1. 首先构造原问题的松弛型：

$$y_1 = 10 - x_1 - 2x_2 - 3x_3$$

$$y_2 = 3 - x_1 + x_3$$

$$y_3 = 5 + x_2 - 2x_3$$

Goal: Maximize $z = 1 + 2x_1 + 3x_2 + 6x_3$, while keeping $\forall i.\ x_i \geq 0$ and $\forall i.\ y_i \geq 0$

step1: basic solution

$$x_1 = x_2 = x_3 = 0$$

$$y_1 = 10,\ y_2 = 3,\ y_3 = 5$$

Observation: 保持$x_2$和$x_3$为0，增大$x_1$可使目标函数值增大，可得到以下情况

- $y_1 = 10 - x_1$ : only OK if $x_1 \leq 10$
- $y_2 = 3 - x_1$ : only OK if $x_1 \leq 3$
- $y_3 = 5$ : OK if $x_1$ increases

$\Rightarrow$ $x_1$最大可增大为3，这时$y_2$会变成0

step2: pivot: swap $x_1$ with $y_2$，得到新的目标函数形式和约束：

$$x_1 = 3 + x_3 - y_2$$

$$y_1 = 7 - 2x_2 - 4x_3 + y_2$$

$$y_3 = 5 + x_2 - 2x_3$$

Goal: Maximize $z = 7 + 3x_2 + 8x_3 - 2y_2$, while keeping $\forall i.\ x_i \geq 0$ and $\forall i.\ y_i \geq 0$

Observation: 保持$x_3$和$y_2$为0，增大$x_2$可使目的标函数值增大，有

- $x_1 = 3$ : OK if $x_2$ increases
- $y_1 = 7 - 2x_2$ : only OK if $x_2 \leq \frac{7}{2}$
- $y_3 = 5 + x_2$ : OK if $x_2$ increases

$\Rightarrow x_2$ 最大可增大为 $\frac{7}{2}$，这时 $y_1$ 会变为0

Step3: pivot: swap $x_2$ with $y_1$，得到新的目标函数形式和约束：

$x_1 = 3 + x_3 - y_2$

$x_2 = \frac{7}{2} - 2x_3 + \frac{1}{2}y_2 - \frac{1}{2}y_1$

$y_3 = \frac{17}{2} - 4x_3 + \frac{1}{2}y_2 - \frac{1}{2}y_1$

Goal: Maximize $z = \frac{35}{2} + 2x_3 - \frac{1}{2}y_2 - \frac{3}{2}y_1$, while keeping $\forall i. x_i \geq 0$ and $\forall i. y_i \geq 0$

Observation: 保持 $y_2$ 和 $y_1$ 为0，增大 $x_3$ 可使目标函数值增大，有

- $x_1 = 3 + x_3$: OK if $x_3$ increases
- $x_2 = \frac{7}{2} - 2x_3$: only OK if $x_3 \leq \frac{7}{4}$
- $y_3 = \frac{17}{2} - 4x_3$: only OK if $x_3 \leq \frac{17}{8}$

$\Rightarrow x_3$ 最大可增大为 $\frac{7}{4}$，这时 $x_2$ 会变为0

Step4: pivot: swap $x_3$ with $x_2$，得到新的目标函数形式和约束：

$x_1 = \frac{19}{4} - \frac{3}{4}y_2 - \frac{1}{4}y_1 - \frac{1}{2}x_2$

$x_3 = \frac{7}{4} + \frac{1}{4}y_2 - \frac{1}{4}y_1 - \frac{1}{2}x_2$

$y_3 = \frac{3}{2} - \frac{1}{2}y_2 + \frac{1}{2}y_1 + 2x_2$

Goal: Maximize $z = 21 - 2y_1 - x_2$, while keeping $\forall i. x_i \geq 0$ and $\forall i. y_i \geq 0$

由于 $y_1, x_2 \geq 0$，所以 $z \leq 21$，同时，令 $y_2 = y_1 = x_2 = 0$，$x_1 = \frac{19}{4}$，$x_3 = \frac{7}{4}$，$y_3 = \frac{3}{2}$ 可使 $z = 21$.

2. step1: introduce z for maximizing $-z$:

$$x - y - z \le -3$$

$$2x + y - z \le 7$$

$$-x - 2y - z \le -9$$

step2: Slack form:

$$y_1 = -3 - x + y + z$$

$$y_2 = 7 - 2x - y + z$$

$$y_3 = -9 + x + 2y + z$$

$x, y, z \ge 0$

Goal: Maximize $-z$, while keeping ~~$\forall i.\ x_i \ge 0$~~ and $\forall i.\ y_i \ge 0$

Most negative $b_i$ is $b_3 = -9$, so do pivot on $z$ and $y_3$

step3: pivot swapping $z$ and $y_3$

$$y_1 = 6 - 2x - y + y_3$$

$$y_2 = 16 - 3x - 3y + y_3$$

$$z = 9 - x - 2y + y_3$$

Goal: Maximize $-9 + x + 2y - y_3$, while keeping $x, y, z \ge 0$ and $\forall i.\ y_i \ge 0$

Observation: 增大x，保持y和$y_3$为0，可使目标函数值增大.

- $y_1 = 6 - 2x$: only OK if $x \le 3$
- $y_2 = 16 - 3x$: only OK if $x \le \frac{16}{3}$
- $y_3 z = 9 - x$: only OK if $x \le 9$

$\Rightarrow$ x最大可增大到3，这时$y_1$会变为0

step4: pivot swapping $x$ and $y_1$

$$x = 3 - \frac{1}{2}y_1 - \frac{1}{2}y + \frac{1}{2}y_3$$

$$y_2 = 7 + \frac{3}{2}y_1 - \frac{3}{2}y - \frac{1}{2}y_3$$

$$z = 6 + \frac{1}{2}y_1 - \frac{3}{2}y + \frac{1}{2}y_3$$

Goal: Maximize $-6 - \frac{1}{2}y_1 + \frac{3}{2}y - \frac{1}{2}y_3$, while keeping $x, y, z \ge 0$ and $\forall i.\ y_i \ge 0$

Observation: 保持 $y_1$ 和 $y_3$ 为0，增大 $y$，可使目标函数值增大.

- $x = 3 - \frac{1}{2}y$ : only OK if $y \le 6$
- $y_2 = 7 - \frac{3}{2}y$ : only OK if $y \le \frac{14}{3}$
- $z = 6 - \frac{3}{2}y$ : only OK if $y \le 4$

⇒ $y$ 最大可增大到4，这时 $z$ 变为0

step 5 : pivot swapping $y$ and $z$

$$x = 1 - \frac{2}{3}y_1 + \frac{1}{3}y_3 + \frac{1}{3}z$$

$$y_2 = 1 + y_1 - y_3 + z$$

$$y = 4 + \frac{1}{3}y_1 + \frac{1}{3}y_3 - \frac{2}{3}z$$

Goal: Maximize $-z$, while keeping $x, y, z \ge 0$ and $\forall i. y_i \ge 0$

故取值 $y_1 = y_3 = z = 0$，$x = 1$，$y_2 = 1$，$y = 4$ 可使 $-z$ 取得最大值0.

所以 $x = 1$，$y = 4$ 可使原约束

$$x - y \le -3$$

$$2x + y \le 7$$

$$-x - 2y \le -9$$

得到满足.