Least square
Hilbert Matrix

2016039034 박준형
2018021794 염태은

목차

1. Hilbert Matrix

$$H_{ij} = \frac{1}{i + j - 1}$$

Ex)

$$H = \begin{pmatrix} 1 & \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{3} & \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} & \frac{1}{8} & \frac{1}{9} \end{pmatrix}$$

: 5×5 Hilbert Matrix

2. Condition Number

$$Ax = b$$

$$A(x + \Delta x) = b + \Delta b$$

$$\mathrm{c}(A) = \|A\|\|A^{-1}\|$$

3. Normal Equation NE

Matlab code

```
K= hilb(n);
K(:,[n:n-2])=[];
x = ones(n-3,1);
b = K*x;
xx = (transpose(K)*K)\(transpose(K)*b);
norm(x-xx)
```

4. QR Decomposition QR

Matlab code

```
K= hilb(n);
K(:,[n:n-2])=[];
x = ones(n-3:1);
b = K*x;
[Q,R] = qr(K);
xx =
R\(transpose(Q)*b);
norm(x-xx)
```

Algorithm : QR

$$Kx = b$$

$$\hat{Q}\hat{R}x = b$$

$$\hat{Q}^T\hat{Q}\hat{R}x = \hat{Q}^T b$$

$$\hat{R}x = \hat{Q}^T b$$

5.Singular Value Decomposition SVD

Matlab code

```
K= hilb(n);
K(:,[n:n-2])=[];
x = ones(n-3,1);
b = K*x;
[u,s,v] = svd(K);
xx = pinv(K)*b;
norm(x-xx)
```

Pseudo Inverse

$$K = U\Sigma V^T$$

$$K^+ = V\Sigma^+ U^T$$

결과

Via NE, QR, and SVD

NE

n=10	n=15	n=20
1.00E+00	0.9998979783	0.9992230514
1	1.004258801	1.029339207
1	0.9579749906	0.7624567277
1	1.15438534	1.52969222
1	0.7886971386	1.470211173
1	1.0938177	-1.721950643
1	0.4594932254	3.496942644
	3.103631247	-1.200280308
	-2.00054217	6.491748896
	2.808765268	-2.041015717
	0.6900329718	-3.649292033
	0.9395935275	8.014831742
		-3.391591625
		0.4955144951
		-0.2878591993
		8.066464285
		-3.065310887

QR

n=10	n=15	n=20
1	0.9999999919	0.9999999624
1	1.000000908	1.000003811
0.9999999995	0.9999742286	0.9999356387
1.000000002	1.000321558	0.9995257304
0.9999999967	0.9978163553	1.020856408
1.000000003	1.008970448	0.7750565494
0.9999999991	0.9764528068	2.279098324
	1.040412018	-3.419287814
	0.9548388563	10.64898751
	1.031671798	-11.96191863
	0.9873408729	10.04224116
	1.002200159	1.089710834
		-4.277098676
		4.097572334
		1.440901786
		-0.01288647687
		1.277301591

SVD

n=10	n=15	n=20
1	0.9999999903	1.00000017
1	1.000001116	0.999962297
0.9999999993	0.9999676749	1.001937811
1.000000003	1.000411169	0.9582973985
0.9999999955	0.997157503	1.474805601
1.000000004	1.01187157	-2.217113578
0.9999999988	0.96835766	14.79737957
	1.055078311	-37.38196069
	0.9376371184	69.3083035
	1.044270875	-70.39508699
	0.9821036291	30.43905875
	1.003143386	16.63138637
		-7.982780759
		-28.63353088
		42.1170024
		-20.22443935
		5.106778597

결과

Error of NE, QR, and SVD & Condition number

	n=10	n=15	n=20
normal equation	0.4641	31.5684	59.7937
QR decomposition	4.8221e-09	0.0740	20.0709
SVD	2.6038e-09	0.0014	0.0215
conditioned number	8.4281e+07	1.5230e+15	7.1703e+17