APPENDIX TO CUBIT

A PARALLELIZE PRIOR BITMAP INDEXES

UpBit. We parallelize UpBit by using fine-grained reader-writer latches. Specifically, the <VB, UB> pair of each value v is protected by a reader-writer latch, denoted $latch_v$. A latch $latch_{nor}$ is used to protect the global variables like *number_of_rows*. An insert operation acquires $latch_{nor}$ and the corresponding $latch_v$, before updating the UB of value v and the variable *number_of_rows*. An update acquires the latches for both the old and new values, and a delete acquires the $latch_v$ for the old value v, before updating the corresponding UBs. A query acquires the corresponding $latch_v$ in shared mode, and upgrades it to exclusive mode when a merge operation is required.

UCB. All UDIs of UCB update the only EB, and all queries access this EB simultaneously. We thus use a global reader-writer latch to synchronize concurrent queries and UDIs.

In-place. It would be possible to parallelize In-place by using fine-grained reader-writer latches that work the same way as in UpBit. However, with this mechanism, an insert needs to acquire *cardinality* latches before appending bits to the tail of all of the VBs, increasing the probability of deadlock. Instead, we parallelize In-place by using a global reader-writer latch, the same as in UCB. Surprisingly, the parallelized In-place outperforms UCB for high concurrency (§6.2).

B PROGRAMMING SEMANTICS OF CUBIT

CUBIT complies with the standard specification for database indexes and is aware of the standard database transaction semantics [50]. It provides the following API:

- Update(row, val) retrieves the current value of the specified *row*, updates the value to *val*, and returns true.
- Delete(row) deletes the specified *row*, and returns true.
- Insert(val) appends the value *val* to the tail of the bitmap index, and returns true.
- Query(val) finds whether the specified value *val* exists in the bitmap index, and if so, returns a list of matching positions.
- TransBegin() marks the start of a new transaction T by taking a snapshot, on which T's Queries and UDIs are performed.
- TransCommit() attempts to commit T. It contains the linearization point [35] at which this transaction's UDIs atomically take effect with respect to other concurrent transactions.
- TransAbort() gives up this transaction.

Further, there is an internal Merge operation that propagates changes captured in UBs to VBs and reinitializes the UBs (§4.8).

Write Skew Anomaly. CUBIT satisfies *snapshot isolation* [13], which is not serializable, such that CUBIT permits write skew anomaly [13]. This is the same as most modern MVCC-based database systems [24]. CUBIT can detect write skew anomaly, abort conflicting transactions, and guarantees serializability, by also recording the read set of each transaction and checking read-write conflicts before committing each transaction.

C PSEUDOCODE FOR CUBIT

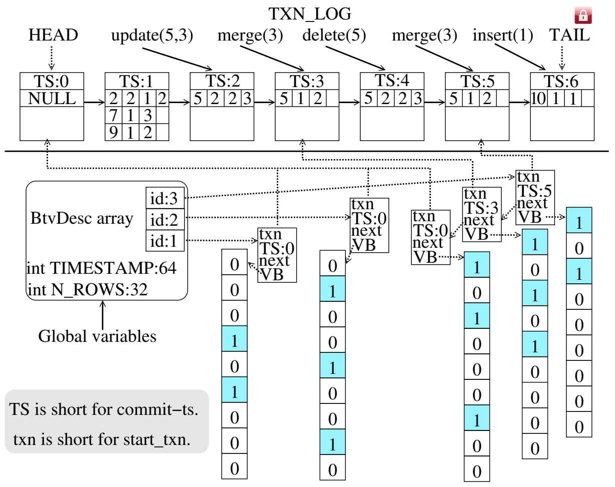


Figure 14: Illustrative architecture of CUBIT. The initial state is equivalent to Figure 2(d), followed by a Update to change the 5th row to value 3, a Merge on value 3, a Delete to eliminate the 5th row, another Merge on value 3, and Insert to append a value 1.

We now present the key data structures and functions of CUBIT based on its overall architecture shown in Figure 14.

C.1 Data Structure

Algorithm 1 lists the shared variables and core data structures of CUBIT. Structure RUB consists of a 32-bit row ID, an 8-bit counter indicating the number of 1s in this RUB, and an array of 12-bit integers keeping track of the positions of the 1s. The size of each field in RUB is pre-defined and configurable. For the configuration example in Algorithm 1, the number of rows, number of 1s in each RUB, and cardinality of the indexed attribute are in maximum 2^{32}, 2^8 and 2^{12}, respectively.

Each transaction allocates an instance of the structure TxnDesc, which contains the timestamps indicating when the transaction started (*start-ts*) and committed (*commit-ts*), the number of rows of this snapshot, and an array of RUBs generated by the UDIs in this transaction.

Each BtvDesc contains the timestamp indicating when this VB is committed, a shortcut pointer referencing to the TxnDesc from where subsequent queries and UDIs start searching related RUBs, and a pointer to the underlying VB,

The global array, *vcs*, contains cardinality pointers, each of which references to a version chain. The global variables TIMESTAMP and N_ROWS record the current timestamp value and the number of rows of the dataset. Pointers HEAD and TAIL points to the head and tail of TXN_LOG. In CUBIT-lk, concurrent updates to the global variables are protected by the latch *bitmap_lk*. Note that queries do not acquire this latch, and that in CUBIT-lf, this latch is removed.

Each active thread contains an instance of *ThreadInfo*, which points to the TxnDesc of the active transaction, or is set to NULL if this thread currently does not involve in any active transaction.

C.2 Query and UDI

The pseudocode for the query operation is shown in Algorithm 2. Assume that a query is in a transaction T. It first traverses the

Algorithm 1: Structures and shared variables of CUBIT.

```
1  struct RUB
2      row_id : 32;   n_ones : 8;   pos[];
3  struct TxnDesc
4      int start-ts: 64;   int n_rows: 32;   int commit-ts: 64;
5      RUB rubs[];   TxnDesc *next;
6  struct BtvDesc
7      int commit_ts: 64;   TxnDesc *start_txn;
8      Bitvector *vb;   BtvDesc *next;
9  struct ThreadInfo
10     TxnDesc *txn;
11 Global variables:
12     BtvDesc *vcs[cardinality] ;                    /* Version chains */
13     int TIMESTAMP : 64 ;                        /* Timestamp counter */
14     int N_ROWS : 32 ;                       /* Number of rows counter */
15     TxnDesc *HEAD,  *TAIL ;                    /* Pointers to TXN_LOG */
16     mutex bitmap_lk ;                     /* Global latch for CUBIT-lk */
17 Shared variables for thread i:
18     ThreadInfo  *th[i];
```

corresponding version chain and locates the BtvDesc *btv* with the largest *commit-ts* that is less than or equal to *T.start-ts* (Line 22). The query then collects a RUB set by traversing TxnDescs with their *commit-ts* ∈ (*btv.commit-ts*, *T.start-ts*] (Line 24). If the return set is empty, which indicates that *btv* is the latest version when *T* took the snapshot, we reuse this VB for evaluation (Line 27). Otherwise, the query makes a private copy of the underlying VB (Line 30), applies the RUBs to the private copy (Line 32), and then evaluate on the private copy (Line 33). If the number of RUBs merged into the private copy is larger than a pre-defined threshold, the query sends a merge request to the background maintenance threads which are in charge of inserting the newly-generated VB into the version chain (§C.4).

Algorithm 2: Query of CUBIT.

```
19 function Query(ThreadInfo *th, int val)
20     BtvDesc *btv ← btvs[val];
21     int tsp_end ← th->txn->start-ts;
22     Traverse version chain until btv->commit-ts ≤ tsp_end ;
23     int tsp_begin ← btv->commit-ts;
24     rubs ← rubs_involving_val(val, tsp_begin, tsp_end) ;
25     Bitvector * vb_new;
26     if rubs is empty:
27         vb_new ← btv.vb ;
28     else:
29         vb_new ← allocate Bitvector;
30         vb_new->copy(btv.vb) ;
31         for each rub in rubs:
32             vb_new[rub.row_id] ← ¬ vb_new[rub.row_id] ;
33     Evaluate(vb_new) ;
34     if (rubs.size() > MERGE_THRESHOLD):
35         register_merge_request(th, val, btv, vb_new, rubs) ;
36 function rubs_involving_val(val, tsp_begin, tsp_end)
37     map<int:32, int:64> rubs;
38     for each TxnDesc T in TXN_LOG(tsp_begin, tsp_end]:
39         for each rub in T->rubs:
40             if val ∈ rub.pos[]:
41                 rubs[rub.row_id] ← rub;
42             else if exists rubs[rub.row_id];
43                 rubs.erase(rub.row_id);
44     return rubs;
```

The helper function *rubs_involving_val* checks a list of TxnDescs and returns a RUB set that will be applied to the corresponding VB. By traversing the TxnDescs in the order in which they are committed, *rubs_involving_val* can always select the latest version of a RUB, if there are several versions in different TxnDescs.

UDIs of transaction *T* do not modify VBs. In contrast, they append RUBs reflecting the modifications to *T's* TxnDesc. UDIs take effect when *T* is committed, which we will discuss §C.3.

C.3 Transaction Semantics

Algorithm 3 presents the pesudocode for the transaction semantics. Programmers can explicitly start a new transaction by invoking the function TxnBegin. Otherwise, CUBIT runs in the *autocommit* mode. Each TxnBegin allocates an instance of TxnDesc (line 47) and retrieves the current TIMESTAMP and N_ROWS (line 48) which essentially takes a snapshot of the bitmap index.

The function TxnCommit pairs with TxnBegin. We present the pseudocode for CUBIT-lk in Algorithm 3 and will discuss the non-blocking version in §4.9. Specifically, CUBIT commits a transaction by appending its TxnDesc to the tail of TXN_LOG and updating the global variables accordingly, which must be atomic with respect to other concurrent transactions. To that end, a TxnCommit acquires the global latch *bitmap_lk* (line 53) and then checks if the RUBs generated in this transaction conflict with those in the TxnDescs committed by other concurrent threads (line 66). If there is no conflict, TxnCommit sets the TxnDesc's commit-ts to the current value of TIMESTAMP in addition to 1 (Line 58), and fills the *row_id* fields of the RUBs generated by the insert operations according to the current value of N_ROWS (Line 59). [1] TxnCommit then appends the TxnDesc of this transaction to the tail of TXN_LOG (line 60), and updates the global variables accordingly (Line 62), before releasing the latch (line 64).

Note that incrementing the global TIMESTAMP makes *T* visible to other transactions. To prevent concurrent queries from retrieving incorrect <TIMESTAMP, N_ROWS> pair, for ease of presentation, we place the two variables TIMESTAMP and N_ROWS in an aligned 128-bit word that can be atomically read (line 48) and write (line 62) on modern 64-bit architectures. Accessing these two variables without using 128-bit words demands strict orders when accessing/updating these two variables, which can be found in the source code of CUBIT.

C.4 Merge Operation

A query generates a merge request by inserting the request into an associated non-blocking first-in-first-out (FIFO) queues [30] (Line 35). CUBIT manipulates a group of background maintenance threads that repeatedly check if there are merge requests from worker threads via the FIFO queues. Once a merge request arrives, a maintenance thread first extracts the request out of the FIFO queue and then invokes the function Merge.

Merge operations must be synchronized to avoid conflicts with TxnCommit and with each other. In CUBIT-lk, we reuse the global latch *bitmap_lk* to serialize the attempts to append TxnDescs to TXN_LOG. In CUBIT-lf, we use a *helping* mechanism to remove this latch (§4.9).

[1] Deferring the assignment of *row_ids* avoids conflicts among concurrent transactions that include insert operations.

Algorithm 3: Transaction semantics of CUBIT.

```
function TxnBegin(ThreadInfo *th)
  if th->txn $\neq$ null return th->txn;
  th->txn $\leftarrow$ allocate TxnDesc ;
  th->txn-><start_ts, n_rows> $\leftarrow$ <TIMESTAMP, N_ROWS> ;
  return th->txn;
function TxnCommit(ThreadInfo *th)
  if th->txn = null   return -EPERM;
  if th->txn->rubs[] is empty   return -ENOENT;
  mutex_lock(bitmap_lk) ;
    tsp_begin $\leftarrow$ th->txn->start_ts;
    tsp_end $\leftarrow$ TAIL->commit_ts;
    if (check_conflict(th->txn, tsp_begin, tsp_end) $\neq$ 0):
      return -ERETRY;
    th->txn->commit_ts = TIMESTAMP + 1 ;
    Fill row_id field of RUBs generated by Inserts ;
    Append th->txn to the tail of TXN_LOG ;
    TAIL $\leftarrow$ th->txn;
    <TIMESTAMP, N_ROWS> $\leftarrow$ <TIMESTAMP + 1, N_ROWS + #Inserts
   in this transaction>;
    th->txn $\leftarrow$ NULL;
  mutex_unlock(bitmap_lk) ;
function check_conflict(txn, tsp_begin, tsp_end)
  for each TxnDesc T in TXN_LOG(tsp_begin, tsp_end]:
    if (T->rubs.row_ids $\wedge$ txn->rubs.row_ids) $\neq \emptyset$:
      return -EINVAL;
  return 0;
```

Algorithm 4: Merge operation.

```
function Merge(ThreadInfo *th, val, btv_old, btv_new)
  TxnDesc *txn_merge $\leftarrow$ generate synthetic TxnDesc;
  mutex_lock(bitmap_lk);
    if (btvs[val] != btv_old)   return -ERETRY;
    tsp_begin $\leftarrow$ th->txn->start_ts;
    tsp_end$\leftarrow$ TAIL->commit_ts;
    if (check_conflict(txn_merge, tsp_begin, tsp_end) $\neq$ 0):
      return -ERETRY;
    BtvDesc *btv_merge $\leftarrow$ allocate BtvDesc;
    btv_merge->commit_ts $\leftarrow$ TIMESTAMP + 1;
    txn_merge->commit_ts $\leftarrow$ btv_merge->commit_ts;
    btv_merge->start_txn $\leftarrow$ txn_merge;
    btv_merge->btv $\leftarrow$ btv_new;
    Insert btv_merge to the head of btvs[val];
    Insert txn_merge to the tail of TXN_LOG;
    Increment TIMESTAMP by 1;
  mutex_unlock(bitmap_lk);
```

The function Merge creates a synthetic TxnDesc, by making a copy of the received RUB set and cleaning the 1s that have been merged into the newly-generated VB (§4.8). The remaining of Merge is similar to TxnCommit. Specifically, Merge first employs the classic check-after-locking mechanism to check (1) if other concurrent Merges have inserted other new VBs into the same version chain, and (2) if there are any conflicts between the synthetic TxnDesc and the TxnDescs committed by other concurrent transactions, the same as in TxnCommit. If there is no conflict, Merge sets the *commit_ts* in both the new VB and the synthetic TxnDesc, sets the new VB's *start_txn* to point to the synthetic TxnDesc, and then inserts the new VB and synthetic TxnDesc into the corresponding version chain and TXN_LOG, respectively. Merge then increments the global TIMESTAMP, and finally releases the latch.

D LOCK-FREE MVCC

We now present how we provide a lock-free MVCC in CUBIT.

Lock-free TXN_LOG. We first implement a lock-free TXN_LOG by leveraging a *helping* mechanism [6, 11]. Under the hood, we choose Michael and Scott's classic lock-free first-in-first-out linked list [43], denoted *MS-Queue*, as the implementation of TXN_LOG. MS-Queue provides lock-free insert and delete operations that do not block each other. Specifically, an insert operation A sets the *next* pointer of the last node of the list to a new node n, by using an atomic *compare-and-swap (CAS)* instruction. If successful, this is the linearization point of A [34], implying that n has been successfully inserted into the list with respect to other concurrent operations. The operation A then attempts to set the global pointer TAIL to point to n by using another *CAS* instruction. If A is suspended before executing the second *CAS*, other insert operation B, which failed because of the successful insertion of the node n, first *helps* A complete by setting the global pointer TAIL by also using *CAS* instructions. The operations B then restarts from scratch. Delete operations synchronize with each other similarly.

Lock-free MVCC. The update operations of a MVCC, however, are more complex than that in MS-Queue. In CUBIT, the function TransCommit needs to update TAIL, TIMESTAMP, and N_ROWS, and Merge needs to update TAIL, TIMESTAMP, and the head pointer of the corresponding version chain. We thus extend the *helping* mechanism to atomically update a group of variables.

Helping Mechanism in CUBIT. A TransCommit or Merge operation A first records the old values and the new values of the variables to be updated in its TxnDesc. It linearizes by appending its TxnDesc to the tail of the MS-Queue, by using *CAS* instructions, the same as MS-Queue. Once successful, A's TxnDesc becomes reachable to other transactions via the next pointers of the TxnDescs in TXN_LOG. Another TransCommit or Merge B, which failed to append its TxnDesc, helps A complete by (1) reading the old and the new values of each variables to be updated in TxnDesc, and (2) attempting to change the variable to the new value by using a *CAS* instruction. Once any *CAS* fails, which indicates that this variable has been updated by either A or other helpers, B simply skips updating this variable. After helping update all variables in A's TxnDesc, B starts over.

If TIMESTAMP and N_ROWS are placed in an aligned 128-bit long variable, we update them by using a single *double-word CAS* instruction that has been widely supported on modern 64-bit architectures.

Correctness. In theory, the ABA problem [34] may arise in updating TIMESTAMP and N_ROWS. However, it takes TIMESTAMP more than one million years to wraparound, if there are 500K UDIs per second. Similarly, N_ROWS monotonically increases and can be 64-bit long. Moreover, no ABA problem can arise in updating other variables (e.g., TAIL and the pointer to the version chain), because of the epoch-based reclamation mechanism used in CUBIT(§5), which guarantees that no memory space can be reclaimed (and in turn, reused) if any thread holds a reference to it. We thus get the conclusion that in practice, CUBIT-lf is immune to the ABA problem.

We use the term *shared variables* to describe the global variables updated by TransCommit and Merge. CUBIT-lf is correct

Algorithm 5: TPC-H Q6.

```
SELECT sum(l_extendeprice × l_discount) as revenue
FROM LIMEITEM
WHERE l_shipdate >= date'[DATE]'
    and l_shipdate < date'[DATE]' + interval '1' year
    and l_discount between [DISCOUNT] ± 0.01
    and l_quantity < [QUANTITY];
```

Algorithm 6: TPC-H RF1.

```
INSERT a new row into the ORDERS table
LOOP random[1, 7] times
    INSERT a new row into the LINEITEM table
END LOOP
```

Algorithm 7: TPC-H RF2.

```
DELETE from ORDERS where o_orderkey = [VALUE]
DELETE from LINEITEM where l_orderkey = [VALUE]
```

because of the following facts. (1) Shared variables can only be updated after a TxnDesc has been successfully appended to the tail of TXN_LOG. (2) How shared variables are updated is pre-defined in this TxnDesc by specifying the old and the new values of each variable. (3) Updating shared variables can be performed by any active threads, such that concurrent threads can help each other complete. (4) Shared variables are updated by only using *CAS* instructions. (5) No ABA problem can arise. Overall, CUBIT-lf guarantees that when a TxnCommit or Merge owning a TxnDesc completes, each shared variable (a) has been updated to the specified new value, and (b) has been updated only once.

E TPC-H

In this appendix, we discuss in detail how we generate the TPC-H benchmark and how CUBIT is maintained to support full queries. Our DBMS provides snapshot isolation for transactions including queries and real-time modifications.

Dataset. Our prototype DBMS maintains two tables, *ORDERS* and *LINEITEM*. The dates of the tuples in *LINEITEM* span the range of years [1992, 1998], the discounts are distributed in the range [0, 0.1] with increments of 0.01, and the quantities are in the range [1, 50].

Transactions. We use the Forecasting Revenue Change Query (Q6) as the query workload. The SQL code for Q6 is listed in Algorithm 5. The value of the first parameter *DATE* is the first of January of a randomly selected year in between [1993, 1997], the parameter *DISCOUNT* is randomly selected within [0.02, 0.09], and the parameter *QUANTITY* is randomly selected within [24, 25]. The DBMS creates three CUBIT instances, respectively on the attributes *l_shipdate*, *l_discount*, and *l_quantity*. As a result, each Q6 selects the bitvectors corresponding to 1 of the 7 possible years, 3 of the 11 possible discounts, and 24 or 25 of 50 possible quantities, leading to an average selectivity $\frac{1}{7} \times \frac{3}{11} \times \frac{24.5}{50} \approx 2\%$. Note that we use binning to reduce the number of bitvectors for the attribute Quantity from 25 to 3. Values less than, equal to, and larger than 24 go to one of the three bitvectors, respectively. For each Q6 with CUBIT, the DBMS make a private copy of one bitvector and then performs bitwise OR/AND operations among 5 (1+3+1) or 6 (1+3+2) bitvectors, to retrieve a list of tuple IDs in *LINEITEM*. The Q6 then fetches these tuples to calculate the final revenue result.

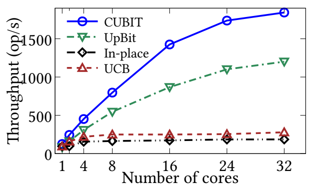
We use a modified New Sales Refresh Function (RF1) and a modified Old Sales Refresh Function (RF2) as the workload of updates. Since our DBMS with CUBIT supports real-time updates to data, it is not necessary for RF1 and RF2 to batch together a number of modifications and then apply them in a batch mode. In contrast, each RF1 and RF2 modifies a single tuple in the table *ORDERS* and the corresponding a few (1-7) tuples in the table *LINEITEM*, and then updates the three CUBIT instances accordingly, altogether in one transaction. The SQL code for RF1 and RF2 is listed in Algorithms 6 and 7. In the experiments, the operation distribution of each worker thread is set to 98, 1, and 1 for Q6, RF1, and RF2, respectively.

Unified CC Mechanism. Integrating CUBIT into a DBMS demands a unified CC mechanism to synchronize concurrent UDIs on both the indexes and the underlying tuples, while allowing wait-free queries. To this end, we choose the optimistic version of Hekaton [24, 37] (an MVCC with timestamping) as the CC mechanism for the underlying tuples, and refer CUBIT-lk to the global variables (e.g. *TIMESTAMP* and the number of rows) maintained by Hekaton. Specifically, on the entry of every transaction, the current *TIMESTAMP* is fetched and is used by queries to retrieve the corresponding versions of the bitmap indexes and the underlying tuples. For UDIs, new versions of the tuples are created, and at the same time, the *End* fields of the old versions are set to the transaction's ID. The *End* fields also serve as the latches for these tuples to prevent write-write conflicts. Corresponding RUBs are then generated. To commit, a transaction containing UDIs (1) grabs a global commit latch, (2) checks if there are write-write conflicts on both the bitmap indexes and the underlying tuples, (3) appends the RUBs to the tail of the TXN_LOGs of the bitmap indexes (Algorithm 3), (4) increments the global *TIMESTAMP*, (5) release the global commit latch, and then (6) sets the timestamp fields of the corresponding tuples. There is a time gap in between the steps 4 and 6, during which concurrent queries may retrieve the latest timestamp but the corresponding versions of the tuples are not ready yet. In this case, queries perform *speculative reads* [37] by proactively (1) checking the transaction's private workspace via the *End* fields of the old versions of the tuples and (2) reading the new versions if applicable. The whole system implements snapshot isolation.

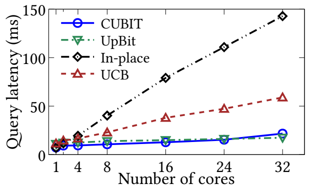
The primary benefit of the unified CC mechanism is that queries are wait-free, even if UDIs are in progress, because once queries get a start timestamp, the corresponding versions of the bitmap indexes and the underlying tuples are guaranteed to be accessible. Concurrent UDIs may contend for the global commit latch, but it is acceptable because the critical section is light-weight and the contention is low for OLAP workloads.

F ZIPFIAN DISTRIBUTION

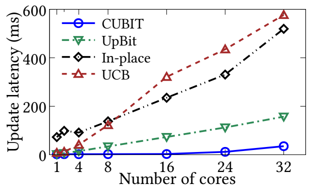
The distribution of data among bitvectors is critical to the performance of updatable bitmap indexes for the following two reasons. First, biased distributions may lead to a situation where a few *target* bitvectors contain many more 1s than others, such that they are less compressible. Second, the target bitvectors face higher contention levels among concurrent UDIs. We thus evaluate how a non-uniform distribution affects the performance of updatable bitmap indexes. We use the same testbed configuration as in §6.2, except that the dataset follows the Zipfian distribution with the distribution parameter α being set to 1.5, which implies that about 40% of the entries have the two most popular values, and the remaining entries are uniformly distributed in the entire value domain.



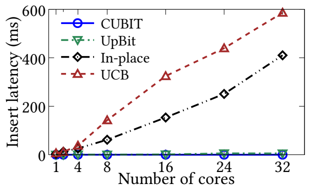
(a) Overall throughput



(b) Query latency



(c) Update latency



(d) Insert latency

Figure 15: Overall throughput and mean latency of bitmap indexes with Zipfian distribution (α = 1.5) as a function of the number of cores.

The results are shown in Figure 15. Both the overall throughput and mean query latency of all algorithms are improved by about 2×, compared to the case with uniform data distribution. The reason is that most bitvectors contain very few 1s, and are thus highly compressible, and queries on these bitvectors are very fast. Since queries dominate 90% of the total operations, the overall throughput of all bitmap indexes also increases by about 2×.