Reconstruction of Flow Lines of Flow Past a Sphere

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1 Point force (Stokeslet)

$$\begin{gathered} \mu \Delta \mathbf{u} - \nabla p + \mathbf{F}\delta_{x_0} = 0 \\ \nabla \cdot \mathbf{u} = 0 \end{gathered} \tag{1}$$

For a point force $\mathbf{F}$ acting at the origin, the velocity field of (1) is given by

$$\mathbf{u}(\mathbf{x}) = G(\mathbf{x})\frac{\mathbf{F}}{8\pi\mu}, \quad \text{where } G(\mathbf{x}) = \frac{1}{r}I + \frac{1}{r^3}\mathbf{x}\otimes\mathbf{x} \text{ and } r = |\mathbf{x}|.$$

When $\mathbf{F} = (F, 0, 0)$,

$$\mathbf{u}(\mathbf{x}) = \frac{F}{8\pi\mu}\left(\frac{1}{r}I + \frac{1}{r^3}\begin{bmatrix} x^2 & xy & xz \\ xy & y^2 & yz \\ xz & yz & z^2 \end{bmatrix}\right)\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \frac{F}{8\pi\mu}\left(\frac{x^2+r^2}{r^3}, \frac{xy}{r^3}, \frac{xz}{r^3}\right). \tag{2}$$

2 Source dipole

The velocity field generated by a dipole of strength $(G, 0, 0)$ is

$$\mathbf{u} = \frac{G}{4\pi}\left(\frac{r^2 - 3x^2}{r^5}, -\frac{3xy}{r^5}, -\frac{3xz}{r^5}\right). \tag{3}$$

3 Flow past a sphere

The velocity field of a sphere of radius a moving at a constant velocity can be produced by a combination of a stokeslet and a dipole. By setting $G = Fa^2/6\mu$ in (3) and adding it to (2), we have

$$\mathbf{u}(\mathbf{x}) = \frac{F}{8\pi\mu}\left(\left(\frac{x^2+r^2}{r^3}, \frac{xy}{r^3}, \frac{xz}{r^3}\right) + \frac{a^2}{3}\left(\frac{r^2-3x^2}{r^5}, -\frac{3xy}{r^5}, -\frac{3xz}{r^5}\right)\right), \tag{4}$$

and then,

$$\mathbf{u}(\mathbf{x})|_{|\mathbf{x}|=a} = \frac{F}{8\pi\mu}(\frac{4}{3a}, \quad 0, \quad 0) = \frac{F}{6\pi\mu a}(1, \quad 0, \quad 0).$$

In reconstructing the flow lines for flow past a sphere, we set $\mathbf{u}|_\infty = \frac{F}{8\pi\mu}(\frac{4}{3a}, 0, 0)$ and plot the velocity $\mathbf{u}|_\infty$ relative to $\mathbf{u}(\mathbf{x})$ in (4). This fulfills the condition that the (relative) velocity of the fluid at the sphere surface is $\mathbf{0}$.

See code at https://github.com/juneshuoyang/Creeping-Flow

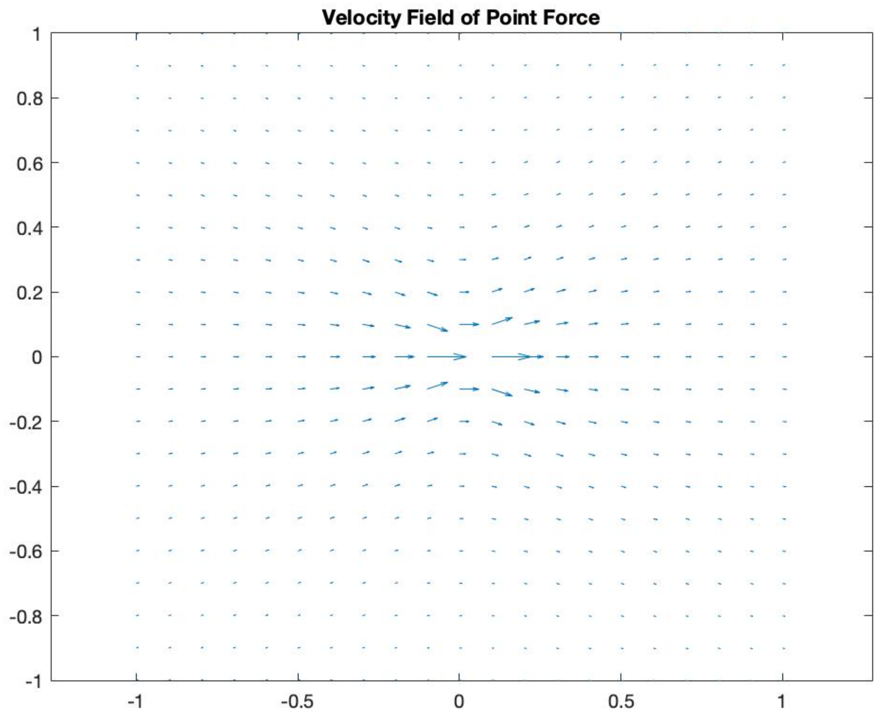

Figure 1: A stokeslet of strength $(F, 0, 0)$ at the origin.

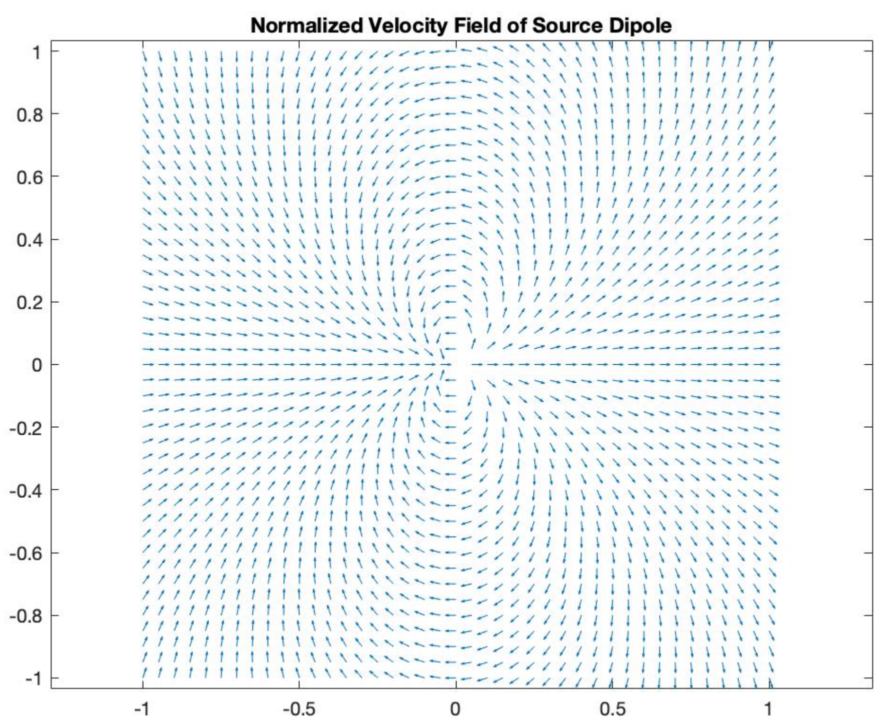

Figure 2: A dipole of strength $(G, 0, 0)$ at the origin.

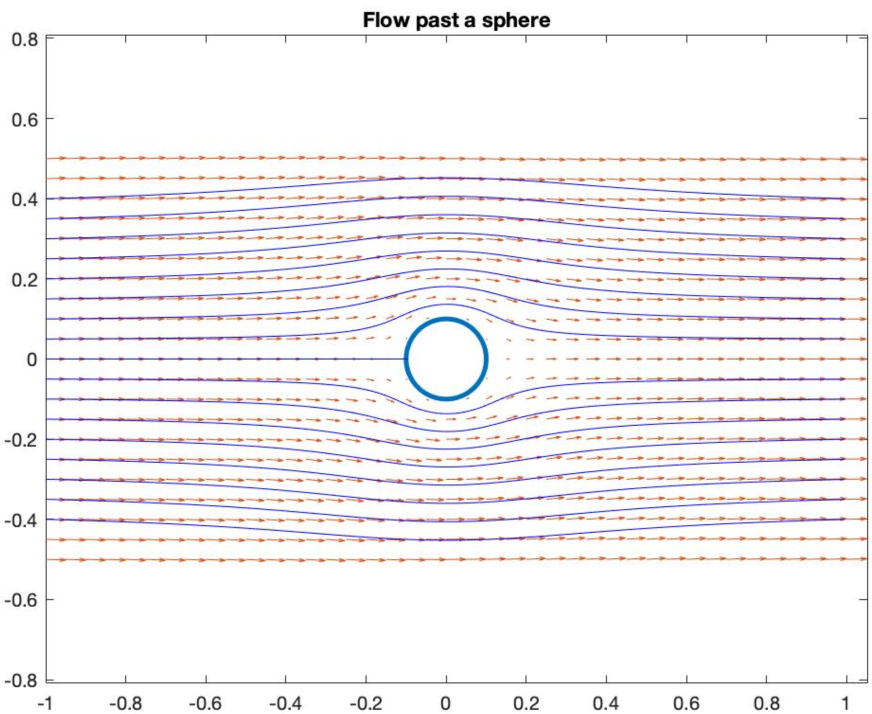

Figure 3: Steady flow at low Reynolds numbers.