Stochastic Volatility Quant Research Ideas

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February 19, 2020 Winter Quarter

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Overall Thinking

We are constructing **Stochastic Volatility Inspired** (SVI) model to describe the volatility smile.
Functional PCA (fPCA) is a good idea to conclude the features of functional data (shape of many curves).

Definition: Functional data is multivariate data with an ordering on dimensions. (Muller, 2006)

Math Background, fPCA for Volatility Surface Modeling:

1. stochastic process: Karhunen-Loève expansion
2. integral operator: Mercer's theorem
3. reproducing kernel Hilbert space

What are we interested in fPCA

1. Basic Expansions
   - Good basis systems approximate any (sufficiently smooth) volatility smile curve arbitrarily well, which helps us gain more effective features of the curves
   - Fourier Basis, B-spline Bases, Wavelets Bases
2. Smoothing Penalties
   - to penalize roughness of the result coming from SVI
   - Ordinary Least-squared Estimates, constraint on Time-warping (registration) functions
3. Tests and Boostrap
   - Theoretical results on asymptotic normality of test statistics, especially with fPCA
   - Still requires bootstrap/permutation procedures to evaluate

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Stochastic Volatility Inspired (SVI) Volatility Surface

Heuristic Intuition

It's a 5-parameter hyperbola for implied variance in terms of log-strike (a, b, ρ, k, m, σ)

1. Notes
   - To exclude maturity-direction arbitrage, it is necessary (but not sufficient) to eliminate the intersections of the variance curves.
   - But this interpolation has two issues:
     (1) the discontinuity of the time derivative of the call values;
     (2) a problem for ATM values before the first maturity due to the intrinsic value, which we use to interpolate, having a kink at the spot.
   - These issues can be mitigated by choosing a smoother monotone spline, e.g., a monotone cubic spline and inserting a tiny extra maturity to handle the ATM kink.

Ensemble Carr-Pelts (ECP) Volatility Surface

Intuition

- We note that although the Carr-Pelts surface is fairly flexible, it is trying to fit a surface with two curves, each roughly corresponding to each dimension of the surface (z does not correspond to K-direction exactly, since solving for z involves T).

  Taking as inspiration the tensor product basis, we can make the surface more flexible by taking a mixture of regular Carr-Pelts surfaces with some positive weights summing up to one.

Comparison of SVI and ECP

Shortcomings of SVI

- lack of flexibility to fit rich options data when arbitrage conditions are enforced
- time-direction interpolation in price terms, whereas the strike interpolation at input maturities is done in volatility terms. The latter may lead to some **unintuitive shapes** for the interpolated volatilities.

Ensemble Carr-Pelts (ECP) surface

- allows both a guarantee of no arbitrage and a reasonable shape of the implied volatility surface at the same time.
- Moreover, both the implied and local volatilities are known essentially in closed form.

Summary

1. SVI
   - SVI is simpler and allows for fast bootstrapping in time calibration, because it is simple (fewer parameters).
   - Whereas ECP does not, unless the h-function parameters are fixed, and only *sigma* values are calibrated.
2. ECP
   - ECP is usually better than SVI in the Dupire test, and the LSV test to fit European options without model calibration.
   - If SVI is used with no-arbitrage conditions enforced, the fit becomes quite unsatisfactory.

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Zumbach effect

Heuristic Definition

- Zumbach effect (2017): price trends induce an increase of volatility.
- Financial time series is the feedback of price returns on volatility. Volatility clustering: past trends in returns convey significant information on future volatility

Natural way to model Zumbach effect: quadratic Hawkes process

- Hawkes process is a point process that they 'self-excite': each arrival increases the rate of future arrivals for a period of time
- Hawkes process is a non-Markovian extension of the Poisson process.
- We can construct a microscopic model, encoding Zumbach effect and leading naturally to super-Heston rough volatility, is to use a quadratic Hawkes based price process.
  - Jim Gatheral (2017) rought volatility: log-volatility behaves as fractional Brownian motion with Hurst parameter of order 0.1

Microstructural foundation for Rough Volatility

1. Behavior of Rough Volatility
   - Assuming only that the order flow is driven by a linear Hawkes process, the price necessarily follows a rough Heston model
   - The rough behavior is due to the singular kernel, which is the same as Mandelbrot-van Ness representation of a fractional Brownian motion, with Hurst parameter $\alpha - 1/2$
2. Our goal
   - We want to establish connections between micro-parameters of the quadratic Hawkes dynamic and macro-phenomena such as the roughness of the volatility and the strong Zumbach effect.
   - Based on PDE techniques, we may obtain a diffusion process with power-law marginal distributions and strong Zumbach effect for the asymptotic volatility.

Mathematical meaning of Zumbach effect

- the volatility is a functional of past price returns through Z
- Z term convolves with a power-law kernel
- Mittag-Leffler type kernels is commonly seen in rough volatility literature

References I

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The Volatility Surface. Wiley, 2006.

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Arbitrage-free SVI volatility surfaces (2012).

Alexandre Antonov, Michael Konikov, and Michael Spector
A New Arbitrage-Free Parametric Volatility Surface (June 2019).

Jim Gatheral
The Volatility Surface: Statics and Dynamics
Bloomberg Quant Seminar (2013).

Giles Hooker
slides on FDA (2017), Cornell.

fda Package

There is special fda package in R, see Gile's slides

References II

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Thanks!

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