形式化方法导引

第 5 章 模型检测

5.1 应用

黄文超

http://staff.ustc.edu.cn/~huangwc/fm.html

回顾: 定义: Verification in Logics

Most logics used in the design, specification and verification of computer systems fundamentally deal with a *satisfaction relation*:

$$\mathcal{M} \models \phi$$

- $\mathcal{M}$ is some sort of situation or *model* of a system
- ϕ is a *specification*, a formula of that logic, expressing what should be true in situation $\mathcal{M}$.
- At the *heart* of this set-up is that one can often *specify and implement algorithms* for computing $\models$.

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- 问: 如何统一化定义 $\mathcal{M}$ 和 ϕ? 答: 一种方案: $\mathcal{M}$ 和 ϕ 均用 *Logics*
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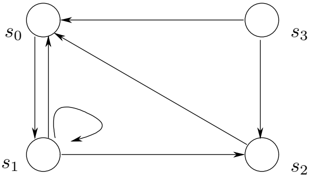

反例：How to define Reachability as ϕ

Given nodes n and n' in a directed graph, is there a finite path of transitions from n to n'?

反例：一种答案

$(u = v) \vee \exists x(R(u,x) \wedge R(x,v)) \vee \exists x_1 \exists x_2(R(u,x_1) \wedge R(x_1,x_2) \wedge R(x_2,v)) \vee \ldots$

- This is infinite, so it's *not* a well-formed formula.
- Can we find a well-formed formula with the same meaning? *No!*

回顾: 另一种答案: Second-order Logic

$$\neg\exists P \forall x \forall y \forall z\ (C_1 \wedge C_2 \wedge C_3 \wedge C_4)$$

where

$$C_1 \stackrel{\text{def}}{=} P(x, x)$$

$$C_2 \stackrel{\text{def}}{=} P(x, y) \wedge P(y, z) \rightarrow P(x, z)$$

$$C_3 \stackrel{\text{def}}{=} P(u, v) \rightarrow \perp$$

$$C_4 \stackrel{\text{def}}{=} R(x, y) \rightarrow P(x, y)$$

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- 难以理解ϕ，难以构建ϕ
- 如何自动验证?

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- 模型检测 (重新定义问题):
  - 重新定义ϕ: LTL, CTL, ...
  - 重新定义$\mathcal{M}$: Transition System 等
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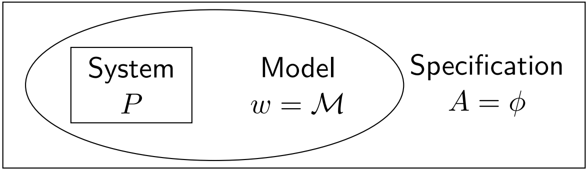
A *verifier* for a language A is an algorithm V, where

$$A = \{w \mid V \text{ accepts } \langle w, c\rangle \text{ for some string } c\}.$$

再往前回顾：验证过程

(1) 构建模型 w. **(2)** 设计规约 A. **(3)** (手动或自动) 构建证明 c
(4) 使用验证器 V，输入 c，输出是否$w \in A$



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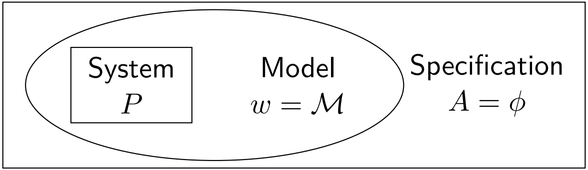

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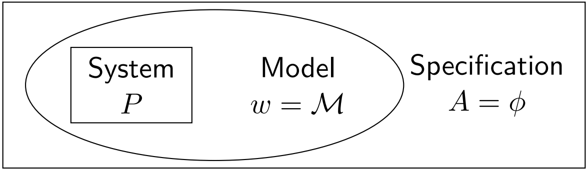

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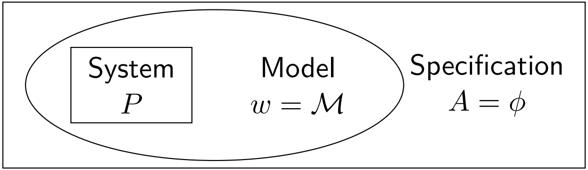

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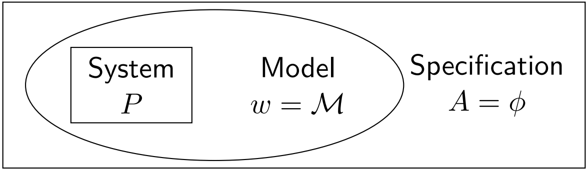

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- $\mathcal{M}$ is an *appropriate* model of the system under consideration.
- s is a *state* of that model
- $\vDash$ is the underlying satisfaction relation
- ϕ is a formula of one of the following *temporal logics*:
  - *Linear-time Temporal Logic (LTL)*
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Linear-time temporal logic (*LTL*) has following syntax given in BNF:

$$\begin{aligned}\phi ::= & \top \mid \bot \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \to \phi) \\ & \mid (\mathrm{X}\ \phi) \mid (\mathrm{F}\ \phi) \mid (\mathrm{G}\ \phi) \mid (\phi\ \mathrm{U}\ \phi) \mid (\phi\ \mathrm{W}\ \phi) \mid (\phi\ \mathrm{R}\ \phi)\end{aligned}$$

where p is any propositional atom from some set **Atoms**.

Convention: The unary connectives (consisting of $\neg$ and the temporal connectives X, F and G) bind most tightly. Next in the order come U, R and W; then come $\wedge$ and $\vee$; and after that comes $\to$. For example:

- $(((\mathrm{F}\ p) \wedge (\mathrm{G}\ q)) \to (p\ \mathrm{W}\ r)) \equiv \mathrm{F}\ p \wedge \mathrm{G}\ q \to p\ \mathrm{W}\ r$
- $(\mathrm{F}\ (p \to (\mathrm{G}\ r)) \vee ((\neg q)\ \mathrm{U}\ p)) \equiv \mathrm{F}\ (p \to \mathrm{G}\ r) \vee \neg q\ \mathrm{U}\ p$
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定义: Linear-time temporal logic (LTL)

Linear-time temporal logic (*LTL*) has following syntax given in BNF:

$$\begin{aligned}\phi ::= & \top \mid \bot \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \to \phi) \\ & \mid (\mathrm{X}\ \phi) \mid (\mathrm{F}\ \phi) \mid (\mathrm{G}\ \phi) \mid (\phi\ \mathrm{U}\ \phi) \mid (\phi\ \mathrm{W}\ \phi) \mid (\phi\ \mathrm{R}\ \phi)\end{aligned}$$

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思路：先定义一种最简化的模型，然后利用这个模型来定义符号

回顾：定义：finite automaton

A *finite automaton* is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

1. Q is a finite set called the *states*,
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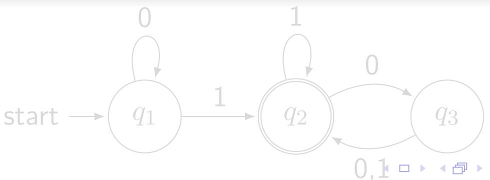

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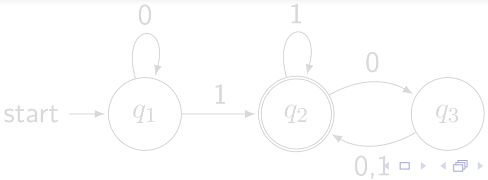

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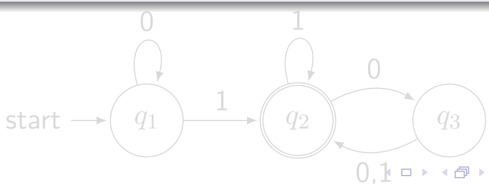

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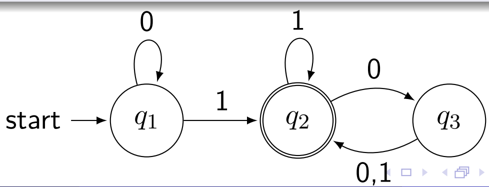

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A transition system $\mathcal{M} = (S, \to, L)$ is

- S: a set of states
- $\to$: a transition relation.
  - every $s \in S$ has some $s' \in S$ with $s \to s'$
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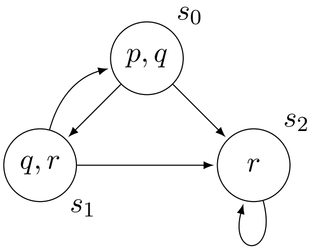

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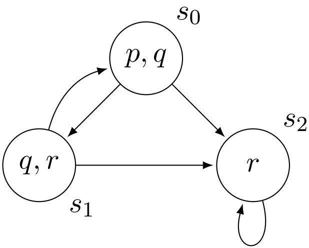

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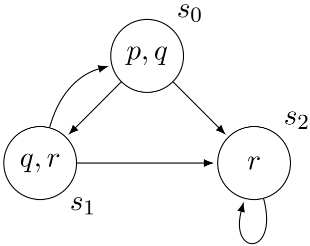

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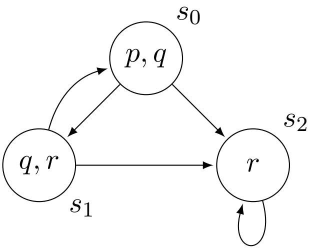

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定义: path

A *path* in a model $\mathcal{M} = (S, \to, L)$ is an infinite sequence of states $s_1, s_2, s_3, \ldots$ in S such that, for each $i \geq 1, s_i \to s_{i+1}$. We write the path as $s_1 \to s_2 \to \ldots$

定义: π^i

Consider the path $\pi = s_1 \to s_2 \to \ldots$.

- It represents a possible future of our system: first it is in state s_1, then it is in state s_2, and so on.

We write π^i for the *suffix* starting at s_i, e.g., $s_3 \to s_4 \to \ldots$

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定义: Semantic of LTL (for $\pi \vDash \phi$)

Let $\mathcal{M} = (S, \rightarrow, L)$ be a model and $\pi = s_1 \rightarrow s_2 \rightarrow \dots$ be a path in $\mathcal{M}$. Whether π satisfies an LTL formula is defined by the satisfaction relation $\vDash$ as follows:

1. $\pi \vDash \top$
2. $\pi \nvDash \bot$
3. $\pi \vDash p$ iff $p \in L(s_1)$
4. $\pi \vDash \neg\phi$ iff $\pi \nvDash \phi$
5. $\pi \vDash \phi_1 \wedge \phi_2$ iff $\pi \vDash \phi_1$ and $\pi \vDash \phi_2$
6. $\pi \vDash \phi_1 \vee \phi_2$ iff $\pi \vDash \phi_1$ or $\pi \vDash \phi_2$
7. $\pi \vDash \phi_1 \rightarrow \phi_2$ iff $\pi \vDash \phi_2$ whenever $\pi \vDash \phi_1$

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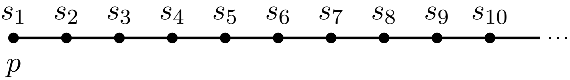
8. $\pi \vDash \mathrm{X}\ \phi$ iff $\pi^2 \vDash \phi$
9. $\pi \vDash \mathrm{G}\ \phi$ iff for all $i \geq 1, \pi^i \vDash \phi$
10. $\pi \vDash \mathrm{F}\ \phi$ iff there is some $i \geq 1$ such that $\pi^i \vDash \phi$
11. $\pi \vDash \phi\ \mathrm{U}\ \psi$ iff there is some $i \geq 1$ such that $\pi^i \vDash \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \vDash \phi$
12. $\pi \vDash \phi\ \mathrm{W}\ \psi$ iff either there is some $i \geq 1$ such that $\pi^i \vDash \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \vDash \phi$; or for $k \geq 1$ we have $\pi^k \vDash \phi$
13. $\pi \vDash \phi\ \mathrm{R}\ \psi$ iff either there is some $i \geq 1$ such that $\pi^i \vDash \phi$ and for all $j = 1, \ldots, i$, we have $\pi^j \vDash \psi$, or for all $k \geq 1$ we have $\pi^k \vDash \psi$

3. $\pi \vDash p$ iff $p \in L(s_1)$

For example, $\pi \vDash p$:



8. $\pi \vDash \mathrm{X}\,\phi$ iff $\pi^2 \vDash \phi$

For example, $\pi \vDash \mathrm{X}\,p$:

3. $\pi \models p$ iff $p \in L(s_1)$

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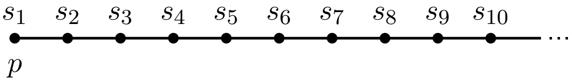

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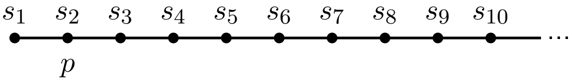
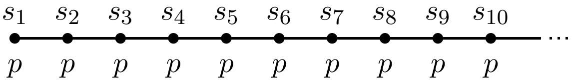


9. $\pi \vDash \mathrm{G}\ \phi$ iff for all $i \geq 1, \pi^i \vDash \phi$

For example, $\pi \vDash \mathrm{G}\ p$:



10. $\pi \vDash \mathrm{F}\ \phi$ iff there is some $i \geq 1$ such that $\pi^i \vDash \phi$

For example, $\pi \vDash \mathrm{F}\ p$:

1. 应用

1.2 Linear-time temporal Logic (LTL) | Semantics

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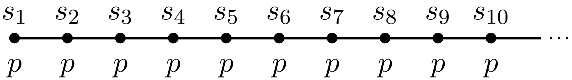

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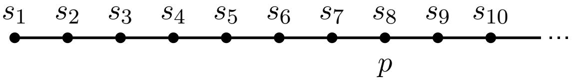

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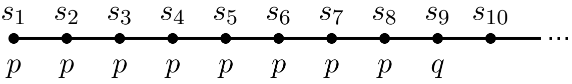

12. $\pi \vDash \phi \text{ W } \psi$ iff either there is some $i \geq 1$ such that $\pi^i \vDash \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \vDash \phi$; or for $k \geq 1$ we have $\pi^k \vDash \phi$
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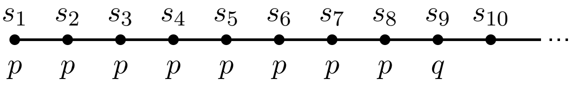
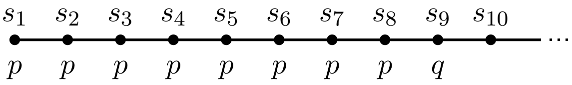
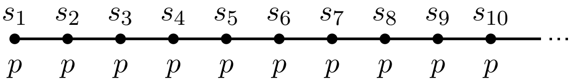


12. $\pi \vDash \phi \ \mathrm{W} \ \psi$ iff either there is some $i \geq 1$ such that $\pi^i \vDash \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \vDash \phi$; or for $k \geq 1$ we have $\pi^k \vDash \phi$

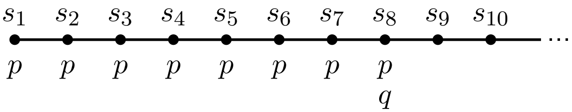
For example, $\pi \vDash p \ \mathrm{W} \ q$:

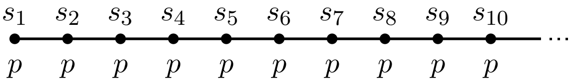




11. $\pi \vDash \phi \mathrm{\ R\ } \psi$ iff either there is some $i \geq 1$ such that $\pi^i \vDash \phi$ and for all $j = 1, \ldots, i$, we have $\pi^j \vDash \psi$, or for all $k \geq 1$ we have $\pi^k \vDash \psi$

For example, $\pi \vDash q \mathrm{\ R\ } p$:





性质: $\phi \mathrm{\ R\ } \psi \equiv \neg(\neg\phi \mathrm{\ U\ } \neg\psi)$

11. $\pi \vDash \phi \ \mathrm{R} \ \psi$ iff either there is some $i \geq 1$ such that $\pi^i \vDash \phi$ and for all $j = 1, \ldots, i$, we have $\pi^j \vDash \psi$, or for all $k \geq 1$ we have $\pi^k \vDash \psi$

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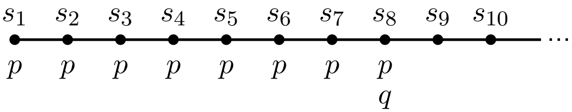
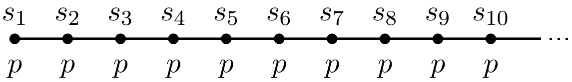




性质: $\phi \ \mathsf{R} \ \psi \equiv \neg(\neg\phi \ \mathsf{U} \ \neg\psi)$

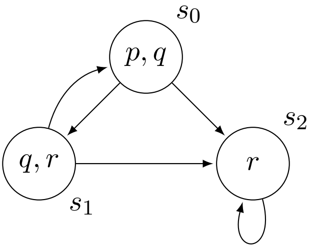
回顾定义: Semantic of LTL (for $\pi \vDash \phi$)

Let $\mathcal{M} = (S, \rightarrow, L)$ be a model and $\pi = s_1 \rightarrow s_2 \rightarrow \ldots$ be a path in $\mathcal{M}$. Whether π satisfies an LTL formula is defined by the satisfaction relation $\vDash$ as follows:

...

定义: Semantic of LTL (for $\mathcal{M}, s \vDash \phi$)

Suppose $\mathcal{M} = (S, \rightarrow, L)$ is a model, $s \in S$, and ϕ an LTL formula. We write $\mathcal{M}, s \vDash \phi$ if, for every execution path π of $\mathcal{M}$ starting at s, we have $\pi \vDash \phi$.



- $\mathcal{M}, s_0 \models p \wedge q$ holds
- $\mathcal{M}, s_0 \models \neg r$ holds
- $\mathcal{M}, s_0 \models \top$ holds
- $\mathcal{M}, s_0 \models \mathrm{X}\ r$ holds
- $\mathcal{M}, s_0 \models \mathrm{X}\ (q \wedge r)$ does not hold
- $\mathcal{M}, s_0 \models \mathrm{G}\ \neg(p \wedge r)$ holds
- $\mathcal{M}, s_2 \models \mathrm{G}\ r$ holds
- For any state s of $\mathcal{M}$, we have $\mathcal{M}, s \models \mathrm{F}\ (\neg q \wedge r) \rightarrow \mathrm{F}\ \mathrm{G}\ r$

- Which π satisfies $\pi \models \mathrm{G}\ \mathrm{F}\ p$?
  - $\pi_1 = s_0 \rightarrow s_1 \rightarrow s_0 \rightarrow s_1 \rightarrow \ldots$ Yes
  - $\pi_2 = s_0 \rightarrow s_2 \rightarrow s_2 \rightarrow s_2 \rightarrow \ldots$ No
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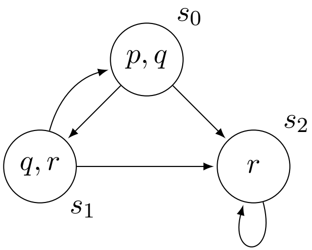

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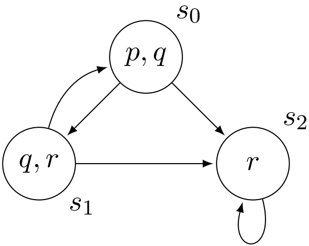

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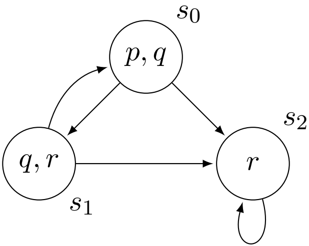

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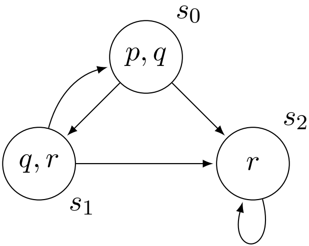

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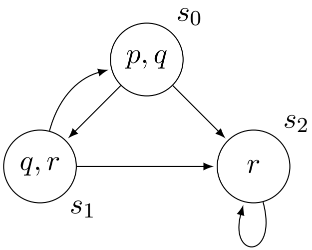

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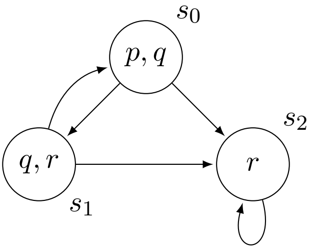

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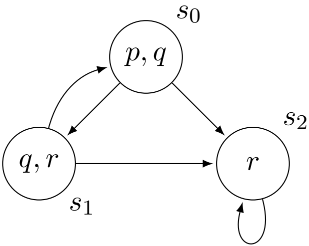

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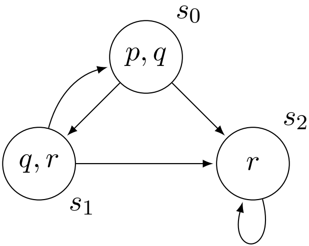

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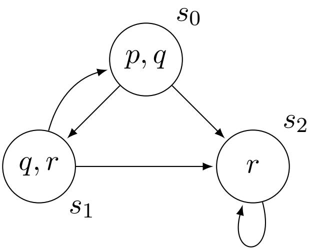

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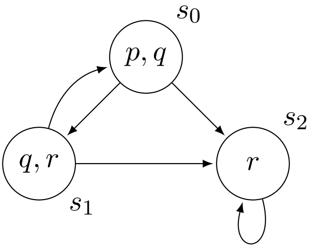

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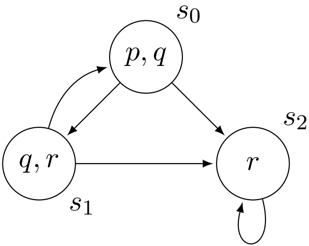

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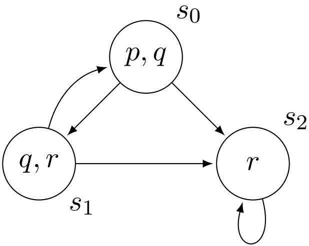

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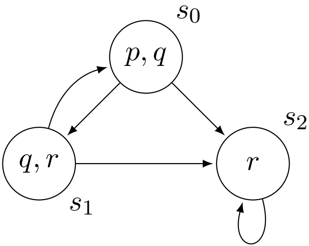

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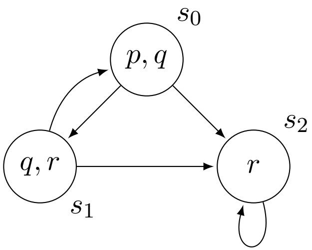

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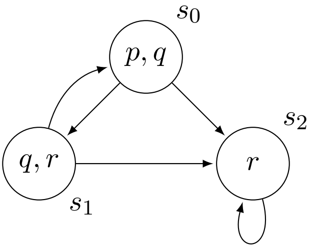

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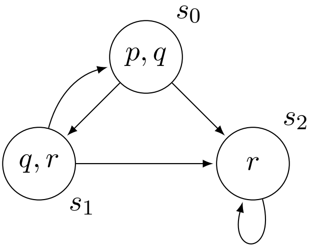

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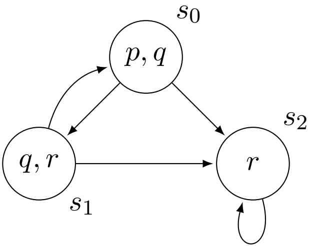
1. 应用

1.2 Linear-time temporal Logic (LTL) | Semantics



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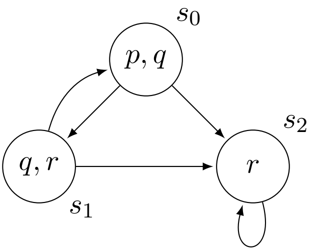

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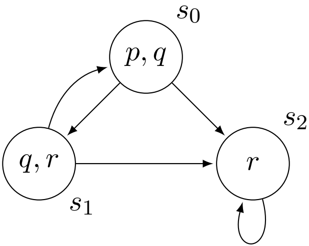

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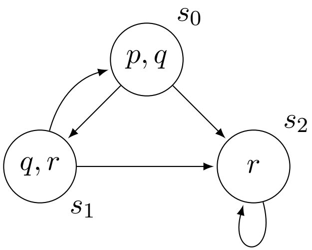

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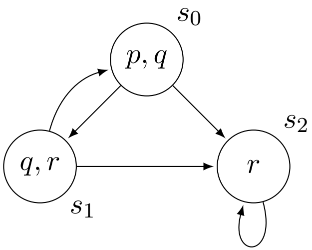

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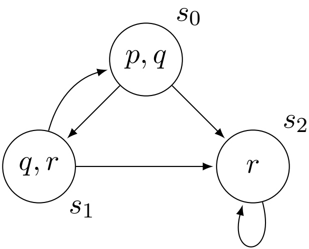

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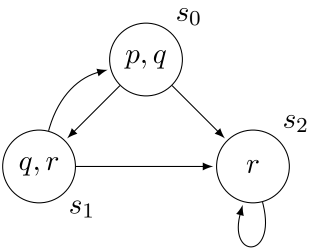

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问题: 怎样将 LTL 用于常见的 Specification 的设计?
答: 看如下案例

- It is impossible to get to a state where *started* holds, but *ready* does not hold:

$$G\neg(\text{started} \wedge \neg\text{ready})$$

- For any state, if a *request* (of some resource) occurs, then it will eventually be *acknowledged*:

$$G\ (\text{requested} \rightarrow F\ \text{acknowledged})$$

- A certain process is *enabled* infinitely often on every computation path:

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- Whatever happens, a certain process will eventually be permanently *deadlocked*:

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- An upwards travelling lift at the second floor does not change its direction when it has passengers wishing to go to the fifth floor:

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新的问题：哪些 Specification不能用 LTL 来设计?
答：看如下案例

- From any state it is possible to get to a restart state
  - i.e., there is a path from all states to a state satisfying restart
- The lift can remain idle on the third floor with its doors closed
  - i.e., from the state in which it is on the third floor, there is a path along which it stays there

LTL can't express these because it *cannot* directly assert the *existence* of paths.
怎么办?
另一种选择：CTL

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$$\neg(\phi \wedge \psi) \equiv \neg\phi \vee \neg\psi \qquad \neg(\phi \vee \psi) \equiv \neg\phi \wedge \neg\psi$$

$$\neg \mathrm{G}\ \phi \equiv \mathrm{F}\ \neg\phi \qquad \neg \mathrm{F}\ \phi \equiv \mathrm{G}\ \neg\phi \qquad \neg \mathrm{X}\ \phi \equiv \mathrm{X}\ \neg\phi$$

$$\neg(\phi\ \mathrm{U}\ \psi) \equiv \neg\phi\ \mathrm{R}\ \neg\psi \qquad \neg(\phi\ \mathrm{R}\ \psi) \equiv \neg\phi\ \mathrm{U}\ \neg\psi$$

$$\mathrm{F}\ (\phi \vee \psi) \equiv \mathrm{F}\ \phi \vee \mathrm{F}\ \psi \qquad \mathrm{G}\ (\phi \wedge \psi) \equiv \mathrm{G}\ \phi \wedge \mathrm{G}\ \psi$$

$$\mathrm{F}\ \phi \equiv \top\ \mathrm{U}\ \phi \qquad \mathrm{G}\ \phi \equiv \bot\ \mathrm{R}\ \phi$$

$$\phi\ \mathrm{U}\ \psi \equiv \phi\ \mathrm{W}\ \psi \wedge \mathrm{F}\ \psi \qquad \phi\ \mathrm{W}\ \psi \equiv \phi\ \mathrm{U}\ \psi \vee \mathrm{G}\ \psi$$

$$\phi\ \mathrm{W}\ \psi \equiv \psi\ \mathrm{R}\ (\phi \vee \psi) \qquad \phi\ \mathrm{R}\ \psi \equiv \psi\ \mathrm{W}\ (\phi \wedge \psi)$$

回顾: 定义: Linear-time temporal logic (LTL)

Linear-time temporal logic (*LTL*) has following syntax given in BNF:

$$\begin{aligned}\phi ::= & \top \mid \bot \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \rightarrow \phi) \\ & \mid (\mathrm{X}\ \phi) \mid (\mathrm{F}\ \phi) \mid (\mathrm{G}\ \phi) \mid (\phi\ \mathrm{U}\ \phi) \mid (\phi\ \mathrm{W}\ \phi) \mid (\phi\ \mathrm{R}\ \phi)\end{aligned}$$

where p is any propositional atom from some set **Atoms**.

定义: Computation Tree Logic (CTL)

Computation Tree logic (*CTL*) has following syntax given in BNF:

$$\begin{aligned}\phi ::= & \top \mid \bot \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \rightarrow \phi) \\ & \mid (\mathrm{AX}\ \phi) \mid (\mathrm{EX}\ \phi) \mid (\mathrm{AF}\ \phi) \mid (\mathrm{EF}\ \phi) \mid (\mathrm{AG}\ \phi) \mid (\mathrm{EG}\ \phi) \\ & \mid \mathrm{A}[\phi\ \mathrm{U}\ \phi] \mid \mathrm{E}[\phi\ \mathrm{U}\ \phi]\end{aligned}$$

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回顾: 定义: Linear-time temporal logic (LTL)

Linear-time temporal logic (*LTL*) has following syntax given in BNF:

$$\begin{aligned}\phi ::= & \top \mid \bot \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \rightarrow \phi) \\ & \mid (\mathrm{X}\ \phi) \mid (\mathrm{F}\ \phi) \mid (\mathrm{G}\ \phi) \mid (\phi\ \mathrm{U}\ \phi) \mid (\phi\ \mathrm{W}\ \phi) \mid (\phi\ \mathrm{R}\ \phi)\end{aligned}$$

where p is any propositional atom from some set **Atoms**.

定义: Computation Tree Logic (CTL)

Computation Tree logic (*CTL*) has following syntax given in BNF:

$$\begin{aligned}\phi ::= & \top \mid \bot \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid (\phi \vee \phi) \mid (\phi \rightarrow \phi) \\ & \mid (\mathrm{AX}\ \phi) \mid (\mathrm{EX}\ \phi) \mid (\mathrm{AF}\ \phi) \mid (\mathrm{EF}\ \phi) \mid (\mathrm{AG}\ \phi) \mid (\mathrm{EG}\ \phi) \\ & \mid \mathrm{A}[\phi\ \mathrm{U}\ \phi] \mid \mathrm{E}[\phi\ \mathrm{U}\ \phi]\end{aligned}$$

where p is any propositional atom from some set **Atoms**.

定义: Semantic of CTL (for $\mathcal{M}, s \vDash \phi$)

Let $\mathcal{M} = (S, \rightarrow, L)$ be a model for CTL, s in S, ϕ a CTL formula. The relation $\mathcal{M}, s \vDash \phi$ is defined by structural induction on ϕ

1. $\mathcal{M}, s \vDash \top$
2. $\mathcal{M}, s \nvDash \bot$
3. $\mathcal{M}, s \vDash p$ iff $p \in L(s)$
4. $\mathcal{M}, s \vDash \neg\phi$ iff $\mathcal{M}, s \nvDash \phi$
5. $\mathcal{M}, s \vDash \phi_1 \wedge \phi_2$ iff $\mathcal{M}, s \vDash \phi_1$ and $\mathcal{M}, s \vDash \phi_2$
6. $\mathcal{M}, s \vDash \phi_1 \vee \phi_2$ iff $\mathcal{M}, s \vDash \phi_1$ or $\mathcal{M}, s \vDash \phi_2$
7. $\mathcal{M}, s \vDash \phi_1 \rightarrow \phi_2$ iff $\mathcal{M}, s \vDash \phi_2$ whenever $\mathcal{M}, s \vDash \phi_1$

定义: Semantic of CTL (for $\mathcal{M}, s \vDash \phi$)

Let $\mathcal{M} = (S, \rightarrow, L)$ be a model for CTL, s in S, ϕ a CTL formula. The relation $\mathcal{M}, s \vDash \phi$ is defined by structural induction on ϕ

8. $\mathcal{M}, s \vDash \mathrm{AX}\ \phi$ iff *for all* s_1 such that $s \rightarrow s_1$ we have $\mathcal{M}, s_1 \vDash \phi$
9. $\mathcal{M}, s \vDash \mathrm{EX}\ \phi$ iff *for some* s_1 such that $s \rightarrow s_1$, we have $\mathcal{M}, s_1 \vDash \phi$
10. $\mathcal{M}, s \vDash \mathrm{AG}\ \phi$ iff *for all paths* $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$, where s_1 equals s, and *for all* s_i along the path, we have $\mathcal{M}, s_i \vDash \phi$
11. $\mathcal{M}, s \vDash \mathrm{EG}\ \phi$ iff *there is a path* $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$, where s_1 equals s, and *for all* s_i along the path, we have $\mathcal{M}, s_i \vDash \phi$
12. $\mathcal{M}, s \vDash \mathrm{AF}\ \phi$ iff *for all* paths $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$, where s_1 equals s, and *there is* some s_i such that $\mathcal{M}, s_i \vDash \phi$
13. $\mathcal{M}, s \vDash \mathrm{EF}\ \phi$ iff *there is a path* $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$, where s_1 equals s, and *there is* some s_i such that $\mathcal{M}, s_i \vDash \phi$

定义: Semantic of CTL (for $\mathcal{M}, s \vDash \phi$)

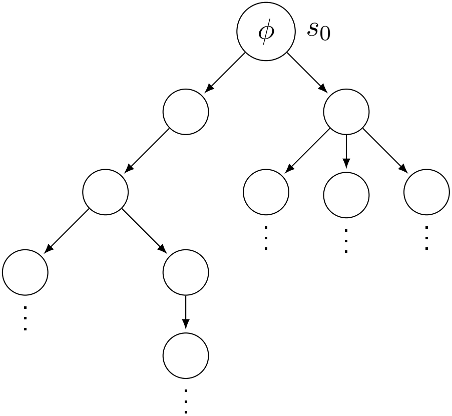
Let $\mathcal{M} = (S, \rightarrow, L)$ be a model for CTL, s in S, ϕ a CTL formula. The relation $\mathcal{M}, s \vDash \phi$ is defined by structural induction on ϕ

14. $\mathcal{M}, s \vDash A[\phi_1 \text{ U } \phi_2]$ iff *for all paths* $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$, where s_1 equals s, that path satisfies $\phi_1 \text{ U } \phi_2$, i.e., there is some s_i along the path, such that $\mathcal{M}, s_i \vDash \phi_2$, and, for each $j < i$, we have $\mathcal{M}, s_i \vDash \phi_1$
15. $\mathcal{M}, s \vDash E[\phi_1 \text{ U } \phi_2]$ iff *there is a path* $s_1 \rightarrow s_2 \rightarrow s_3 \rightarrow \ldots$, where s_1 equals s, that path satisfies $\phi_1 \text{ U } \phi_2$, i.e., there is some s_i along the path, such that $\mathcal{M}, s_i \vDash \phi_2$, and, for each $j < i$, we have $\mathcal{M}, s_i \vDash \phi_1$

3. $\mathcal{M}, s \vDash p$ iff $p \in L(s)$

For example,

$$\mathcal{M}, s_0 \vDash \phi$$



$\mathcal{M}, s \vDash \mathrm{AX}\ \phi$ iff *for all* s_1 such that $s \to s_1$ we have $\mathcal{M}, s_1 \vDash \phi$

For example,

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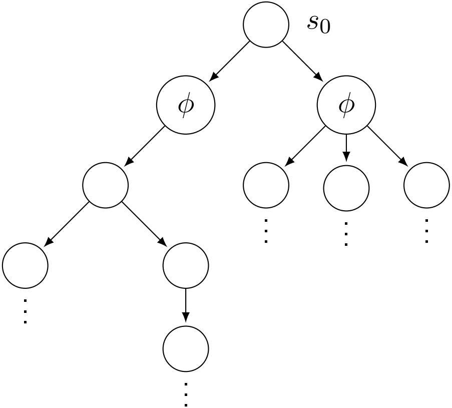

$\mathcal{M}, s \vDash \mathrm{EX}\ \phi$ iff *for some* s_1 such that $s \to s_1$, we have $\mathcal{M}, s_1 \vDash \phi$

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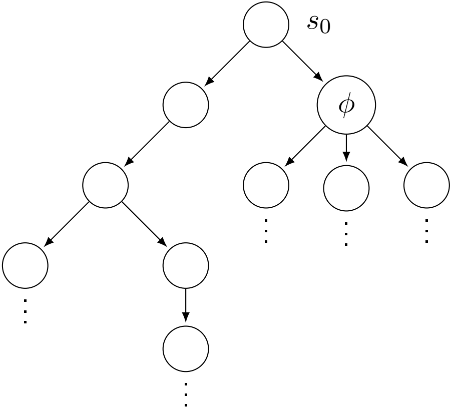

$\mathcal{M}, s \vDash \text{AG } \phi$ iff *for all paths* $s_1 \to s_2 \to s_3 \to \ldots$, where s_1 equals s, and *for all* s_i along the path, we have $\mathcal{M}, s_i \vDash \phi$

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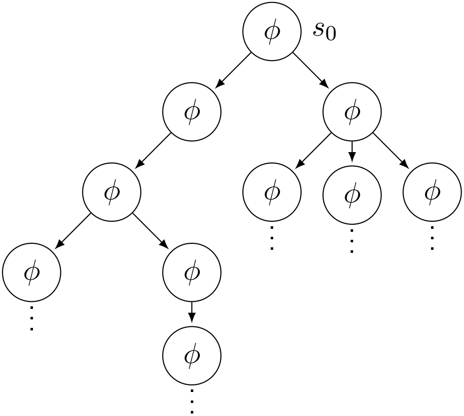

$\mathcal{M}, s \vDash \mathrm{EG}\ \phi$ iff *there is a path* $s_1 \to s_2 \to s_3 \to \ldots$, where s_1 equals s, and *for all* s_i along the path, we have $\mathcal{M}, s_i \vDash \phi$

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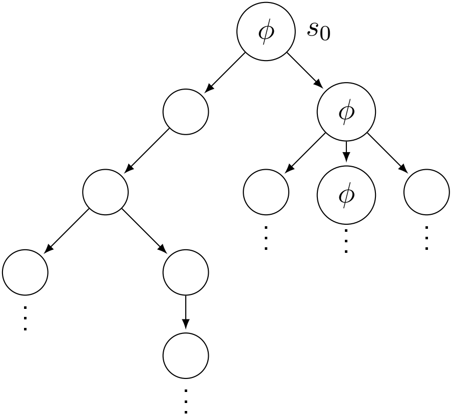

$\mathcal{M}, s \vDash \mathrm{AF}\ \phi$ iff *for all* paths $s_1 \to s_2 \to s_3 \to \ldots$, where s_1 equals s, and *there is* some s_i such that $\mathcal{M}, s_i \vDash \phi$

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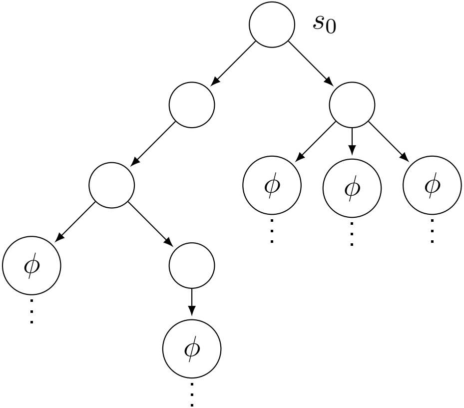

$\mathcal{M}, s \vDash \mathrm{EF}\ \phi$ iff *there is a path* $s_1 \to s_2 \to s_3 \to \ldots,$ where s_1 equals s, and *there is* some s_i such that $\mathcal{M}, s_i \vDash \phi$

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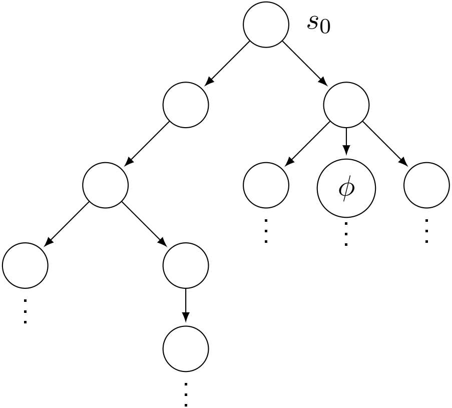

$\mathcal{M}, s \vDash A[\phi_1 \text{ U } \phi_2]$ iff *for all paths* $s_1 \to s_2 \to s_3 \to \ldots$, where s_1 equals s, that path satisfies $\phi_1 \text{ U } \phi_2$, i.e., there is some s_i along the path, such that $\mathcal{M}, s_i \vDash \phi_2$, and, for each $j < i$, we have $\mathcal{M}, s_i \vDash \phi_1$

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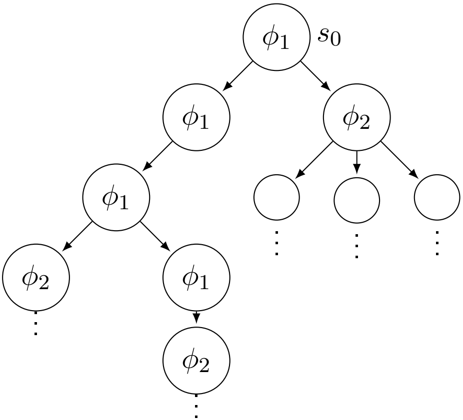

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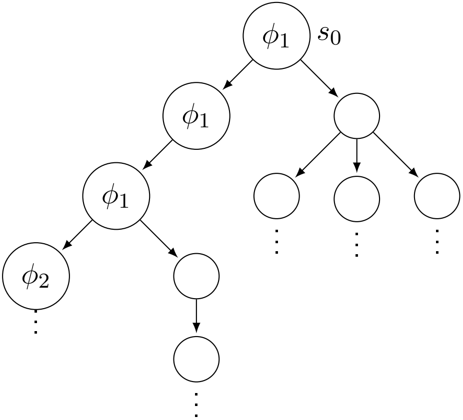

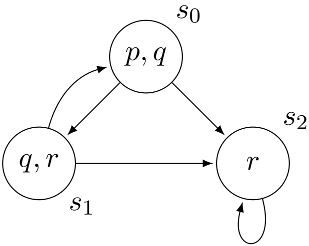

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- $\mathcal{M}, s_0 \vDash \mathrm{EX}\ (q \wedge r)$ holds
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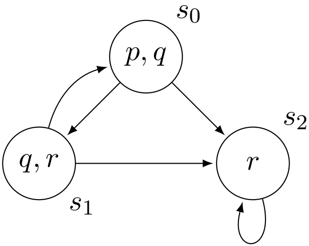

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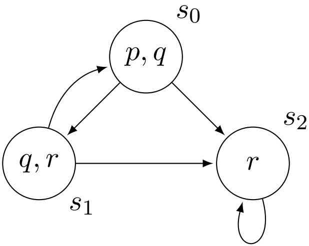

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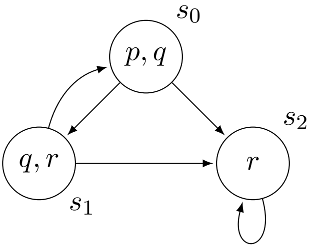

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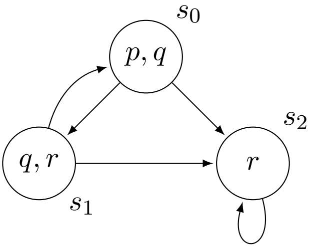

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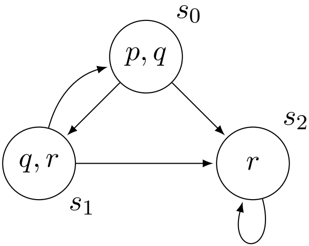

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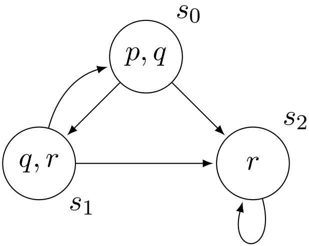

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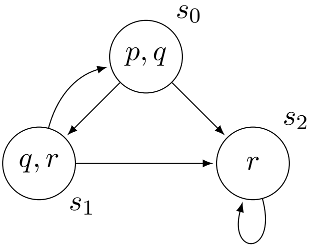

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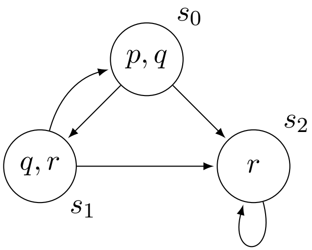

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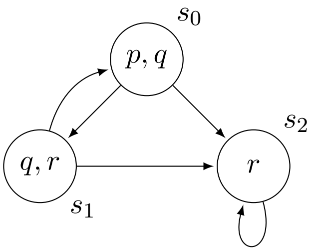

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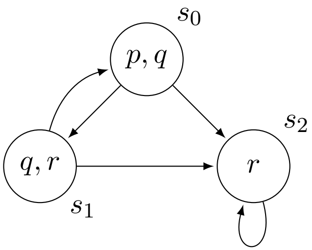

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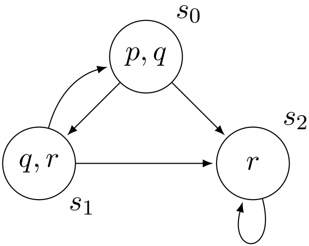

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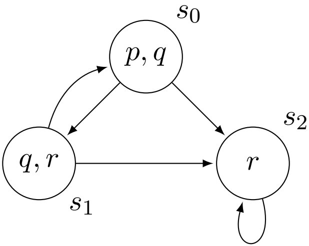
- $\mathcal{M}, s_0 \vDash \mathrm{A}[p\ \mathrm{U}\ r]$ holds
- $\mathcal{M}, s_0 \vDash \mathrm{AG}\ (p \vee q \vee r \rightarrow \mathrm{EF}\ \mathrm{EG}\ r)$ holds



- $\mathcal{M}, s_0 \vDash p \wedge q$ holds
- $\mathcal{M}, s_0 \vDash \neg r$ holds
- $\mathcal{M}, s_0 \vDash \top$ holds
- $\mathcal{M}, s_0 \vDash \mathrm{EX}\ (q \wedge r)$ holds
- $\mathcal{M}, s_0 \vDash \neg \mathrm{AX}\ (q \wedge r)$ holds
- $\mathcal{M}, s_0 \vDash \neg \mathrm{EF}\ (p \wedge r)$ holds
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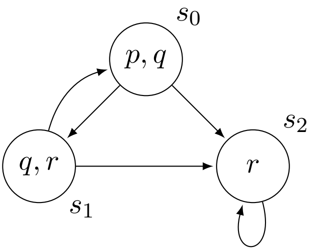

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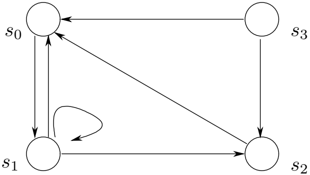
回顾: 反例:

Given a set of states $A = \{s_0, s_1, s_2, s_3\}$, let $R^{\mathcal{M}}$ be the set $\{(s_0, s_1), (s_1, s_0), (s_1, s_1), (s_1, s_2), (s_2, s_0), (s_3, s_0), (s_3, s_2)\}$. We may depict this model as *a directed graph* in a figure, where an edge (a transition) leads from a node s to a node s' iff $(s, s') \in R^{\mathcal{M}}$.



回顾: 反例: How to define Reachability as ϕ

Given nodes n and n' in a directed graph, is there a finite path of transitions from n to n'?

回顾: 反例：How to define Reachability as ϕ

Given nodes n and n' in a directed graph, is there a finite path of transitions from n to n'?

回顾: 反例：一种答案

$$(u = v)\vee\exists x(R(u,x)\wedge R(x,v))\vee\exists x_1\exists x_2(R(u,x_1)\wedge R(x_1,x_2)\wedge R(x_2,v))\vee\ldots$$

- This is infinite, so it's *not* a well-formed formula.
- Can we find a well-formed formula with the same meaning? *No!*

回顾：反例：How to define Reachability as ϕ

Given nodes n and n' in a directed graph, is there a finite path of transitions from n to n'?

回顾：另一种答案：Second-order Logic

$$\neg\exists P \forall x \forall y \forall z \ (C_1 \wedge C_2 \wedge C_3 \wedge C_4)$$

where

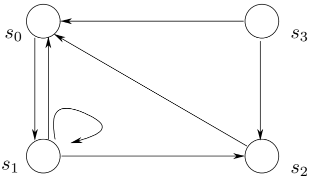
$$C_1 \stackrel{\text{def}}{=} P(x,x)$$
$$C_2 \stackrel{\text{def}}{=} P(x,y) \wedge P(y,z) \to P(x,z)$$
$$C_3 \stackrel{\text{def}}{=} P(u,v) \to \bot$$
$$C_4 \stackrel{\text{def}}{=} R(x,y) \to P(x,y)$$

回顾：反例：How to define Reachability as ϕ

Given nodes n and n' in a directed graph, is there a finite path of transitions from n to n'?

新答案：使用 CTL

$$\mathcal{M}, n \vDash \mathrm{EF}\ (s = n')$$



$$\neg \mathrm{AF}\ \phi \equiv \mathrm{EG}\ \neg\phi$$

$$\neg \mathrm{EF}\ \phi \equiv \mathrm{AG}\ \neg\phi$$

$$\neg \mathrm{AX}\ \phi \equiv \mathrm{EX}\ \neg\phi$$

$$\mathrm{AF}\ \phi \equiv \mathrm{A}[\top\ \mathrm{U}\ \phi]$$

$$\mathrm{EF}\ \phi \equiv \mathrm{E}[\top\ \mathrm{U}\ \phi]$$

回顾: LTL cannot express these because it *cannot* directly assert the *existence* of paths.

- CTL can express the *existence* of paths.

新的问题: Is CTL better than LTL? i.e., Is LTL a subset of CTL?
答案: *No*

例:
An LTL formula:

$$\mathrm{FG}\ p$$

How to express it in CTL? $\mathrm{AFAG}\ p$? *No*
Another LTL formula?

$$\mathrm{F}\ p \rightarrow \mathrm{F}\ q$$

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怎么办?

*CTL** is a logic which *combines* the expressive powers of *LTL* and *CTL*, by *dropping* the CTL *constraint* that every temporal operator (X, U, F, G) has to be associated with a unique path quantifier (A, E). For example:

- A[(p U r) $\vee$ (q U r)]: along all paths, either p is true until r, or q is true until r.
- A[X p $\vee$ XX p]: along all paths, p is true in the next state, or the next but one.
- E[G F p]: there is a path along which p is infinitely often true.

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定义: CTL*

The syntax of CTL* involves two classes of formulas:

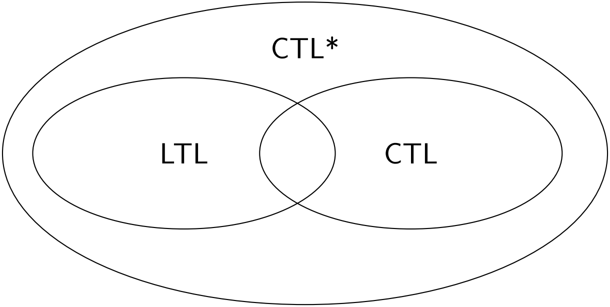
- *state formulas*, which are evaluated in states:

$$\phi ::== \top \mid p \mid (\neg\phi) \mid (\phi \wedge \phi) \mid \mathrm{A}[\alpha] \mid \mathrm{E}[\alpha]$$

  where p is any atomic formula and α any path formula

- *path formulas*, which are evaluated along paths:

$$\alpha ::== \phi \mid (\neg\alpha) \mid (\alpha \wedge \alpha) \mid (\alpha \ \mathrm{U} \ \alpha) \mid (\mathrm{G} \ \alpha) \mid (\mathrm{F} \ \alpha) \mid (\mathrm{X} \ \alpha)$$



CTL*
LTL
CTL

Concluding:

- LTL, CTL, CTL* can be used to model ϕ, instead of propositional logics, first-order logics, higher-order logics
- Transition system can be used to model $\mathcal{M}$, instead of logics

剩下的问题:

- How to program using LTL, CTL, CTL*, transition system
  - Using NuSMV (见第 1.4 节)
- How to implement algorithms for NuSMV
  - (见第 2 节)

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作业

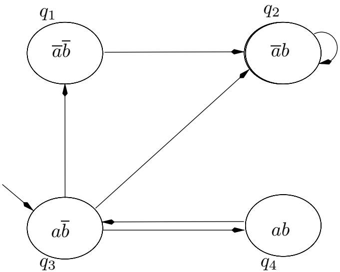

Figure 3.39. A model $\mathcal{M}$.

2. Consider the system of Figure 3.39. For each of the formulas ϕ:
   (a) $\mathrm{G}\, a$
   (b) $a \;\mathrm{U}\; b$
   (c) $a \;\mathrm{U}\; \mathrm{X}\,(a \wedge \neg b)$
   (d) $\mathrm{X}\, \neg b \wedge \mathrm{G}\,(\neg a \vee \neg b)$
   (e) $\mathrm{X}\,(a \wedge b) \wedge \mathrm{F}\,(\neg a \wedge \neg b)$
      (i) Find a path from the initial state q_3 which satisfies ϕ.
      (ii) Determine whether $\mathcal{M}, q_3 \vDash \phi$.

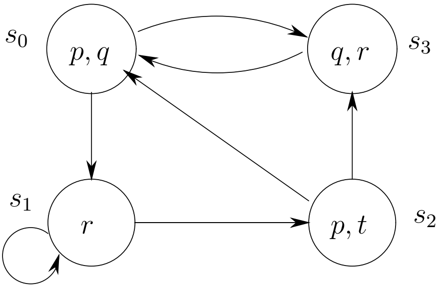

Figure 3.41. Another model with four states.

8. Consider the model $\mathcal{M}$ in Figure 3.41. Check whether $\mathcal{M}, s_0 \vDash \phi$ and $\mathcal{M}, s_2 \vDash \phi$ hold for the CTL formulas ϕ:
   (a) $\mathrm{AF}\, q$
   (b) $\mathrm{AG}\,(\mathrm{EF}\,(p \vee r))$
   (c) $\mathrm{EX}\,(\mathrm{EX}\, r)$
   (d) $\mathrm{AG}\,(\mathrm{AF}\, q)$.