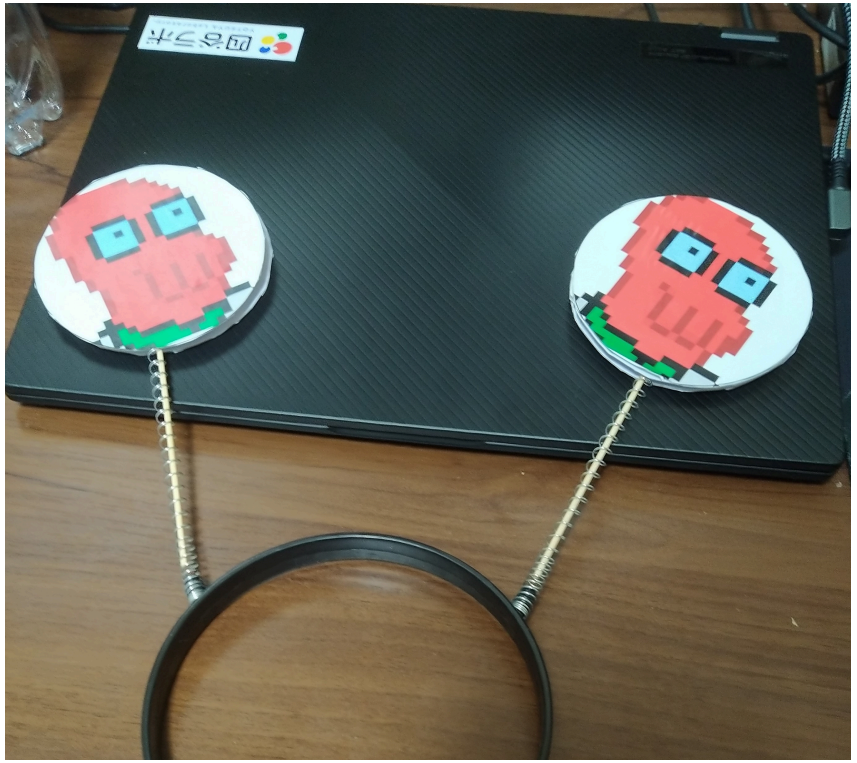
Does Nostr Scale...?

Investigation of Nostr's follower network as "Complex Network"

Self Introduction

- Working as an engineer but in a different field from software
- No background as software engineer, recently
- Math-loving person



Motivation - Is Nostr an efficient social network?

What are the characteristics of "Efficient Networks"?

From a point of view of "Complex Network" ...

- **Scale Free Network**
- **Small World Property**
- Form Clusters

I am going to focus only on "Scale Free Network" and "Small World Property" today.

What is "Complex Network"?

(From Wikipedia https://en.wikipedia.org/wiki/Complex_network)

In the context of network theory, a complex network is a graph (network) with non-trivial topological features—features that do not occur in simple networks such as lattices or random graphs but often occur in networks representing real systems.

(end)

- Not random
- Not simple such as lattice
- Representing real systems

Reference books and readings about "Complex Network"

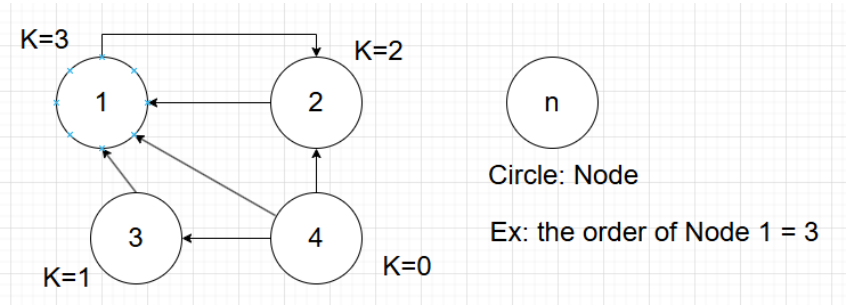
- 「複雑ネットワーク」とは何か　複雑な関係を読み解く新しいアプローチ (増田直紀, 今野紀雄/講談社ブルーバックス)
- 複雑ネットワーク : 基礎から応用まで (増田 直紀, 今野 紀雄/近代科学社)
- 複雑ネットワークの統計的性質(北海道大学 工学研究科 応用物理学専攻 矢久保 考介)

Wat is "Scale Free Network"?

The degree distribution of the **order of nodes** has a power-low.

$$P(k) = P_0 k^{-\gamma}$$

- $P(k)$: Probability(degree) distribution function of the order k
- k : The order nodes - The number of link on node
- γ: Power-law value



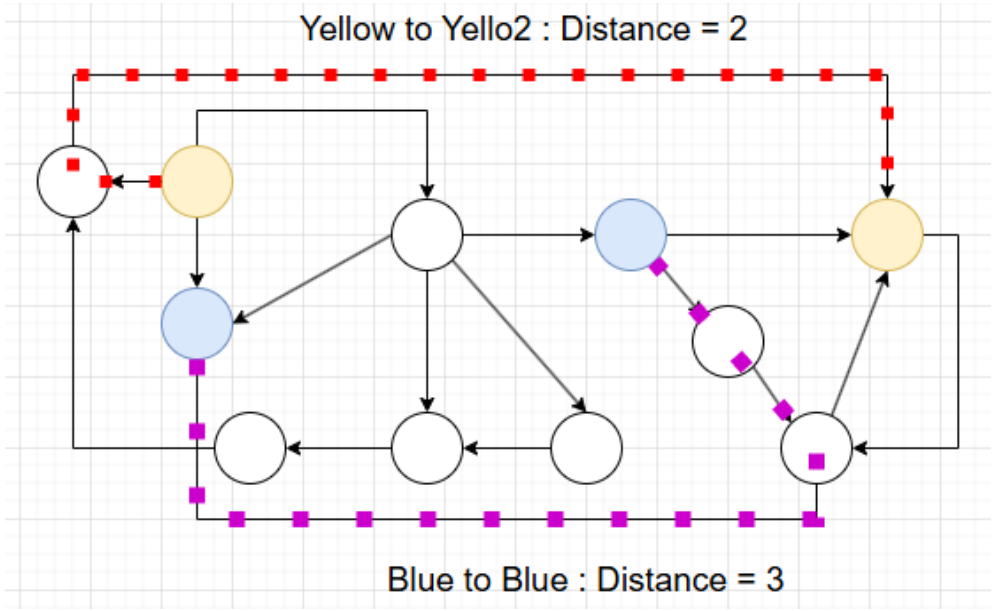
A scale-free network has a long-tail degree distribution, meaning some of nodes(users) have a large number of connection(followers)

What is "Small World Property"?

The average **path length between nodes** are very small compared to the network size.

$$AveragePathLength \propto \log(NetworkSize)$$

Path length between nodes:



Is the real world small?

The idea of "Six degrees of separation"

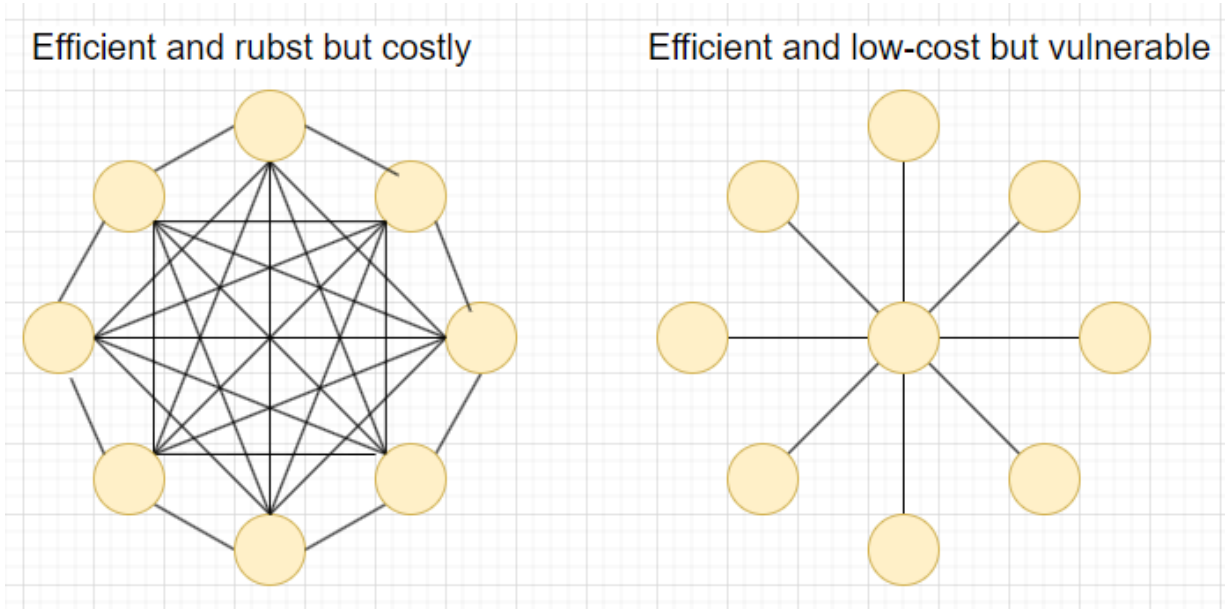
(From wikipedia https://en.wikipedia.org/wiki/Six_degrees_of_separation)

Six degrees of separation is the idea that all people are six or fewer social connections away from each other. As a result, a chain of "friend of a friend" statements can be made to connect any two people in a maximum of six steps. It is also known as the six handshakes rule.
(End)

In real world the distance between a human and a human in a social network can be surprisingly small even the two person are picked randomly.

Why are the concepts important?

"Scale-Free" and "Small World" properties can be good indicators about efficiency and robustness



A key to the scalable network is balance between "efficiency" and "robustness."

Investigation - Is Nostr "Scale-Free" and "Small World"?

Investigated the social graph of Nostr by...

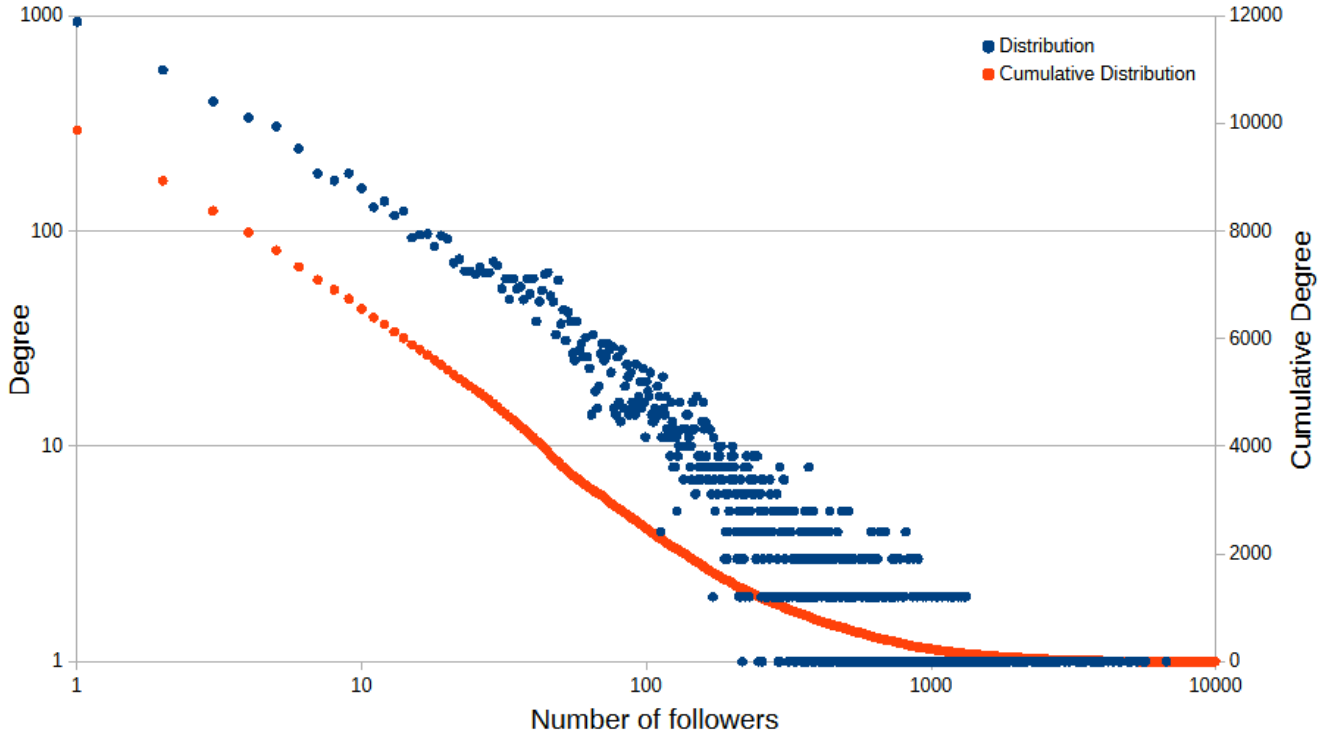
- Collect kind-1 from Aug 1st to 31st, 2024 to identify active users' npub
- Collect kind-3 for all the collected npubs
- Identify all the followers between the active npubs
- Derived the degree distribution of numbers of followers
- Calculated the degree distribution of distances between npubs in "follower-network"

Investigation - Is Nostr "Scale-Free"?

Condition

- Active npubs from Aug 1st to 31st in 2024
- Relays
  nos.lol, relay.snort.social, nostr.fmt.wiz.biz,
  nostr-pub.wellorder.net, nostr.mom, nostr.oxtr.dev,
  nostr.semisol.dev, relay.damus.io, relay.nostr.bg, soloco.nl,
  nostr.bitcoiner.social", **nostr.einundzwanzig.space**
- The number of npubs: 67217
- **The number of npubs with at least one follower: 9868**

Investigation - Is Nostr "Scale-Free"?



γ = 1.27 (For X/Twitter $\approx$ 2 to 3 in various surveys)

γ is smaller in Nostr. Longer-tail distribution compared to X. Users are more equally followed.

Investigation - Is Nostr "Scale-Free"

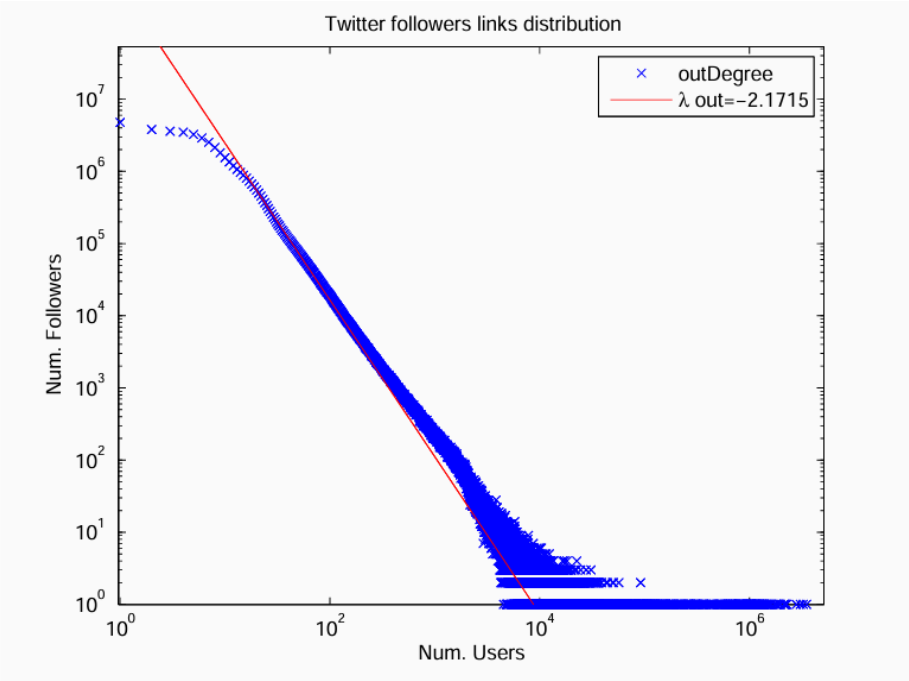


Figure 1. Outgoing degree distribution of Twitter's network. As the figure shows, there are a few users with an enormous degree (number of friends). On the contrary, the majority of them have just at most 1000 friends.

Example Twitter shows "scale-free" property with γ of 2.17

From: https://www.mdpi.com/1099-4300/17/8/5848

Investigation - Is Nostr "Scale-Free"?

Conlusion

- Follower network is "scale-free". $P(k)$ Log-log plot showed Power-law.
  $P\left(k\right) = P_0 k^{-\gamma} \rightarrow \left(\log P\left(k\right)\right) = \log P_0 - \gamma(\log k)$
- $\gamma \approx 1.27$, relatively smaller compared to X/Twitter (2 to 3)
- Suggesting...
  - Users are equally followed compared to X/Twitter
  - May not be efficient as X/Twitter network
  - The network might not be large enough??
  - The effect of relay has not been taken in account

Appendix - Correlation between "follows" and "followers"

Investigation - Is Nostr a"Small-World?"

Condition

- Used the same npubs obtained from "Scale-free" study
  - npubs at least have one follower
- The follower network is represented as directed-unweighted graph
- Calculated the shortest paths between nodes for all the combination utilizing Dijkstra algorithm
- Library: gonum/graph

Investigation - Is Nostr a"Small-World?"

Result

Nostr

Path Length	Degree	In %
No Path	22,215,772	
1	1,274,161	1%
2	32,224,368	37%
3	43,138,408	49%
4	9,965,692	11%
5	986,126	1%
6	75,324	0%
7	2,873	0%
8	82	0%
9	0	0
10	0	0
Total existing path	87,667,034	
Average Path Length	2.74	

X/Twitter

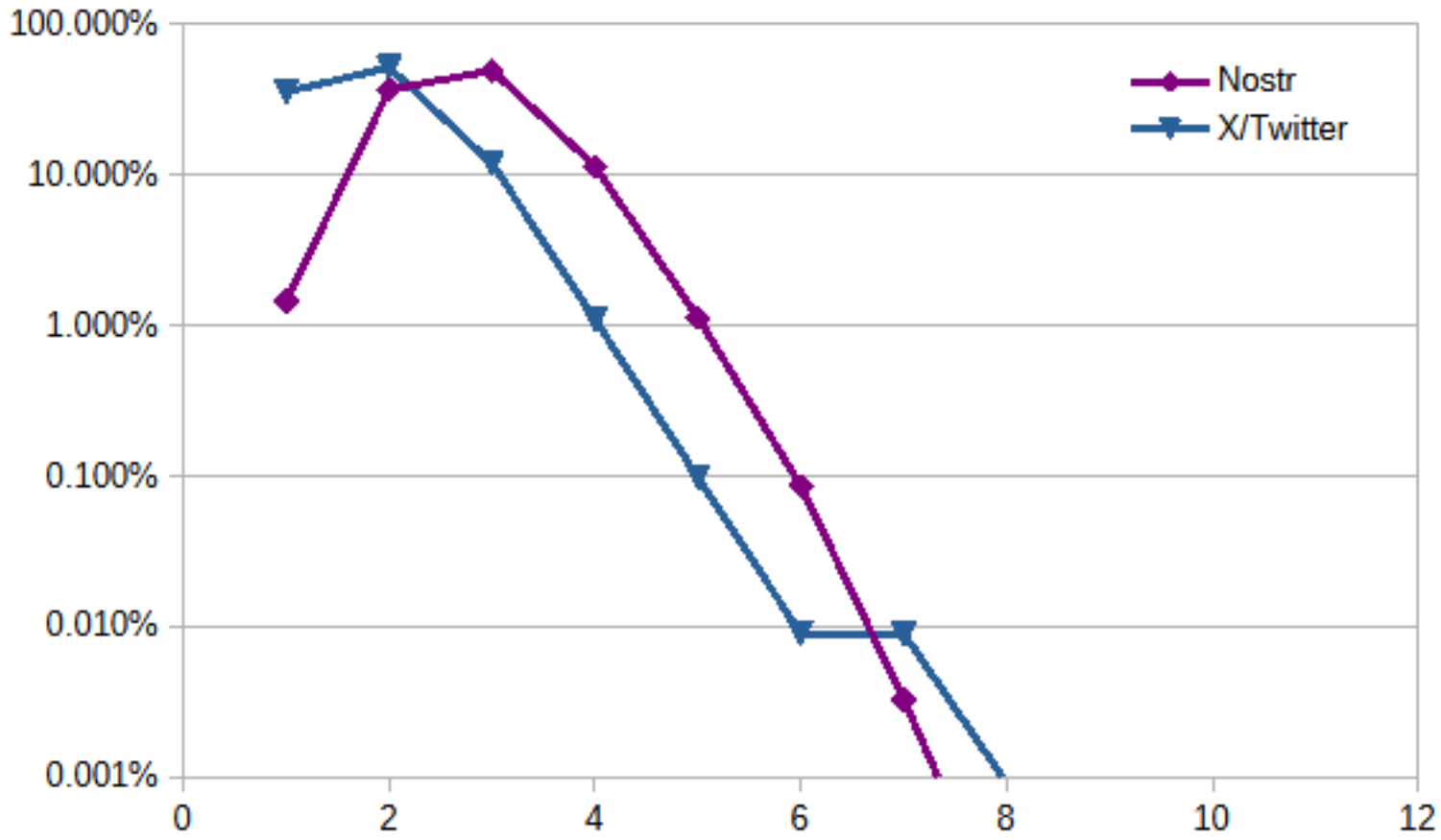
Path Length	Degree	In %
No Path	??	
1	14,951,325	35%
2	21,755,660	52%
3	4,998,876	12%
4	461,414	1%
5	41,337	0%
6	3,789	0%
7	3,789	0%
8	387	0%
9	51	0%
Total existing path	42216628	
Average Path Length	1.79	

- Average distance: 2.74
- Twitter is much smaller (1.79)

From: https://www.mdpi.com/1099-4300/17/8/5848

Investigation - Is Nostr a"Small-World?"

Result



Investigation - Is Nostr a "Small-World"?

Conclusion

- Nosrt is small world
  - About 87% of all the combination have the distances equal or less than 3
- X/Twitter is much smaller

Conclusion - Does Nostr Scale?

Fact

- Nostr's follower network is scale-free
  - $gamma \approx 1.27$, which is smaller than X/Twitter
- Nostr is a "Small World"
  - Average path length: 2.74, larger than X/Twitter
- Twitter is more "scale-free" and "smaller" for the size of the network!

Conclusion - Does Nostr Scale?

Thoughts

- Nostr likely can scale to a larger network as a social media because Nostr shows both "Scale Free" and "Small World" properties
- However, X/Twitter is probably more efficient
- Why Nostr is less efficient than X/Twitter?
  - Users tend to follow each other more often than X/Twitter
  - Relays may separate a group from other groups
  - May need to follow a large number of relays to be a hub user