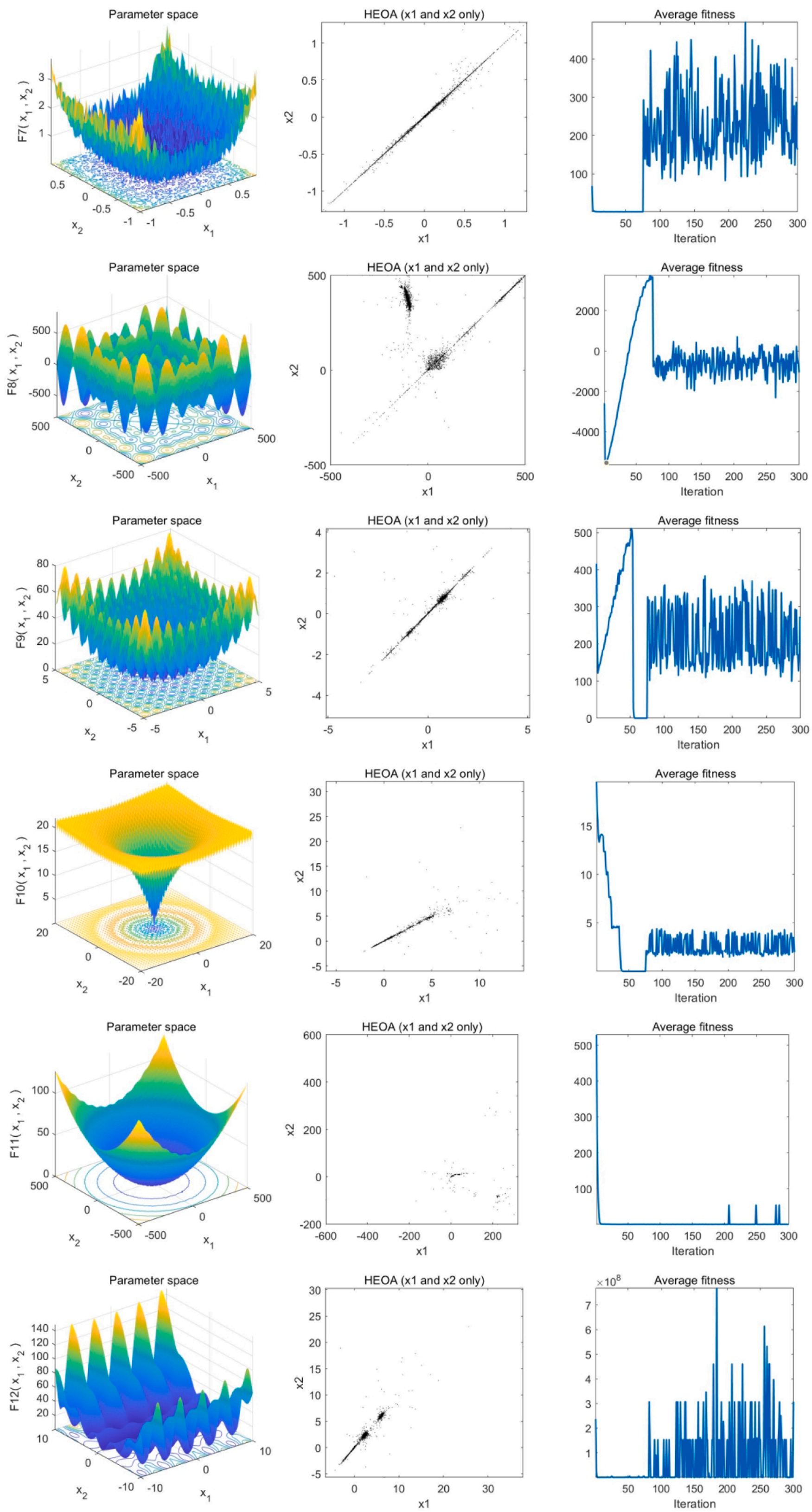

Fig. 5. (*continued*).

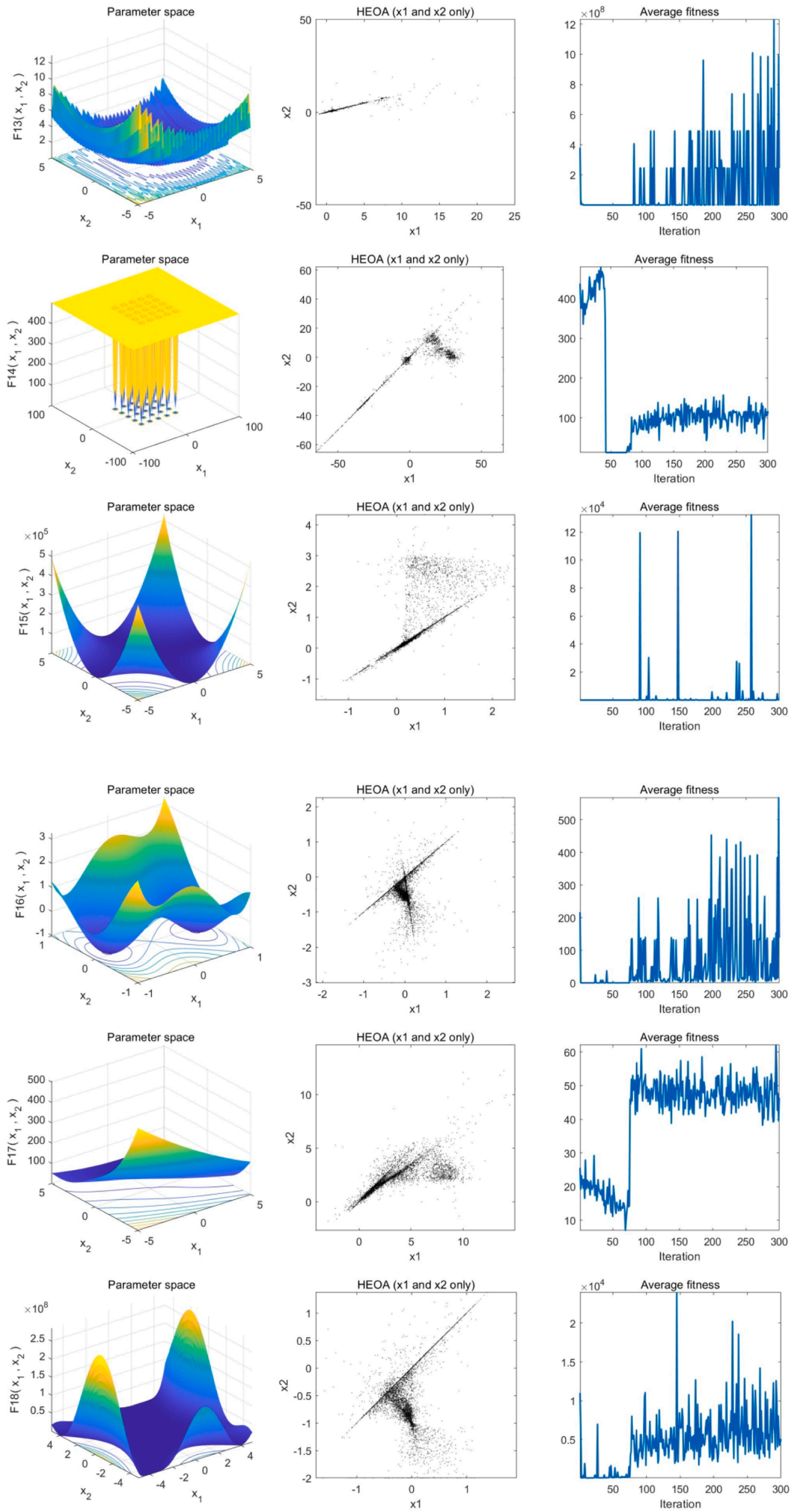

Fig. 5. (*continued*).

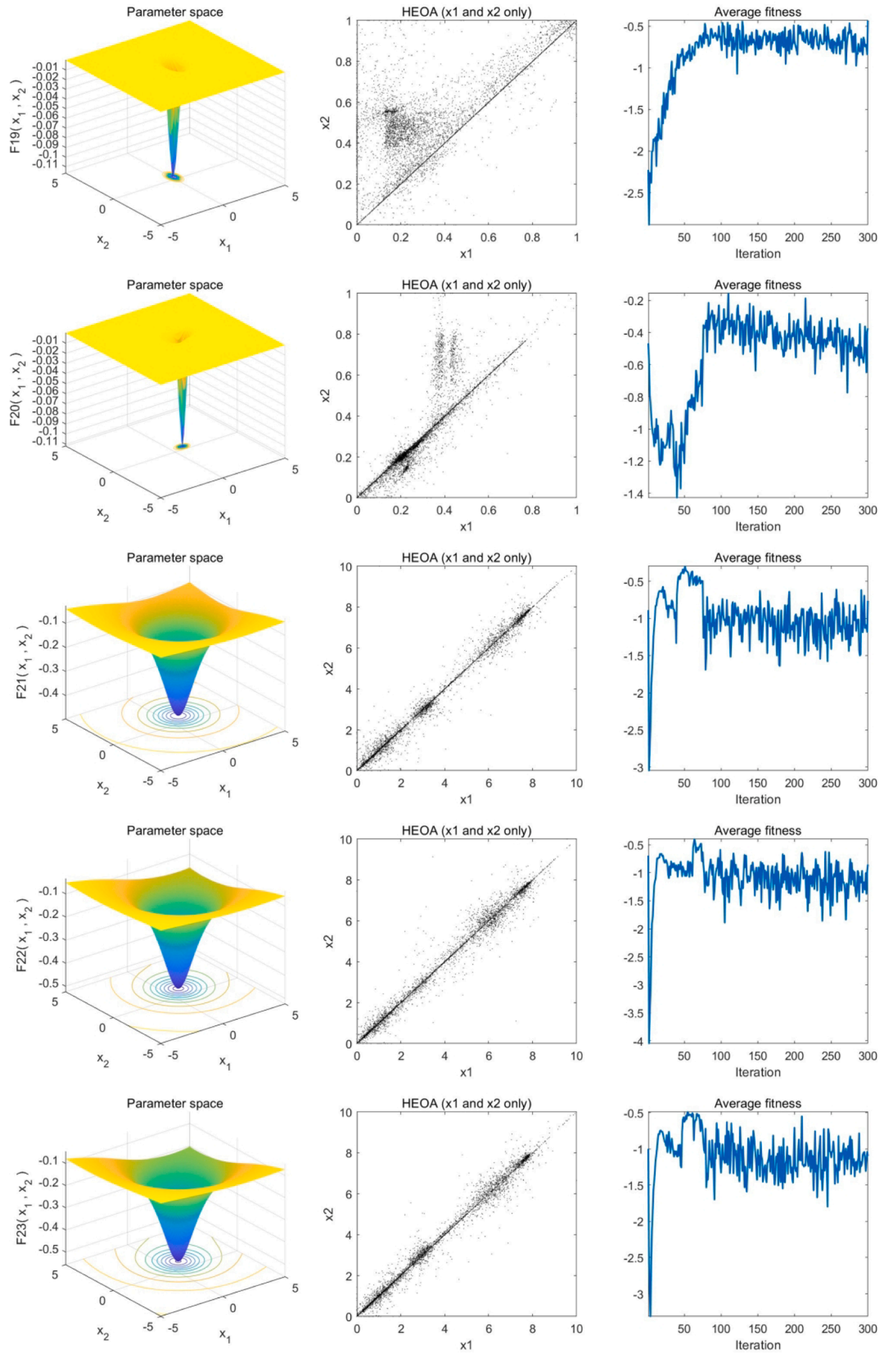

Fig. 5. (*continued*).

leveraging their existing knowledge. This exploration process is captured by the equation expressed in the Eq. (8).

$$X_i^{t+1} = \begin{cases} \omega \cdot X_i^t \cdot \exp\left(\dfrac{-t}{rand \cdot Max_{iter}}\right), R < A \\ \omega \cdot X_i^t + Rn \cdot ones(1, dim), R \geq A \end{cases} \tag{8}$$

In the Eq. (8), Rn represents a random number that follows a normal distribution. The function $ones(1, dim)$ generate a row vector with dim elements, where each element is set to 1. R is a random number in the range [0, 1] that represents the complexity of the situation associated with the leaders. A denotes the evaluation value of the situation. In the conducted experiments, the value of A is 0.6. Based on the complexity of the situation at a particular position, the leader will select an appropriate search strategy. The knowledge acquisition ease coefficient, denoted as ω, gradually decreases as development progresses. The expression for the coefficient ω is as follows:

$$\omega = 0.2\cos\left(\frac{\pi}{2}\left(1 - \frac{t}{Max_{iter}}\right)\right) \tag{9}$$

Explorers: Explorers play a crucial role in venturing into uncharted territory to discover the global optimal solution. In the conducted experiment, individuals ranking in the top 40 % to 80 % of the population in terms of fitness are designated as explorers. The search strategy employed by explorers is represented by Eq. (10).

$$X_i^{t+1} = Rn \cdot \exp\left(\frac{X_{worst}^{t2} - X_i^{t2}}{i^2}\right) \tag{10}$$

In the Eq. (10), X_{worst}^t represents the location of the least adapted

Table 6
Result of multimodal benchmark functions (different population).

Function	Item	N = 10	N = 20	N = 30	N = 40	N = 50
F8	Best	**−1.26E + 04**	**−1.26E + 04**	**−1.26E + 04**	**−1.26E + 04**	**−1.26E + 04**
	Median	**−1.25E + 04**	**−1.26E + 04**	**−1.26E + 04**	**−1.26E + 04**	**−1.26E + 04**
	Mean	**−1.17E + 04**	**−1.24E + 04**	**−1.25E + 04**	**−1.26E + 04**	**−1.26E + 04**
	Worst	**−9.32E + 03**	**−9.65E + 03**	**−1.07E + 04**	**−1.26E + 04**	**−1.26E + 04**
	STD	**1.51E + 06**	**4.25E + 05**	**1.83E + 05**	**5.57E + 00**	**1.96E + 00**
F9	Best	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
	Median	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
	Mean	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
	Worst	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
	STD	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
F10	Best	**4.44E-16**	**4.44E-16**	**4.44E-16**	**4.44E-16**	**4.44E-16**
	Median	**4.44E-16**	**4.44E-16**	**4.44E-16**	**4.44E-16**	**4.44E-16**
	Mean	**4.44E-16**	**4.44E-16**	**4.44E-16**	**4.44E-16**	**4.44E-16**
	Worst	**4.44E-16**	**4.44E-16**	**4.44E-16**	**4.44E-16**	**4.44E-16**
	STD	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
F11	Best	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
	Median	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
	Mean	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
	Worst	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
	STD	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**	**0.00E + 00**
F12	Best	**3.36E-12**	**2.38E-12**	**1.14E-14**	**1.38E-14**	**1.07E-15**
	Median	**2.97E-10**	**7.14E-11**	**4.99E-12**	**2.14E-12**	**2.24E-13**
	Mean	**1.06E-08**	**3.08E-10**	**5.14E-11**	**4.69E-11**	**6.14E-12**
	Worst	**1.16E-07**	**1.66E-09**	**5.91E-10**	**4.76E-10**	**5.26E-11**
	STD	**7.32E-16**	**2.27E-19**	**1.75E-20**	**1.41E-20**	**1.65E-22**
F13	Best	**8.72E-13**	**1.86E-12**	**2.53E-14**	**4.74E-14**	**1.80E-14**
	Median	**3.45E-09**	**1.92E-10**	**8.35E-11**	**5.61E-12**	**4.71E-12**
	Mean	**5.35E-08**	**3.15E-09**	**6.56E-10**	**2.68E-10**	**7.98E-11**
	Worst	**6.48E-07**	**1.43E-08**	**4.69E-09**	**2.16E-09**	**8.31E-10**
	STD	**2.02E-14**	**1.76E-17**	**1.30E-18**	**3.40E-19**	**3.53E-20**

individual within the population at the t^{th} iteration.

Followers: Followers, in the conducted experiment, adhere to the guidance of the most adaptable leader and follow their footsteps. Specifically, the top 80 % to 90 % of the human population, based on their level of adaptation, are assigned the role of followers. The search strategy employed by followers can be expressed as follows:

$$X_i^{t+1} = X_i^t + \omega \cdot Rd \cdot \left(X_{best}^t - X_i^t\right) \tag{11}$$

In the Eq. (11), X_{best}^t represents the location of the most adapted individual within the human population at iteration *t.Rd* denotes a random number within the range [1, dim].

Losers: The individuals who are not well adapted and remain in the population are referred to as losers. These under-adapted losers that do not fit into the society will be eliminated, and the population will be replenished through reproduction in areas suitable for human development. The equation for population replenishment is as follows:

$$X_i^{t+1} = X_{best} + \left(X_{best} - X_i^t\right) \cdot Rn \tag{12}$$

2.5. Pseudo-code of the HEOA

To recap, the HEOA begins optimization with a pre-determined set of candidate solutions, called the population. This population is randomly generated using a Logistic Chaos mapping, which enhances the optimization procedure. During the iterative process, the search strategy of HEOA explores the vicinity of the near-optimal or best solution obtained so far. Each solution in HEOA updates its position based on the collective information gathered from all the current search results. HEOA provides five different search strategies, including one for the exploration phase and four for the development phase. These strategies are designed to strike a balance between exploration and exploitation, ensuring an effective search process. The search process of HEOA terminates when a specific end condition is satisfied. This condition can involve reaching a maximum number of iterations or achieving a satisfactory solution. For a more comprehensive understanding of HEOA, the pseudo-code of the algorithm is detailed in Fig. 3.

2.6. Computational complexity of the ECO

In this section, the general computational complexity of the HEOA is presented. The computational complexity of HEOA typically relies on three rules: solution initialization, fitness function calculation, and solution updating. Assume that N is the number of solutions and $O(N)$ is the computational complexity of the solutions' initialization processes. The computational complexity of the solutions' updating processes is $O(T \times N) + O(T \times N \times dim) + O(T \times N \times \log N)$, which consists of exploring for the best positions and updating the positions of all solutions, where the total number of iterations is called T and the dimension size of the given problem is called *dim*. Accordingly, the total computational complexity of the proposed ECO is $O(T \times N) + O(T \times N \times dim) + O(T \times N \times \log N)$.

3. Results and discussion

In this evaluation study, the performance of the HEOA is assessed using various benchmark functions and engineering problems. The evaluation involves comparing the results of HEOA with ten other population-based metaheuristic algorithms commonly found in the literature.

The algorithms used for comparison are as follows:

1. Ant Lion Optimizer (ALO) (Mirjalili, 2015b)
2. Grey Wolf Optimizer (GWO) (Saremi et al., 2015)
3. Whale Optimization Algorithm (WOA) (Mirjalili et al., 2016)
4. Salp Swarm Algorithm (SSA) (Mirjalili et al., 2017)
5. Arithmetic Optimization Algorithm (AOA) (Agushaka et al., 2021)
6. Harris Hawks Optimization (HHO) (Heidari et al., 2019)
7. Grasshopper Optimization Algorithm (GOA) (Saremi et al., 2017)
8. Sine Cosine Algorithm (SCA) (Mirjalili et al., 2016a)
9. Multi-Verse Optimizer (MVO) (Mirjalili et al., 2016c)
10. Equilibrium optimizer (EO) (Faramarzi et al., 2020a)

Table 7
Result of fixed-dimension multimodal benchmark functions (different population).

Function	Item	N = 10	N = 20	N = 30	N = 40	N = 50
F14	Best	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**
	Median	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**
	Mean	**3.00E + 00**	**1.20E + 00**	**1.05E + 00**	**9.98E-01**	**9.98E-01**
	Worst	**1.27E + 01**	**4.95E + 00**	**1.99E + 00**	**9.98E-01**	**9.98E-01**
	STD	**1.69E + 01**	**7.42E-01**	**4.69E-02**	**5.85E-26**	**1.55E-31**
F15	Best	**3.09E-04**	**3.09E-04**	**3.10E-04**	**3.08E-04**	**3.08E-04**
	Median	**3.39E-04**	**3.21E-04**	**3.22E-04**	**3.20E-04**	**3.14E-04**
	Mean	**3.50E-04**	**3.31E-04**	**3.26E-04**	**3.22E-04**	**3.17E-04**
	Worst	**5.07E-04**	**4.08E-04**	**3.61E-04**	**3.67E-04**	**3.35E-04**
	STD	**1.82E-09**	**5.70E-10**	**2.05E-10**	**1.70E-10**	**7.21E-11**
F16	Best	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**
	Median	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**
	Mean	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**
	Worst	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**
	STD	**2.16E-11**	**2.68E-14**	**1.39E-16**	**5.11E-17**	**1.20E-19**
F17	Best	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**
	Median	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**
	Mean	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**
	Worst	**3.99E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**
	STD	**3.09E-08**	**8.90E-11**	**1.19E-12**	**1.04E-14**	**8.06E-15**
F18	Best	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**
	Median	**3.58E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**
	Mean	**1.52E + 01**	**5.70E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**
	Worst	**3.00E + 01**	**3.00E + 01**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**
	STD	**1.79E + 02**	**6.56E + 01**	**2.44E-09**	**9.32E-13**	**1.38E-14**
F19	Best	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**
	Median	**−3.76E + 00**	**−3.75E + 00**	**−3.78E + 00**	**−3.86E + 00**	**−3.86E + 00**
	Mean	**−3.75E + 00**	**−3.76E + 00**	**−3.79E + 00**	**−3.83E + 00**	**−3.84E + 00**
	Worst	**−3.55E + 00**	**−3.60E + 00**	**−3.73E + 00**	**−3.74E + 00**	**−3.75E + 00**
	STD	**6.10E-03**	**3.43E-03**	**2.13E-03**	**2.02E-03**	**1.24E-03**
F20	Best	**−3.15E + 00**	**−3.29E + 00**	**−3.30E + 00**	**−3.31E + 00**	**−3.32E + 00**
	Median	**−2.88E + 00**	**−3.09E + 00**	**−3.15E + 00**	**−3.15E + 00**	**−3.26E + 00**
	Mean	**−2.91E + 00**	**−3.09E + 00**	**−3.16E + 00**	**−3.17E + 00**	**−3.23E + 00**
	Worst	**−2.67E + 00**	**−2.93E + 00**	**−2.92E + 00**	**−2.95E + 00**	**−3.08E + 00**
	STD	**1.95E-02**	**1.15E-02**	**9.67E-03**	**9.49E-03**	**7.65E-03**
F21	Best	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**
	Median	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**
	Mean	**−1.01E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**
	Worst	**−1.01E + 01**	**−1.01E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.01E + 01**
	STD	**3.84E-04**	**5.22E-06**	**5.87E-08**	**8.52E-08**	**4.81E-07**
F22	Best	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**
	Median	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**
	Mean	**−1.03E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**
	Worst	**−9.77E + 00**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**
	STD	**3.67E-02**	**1.54E-05**	**4.49E-06**	**8.13E-08**	**9.94E-08**
F23	Best	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**
	Median	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**
	Mean	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**
	Worst	**−1.03E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**
	STD	**3.25E-03**	**1.74E-05**	**1.15E-07**	**4.42E-08**	**2.36E-09**

It is important to highlight that the parameters used in algorithms can significantly influence the search results. For the experiments conducted, the parameters employed were the original values set by the authors of the respective algorithms in their research papers. Specifically, in EO, the parameters a_1 and a_2 were set to 2 and 1, respectively, while GP was set to 0.5. In GOA, the parameter $c_{\max}$ was assigned a value of 1, and $c_{\min}$ was set to 0.00004. Similarly, in MVO, the parameters $WEP_{\max}$ and $WEP_{\min}$ were set to 1 and 0.2, respectively. The implementation of the algorithms, benchmark functions (23 famous test functions), and engineering design problems was done using MATLAB R2019b.

To ensure robustness and reliability of the results, each algorithm was executed independently for 20 runs. Five performance indicators, namely Best, Worst, Average, Standard Deviation (SD), and Median values, were used to present the quality of solutions obtained by the HEOA.

3.1. Optimal population size

Determining the optimal population size is a crucial aspect for a newly developed algorithm. To evaluate the performance and capabilities of optimization algorithms, researchers commonly employ a set of 23 classical test functions (Yao et al., 1999). These test functions have been extensively used in various optimization algorithm studies (Saremi et al., 2017; Faramarzi et al., 2020a; Chopra et al., 2022; MiarNaeimi et al., 2021; Dong et al., 2021). We have selected this test set as well.

The 23 classical test functions are categorized into three types: single-peak, multi-peak, and fixed-dimension multi-peak functions. In Tables 2-4, these functions are presented along with their specific details, such as function types, search ranges, and theoretical optimal values. These test functions serve as benchmarks to assess the algorithm's ability to handle different function types and optimize within specific ranges. By evaluating the performance of the algorithm across these 23 test functions, we can gain insights into its effectiveness and efficiency in solving optimization problems.

Table 5 provides a comprehensive overview of the search results

Table 8
Results of unimodal benchmark functions (different algorithms).

Function	Item	HEOA	ALO	GWO	WOA	SSA	AOA	HHO	GOA	SCA	MVO	EO
F1	Best	**0.00E + 00**	3.29E-03	1.03E-19	3.71E-56	2.19E-06	3.67E-136	3.82E-80	1.94E + 00	1.63E + 00	1.10E + 00	6.20E-30
	Median	**0.00E + 00**	1.90E-03	3.30E-36	1.11E-101	2.64E-06	2.37E-39	4.21E-123	5.55E + 01	1.29E + 04	3.05E-01	2.35E-53
	Mean	**0.00E + 00**	4.23E-02	1.90E-18	1.59E-51	1.03E-03	1.12E-20	1.52E-62	1.17E + 01	8.42E + 01	1.77E + 00	1.67E-27
	Worst	**0.00E + 00**	1.45E-01	6.99E-18	1.39E-50	6.56E-03	2.23E-19	2.98E-61	2.65E + 01	4.30E + 02	3.57E + 00	2.24E-26
	STD	**0.00E + 00**	1.90E-03	3.30E-36	1.11E-101	2.64E-06	2.37E-39	4.21E-123	5.55E + 01	1.29E + 04	3.05E-01	2.35E-53
F2	**Best**	**0.00E + 00**	1.15E + 02	4.40E-11	3.03E-32	5.95E + 00	**0.00E + 00**	4.34E-33	1.10E + 02	7.35E-01	5.92E + 01	6.65E-16
	Median	**0.00E + 00**	1.72E + 01	1.80E-11	5.21E-34	1.47E + 00	**0.00E + 00**	2.22E-36	7.28E + 00	1.24E-01	1.03E + 00	1.38E-16
	Mean	**0.00E + 00**	4.44E + 01	1.91E-11	3.39E-33	2.08E + 00	**0.00E + 00**	3.27E-34	1.95E + 01	1.59E-01	6.06E + 00	2.13E-16
	Worst	**0.00E + 00**	1.15E + 02	4.40E-11	3.03E-32	5.95E + 00	**0.00E + 00**	4.34E-33	1.10E + 02	7.35E-01	5.92E + 01	6.65E-16
	STD	**0.00E + 00**	1.56E + 03	1.07E-22	4.90E-65	2.72E + 00	**0.00E + 00**	9.00E-67	9.55E + 02	2.32E-02	2.21E + 02	3.20E-32
F3	**Best**	**0.00E + 00**	1.50E + 03	3.04E-06	7.74E + 03	3.84E + 02	1.91E-132	4.98E-69	3.81E + 02	2.93E + 03	1.69E + 02	3.13E-09
	Median	**0.00E + 00**	3.32E + 03	5.10E-04	4.45E + 04	1.19E + 03	2.67E-47	5.42E-59	1.48E + 03	8.18E + 03	3.26E + 02	1.64E-04
	Mean	**0.00E + 00**	3.54E + 03	8.05E-04	4.44E + 04	1.66E + 03	4.72E-03	3.05E-50	2.10E + 03	9.10E + 03	3.25E + 02	9.70E-06
	Worst	**0.00E + 00**	6.85E + 03	4.92E-03	6.37E + 04	3.79E + 03	2.43E-02	4.94E-49	5.56E + 03	1.79E + 04	4.79E + 02	1.22E-04
	STD	**0.00E + 00**	1.73E + 06	1.18E-06	1.55E + 08	1.27E + 06	5.63E-05	1.16E-98	1.72E + 06	2.19E + 07	8.73E + 03	8.38E-10
F4	**Best**	**0.00E + 00**	1.12E + 01	3.54E-05	2.00E-03	3.21E + 00	8.03E-41	4.69E-37	4.14E + 00	2.04E + 01	8.83E-01	5.09E-08
	Median	**0.00E + 00**	1.61E + 01	9.75E-05	5.36E + 01	1.08E + 01	4.15E-02	3.02E-34	8.68E + 00	4.60E + 01	1.76E + 00	1.56E-07
	Mean	**0.00E + 00**	1.61E + 01	1.24E-04	4.88E + 01	1.06E + 01	3.44E-02	2.14E-32	8.69E + 00	4.53E + 01	1.85E + 00	3.93E-07
	Worst	**0.00E + 00**	2.14E + 01	2.83E-04	8.80E + 01	1.76E + 01	4.88E-02	2.38E-31	1.18E + 01	6.80E + 01	2.95E + 00	2.20E-06
	STD	**0.00E + 00**	6.68E + 00	5.60E-09	8.77E + 02	1.23E + 01	2.40E-04	3.50E-63	4.35E + 00	1.73E + 02	3.01E-01	3.23E-13
F5	**Best**	**5.04E-12**	8.96E + 01	2.60E + 01	2.75E + 01	2.87E + 01	2.78E + 01	4.61E-04	2.13E + 02	1.48E + 03	4.23E + 01	2.50E + 01
	Median	**6.90E-09**	2.14E + 02	2.64E + 01	2.78E + 01	1.38E + 02	2.84E + 01	1.06E-02	6.29E + 02	2.69E + 04	1.15E + 02	2.54E + 01
	Mean	**1.48E-07**	4.45E + 02	2.66E + 01	2.79E + 01	3.11E + 02	2.84E + 01	2.45E-02	1.42E + 03	1.17E + 05	2.80E + 02	2.54E + 01
	Worst	**1.33E-06**	1.68E + 03	2.80E + 01	2.87E + 01	1.33E + 03	2.88E + 01	1.49E-01	1.02E + 04	5.69E + 05	1.28E + 03	2.56E + 01
	STD	**1.50E-13**	1.79E + 05	3.29E-01	1.14E-01	1.61E + 05	7.30E-02	1.55E-03	4.64E + 06	2.60E + 10	9.88E + 04	2.30E-02
F6	**Best**	**1.15E-16**	2.39E-03	1.09E-04	7.54E-02	2.70E-06	2.77E + 00	2.68E-08	1.31E + 00	6.51E + 00	1.19E + 00	4.77E-06
	Median	**9.59E-12**	3.33E-02	5.04E-01	2.91E-01	1.79E-04	3.27E + 00	6.28E-05	6.94E + 00	5.63E + 01	1.90E + 00	1.37E-05
	Mean	**3.50E-10**	7.70E-02	5.13E-01	2.94E-01	7.94E-04	3.27E + 00	2.02E-04	8.75E + 00	1.01E + 02	1.90E + 00	1.57E-05
	Worst	**2.70E-09**	6.82E-01	1.26E + 00	5.35E-01	7.58E-03	3.98E + 00	1.10E-03	2.72E + 01	4.09E + 02	3.15E + 00	3.10E-05
	STD	**4.75E-19**	2.14E-02	1.16E-01	1.90E-02	2.80E-06	7.28E-02	8.62E-08	4.95E + 01	1.22E + 04	2.56E-01	5.98E-11
F7	**Best**	**1.32E-06**	1.02E-01	4.58E-04	2.64E-05	5.95E-02	1.60E-06	2.04E-05	3.27E-02	3.13E-02	8.00E-03	2.94E-04
	Median	**4.40E-06**	2.27E-01	1.61E-03	2.56E-03	1.60E-01	6.77E-05	1.25E-04	6.32E-02	1.82E-01	2.95E-02	1.25E-03
	Mean	**4.50E-06**	2.31E-01	1.77E-03	3.83E-03	1.54E-01	8.47E-05	2.01E-04	6.77E-02	2.11E-01	3.19E-02	1.50E-03
	Worst	**8.04E-06**	4.10E-01	4.06E-03	2.00E-02	2.35E-01	2.80E-04	6.84E-04	1.41E-01	7.00E-01	6.66E-02	3.71E-03
	STD	**6.00E-12**	5.37E-03	1.07E-06	2.42E-05	2.29E-03	6.44E-09	3.57E-08	8.78E-04	2.64E-02	1.62E-04	6.87E-07

obtained by the HEOA after 300 iterations for different population sizes, specifically N = 10, 20, 30, 40, and 50. The table includes detailed information on the algorithm's performance, showcasing its effectiveness in optimizing the solutions.

Fig. 4 presents the convergence curve of the fitness values achieved by the HEOA when searching for optimal solutions of F1-F23 across various population sizes. This visual representation allows for a comparison of the algorithm's performance across different function types and population sizes.

Additionally, Fig. 5 displays the function images of F1-F23, along with the convergence curves of historical search points and the average fitness change within the population achieved by the HEOA when searching for optimal solutions at N = 50. This visualization aids in understanding the algorithm's search trajectory and the progression of

Table 9
Result of multimodal benchmark functions (different algorithms).

Function	Item	HEOA	ALO	GWO	WOA	SSA	AOA	HHO	GOA	SCA	MVO	EO
F8	Best	−6.96E + 03	−5.42E + 03	−3.60E + 03	−7.80E + 03	−6.74E + 03	−4.45E + 03	**−1.26E + 04**	−6.14E + 03	−3.29E + 03	−6.54E + 03	−8.25E + 03
	Median	−8.04E + 03	−5.42E + 03	−6.19E + 03	−9.04E + 03	−7.30E + 03	−5.39E + 03	**−1.26E + 04**	−7.28E + 03	−3.82E + 03	−7.60E + 03	−8.76E + 03
	Mean	−8.22E + 03	−5.77E + 03	−6.03E + 03	−1.01E + 04	−7.50E + 03	−5.26E + 03	**−1.26E + 04**	−7.42E + 03	−3.79E + 03	−7.51E + 03	−8.91E + 03
	Worst	−1.01E + 04	−8.98E + 03	−7.34E + 03	**−1.26E + 04**	−8.76E + 03	−5.70E + 03	**−1.26E + 04**	−9.03E + 03	−4.24E + 03	−9.12E + 03	−9.96E + 03
	STD	5.31E + 05	7.16E + 05	5.07E + 05	2.98E + 06	3.43E + 05	1.47E + 05	**1.91E + 00**	6.20E + 05	7.05E + 04	4.08E + 05	1.99E + 05
F9	Best	**0.00E + 00**	4.38E + 01	3.98E-13	**0.00E + 00**	1.89E + 01	**0.00E + 00**	**0.00E + 00**	3.51E + 01	2.42E + 00	6.86E + 01	**0.00E + 00**
	Median	**0.00E + 00**	7.37E + 01	5.79E + 00	**0.00E + 00**	3.33E + 01	**0.00E + 00**	**0.00E + 00**	8.59E + 01	5.17E + 01	1.12E + 02	**0.00E + 00**
	Mean	**0.00E + 00**	7.37E + 01	5.15E + 00	5.68E-15	3.62E + 01	**0.00E + 00**	**0.00E + 00**	8.56E + 01	6.32E + 01	1.19E + 02	**0.00E + 00**
	Worst	**0.00E + 00**	9.85E + 01	1.34E + 01	5.68E-14	7.46E + 01	**0.00E + 00**	**0.00E + 00**	1.22E + 02	2.10E + 02	1.84E + 02	**0.00E + 00**
	STD	**0.00E + 00**	2.16E + 02	1.84E + 01	2.91E-28	2.06E + 02	**0.00E + 00**	**0.00E + 00**	6.27E + 02	2.87E + 03	9.15E + 02	**0.00E + 00**
F10	Best	**4.44E-16**	1.84E + 00	3.84E-11	**4.44E-16**	1.65E + 00	**4.44E-16**	**4.44E-16**	2.38E + 00	1.41E + 00	1.09E + 00	2.18E-14
	Median	**4.44E-16**	5.98E + 00	2.11E-10	4.00E-15	2.60E + 00	**4.44E-16**	**4.44E-16**	4.48E + 00	8.53E + 00	1.95E + 00	3.95E-14
	Mean	**4.44E-16**	6.10E + 00	2.52E-10	4.88E-15	2.69E + 00	**4.44E-16**	**4.44E-16**	4.07E + 00	1.12E + 01	1.98E + 00	3.77E-14
	Worst	**4.44E-16**	1.19E + 01	5.74E-10	7.55E-15	4.30E + 00	**4.44E-16**	**4.44E-16**	5.07E + 00	2.04E + 01	3.13E + 00	5.02E-14
	STD	**0.00E + 00**	6.58E + 00	2.60E-20	6.15E-30	6.11E-01	**0.00E + 00**	**0.00E + 00**	6.57E-01	6.72E + 01	2.30E-01	5.11E-29
F11	Best	**0.00E + 00**	4.71E-02	**0.00E + 00**	**0.00E + 00**	1.34E-02	3.53E-02	**0.00E + 00**	6.41E-01	1.06E + 00	8.58E-01	**0.00E + 00**
	Median	**0.00E + 00**	1.69E-01	1.11E-16	**0.00E + 00**	7.72E-02	1.77E-01	**0.00E + 00**	9.72E-01	1.33E + 00	9.38E-01	**0.00E + 00**
	Mean	**0.00E + 00**	1.64E-01	3.95E-03	3.14E-02	7.30E-02	1.91E-01	**0.00E + 00**	9.48E-01	1.78E + 00	9.41E-01	**0.00E + 00**
	Worst	**0.00E + 00**	2.87E-01	3.04E-02	2.86E-01	1.46E-01	5.19E-01	**0.00E + 00**	1.17E + 00	4.80E + 00	1.02E + 00	**0.00E + 00**
	STD	**0.00E + 00**	4.46E-03	7.41E-05	6.14E-03	1.15E-03	1.29E-02	**0.00E + 00**	1.87E-02	1.16E + 00	2.65E-03	**0.00E + 00**
F12	Best	**2.04E-14**	4.81E + 00	6.58E-03	3.10E-03	1.41E + 06	2.51E + 00	4.40E-01	3.19E-07	2.81E + 00	1.44E + 00	5.48E-01
	Median	**1.32E-12**	1.08E + 01	3.28E-02	1.48E-02	2.33E + 06	5.13E + 00	5.27E-01	3.70E-06	5.16E + 00	1.61E + 01	1.79E + 00
	Mean	**2.95E-11**	1.21E + 01	3.72E-02	2.38E-02	2.93E + 06	5.77E + 00	5.25E-01	6.74E-06	5.25E + 00	1.79E + 05	2.31E + 00
	Worst	**1.79E-10**	2.13E + 01	8.83E-02	9.54E-02	6.74E + 06	1.28E + 01	6.14E-01	2.58E-05	7.95E + 00	1.63E + 06	4.41E + 00
	STD	**2.41E-21**	1.48E + 01	4.58E-04	5.29E-04	2.27E + 12	6.75E + 00	2.91E-03	5.14E-11	2.09E + 00	1.55E + 11	1.54E + 00
F13	Best	**3.70E-14**	2.77E + 00	2.02E-01	9.55E-02	1.85E + 00	2.56E + 00	2.29E-07	1.78E + 00	1.44E + 00	6.52E-02	4.07E-06
	Median	**6.89E-12**	3.87E + 01	3.65E-01	3.59E-01	9.32E + 00	2.83E + 00	1.07E-04	1.95E + 01	3.65E + 04	1.78E-01	2.33E-05
	Mean	**7.98E-11**	4.10E + 01	3.90E-01	4.52E-01	1.27E + 01	2.82E + 00	1.52E-04	1.98E + 01	1.49E + 06	2.16E-01	5.99E-03
	Worst	**8.31E-10**	7.96E + 01	7.48E-01	1.67E + 00	3.08E + 01	2.94E + 00	5.99E-04	4.44E + 01	1.68E + 07	6.23E-01	9.74E-02
	STD	**3.53E-20**	4.84E + 02	2.14E-02	1.27E-01	7.99E + 01	9.11E-03	2.28E-08	1.72E + 02	1.47E + 13	1.48E-02	4.50E-04

the fitness values.

The combination of Table 5-7 with Figs. 4-5 offers a comprehensive view of the search performance and convergence behavior of the HEOA across different population sizes and function types. The results indicate that, at N = 50, the HEOA exhibits faster convergence and greater stability. It is noteworthy that in the case of F5 and F6, the algorithm demonstrates stronger stability at N = 40.

Furthermore, it is observed that for functions F1-F4, F9-F11, F15, and F17, the HEOA achieves convergence during the human exploration stage, successfully finding optimal solutions. For the remaining eight functions, except for F15 which yields a near-optimal solution, the HEOA discovers optimal solutions for F8, F21-F23, and near-optimal solutions for the others. This showcases the effectiveness of the algorithm in the human exploration stage, particularly in searching for single-peak functions.

However, for functions F5-F6, F8, F12-F13, F20-F23, the HEOA experiences mutations during both the human exploration stage and the human development stage. Notably, F8 and F21-F23 yield optimal solutions, while the remaining functions produce near-optimal solutions. This demonstrates the rationality of the algorithm's approach during the human development stage and the effectiveness of combining the human exploration and human development stages.

Table 10
Result of fixed-dimension multimodal benchmark functions (different algorithms).

Function	Item	HEOA	ALO	GWO	WOA	SSA	AOA	HHO	GOA	SCA	MVO	EO
F14	Best	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**	**9.98E-01**
	Median	7.37E + 00	1.99E + 00	2.49E + 00	9.98E-01	9.98E-01	1.27E + 01	9.98E-01	9.98E-01	9.99E-01	**9.98E-01**	**9.98E-01**
	Mean	7.28E + 00	2.28E + 00	2.48E + 00	2.67E + 00	1.35E + 00	8.84E + 00	1.49E + 00	9.98E-01	1.59E + 00	9.98E-01	**9.98E-01**
	Worst	1.27E + 01	7.87E + 00	1.08E + 01	1.08E + 01	3.97E + 00	1.27E + 01	5.93E + 00	9.98E-01	2.98E + 00	9.98E-01	**9.98E-01**
	STD	2.78E + 01	3.30E + 00	4.45E + 00	8.70E + 00	6.16E-01	2.32E + 01	1.22E + 00	1.55E-31	8.26E-01	1.11E-21	**3.45E-32**
F15	Best	**3.08E-04**	5.11E-04	**3.08E-04**	3.18E-04	4.23E-04	3.61E-04	3.08E-04	6.54E-04	4.34E-04	4.44E-04	**3.08E-04**
	Median	**3.18E-04**	1.23E-03	3.96E-04	5.76E-04	8.47E-04	7.20E-03	3.31E-04	1.24E-03	8.63E-04	9.91E-04	3.19E-04
	Mean	**3.30E-04**	5.93E-03	2.46E-03	6.39E-04	2.85E-03	1.74E-02	4.47E-04	3.68E-03	1.03E-03	6.69E-03	3.34E-03
	Worst	**3.69E-04**	2.09E-02	2.04E-02	1.97E-03	2.04E-02	9.61E-02	1.30E-03	2.04E-02	1.52E-03	2.04E-02	2.04E-02
	STD	**2.27E-10**	7.06E-05	3.57E-05	1.40E-07	3.42E-05	5.72E-04	8.05E-08	4.05E-05	1.27E-07	8.02E-05	5.12E-05
F16	Best	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**
	Median	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**
	Mean	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**
	Worst	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**	**−1.03E + 00**
	STD	2.53E-18	8.68E-27	2.23E-15	5.07E-18	3.50E-27	1.78E-14	1.62E-19	1.09E-23	4.58E-09	3.50E-13	**2.71E-32**
F17	Best	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**
	Median	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	4.05E-01	**3.98E-01**	**3.98E-01**	3.99E-01	**3.98E-01**	**3.98E-01**
	Mean	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	4.08E-01	**3.98E-01**	**3.98E-01**	4.00E-01	**3.98E-01**	**3.98E-01**
	Worst	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	**3.98E-01**	4.31E-01	**3.98E-01**	**3.98E-01**	4.07E-01	**3.98E-01**	**3.98E-01**
	STD	7.83E-13	2.71E-26	9.74E-09	1.18E-10	1.80E-27	7.11E-05	7.04E-10	1.01E-20	4.88E-06	3.01E-12	**0.00E + 00**
F18	Best	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**
	Median	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**
	Mean	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	1.02E + 01	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**
	Worst	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	4.87E + 01	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**	**3.00E + 00**
	STD	1.87E-13	1.92E-25	2.43E-09	8.65E-10	3.31E-25	2.24E + 02	2.53E-14	1.74E-21	4.79E-08	9.09E-11	**2.15E-30**
F19	Best	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**
	Median	−3.78E + 00	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	−3.86E + 00	−3.85E + 00	**−3.86E + 00**	−3.86E + 00	−3.85E + 00	**−3.86E + 00**	**−3.86E + 00**
	Mean	−3.80E + 00	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	**−3.86E + 00**	−3.85E + 00	**−3.86E + 00**	−3.86E + 00	−3.85E + 00	**−3.86E + 00**	**−3.86E + 00**
	Worst	−3.74E + 00	**−3.86E + 00**	**−3.86E + 00**	−3.83E + 00	**−3.86E + 00**	−3.84E + 00	−3.85E + 00	−3.84E + 00	−3.85E + 00	**−3.86E + 00**	**−3.86E + 00**
	STD	2.23E-03	9.58E-25	1.07E-06	1.18E-04	5.46E-15	1.88E-05	8.40E-06	3.52E-05	5.47E-06	3.32E-12	**3.78E-30**
F20	Best	−3.31E + 00	**−3.32E + 00**	**−3.32E + 00**	**−3.32E + 00**	**−3.32E + 00**	−3.22E + 00	−3.23E + 00	**−3.32E + 00**	−3.13E + 00	**−3.32E + 00**	**−3.32E + 00**
	Median	−3.22E + 00	**−3.32E + 00**	−3.26E + 00	−3.27E + 00	−3.20E + 00	−3.09E + 00	−3.09E + 00	−3.20E + 00	−3.04E + 00	**−3.32E + 00**	**−3.32E + 00**
	Mean	−3.19E + 00	**−3.27E + 00**	−3.25E + 00	−3.24E + 00	−3.23E + 00	−3.08E + 00	−3.08E + 00	−3.25E + 00	−3.02E + 00	**−3.27E + 00**	**−3.27E + 00**
	Worst	−3.04E + 00	−3.19E + 00	−3.09E + 00	−3.06E + 00	−3.16E + 00	−2.97E + 00	−2.90E + 00	−3.19E + 00	−2.75E + 00	**−3.20E + 00**	−3.14E + 00
	STD	7.06E-03	3.65E-03	5.96E-03	7.72E-03	4.09E-03	3.51E-03	9.89E-03	3.64E-03	8.26E-03	**3.43E-03**	4.10E-03
F21	Best	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	**−1.02E + 01**	−7.25E + 00	−1.01E + 01	**−1.02E + 01**	−5.83E + 00	**−1.02E + 01**	**−1.02E + 01**
	Median	**−1.02E + 01**	−5.06E + 00	−1.01E + 01	−1.01E + 01	**−1.02E + 01**	−3.31E + 00	−5.05E + 00	−5.10E + 00	−4.38E + 00	−7.63E + 00	**−1.02E + 01**
	Mean	**−1.01E + 01**	−5.88E + 00	−9.64E + 00	−8.23E + 00	−8.14E + 00	−3.83E + 00	−5.79E + 00	−6.39E + 00	−3.58E + 00	−7.01E + 00	−8.25E + 00
	Worst	**−1.01E + 01**	−2.63E + 00	−5.10E + 00	−2.63E + 00	−2.68E + 00	−1.71E + 00	−5.05E + 00	−2.63E + 00	−4.97E-01	−2.63E + 00	−2.63E + 00
	STD	**8.13E-05**	1.07E + 01	2.29E + 00	7.05E + 00	7.94E + 00	2.12E + 00	3.12E + 00	1.04E + 01	2.90E + 00	1.06E + 01	7.00E + 00
F22	Best	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	**−1.04E + 01**	−7.75E + 00	−5.09E + 00	**−1.04E + 01**	−5.02E + 00	**−1.04E + 01**	**−1.04E + 01**
	Median	**−1.04E + 01**	−5.13E + 00	**−1.04E + 01**	−5.09E + 00	**−1.04E + 01**	−4.27E + 00	−5.09E + 00	**−1.04E + 01**	−4.51E + 00	**−1.04E + 01**	**−1.04E + 01**
	Mean	**−1.04E + 01**	−6.54E + 00	−1.00E + 01	−7.12E + 00	−9.37E + 00	−4.03E + 00	−5.08E + 00	−7.75E + 00	−3.72E + 00	−8.31E + 00	−9.49E + 00

(*continued on next page*)

Table 10 (*continued*)

Function	Item	HEOA	ALO	GWO	WOA	SSA	AOA	HHO	GOA	SCA	MVO	EO
	Worst	**−1.04E + 01**	−2.75E + 00	−2.65E + 00	−1.84E + 00	−2.75E + 00	−1.24E + 00	−5.08E + 00	−1.84E + 00	−9.07E-01	−2.77E + 00	−2.77E + 00
	STD	**7.56E-06**	8.69E + 00	2.85E + 00	9.39E + 00	6.18E + 00	2.69E + 00	8.06E-06	1.11E + 01	1.83E + 00	8.49E + 00	4.91E + 00
F23	Best	**−1.05E + 01**	−1.05E + 01	−1.05E + 01	−1.05E + 01	−1.05E + 01	−9.23E + 00	−1.05E + 01	−1.05E + 01	−9.79E + 00	−1.05E + 01	−1.05E + 01
	Median	**−1.05E + 01**	−4.48E + 00	**−1.05E + 01**	**−1.05E + 01**	**−1.05E + 01**	−4.40E + 00	−5.13E + 00	**−1.05E + 01**	−4.47E + 00	**−1.05E + 01**	**−1.05E + 01**
	Mean	**−1.05E + 01**	−5.55E + 00	−1.01E + 01	−8.11E + 00	−8.49E + 00	−4.62E + 00	−5.89E + 00	−8.14E + 00	−4.10E + 00	−9.20E + 00	−9.80E + 00
	Worst	**−1.05E + 01**	−1.86E + 00	−2.42E + 00	−1.68E + 00	−2.43E + 00	−1.91E + 00	−5.10E + 00	−2.42E + 00	−9.45E-01	−2.43E + 00	−2.42E + 00
	STD	**2.36E-09**	9.17E + 00	3.12E + 00	9.06E + 00	1.01E + 01	2.84E + 00	3.30E + 00	1.34E + 01	3.55E + 00	7.42E + 00	4.99E + 00

In summary, the results indicate the HEOA's rationality in both the human exploration and human development stages, showcasing its powerful search capabilities for single-peak functions and its effectiveness in finding optimal or near-optimal solutions across various function types.

3.2. Comparison of different algorithms on classical test functions

To assess and compare the search capabilities of the HEOA, we have chosen ten different algorithms to conduct a comparative analysis on a classical test function. In order to ensure a fair comparison, all the considered algorithms were implemented using the same number of iterations (300) and population size (50) as the HEOA.

This approach enables us to evaluate the relative performance and efficiency of the HEOA against the selected algorithms, using consistent experimental settings. By applying this standardized evaluation framework, we can effectively compare and analyze the search abilities of the algorithms under the same conditions.

Table 8-10 provides a comprehensive comparison of the search results of the HEOA and ten popular optimization algorithms on F1-F23, utilizing five evaluation metrics. The average convergence curves of the eleven algorithms are illustrated in Fig. 6. The results demonstrate that the HEOA outperforms the other algorithms in the majority of the tested functions, except for F8, F14, and F19-20, where all algorithms achieve optimal or near-optimal solutions.

Furthermore, the HEOA exhibits superior convergence ability in most of the test functions. These findings indicate that the HEOA performs well across single-peak, multi-peak, and fixed-dimension multi-peak functions. Thus, the HEOA proves to be a comprehensive optimization algorithm with strong search capabilities.

This comprehensive evaluation demonstrates the effectiveness and competitiveness of the HEOA in comparison to other established optimization algorithms, confirming its robust performance in various function types and its suitability for solving complex optimization problems.

3.3. Comparison of different algorithms on engineering problems

In this section, the newly proposed HEOA is applied to solve four different engineering design problems, and the obtained results are presented. These engineering optimization problems are formulated to find the optimal solutions under specific conditions and constraints.

Typically, metaheuristic algorithms are not directly capable of solving constrained optimization problems. However, by incorporating constraint handling techniques (CHTs), these algorithms can effectively handle both the objective function and associated constraints. In each iteration, the algorithm evaluates the fitness of the candidate population by considering the objective function and constraints. Subsequently, the next generation of candidate populations is evaluated based on the calculated fitness values.

By utilizing the HEOA in these engineering design problems, simultaneous consideration of the objective function and constraints is achieved. This enables the search for optimal solutions that satisfy the design requirements and constraints. The effectiveness of the HEOA in handling complex engineering optimization problems is demonstrated through the integration of CHTs.

3.3.1. The three-bar truss design problem (3-BTD)

The objective of the three-bar truss problem is to minimize the volume of a loaded three-bar truss while satisfying stress (σ) constraints for each bar. This problem involves two design variables: the cross-sectional areas $A_1(=x_1)$ and $A_2(=x_2)$, as depicted in Fig. 7. The mathematical model for the three-bar truss problem is represented by Eq. (13) (Ray et al., 2001).

$$\min f(X) = (2\sqrt{2}x_1 + x_2) \times l \tag{13}$$

Subject to

$$g_1(X) = \frac{\sqrt{2}x_1 + x_2}{\sqrt{2}x_1^2 + 2x_1x_2}P - \sigma \leq 0,$$

$$g_2(X) = \frac{x_2}{\sqrt{2}x_1^2 + 2x_1x_2}P - \sigma \leq 0,$$

$$g_3(X) = \frac{1}{\sqrt{2}x_2 + x_1}P - \sigma \leq 0$$

$$l = 100cm, P = 2KN/cm^3, \sigma = 2KN/cm^3$$

Range

$$0 \leq x_1, x_2 \leq 1$$

To compare the performance of the 3-BTD solution across different algorithms, the same ten popular algorithms mentioned earlier were employed. The comparative analysis of their results is presented in Table 11. Fig. 8 displays the search history of the HEOA, the variation of the average fitness value, and the comparison of the average convergence curves of multiple algorithms. It can be observed that all algorithms, except WOA, AOA, HHO, and SCA, return the optimal solution for the 3-BTD. Notably, the HEOA exhibits the smallest standard deviation value, indicating its superior stability and performance.

These results substantiate the effectiveness and competitiveness of the proposed HEOA in solving the 3-BTD, showcasing its stability and superiority over other algorithms.

3.3.2. The optimal design of i-shaped beam (IBD)

The I-beam design problem (IBD), depicted in Fig. 9 and derived from Eq. (14), focuses on minimizing the vertical deflection of a beam. It is subject to constraints related to the loaded cross-sectional area and stresses. The design variables for this problem consist of the flange width

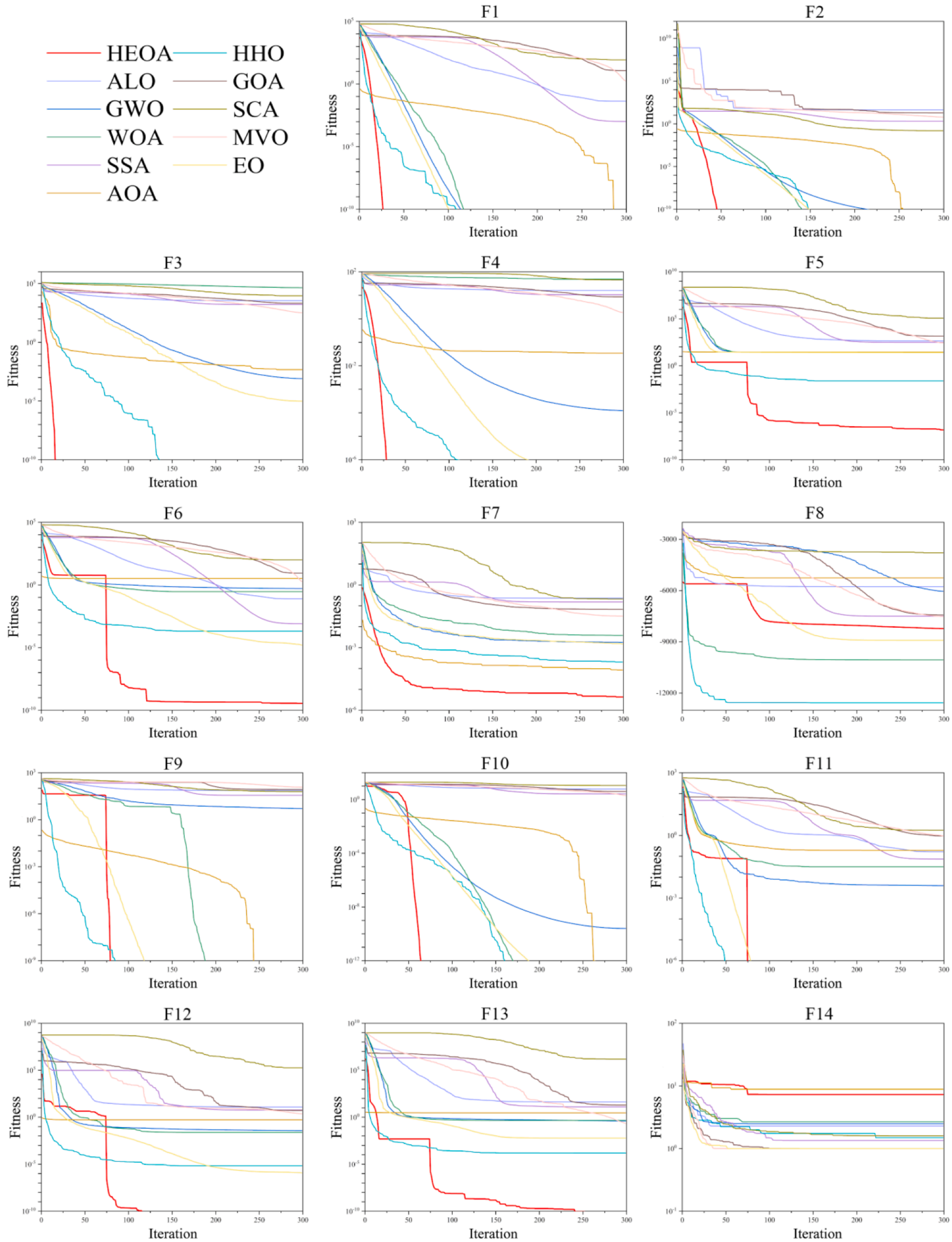

Fig. 6. Comparison of convergence rates for different algorithms.

x_1, section height x_2, web thickness x_3, and flange thickness x_4.

$$\min f(X) = \frac{500}{\frac{x_3(x_2-2x_4)^3}{12} + \left(\frac{x_1 x_4^3}{6}\right) + 2bx_4(x_2-x_4)^2} \tag{14}$$

Subject to

$$g_1(X) = 2x_1x_3 + x_3(x_2-2x_4) \leq 300$$

$$g_2(X) = \frac{18x_2 \times 10^4}{x_3(x_2-2x_4)^3 + 2x_1x_3(4x_4^2 + 3x_2(x_2-2x_4))} + \frac{15x_1 \times 10^3}{(x_2-2x_4)x_3^3 + 2x_3x_1^3} \leq 6$$

Range

$$10 \leq x_1 \leq 50, 10 \leq x_2 \leq 80, 0.9 \leq x_3 \leq 5, 0.9 \leq x_4 \leq 5$$

Based on the results presented in Table 12, all the algorithms successfully obtained the best or near-best solution for solving the IBD problem. Moreover, they yielded similar average values. Notably, HHO and EO demonstrated the highest stability, followed by HEOA, indicating their robust performance in tackling the IBD problem.

Fig. 10 showcases the search history of the HEOA, the variation of the average fitness value, and a comparison of the average convergence

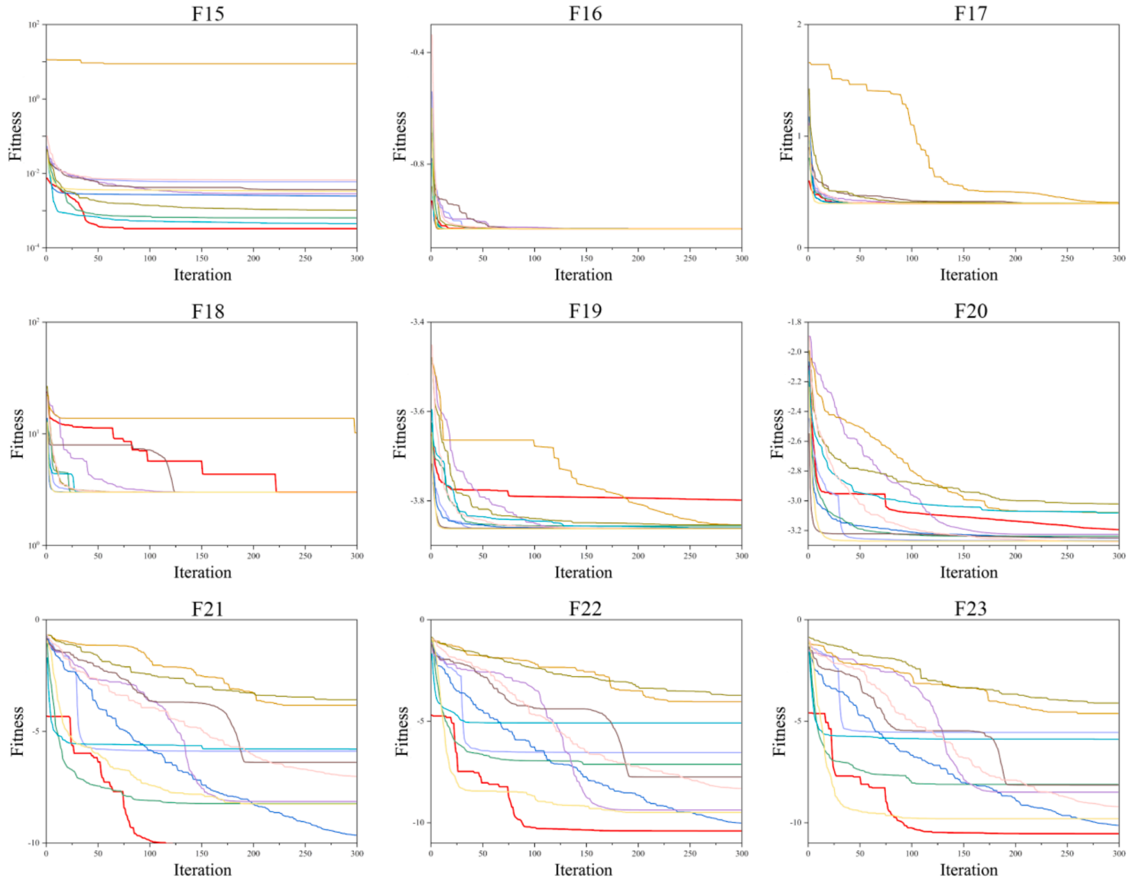

Fig. 6. (*continued*).

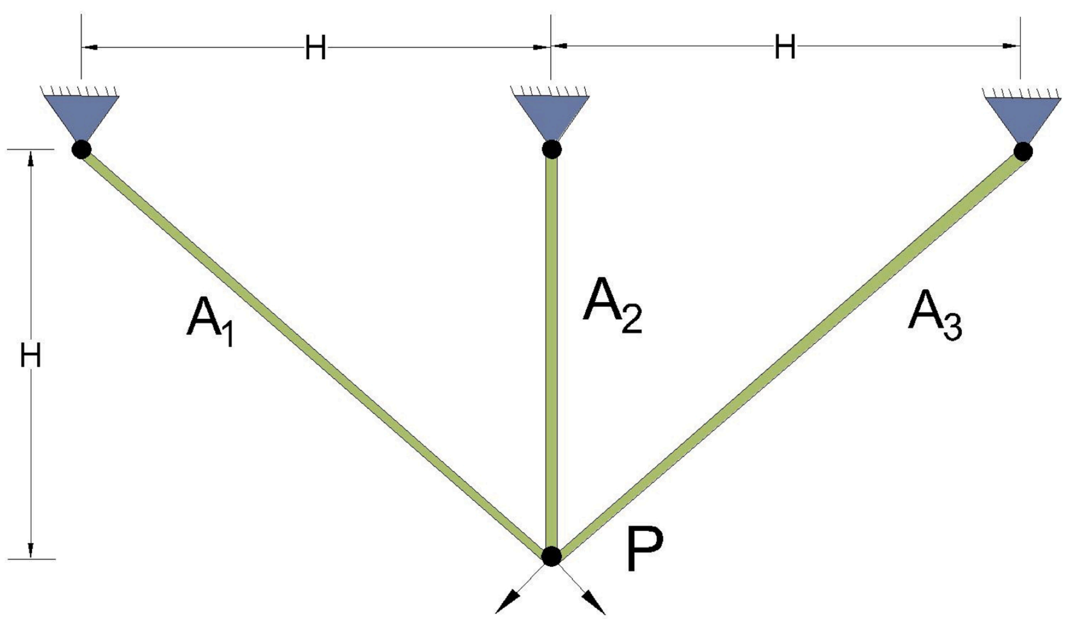

Fig. 7. Schematic illustration of 3-BTD.

curves across multiple algorithms. These visual representations provide insights into the algorithm's search trajectory and convergence behavior during the optimization process.

3.3.3. *The cantilever beam design problem (CBD)*

The objective of the cantilever beam design problem is to minimize the weight of a square section cantilever beam, as illustrated in Fig. 11 (Chickermane et al., 1996). The decision variables in this problem are the heights or widths of five hollow square blocks, with their thicknesses being constant. The mathematical model for the cantilever beam design problem is represented by Eq. (15).

$$\min f(X) = 0.0624(x_1 + x_2 + x_3 + x_4 + x_5) \tag{15}$$

Table 11
Result of 3-BTD problem.

	x1	x2	Best	Worst	Average	SD	Median
HEOA	7.89E-01	4.09E-01	**2.64E + 02**	**2.64E + 02**	**2.64E + 02**	**1.14E-06**	**2.64E + 02**
ALO	7.89E-01	4.08E-01	**2.64E + 02**	**2.64E + 02**	**2.64E + 02**	2.29E-04	**2.64E + 02**
GWO	7.88E-01	4.11E-01	**2.64E + 02**	**2.64E + 02**	**2.64E + 02**	4.03E-05	**2.64E + 02**
WOA	7.91E-01	4.02E-01	**2.64E + 02**	2.80E + 02	2.66E + 02	1.33E + 01	2.65E + 02
SSA	7.89E-01	4.08E-01	**2.64E + 02**	**2.64E + 02**	**2.64E + 02**	1.67E-04	**2.64E + 02**
AOA	7.88E-01	4.10E-01	**2.64E + 02**	2.83E + 02	2.66E + 02	1.55E + 01	2.66E + 02
HHO	7.89E-01	4.08E-01	**2.64E + 02**	2.65E + 02	**2.64E + 02**	1.23E-01	**2.64E + 02**
GOA	7.88E-01	4.10E-01	**2.64E + 02**	2.65E + 02	**2.64E + 02**	1.25E-01	**2.64E + 02**
SCA	7.85E-01	4.19E-01	**2.64E + 02**	2.65E + 02	**2.64E + 02**	2.58E-02	**2.64E + 02**
MVO	7.89E-01	4.06E-01	**2.64E + 02**	**2.64E + 02**	**2.64E + 02**	1.33E-04	**2.64E + 02**
EO	7.89E-01	4.08E-01	**2.64E + 02**	**2.64E + 02**	**2.64E + 02**	2.95E-06	**2.64E + 02**

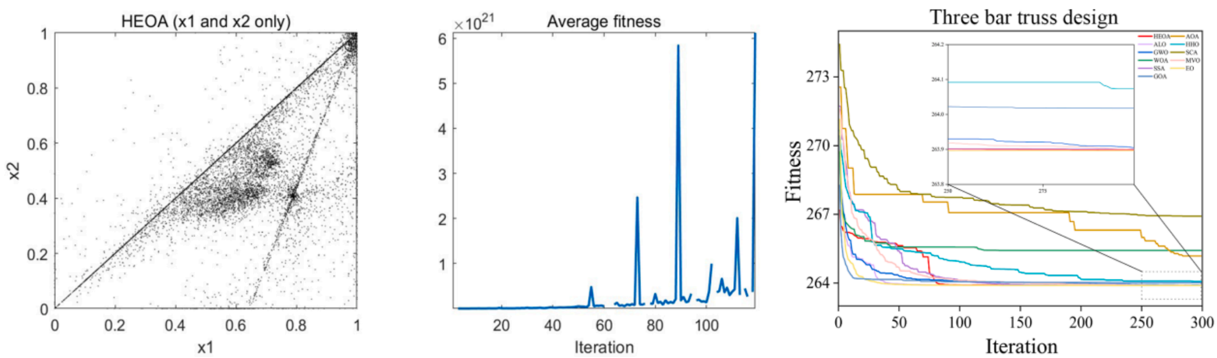

Fig. 8. Result of 3-BTD.

Subject to

$$g(X) = \frac{61}{x_1^3} + \frac{37}{x_2^3} + \frac{19}{x_3^3} + \frac{7}{x_4^3} + \frac{1}{x_5^3} - 1 \le 0$$

Range

$$0.01 < x_1 < 100, \forall i = 1, 2, 3, 4, 5$$

Based on the results presented in Table 13, it is evident that EO demonstrates the highest stability among the considered algorithms. Fig. 12 provides further insights by illustrating the search history of HEO, the variation of the average fitness value, and a comparison of the average convergence curves across multiple algorithms. The Fig. 8 highlights that HEOA exhibits the fastest convergence rate and is considered a feasible method. Although HEOA only achieves an approximate optimal solution, its convergence speed and overall performance make it a competitive approach for solving the CBD problem.

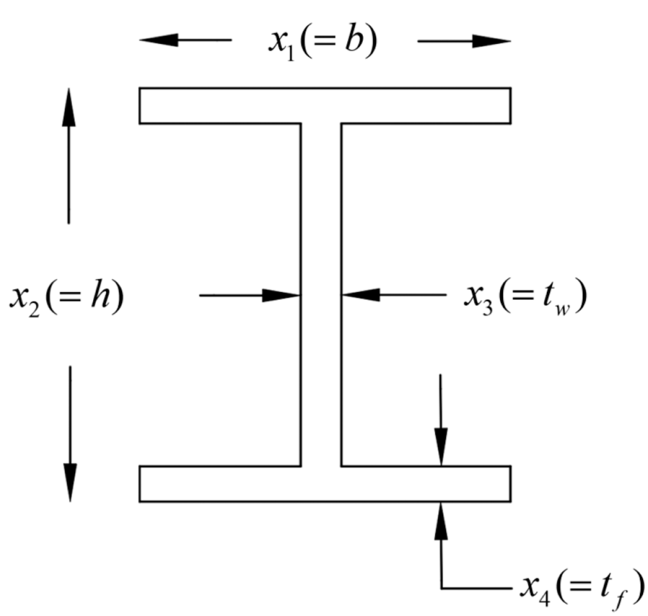

Fig. 9. Schematic illustration of IBD.

3.3.4. Hydro-static thrust bearing design problem (TBD)

The primary objective of this design problem (Kumar et al., 2020) is to optimize bearing power loss, which involves four design variables: oil viscosity μ, bearing radius R, flow rate Q, and recess radius R_0. Additionally, there are seven non-linear constraints associated with inlet oil pressure, load-carrying capacity, oil film thickness, and inlet oil pressure. These constraints play a crucial role in ensuring the feasibility and effectiveness of the optimization process.

The problem is defined as follows, where the goal is to minimize bearing power loss while satisfying the specified non-linear constraints and considering the variations in oil viscosity, bearing radius, flow rate, and recess radius.

$$f(\overline{x}) = \frac{QP_0}{0.7} + E_f \tag{16}$$

Subject to

$$g_1(\overline{x}) = 1000 - P_0 \le 0$$

$$g_2(\overline{x}) = W - 101000 \le 0$$

$$g_3(\overline{x}) = 5000 - \frac{W}{\pi(R^2 - R_0^2)} \le 0$$

$$g_4(\overline{x}) = 50 - P_0 \le 0$$

$$g_5(\overline{x}) = 0.001 - \frac{0.0307}{386.4P_0}\left(\frac{Q}{2\pi Rh}\right) \le 0$$

Table 12
Result of IBD problem.

	x1	x2	x3	x4	Best	Worst	Average	SD	Median
HEOA	5.00E + 01	8.00E + 01	1.76E + 00	5.00E + 00	**1.75E-03**	**1.75E-03**	**1.75E-03**	6.30E-27	**1.75E-03**
ALO	5.00E + 01	8.00E + 01	1.76E + 00	5.00E + 00	**1.75E-03**	**1.75E-03**	**1.75E-03**	2.93E-25	**1.75E-03**
GWO	5.00E + 01	8.00E + 01	1.76E + 00	5.00E + 00	**1.75E-03**	**1.75E-03**	**1.75E-03**	8.64E-18	**1.75E-03**
WOA	5.00E + 01	8.00E + 01	1.76E + 00	5.00E + 00	**1.75E-03**	**1.75E-03**	**1.75E-03**	1.22E-24	**1.75E-03**
SSA	5.00E + 01	8.00E + 01	1.76E + 00	5.00E + 00	**1.75E-03**	**1.75E-03**	**1.75E-03**	2.61E-15	**1.75E-03**
AOA	5.00E + 01	8.00E + 01	1.76E + 00	5.00E + 00	**1.75E-03**	**1.75E-03**	**1.75E-03**	4.14E-15	**1.75E-03**
HHO	5.00E + 01	8.00E + 01	1.76E + 00	5.00E + 00	**1.75E-03**	**1.75E-03**	**1.75E-03**	**1.88E-37**	**1.75E-03**
GOA	NA	NA	NA	NA	NA	NA	NA	NA	NA
SCA	5.00E + 01	8.00E + 01	1.76E + 00	5.00E + 00	**1.75E-03**	**2.02E-03**	**1.77E-03**	5.24E-09	**1.75E-03**
MVO	5.00E + 01	8.00E + 01	1.76E + 00	5.00E + 00	**1.75E-03**	**1.75E-03**	**1.75E-03**	2.22E-17	**1.75E-03**
EO	5.00E + 01	8.00E + 01	1.76E + 00	5.00E + 00	**1.75E-03**	**1.75E-03**	**1.75E-03**	**1.88E-37**	**1.75E-03**

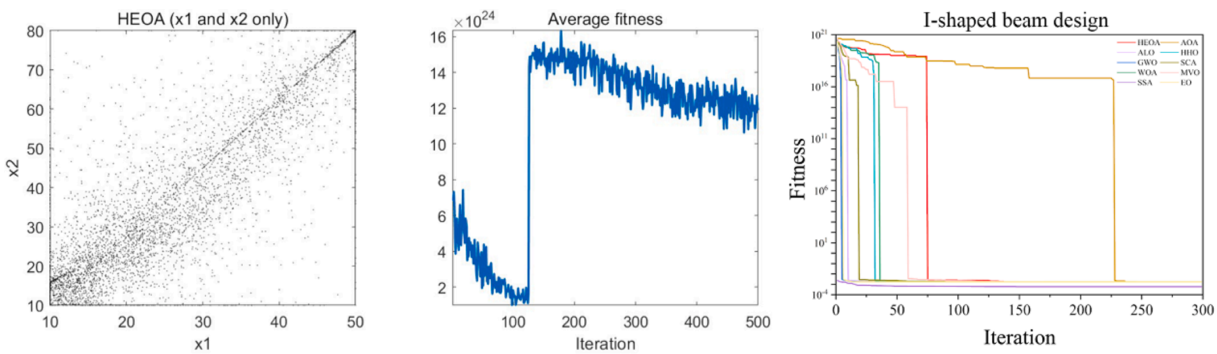

Fig. 10. Result of IBD.

$$g_6(\bar{x}) = R - R_0 \le 0$$

$$g_7(\bar{x}) = h - 0.001 \le 0$$

$$W = \frac{\pi P_0}{2} \frac{R^2 - R_0^2}{\ln\left(\frac{R}{R_0}\right)}, P_0 = \frac{6\mu Q}{\pi h^3} \ln\left(\frac{R}{R_0}\right),$$

$$E_f = 9336Q \times 0.0307 \times 0.5\triangle T, \triangle T = 2(10^P - 559.7),$$

$$P = \frac{\log_{10}\log_{10}(8.122 \times 10^6\mu + 0.8) + 3.55}{10.04},$$

$$h = \left(\frac{2\pi \times 750}{60}\right) \frac{2\pi\mu}{E_f} \left(\frac{R^4}{4} - \frac{R_0^4}{4}\right),$$

Range

$$1 \le R \le 16, 1 \le R_0 \le 16,$$

$$1 \times 10^{-6} \le \mu \le 16 \times 10^{-6}, 1 \le Q \le 16$$

From the results presented in Table 14, it is evident that EO exhibits the highest search capability and stability among the considered algorithms. On the other hand, HEOA may not be as stable as EO, but this instability can be attributed to its stronger potential for searching for the optimal solution.

Fig. 13 provides further insights by illustrating the search history of HEOA, the variation of the average fitness value, and a comparison of the average convergence curves across various algorithms. These visual representations offer a comprehensive view of the algorithm's search trajectory and convergence behavior during the optimization process.

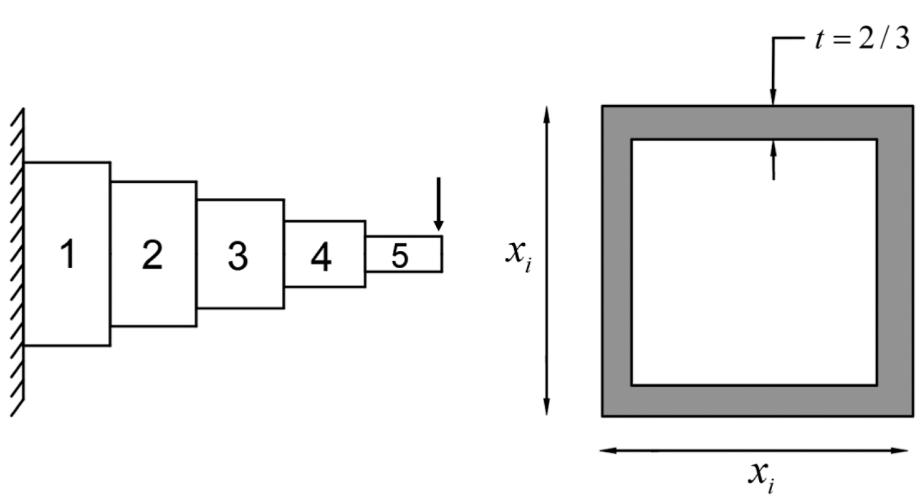

Fig. 11. Schematic illustration of CBD.

Table 13
Result of CBD problem.

	x1	x2	x3	x4	x5	Best	Worst	Average	SD	Median
HEOA	6.12E + 00	5.28E + 00	4.46E + 00	3.51E + 00	2.12E + 00	**1.34E + 00**	1.60E + 01	2.05E + 00	9.73E + 00	1.35E + 00
ALO	6.01E + 00	5.32E + 00	4.50E + 00	3.49E + 00	2.16E + 00	**1.34E + 00**	**1.34E + 00**	**1.34E + 00**	4.23E-07	**1.34E + 00**
GWO	6.02E + 00	5.31E + 00	4.51E + 00	3.51E + 00	2.12E + 00	**1.34E + 00**	**1.34E + 00**	**1.34E + 00**	6.99E-09	**1.34E + 00**
WOA	5.55E + 00	5.27E + 00	4.82E + 00	3.60E + 00	2.40E + 00	1.35E + 00	1.80E + 00	1.54E + 00	1.56E-02	1.50E + 00
SSA	6.03E + 00	5.27E + 00	4.49E + 00	3.52E + 00	2.17E + 00	**1.34E + 00**	**1.34E + 00**	**1.34E + 00**	2.32E-08	**1.34E + 00**
AOA	7.51E + 00	5.21E + 00	5.97E + 00	2.75E + 00	2.97E + 00	1.52E + 00	4.36E + 00	2.63E + 00	4.25E-01	2.43E + 00
HHO	5.96E + 00	5.20E + 00	4.54E + 00	3.66E + 00	2.13E + 00	**1.34E + 00**	1.35E + 00	1.35E + 00	9.14E-06	**1.34E + 00**
GOA	5.99 E + 00	5.34 E + 00	4.46 E + 00	3.56 E + 00	2.12 E + 00	**1.34E + 00**	1.35E + 00	**1.34E + 00**	2.52E-06	**1.34E + 00**
SCA	6.09E + 00	6.78E + 00	4.24E + 00	2.79E + 00	3.12E + 00	1.37E + 00	1.47E + 00	1.41E + 00	7.58E-04	1.41E + 00
MVO	5.95E + 00	5.31E + 00	4.55E + 00	3.51E + 00	2.16E + 00	**1.34E + 00**	**1.34E + 00**	**1.34E + 00**	1.38E-06	**1.34E + 00**
EO	6.02E + 00	5.31E + 00	4.49E + 00	3.51E + 00	2.15E + 00	**1.34E + 00**	**1.34E + 00**	**1.34E + 00**	**7.49E-10**	**1.34E + 00**

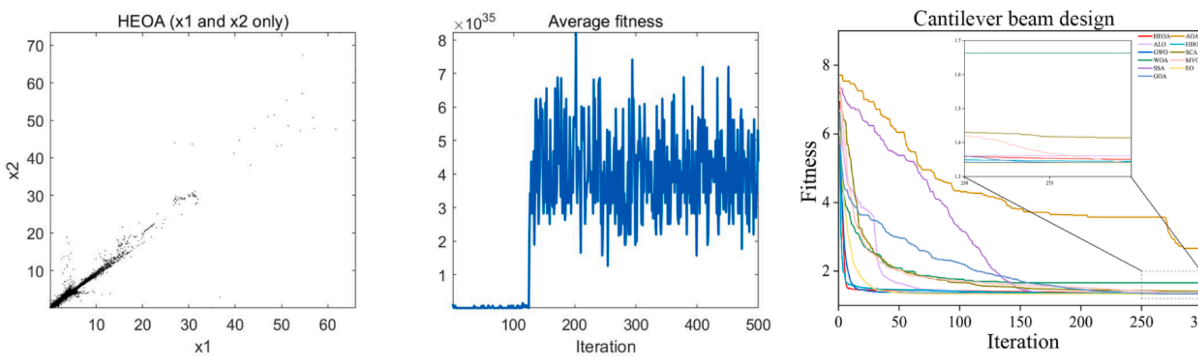

Fig. 12. Result of CBD.

3.4. *Strengths and limitations of HEOA*

The proposed HEOA exhibits strong theoretical potential for finding globally optimal solutions in various optimization problems and outperforms certain algorithms in the literature. HEOA employs human evolutionary strategies to simulate the process of knowledge acquisition and truth-seeking in humans. It incorporates adaptive strategies, jumping strategies, and Levy distributional strategies to achieve accurate search in single-peak, multi-peak, and fixed-dimension multi-peak functions.

However, it is important to note that, like many other algorithms, HEOA does not guarantee the computation of an optimal solution. Specifically, when the initial solution is already close to the optimal solution, as observed in the case of F15-F17, HEOA exhibits a lower

Table 14
Result of TBD problem.

	x1	x2	x3	x4	Best	Worst	Average	SD	Median
HEOA	6.06E + 00	5.50E + 00	6.07E-06	2.95E + 00	**1.73E + 03**	7.25E + 03	3.73E + 03	2.45E + 06	3.29E + 03
ALO	5.96E + 00	5.39E + 00	6.40E-06	3.21E + 00	2.17E + 03	5.77E + 15	4.40E + 14	1.88E + 30	2.96E + 03
GWO	6.11E + 00	5.55E + 00	6.35E-06	3.30E + 00	2.14E + 03	3.12E + 03	2.57E + 03	7.72E + 04	2.47E + 03
WOA	7.00E + 00	6.50E + 00	6.73E-06	4.82E + 00	1.91E + 03	1.16E + 14	5.81E + 12	6.42E + 26	3.15E + 03
SSA	1.59E + 01	1.59E + 01	1.38E + 01	1.17E + 00	1.02E + 30	1.02E + 30	1.02E + 30	6.76E + 51	1.02E + 30
AOA	6.48E + 00	5.79E + 00	8.65E-06	1.10E + 01	3.21E + 03	3.26E + 17	5.49E + 16	7.78E + 33	6.43E + 03
HHO	8.40E + 00	7.97E + 00	6.99E-06	7.23E + 00	2.26E + 03	7.97E + 23	4.00E + 22	3.02E + 46	5.87E + 13
GOA	NA	NA	NA	NA	NA	NA	NA	NA	NA
SCA	6.49E + 00	5.75E + 00	6.91E-06	5.30E + 00	2.80E + 03	8.14E + 03	5.21E + 03	2.56E + 06	4.66E + 03
MVO	6.49E + 00	5.97E + 00	5.41E-06	2.66E + 00	2.09E + 03	3.33E + 03	2.71E + 03	1.51E + 05	2.70E + 03
EO	6.26E + 00	5.72E + 00	5.61E-06	2.68E + 00	1.95E + 03	**2.76E + 03**	**2.36E + 03**	**5.21E + 04**	**2.34E + 03**

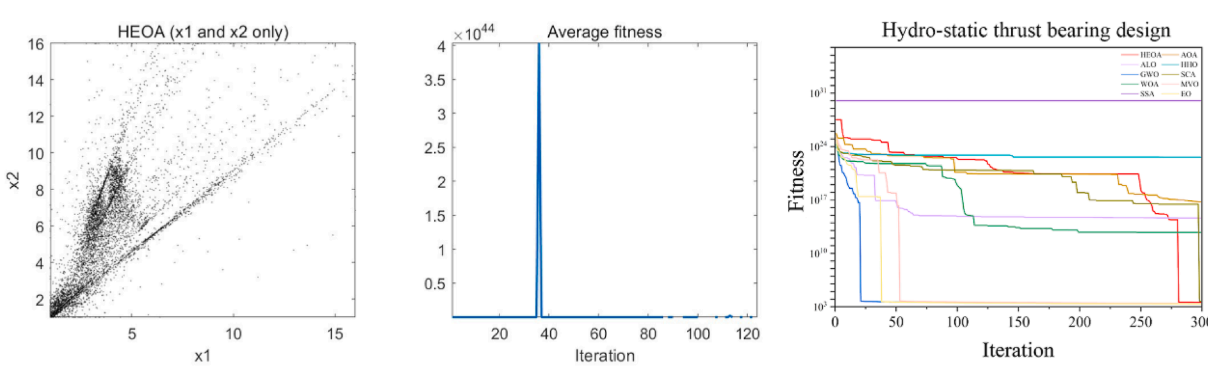

Fig. 13. Result of TBD.

convergence rate. In such scenarios, the algorithm may require more iterations to obtain a better solution. Future research should focus on further evaluating the performance of HEOA on real-world problems, which will provide valuable insights and contribute to its continuous improvement.

4. Conclusion and future work

In summary, this paper introduces the Human Evolutionary Optimization Algorithm (HEOA), a metaheuristic algorithm inspired by the human evolutionary process. HEOA's distinct approach divides the global search into two phases: human exploration and human development, initialized using logistic chaotic mapping. In the exploration phase, global search is followed by the human development phase, categorizing the population into leaders, explorers, followers, and losers, each employing specific search strategies.

Our comprehensive evaluation reveals that HEOA achieves its best performance, measured by five carefully selected performance metrics, when utilizing a population size of 50. Jumping strategies can be effective in improving search capabilities. Interestingly, our experiments indicate that larger population sizes do not necessarily yield better convergence and search capabilities. We rigorously assess HEOA's convergence speed and search accuracy across 23 classical test functions, and its suitability for engineering optimization is confirmed through four engineering problems. Extensive comparative analysis with ten state-of-the-art algorithms consistently demonstrates HEOA's superiority in approximating optimal solutions for complex global optimization problems.

Nevertheless, it is essential to acknowledge the challenges in HEOA's performance, particularly during the transition from the human search phase to the development phase. Balancing these two phases represents a crucial area for future research. Improving the trade-off between search and development phases is key to enhancing HEOA's capabilities. Furthermore, the integration of HEOA with other established algorithms holds promise for creating more robust optimization methods that leverage diverse metaheuristic techniques.

CRediT authorship contribution statement

Junbo Lian: Investigation, Resources, Conceptualization, Methodology, Software, Data curation, Formal analysis, Visualization, Writing – original draft. **Guohua Hui:** Validation, Funding acquisition, Supervision, Project administration, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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References

Abedinpourshotorban, H., Shamsuddin, S. M., Beheshti, Z., & Jawawi, D. N. (2016). Electromagnetic field optimization: A physics-inspired metaheuristic optimization algorithm. *Swarm and Evolutionary Computation, 26*, 8–22. https://doi.org/10.1016/j.swevo.2015.07.002

Abualigah, L., Diabat, A., & Geem, Z. W. (2020). A comprehensive survey of the harmony search algorithm in clustering applications. *Applied Sciences, 10*(11), 3827. https://doi.org/10.3390/app10113827

Abualigah, L., Yousri, D., Abd Elaziz, M., Ewees, A. A., Al-Qaness, M. A., & Gandomi, A. H. (2021). Aquila optimizer: A novel meta-heuristic optimization algorithm. *Computers & Industrial Engineering, 157*, Article 107250. https://doi.org/10.1016/j.cie.2021.107250

Abualigah, L., Diabat, A., Mirjalili, S., Abd Elaziz, M., & Gandomi, A. H. (2021). The arithmetic optimization algorithm. *Computer methods in applied mechanics and engineering, 376*, Article 113609. https://doi.org/10.1016/j.cma.2020.113609

Agushaka, J. O., Ezugwu, A. E., & Abualigah, L. (2022). Dwarf mongoose optimization algorithm. *ComputerMethods in Applied Mechanics and Engineering, 391*, Article 114570. https://doi.org/10.1016/j.cma.2022.114570

Agushaka, J. O., & Ezugwu, A. E. (2021). Evaluation of several initialization methods on arithmetic optimization algorithm performance. *Journal of Intelligent Systems, 31*(1), 70–94. https://doi.org/10.1515/jisys-2021-0164

Barrow, J. D. (1977). A chaotic cosmology. *Nature, 267*(5607), 117–120. https://doi.org/10.1038/267117a0

Barthelemy, P., Bertolotti, J., & Wiersma, D. S. (2008). A Lévy flight for light. *Nature, 453*(7194), 495–498. https://doi.org/10.1038/nature06948

Beni, G., & Wang, J. (1993). Swarm intelligence in cellular robotic systems. In *Robots and biological systems: towards a new bionics?* (pp. 703–712). Berlin, Heidelberg: Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-58069-7_38.

Cheraghalipour, A., Hajiaghaei-Keshteli, M., & Paydar, M. M. (2018). Tree Growth Algorithm (TGA): A novel approach for solving optimization problems. *Engineering Applications of Artificial Intelligence, 72*, 393–414. https://doi.org/10.1016/j.engappai.2018.04.021

Chickermane, H., & Gea, H. C. (1996). Structural optimization using a new local approximation method. *International Journal For Numerical Methods In Engineering, 39*(5), 829–846. https://doi.org/10.1002/(SICI)1097-0207(19960315)39:5<829::AID-NME884>3.0.CO;2-U

Chopra, N., & Ansari, M. M. (2022). Golden jackal optimization: A novel nature-inspired optimizer for engineering applications. *Expert Systems with Applications, 198*, Article 116924. https://doi.org/10.1016/j.eswa.2022.116924

Colorni, A., Dorigo, M., & Maniezzo, V. (1991, December). Distributed optimization by ant colonies. In Proceedings of the first European conference on artificial life (Vol. 142, pp. 134-142). https://www-public.imtbs-tsp.eu/~gibson/Teaching/Teaching-ReadingMaterial/ColorniDorigoManiezzo91.pdf.

Dong, R., Chen, H., Heidari, A. A., Turabieh, H., Mafarja, M., & Wang, S. (2021). Boosted kernel search: Framework, analysis and case studies on the economic emission dispatch problem. *Knowledge-Based Systems, 233*, Article 107529. https://doi.org/10.1016/j.knosys.2021.107529

Eberhart, R., & Kennedy, J. (1995, October). A new optimizer using particle swarm theory. In MHS'95. Proceedings of the sixth international symposium on micro machine and human science (pp. 39-43). IEEE. https://ieeexplore.ieee.org/document/494215.

Erol, O. K., & Eksin, I. (2006). A new optimization method: Big bang–big crunch. *Advances in engineering software, 37*(2), 106–111. https://doi.org/10.1016/j.advengsoft.2005.04.005

Ezugwu, A. E., Shukla, A. K., Nath, R., Akinyelu, A. A., Agushaka, J. O., Chiroma, H., & Muhuri, P. K. (2021). Metaheuristics: A comprehensive overview and classification along with bibliometric analysis. *Artificial Intelligence Review, 54*, 4237–4316. https://doi.org/10.1007/s10462-020-09952-0

Faramarzi, A., Heidarinejad, M., Stephens, B., & Mirjalili, S. (2020). Equilibrium optimizer: A novel optimization algorithm. *Knowledge-Based Systems, 191*, Article 105190. https://doi.org/10.1016/j.knosys.2019.105190

Faramarzi, A., Heidarinejad, M., Mirjalili, S., & Gandomi, A. H. (2020). Marine predators algorithm: A nature-inspired metaheuristic. *Expert systems with applications, 152*, Article 113377. https://doi.org/10.1016/j.eswa.2020.113377

Formato, R. A. (2007). Central force optimization. Prog Electromagn Res, 77(1), 425-491. https://www.academia.edu/download/39993697/CFO_PREPRINT_11-12-2015.pdf.

Gandomi, A. H., & Alavi, A. H. (2012). Krill herd: A new bio-inspired optimization algorithm. *Communications in Nonlinear Science and Numerical Simulation, 17*(12), 4831–4845. https://doi.org/10.1016/j.cnsns.2012.05.010

Hajipour, V., Mehdizadeh, E., & Tavakkoli-Moghaddam, R. (2014). A novel Pareto-based multi-objective vibration damping optimization algorithm to solve multi-objective optimization problems. *Scientia Iranica, 21*(6), 2368–2378. https://scientiairanica.sharif.edu/article_3628.html.

Hajipour, V., Kheirkhah, A., Tavana, M., & Absi, N. (2015). Novel Pareto-based meta-heuristics for solving multi-objective multi-item capacitated lot-sizing problems. *The International Journal of Advanced Manufacturing Technology, 80*, 31–45. https://doi.org/10.1007/s00170-015-6993-6

Haklı, H., & Uğuz, H. (2014). A novel particle swarm optimization algorithm with Levy flight. *Applied Soft Computing, 23*, 333–345. https://doi.org/10.1016/j.asoc.2014.06.034

Hashim, F. A., Houssein, E. H., Mabrouk, M. S., Al-Atabany, W., & Mirjalili, S. (2019). Henry gas solubility optimization: A novel physics-based algorithm. *Future Generation Computer Systems, 101*, 646–667. https://doi.org/10.1016/j.future.2019.07.015

Heidari, A. A., Mirjalili, S., Faris, H., Aljarah, I., Mafarja, M., & Chen, H. (2019). Harris hawks optimization: Algorithm and applications. *Future Generation Computer Systems, 97*, 849–872. https://doi.org/10.1016/j.future.2019.02.028

Jain, M., Singh, V., & Rani, A. (2019). A novel nature-inspired algorithm for optimization: Squirrel search algorithm. *Swarm and evolutionary computation, 44*, 148–175. https://doi.org/10.1016/j.swevo.2018.02.013

Jerebic, J., Mernik, M., Liu, S. H., Ravber, M., Baketarić, M., Mernik, L., & Črepinšek, M. (2021). A novel direct measure of exploration and exploitation based on attraction basins. *Expert Systems with Applications, 167*, Article 114353. https://doi.org/10.1016/j.eswa.2020.114353

Johnson, T., & Husbands, P. (1990, October). System identification using genetic algorithms. In International conference on parallel problem solving from nature (pp. 85-89). Berlin, Heidelberg: Springer Berlin Heidelberg. https://doi.org/10.1007/BFb0029736.

John, H. (1992). Holland. Genetic algorithms. *Scientific American, 267*(1), 44–50. https://www.jstor.org/stable/24939139.

Kaidi, W., Khishe, M., & Mohammadi, M. (2022). Dynamic levy flight chimp optimization. *Knowledge-Based Systems, 235*, Article 107625. https://doi.org/10.1016/j.knosys.2021.107625

Kanso, A., & Smaoui, N. (2009). Logistic chaotic maps for binary numbers generations. *Chaos, Solitons & Fractals, 40*(5), 2557–2568. https://doi.org/10.1016/j.chaos.2007.10.049

Karaboga, D., & Basturk, B. (2007). A powerful and efficient algorithm for numerical function optimization: Artificial bee colony (ABC) algorithm. *Journal of global optimization, 39*, 459–471. https://doi.org/10.1007/s10898-007-9149-x

Kaveh, A., & Dadras, A. (2017). A novel meta-heuristic optimization algorithm: Thermal exchange optimization. *Advances in engineering software, 110*, 69–84. https://doi.org/10.1016/j.advengsoft.2017.03.014

Kumar, A., Wu, G., Ali, M. Z., Mallipeddi, R., Suganthan, P. N., & Das, S. (2020). A test-suite of non-convex constrained optimization problems from the real-world and some baseline results. *Swarm and Evolutionary Computation, 56*, Article 100693. https://doi.org/10.1016/j.swevo.2020.100693

Kumar, M., Kulkarni, A. J., & Satapathy, S. C. (2018). Socio evolution & learning optimization algorithm: A socio-inspired optimization methodology. *Future Generation Computer Systems, 81*, 252–272. https://doi.org/10.1016/j.future.2017.10.052

Liang, J. J., Qu, B. Y., & Suganthan, P. N. (2013). Problem definitions and evaluation criteria for the CEC 2014 special session and competition on single objective real-parameter numerical optimization. *Computational Intelligence Laboratory, Zhengzhou University, Zhengzhou China and Technical Report, Nanyang Technological University, Singapore, 635*(2). http://bee22.com/manual/tf_images/Liang%20CEC2014.pdf.

Nadimi-Shahraki, M. H., Fatahi, A., Zamani, H., Mirjalili, S., & Abualigah, L. (2021). An improved moth–flame optimization algorithm with adaptation mechanism to solve numerical and mechanical engineering problems. *Entropy, 23*(12), 1637. https://doi.org/10.3390/e23121637

MiarNaeimi, F., Azizyan, G., & Rashki, M. (2021). Horse herd optimization algorithm: A nature-inspired algorithm for high-dimensional optimization problems. *Knowledge-Based Systems, 213*, Article 106711. https://doi.org/10.1016/j.knosys.2020.106711

Michalewicz, Z., Krawczyk, J. B., Kazemi, M., & Janikow, C. Z. (1990, December). Genetic algorithms and optimal control problems. In 29th IEEE conference on decision and control (pp. 1664-1666). IEEE. https://ieeexplore.ieee.org/abstract/document/203904.

Mirjalili, S. (2015a). Moth-flame optimization algorithm: A novel nature-inspired heuristic paradigm. *Knowledge-based Systems, 89*, 228–249. https://doi.org/10.1016/j.knosys.2015.07.006

Mirjalili, S. (2015b). The ant lion optimizer. *Advances in Engineering Software, 83*, 80–98. https://doi.org/10.1016/j.advengsoft.2015.01.010

Mirjalili, S. (2016a). SCA: A sine cosine algorithm for solving optimization problems. *Knowledge-based systems, 96*, 120–133. https://doi.org/10.1016/j.knosys.2015.12.022

Mirjalili, S. (2016b). Dragonfly algorithm: A new meta-heuristic optimization technique for solving single-objective, discrete, and multi-objective problems. *Neural Computing and Applications, 27*, 1053–1073. https://doi.org/10.1007/s00521-015-1920-1

Mirjalili, S., Mirjalili, S. M., & Hatamlou, A. (2016). Multi-verse optimizer: A nature-inspired algorithm for global optimization. *Neural Computing and Applications, 27*, 495–513. https://doi.org/10.1007/s00521-015-1870-7

Mirjalili, S., & Lewis, A. (2016). The whale optimization algorithm. *Advances in Engineering Software, 95*, 51–67. https://doi.org/10.1016/j.advengsoft.2016.01.008

Mirjalili, S., Gandomi, A. H., Mirjalili, S. Z., Saremi, S., Faris, H., & Mirjalili, S. M. (2017). Salp Swarm Algorithm: A bio-inspired optimizer for engineering design problems. *Advances in Engineering Software, 114*, 163–191. https://doi.org/10.1016/j.advengsoft.2017.07.002

Moghdani, R., & Salimifard, K. (2018). Volleyball premier league algorithm. *Applied Soft Computing, 64*, 161–185. https://doi.org/10.1016/j.asoc.2017.11.043

Mohammadi-Balani, A., Nayeri, M. D., Azar, A., & Taghizadeh-Yazdi, M. (2021). Golden eagle optimizer: A nature-inspired metaheuristic algorithm. *Computers & Industrial Engineering, 152*, Article 107050. https://doi.org/10.1016/j.cie.2020.107050

Nabil, E. (2016). A modified flower pollination algorithm for global optimization. *Expert Systems with Applications, 57*, 192–203. https://doi.org/10.1016/j.eswa.2016.03.047

Oyelade, O. N., & Ezugwu, A. E. (2021). Ebola Optimization Search Algorithm (EOSA): A new metaheuristic algorithm based on the propagation model of Ebola virus disease. arXiv preprint arXiv:2106.01416. https://arxiv.org/abs/2106.01416.

Pan, W. T. (2012). A new fruit fly optimization algorithm: Taking the financial distress model as an example. *Knowledge-Based Systems, 26*, 69–74. https://doi.org/10.1016/j.knosys.2011.07.001

Qin, A. K., Huang, V. L., & Suganthan, P. N. (2008). Differential evolution algorithm with strategy adaptation for global numerical optimization. *IEEE transactions on Evolutionary Computation, 13*(2), 398–417. https://ieeexplore.ieee.org/abstract/document/4632146.

Rao, R. V., Savsani, V. J., & Vakharia, D. P. (2012). Teaching–learning-based optimization: An optimization method for continuous non-linear large scale problems. *Information Sciences, 183*(1), 1–15. https://doi.org/10.1016/j.ins.2011.08.006

Rashedi, E., Nezamabadi-Pour, H., & Saryazdi, S. (2009). GSA: A gravitational search algorithm. *Information Sciences, 179*(13), 2232–2248. https://doi.org/10.1016/j.ins.2009.03.004

Ray, T., & Saini, P. (2001). Engineering design optimization using a swarm with an intelligent information sharing among individuals. *Engineering Optimization, 33*(6), 735–748. https://doi.org/10.1080/03052150108940941

Salcedo-Sanz, S. (2016). Modern meta-heuristics based on nonlinear physics processes: A review of models and design procedures. *Physics Reports, 655*, 1–70. https://doi.org/10.1016/j.physrep.2016.08.001

Saremi, S., Mirjalili, S. Z., & Mirjalili, S. M. (2015). Evolutionary population dynamics and grey wolf optimizer. *Neural Computing and Applications, 26*, 1257–1263. https://doi.org/10.1007/s00521-014-1806-7

Saremi, S., Mirjalili, S., & Lewis, A. (2017). Grasshopper optimisation algorithm: Theory and application. *Advances in engineering software, 105*, 30–47. https://doi.org/10.1016/j.advengsoft.2017.01.004

Simon, D. (2008). Biogeography-based optimization. *IEEE transactions on evolutionary computation, 12*(6), 702–713. https://ieeexplore.ieee.org/document/4475427.

Storn, R., & Price, K. (1997). Differential evolution–a simple and efficient heuristic for global optimization over continuous spaces. *Journal of global optimization, 11*, 341–359. https://doi.org/10.1023/A:1008202821328

Tang, D., Dong, S., Jiang, Y., Li, H., & Huang, Y. (2015). ITGO: Invasive tumor growth optimization algorithm. *Applied Soft Computing, 36*, 670–698. https://doi.org/10.1016/j.asoc.2015.07.045

Wang, G. G. (2018). Moth search algorithm: A bio-inspired metaheuristic algorithm for global optimization problems. *Memetic Computing, 10*(2), 151–164. https://doi.org/10.1007/s12293-016-0212-3

Wu, G. (2016). Across neighborhood search for numerical optimization. *Information Sciences, 329*, 597–618. https://doi.org/10.1016/j.ins.2015.09.051

Xue, J., & Shen, B. (2020). A novel swarm intelligence optimization approach: Sparrow search algorithm. *Systems science & control engineering, 8*(1), 22–34. https://doi.org/10.1080/21642583.2019.1708830

Yang, X. S. (2010). A new metaheuristic bat-inspired algorithm. In Nature inspired cooperative strategies for optimization (NICSO 2010) (pp. 65-74). Berlin, Heidelberg: Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-12538-6_6.

Yang, X. S. (2009a). Firefly algorithms for multimodal optimization. In International symposium on stochastic algorithms (pp. 169-178). Berlin, Heidelberg: Springer Berlin Heidelberg. https://doi.org/10.1007/978-3-642-04944-6_14.

Yang, X. S., & Deb, S. (2009). Cuckoo search via Lévy flights. In *In 2009 World congress on nature & biologically inspired computing (NaBIC)* (pp. 210–214). IEEE.

Yao, X., Liu, Y., & Lin, G. (1999). Evolutionary programming made faster. *IEEE Transactions on Evolutionary computation, 3*(2), 82–102. https://ieeexplore.ieee.org/document/771163.

Yazdani, M., & Jolai, F. (2016). Lion optimization algorithm (LOA): A nature-inspired metaheuristic algorithm. *Journal of computational design and engineering, 3*(1), 24–36. https://doi.org/10.1016/j.jcde.2015.06.003

Zapata, H., Perozo, N., Angulo, W., & Contreras, J. (2020). A hybrid swarm algorithm for collective construction of 3D structures. *International Journal of Artificial Intelligence, 18*(1), 1–18. https://www.academia.edu/83417198/A_Hybrid_Swarm_Algorithm_for_Collective_Construction_of_3D_Structures.

Zhang, Q., Wang, R., Yang, J., Ding, K., Li, Y., & Hu, J. (2017). Collective decision optimization algorithm: A new heuristic optimization method. *Neurocomputing, 221*, 123–137. https://doi.org/10.1016/j.neucom.2016.09.068

Zheng, R., Jia, H., Abualigah, L., Liu, Q., & Wang, S. (2022). An improved arithmetic optimization algorithm with forced switching mechanism for global optimization problems. *Mathematical Biosciences and Engineering, 19*(1), 473–512. http://www.aimspress.com/aimspress-data/mbe/2022/1/PDF/mbe-19-01-023.pdf.