A. 每一題，請附上原始碼和執行畫面，以及必要之解說。

B. 你的 Word 檔應貼上程式碼的截圖或 M file 內容（非在 word 中重新 keyin 程式碼)。另外，再將各題 M files 與 Word 檔壓縮成 ZIP 後，再上傳。 所有檔名請用學號命名。

C. 每一題輸入值皆應寫入 M file。

1.

15.2 Using the same approach as was employed to derive Eqs. (14.15) and (14.16), derive the least-squares fit of the following model:

$$y = a_1 x + a_2 x^2 + e$$

That is, determine the coefficients that result in the least-squares fit for a second-order polynomial with a zero intercept. Test the approach by using it to fit the data from Table 14.1.

TABLE 14.1 Experimental data for force (N) and velocity (m/s) from a wind tunnel experiment.

v, m/s	10	20	30	40	50	60	70	80
F, N	25	70	380	550	610	1220	830	1450

```
v = [10 20 30 40 50 60 70 80];
F = [25 70 380 550 610 1220 830 1450];
N = [sum(v.^2) sum(v.^3);sum(v.^3) sum(v.^4)];
r = [sum(F.*v) sum(F.* (v.^2))]';
a = N\r;
Sr = 0; St = 0;
for i = 1:size(v)
    Sr = Sr+ (F(i) - a(1)*v(i) - a(2)*(v(i)^2))^2;
    St = St + (F(i)- mean(F))^2;
end
syslashx = sqrt(Sr/(length(v)-3));
r2 = (St -Sr)/St;
xp = linspace(min(v),max(v));
yp = a(1).*xp + a(2).*(xp.^2);
plot(v,F,'o',xp,yp); grid;
disp('r^2:');
disp(r2);
disp('std:');
disp(syslashx);
fprintf('F = %f*v + %f*v^2\n',a(1),a(2));
```

程　　式　　碼

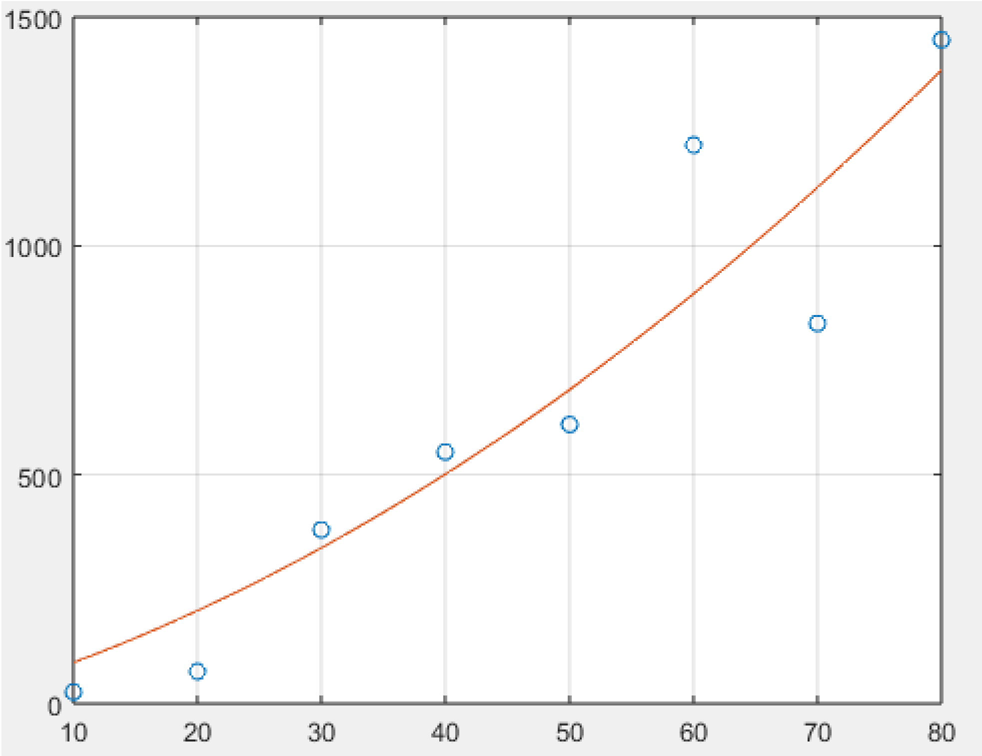


圖　形

```
r^2:
    0.9890

std:
   28.8979

F = 7.771024*v + 0.119075*v^2
```

輸出結果

題目 a0 = 0，於是變成對 a1 和 a2 做偏微分就成，[n sum(x) sum(x^2);……]的 3*3 矩陣剩下右下角的 2*2 矩陣，等號右邊本來有個 3*1 向量也剩 2*1，其餘方法皆套用之前的作業。

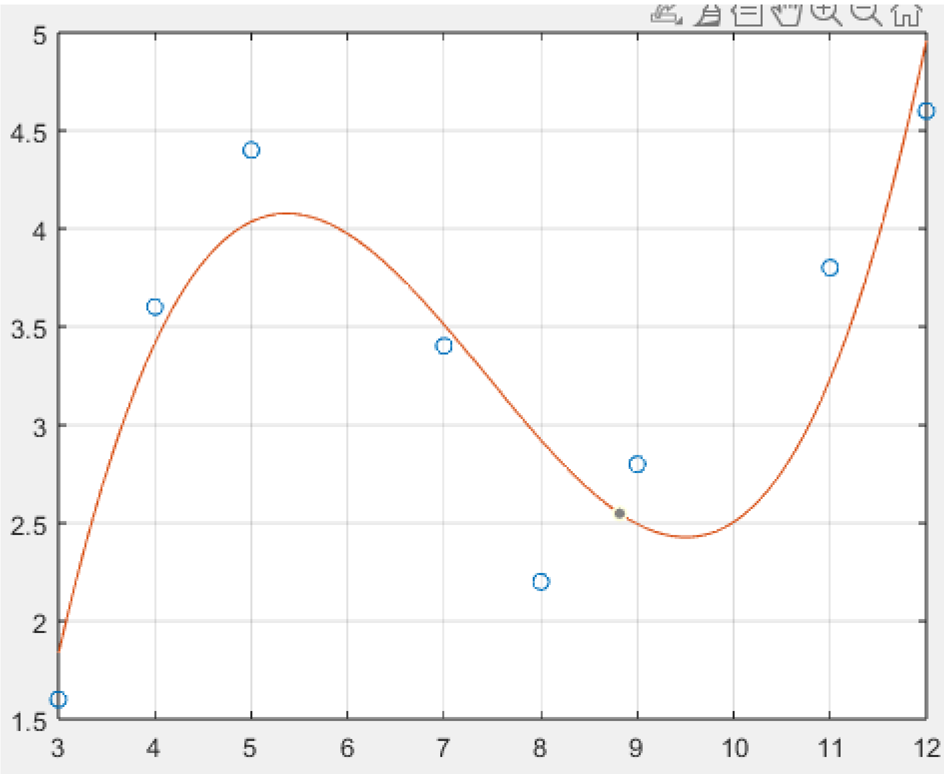
2.

15.3 Fit a cubic polynomial to the following data:

x	3	4	5	7	8	9	11	12
y	1.6	3.6	4.4	3.4	2.2	2.8	3.8	4.6

Along with the coefficients, determine r^2 and $s_{y/x}$.

用一般線性最小平方模型，Z 矩陣為[1 x x^2 x^3]，除了 Sr 的計算有些改動，其餘皆和第一題相似。




```
x = [3 4 5 7 8 9 11 12]';
y= [1.6 3.6 4.4 3.4 2.2 2.8 3.8 4.6]';
z = [ones(size(x)) x x.^2 x.^3];
a = (z'*z)\(z'*y);
Sr = sum((y-z*a).^2);
r2 = 1-Sr/sum((y-mean(y)).^2);
disp('r^2');
disp(r2);
syslashx = sqrt(Sr/(length(x)-4));
disp('std:');
disp(syslashx);
xp = linspace(min(x),max(x));
yp =a(1)+  a(2).*xp + a(3).*(xp.^2)+ a(4).*(xp.^3);
plot(x,y,'o',xp,yp); grid;
fprintf("y= %f + %fx + %fx^2 + %fx^3\n",a(1),a(2),a(3),a(4));
```

```
r^2
    0.8290

std:
    0.5700

y= -11.488707 + 7.143817x + -1.041207x^2 + 0.046676x^3
```

輸出結果

用一般線性最小平方模型，Z 矩陣為[1 x x^2 x^3]，除了 Sr 的計算有些改動，其餘皆和第一題相似。

3. **15.9** The following data were collected for the steady flow of water in a concrete circular pipe:

Experiment	Diameter, m	Slope, m/m	Flow, m^3/s
1	0.3	0.001	0.04
2	0.6	0.001	0.24
3	0.9	0.001	0.69
4	0.3	0.01	0.13
5	0.6	0.01	0.82
6	0.9	0.01	2.38
7	0.3	0.05	0.31
8	0.6	0.05	1.95
9	0.9	0.05	5.66

Use multiple linear regression to fit the following model to this data:

$$Q = \alpha_0 D^{\alpha_1} S^{\alpha_2}$$

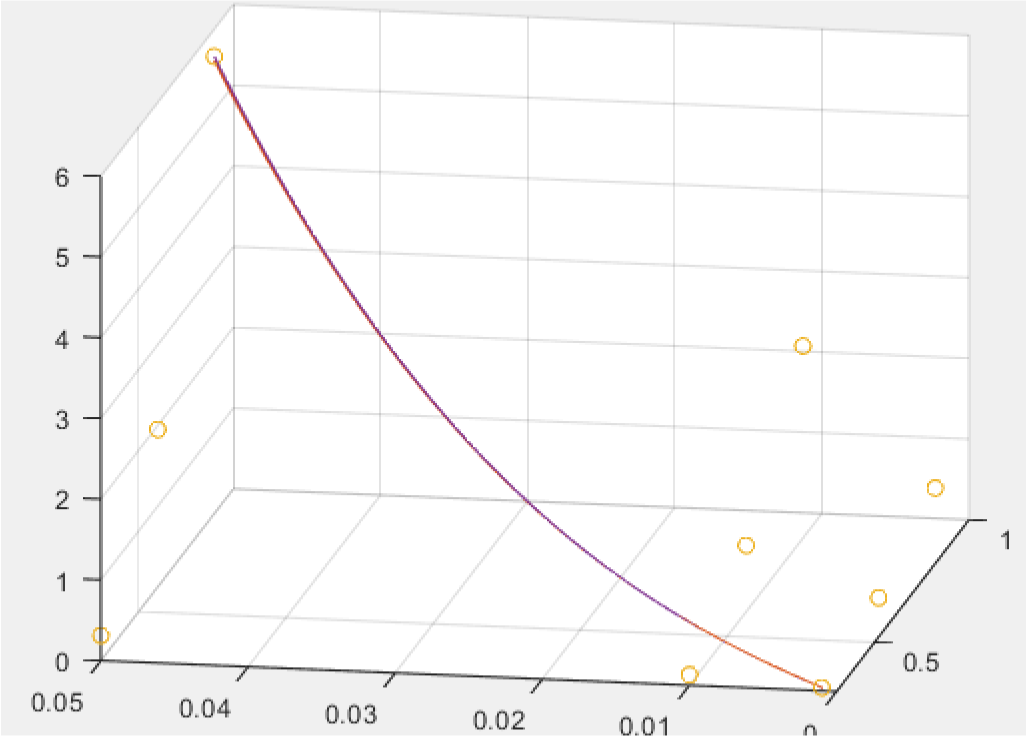
where Q = flow, D = diameter, and S = slope.

```
Q =[0.04 0.24 0.69 0.13 0.82 2.38 0.31 1.95 5.66]';
D=[0.3 0.6 0.9 0.3 0.6 0.9 0.3 0.6 0.9]';
S=[0.001 0.001 0.001 0.01 0.01 0.01 0.05 0.05 0.05]';
logQ = log10(Q); logD = log10(D); logS = log10(S);
z = [ones(size(logQ)) logD logS];
a = (z'*z)\(z'*logQ);

fprintf("logQ = %f + %fD + %fS\n",a(1),a(2),a(3));
disp('or');
fprintf('Q = %f * D^%f * S^%f\n',10^a(1),a(2),a(3));
xp = linspace(min(D),max(D));
yp = linspace(min(S),max(S));
fp = 10^a(1).*xp.^a(2).*yp.^a(3);
plot3(D,S,Q,'o',xp,yp,fp);hold on;
b = fminsearch(@qds,[1 1 1],[],Q,D,S);
disp('or by fminsearch');
fprintf('Q = %f * D^%f * S^%f\n',b(1),b(2),b(3));
fp2 = b(1).*xp.^b(2).*yp.^b(3);
plot3(D,S,Q,'o',xp,yp,fp2); grid;
function f = qds(A,Q,D,S)
    yp = A(1)*D.^A(2).*S.^A(3);
    f = sum((Q-yp).^2);
end
```

程式碼

畫 圖




```
logQ = 1.560879 + 2.627937D + 0.531987S
or
Q = 36.381332 * D^2.627937 * S^0.531987
or by fminsearch
Q = 37.429765 * D^2.630466 * S^0.538055
```
輸出結果

對等式左右取 log，會變 logQ = loga0 + a1logD + a2 logS，之後把 Z = [1 logd logs]，a =(z’ *z)\(z’ *logq)，應該就是對數轉換的一般線性最小平方法，畫圖用 plot3，畫的是轉換後再帶回轉換前係數的圖形和 fminsearch 求出之圖形，並比較和 fminsearch 的方法求出之差別。