Data Mining
Classification: Basic Concepts and Techniques

Lecture Notes for Chapter 3

Introduction to Data Mining, 2nd Edition

by

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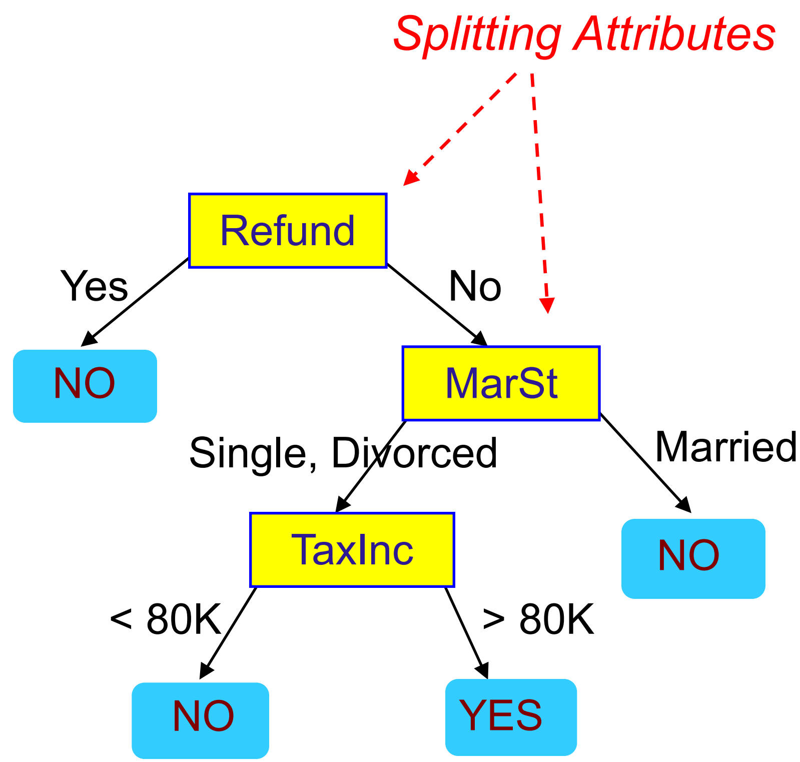
Example of a Decision Tree

categorical categorical continuous class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
2	No	Married	100K	No
3	No	Single	70K	No
4	Yes	Married	120K	No
5	No	Divorced	95K	Yes
6	No	Married	60K	No
7	Yes	Divorced	220K	No
8	No	Single	85K	Yes
9	No	Married	75K	No
10	No	Single	90K	Yes

Training Data



Model: Decision Tree

Another Example of Decision Tree

categorical categorical continuous class

Tid	Refund	Marital Status	Taxable Income	Cheat
1	Yes	Single	125K	No
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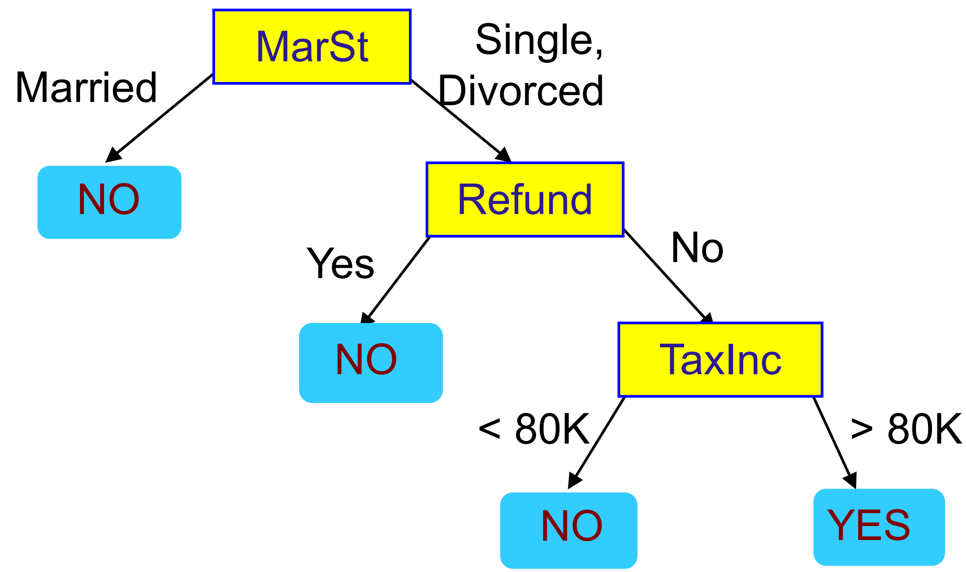

There could be more than one tree that fits the same data!

Decision Tree Classification Task

Tid	Attrib1	Attrib2	Attrib3	Class
1	Yes	Large	125K	**No**
2	No	Medium	100K	**No**
3	No	Small	70K	**No**
4	Yes	Medium	120K	**No**
5	No	Large	95K	**Yes**
6	No	Medium	60K	**No**
7	Yes	Large	220K	**No**
8	No	Small	85K	**Yes**
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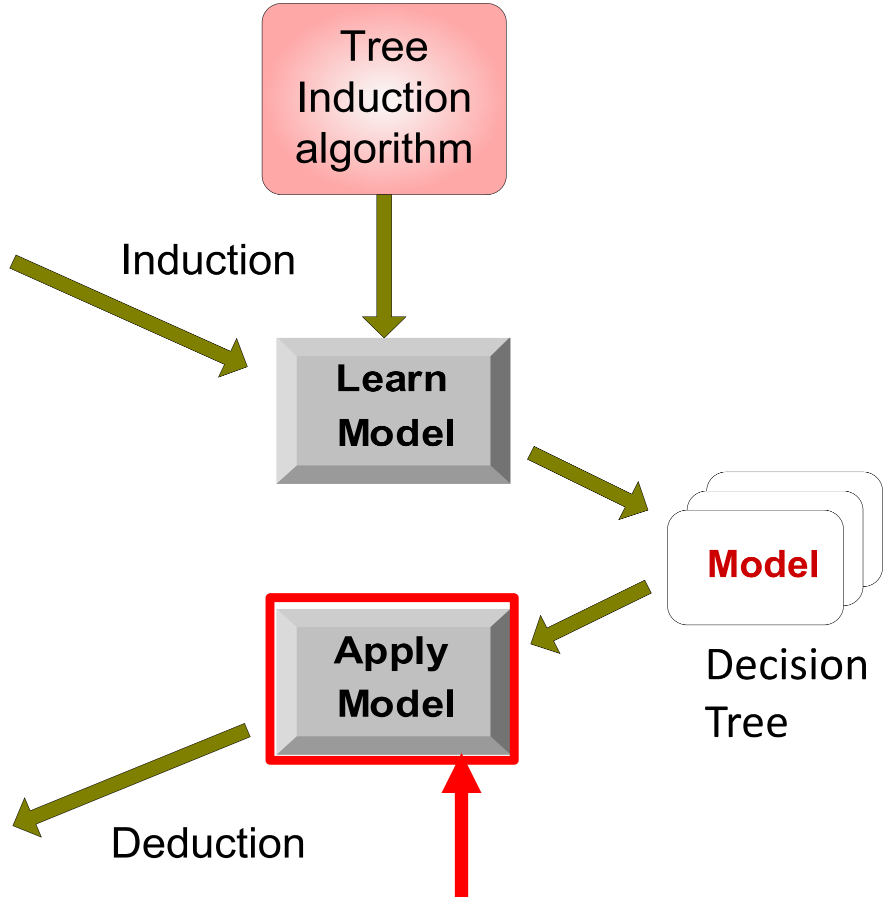
10	No	Small	90K	**Yes**

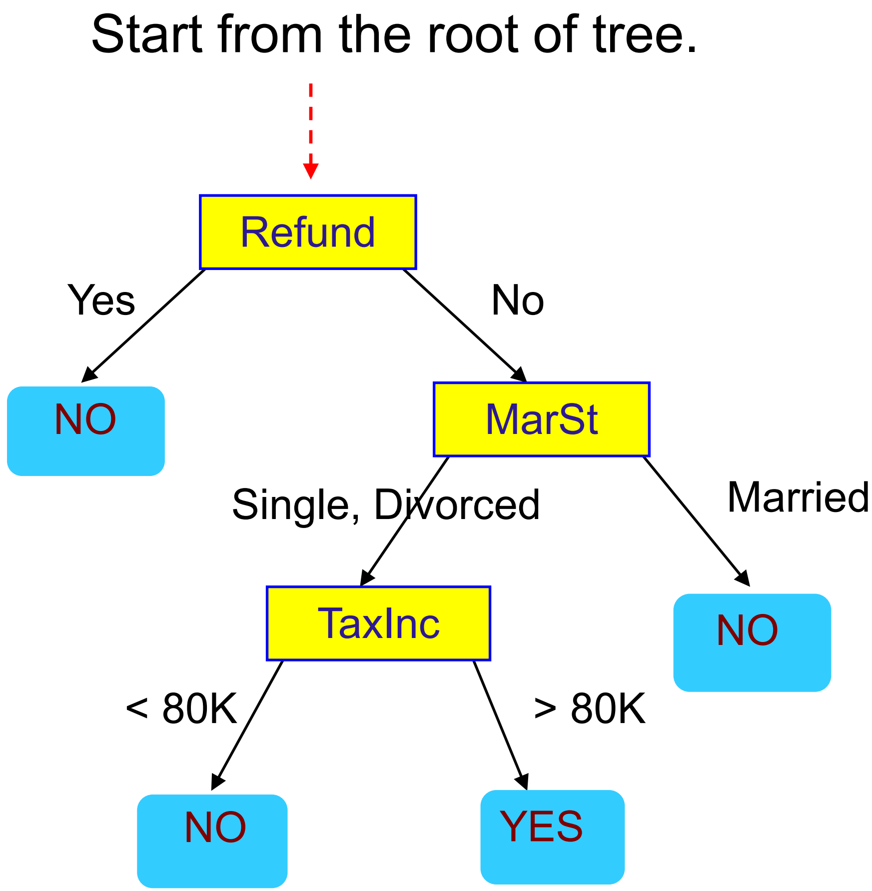
Training Set



Tid	Attrib1	Attrib2	Attrib3	Class
11	No	Small	55K	**?**
12	Yes	Medium	80K	**?**
13	Yes	Large	110K	**?**
14	No	Small	95K	**?**
15	No	Large	67K	**?**

Test Set

Apply Model to Test Data



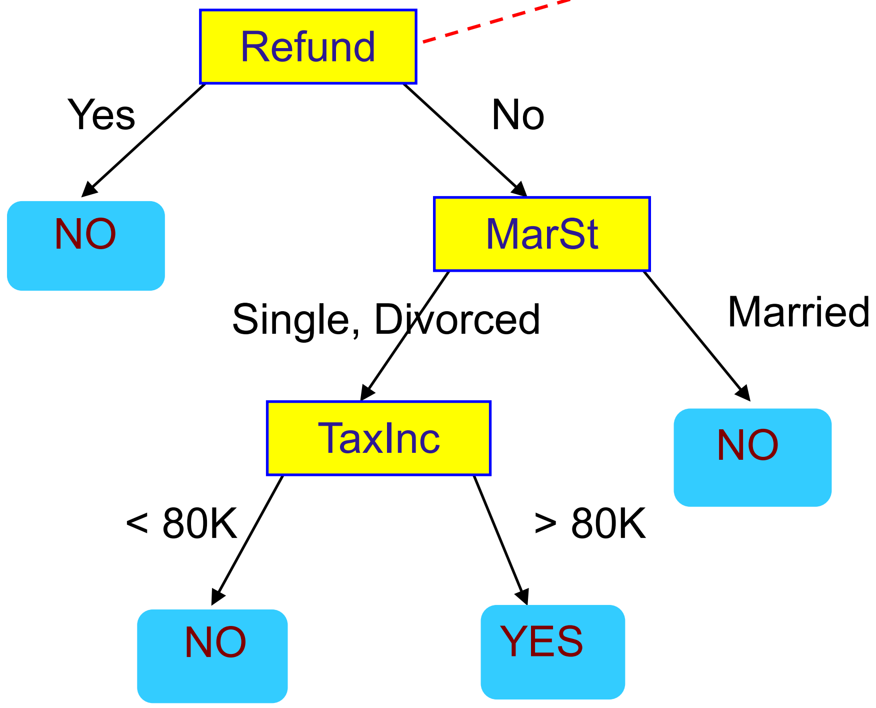
Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

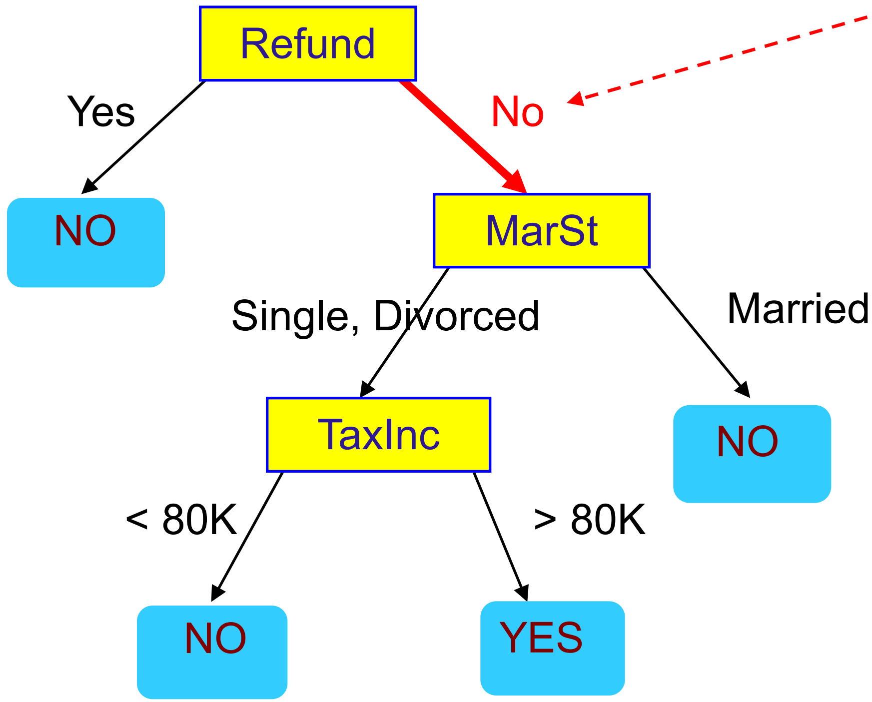
Apply Model to Test Data

Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Apply Model to Test Data



Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

Apply Model to Test Data

Test Data

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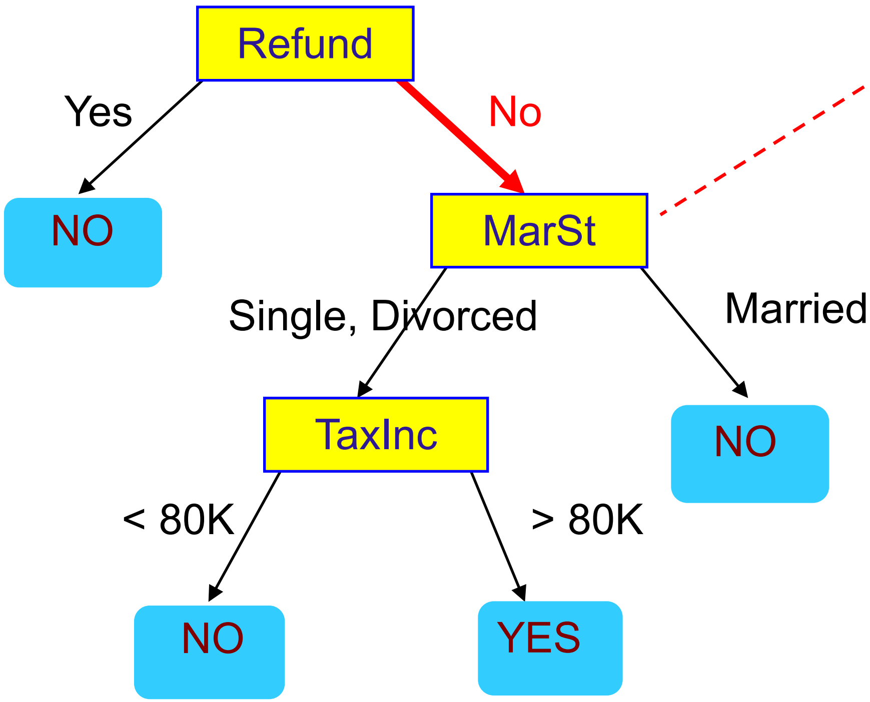
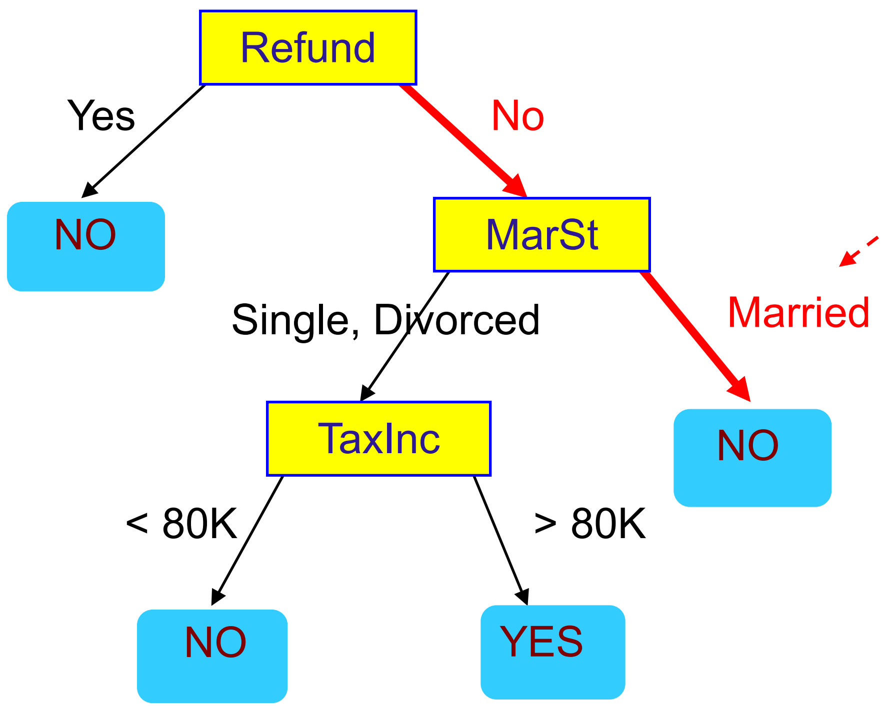
No	Married	80K	?



Apply Model to Test Data



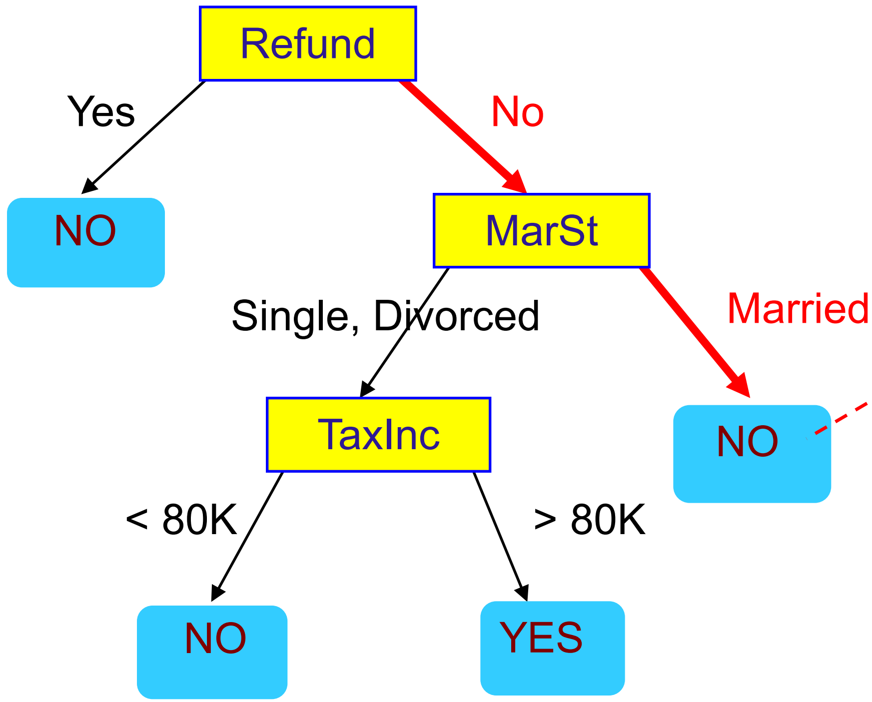
Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?

Apply Model to Test Data

Test Data

Refund	Marital Status	Taxable Income	Cheat
No	Married	80K	?



Assign Cheat to “No”

Decision Tree Classification Task

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Training Set

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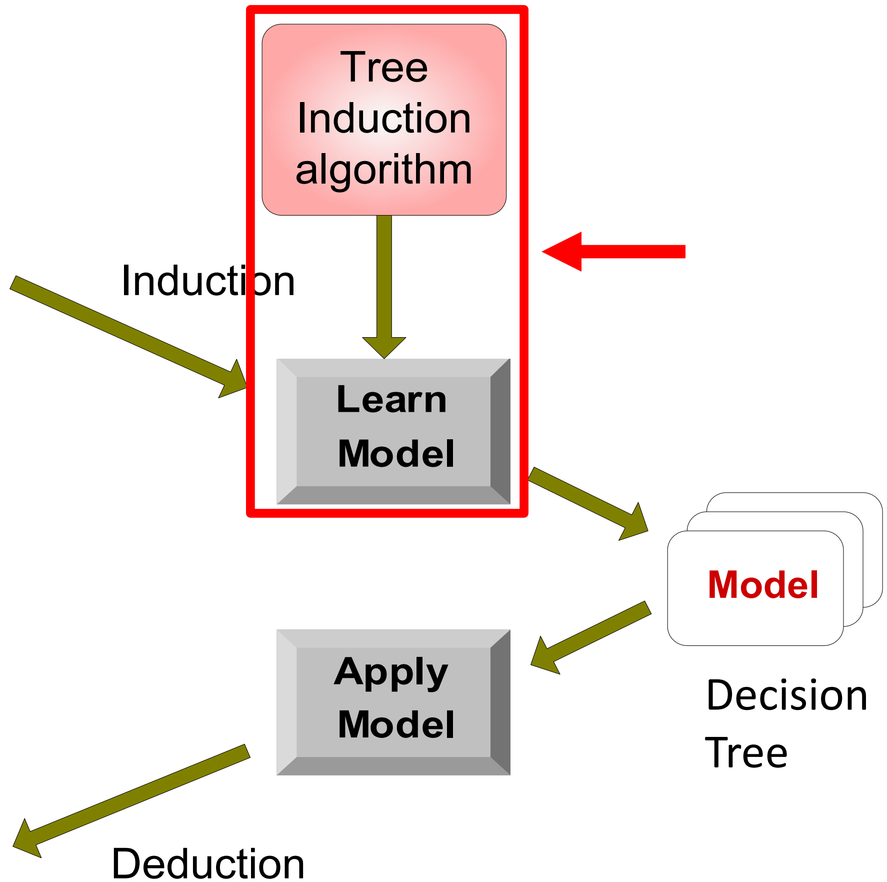
Test Set



General Structure

- Let D_t be the set of training records that reach a node t
- **General Procedure:**
  - If D_t contains records that belong the same class y_t, then t is a leaf node labeled as y_t
  - If D_t contains records that belong to more than one class, use an attribute to split the data into smaller subsets. Recursively apply the procedure to each subset

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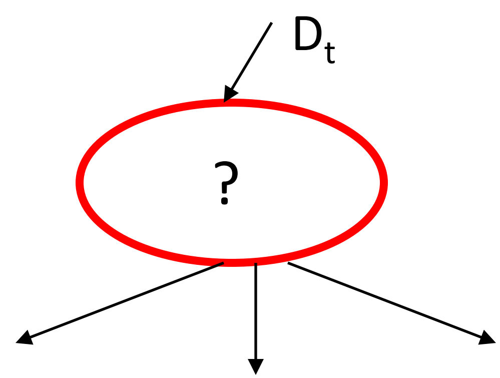
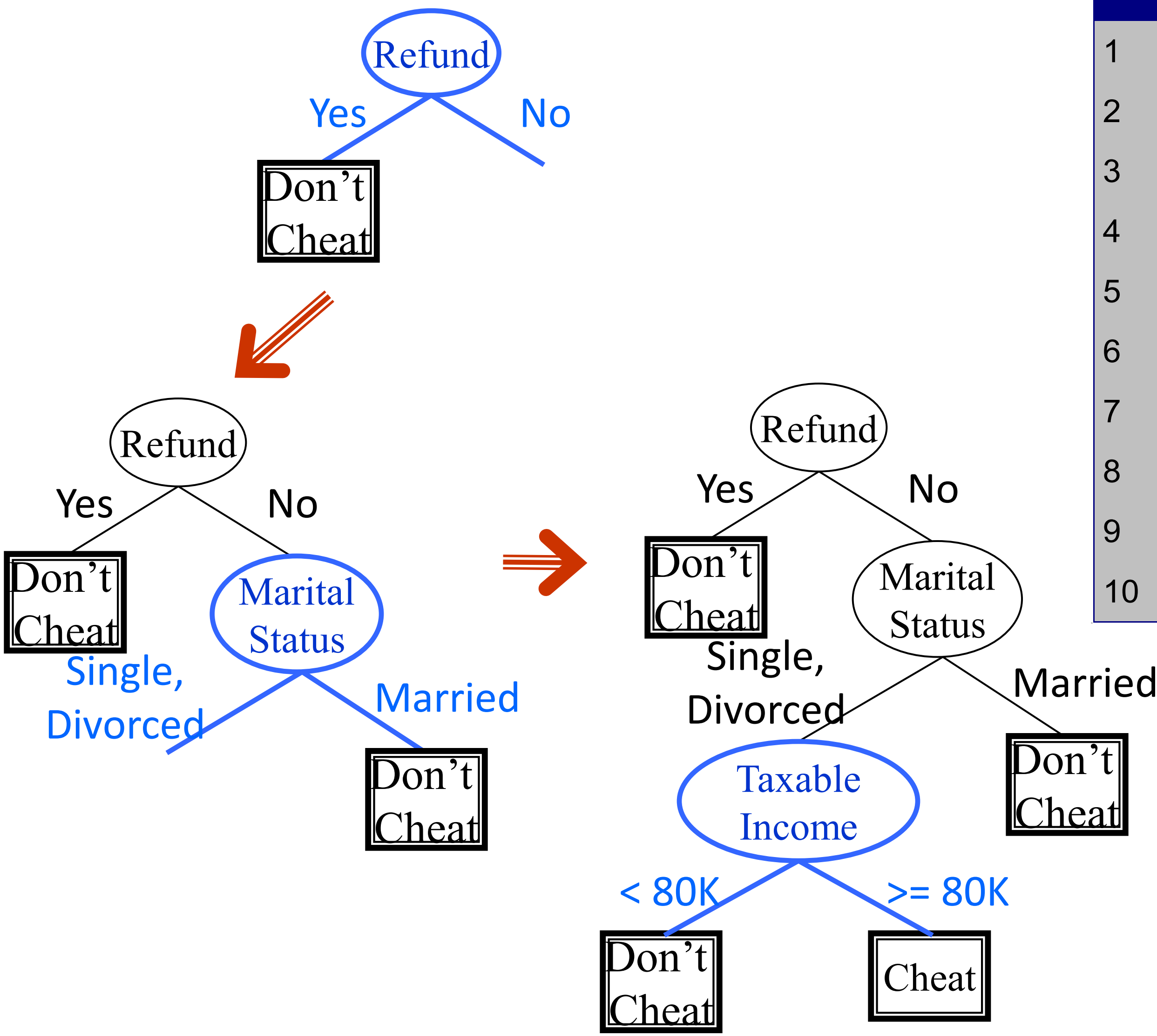


Example



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10	No	Single	90K	**Yes**

Tree Induction

- **Greedy strategy**
  - Split the records based on an attribute test that optimizes certain criterion

- **Issues**
  - Determine how to split the records
    - How to specify the attribute test condition?
    - How to determine the best split?
  - Determine when to stop splitting

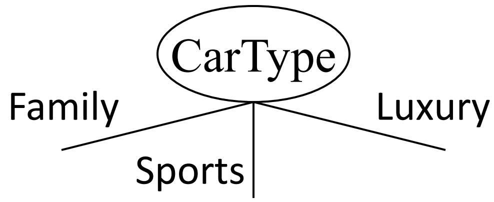
How to Specify Test Condition?

- **Depends on attribute types**
  - Nominal
  - Ordinal
  - Continuous

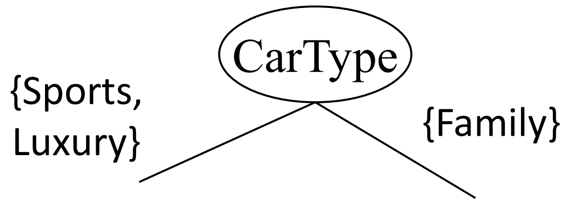
- **Depends on number of ways to split**
  - 2-way split
  - Multi-way split

Splitting Based on Nominal Attributes

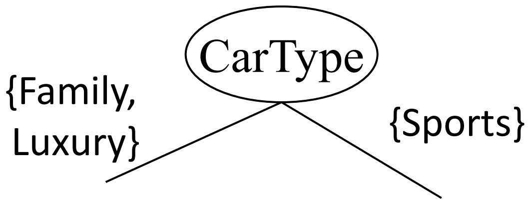
- **Multi-way split:** Use as many partitions as distinct values



- **Binary split:** Divides values into two subsets
  Need to find optimal partitioning

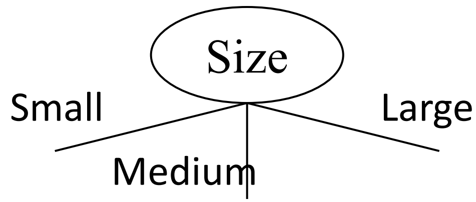


OR

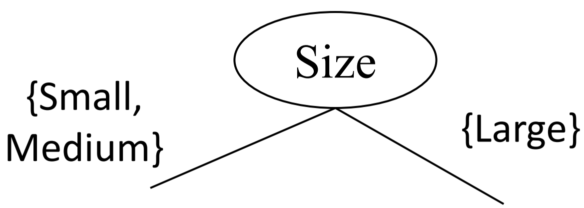


Splitting Based on Ordinal Attributes

- **Multi-way split:** Use as many partitions as distinct values.



- **Binary split:** Divides values into two subsets
  Need to find optimal partitioning



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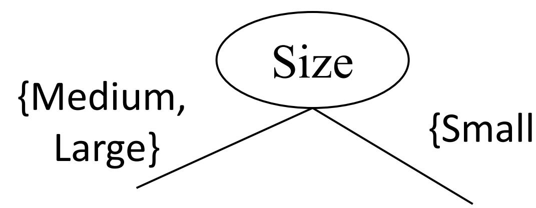
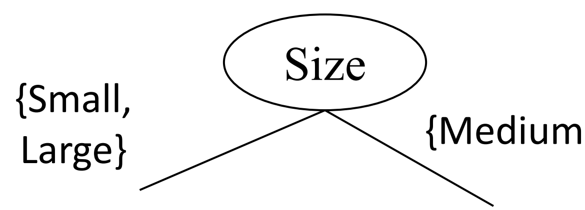


- What about this split?

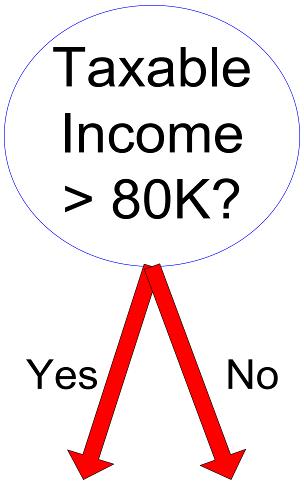


Splitting Based on Continuous Attributes

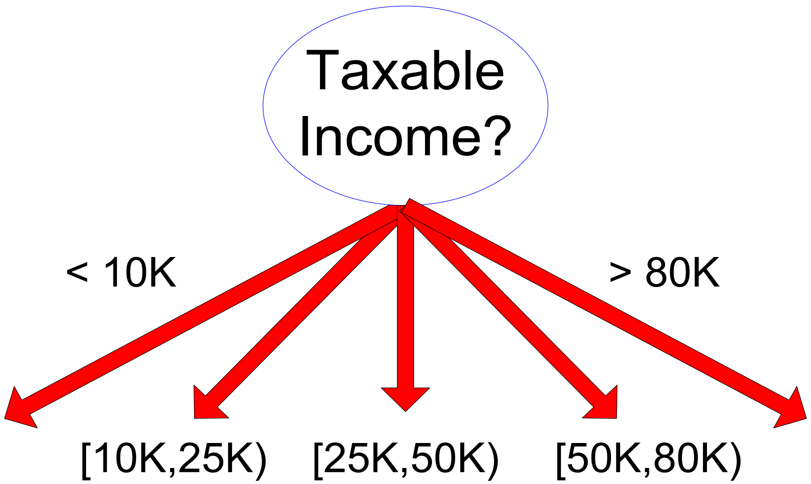
- Different ways of handling
  - **Discretization** to form an ordinal categorical attribute

  - **Binary Decision:** $(A < v)$ or $(A \geq v)$
    - consider all possible splits and finds the best cut
    - can be more computation intensive

Splitting Based on Continuous Attributes



(i) Binary split



(ii) Multi-way split

Tree Induction

- **Greedy strategy**
  - Split the records based on an attribute test that optimizes certain criterion.

- **Issues**
  - Determine how to split the records
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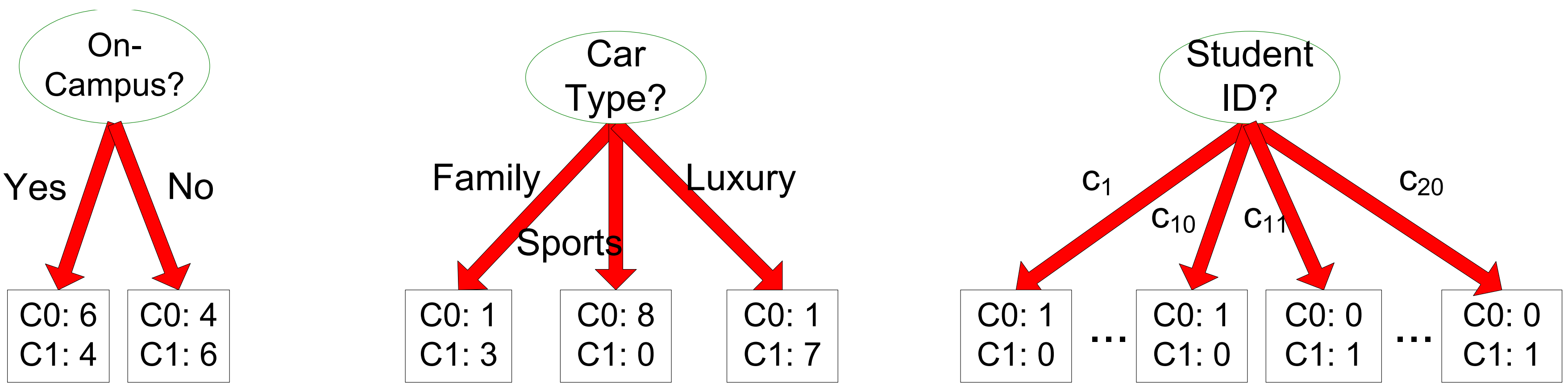
  - Determine when to stop splitting

How to determine the Best Split

Before Splitting: 10 records of class 0,
10 records of class 1



Which test condition is the best?

How to determine the Best Split

- **Greedy approach:**
  - Nodes with homogeneous class distribution are preferred
- **Need a measure of node impurity:**

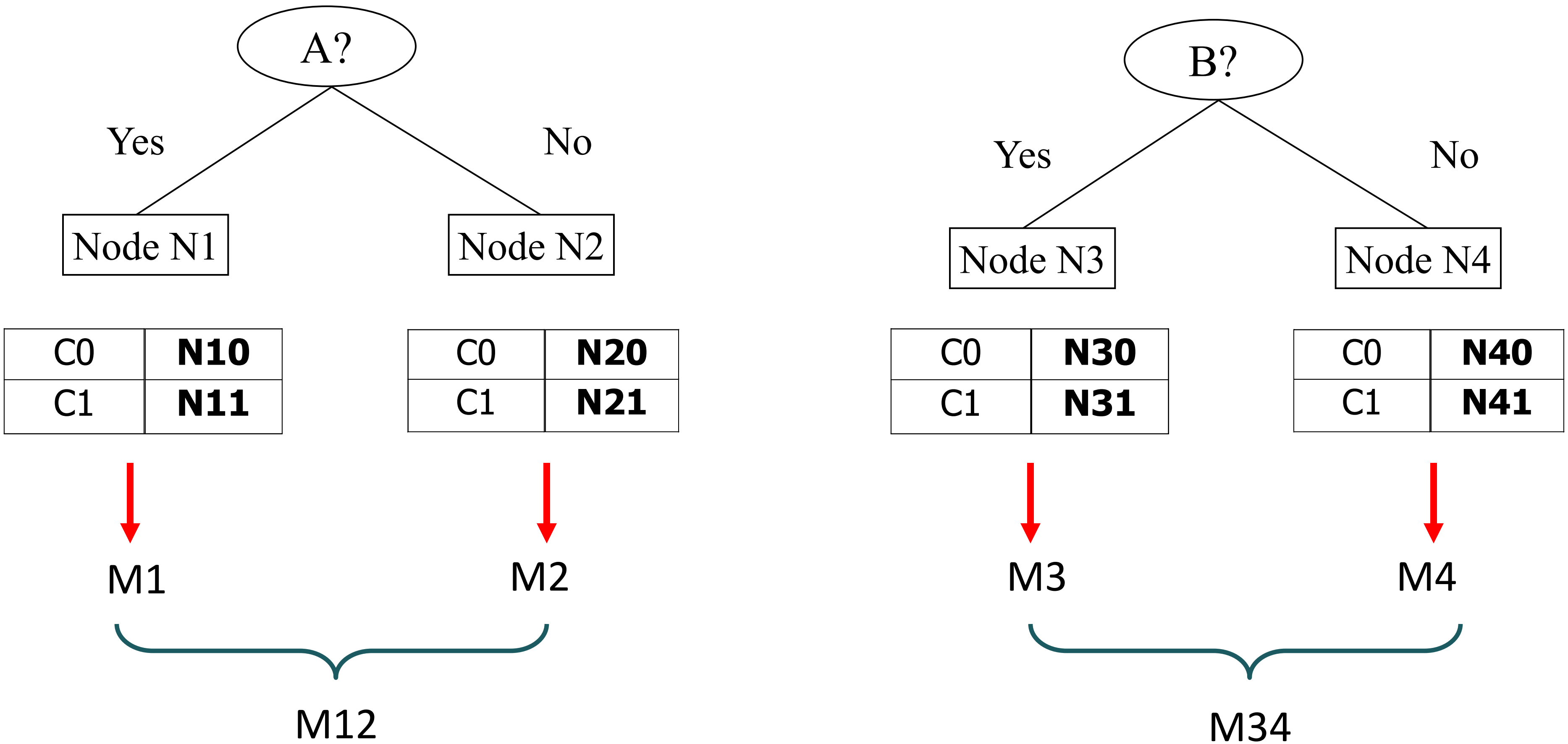
C0: 5 C1: 5	C0: 9 C1: 1
Non-homogeneous, High degree of impurity	Homogeneous, Low degree of impurity

How to Find the Best Split

Before Splitting:

C0	**N00**
C1	**N01**

M0



Gain = M0 – M12 vs M0 – M34

Measures of Node Impurity

- Gini Index
- Entropy
- Misclassification error

Measure of Impurity: GINI

- Gini Index for a given node t :

$$GINI(t) = 1 - \sum_j [p(j \mid t)]^2$$

(NOTE: $p(j \mid t)$ is the relative frequency of class j at node t).

  - Maximum $(1 - 1/n_c)$ when records are equally distributed among all classes, implying least interesting information
  - Minimum (0) when all records belong to one class, implying most useful information

C1	**0**
C2	**6**
Gini=0.000	

C1	**1**
C2	**5**
Gini=0.278	

C1	**2**
C2	**4**
Gini=0.444	

C1	**3**
C2	**3**
Gini=0.500	

Examples for computing GINI

$$GINI(t) = 1 - \sum_j [p(j \mid t)]^2$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0 P(C2) = 6/6 = 1

Gini = 1 – $P(C1)^2$ – $P(C2)^2$ = 1 – 0 – 1 = 0

C1	**1**
C2	**5**

P(C1) = 1/6 P(C2) = 5/6

Gini = 1 – $(1/6)^2$ – $(5/6)^2$ = 0.278

C1	**2**
C2	**4**

P(C1) = 2/6 P(C2) = 4/6

Gini = 1 – $(2/6)^2$ – $(4/6)^2$ = 0.444

Entropy

- Entropy at a given node t:

$$Entropy(t) = -\sum_j p(j\,|\,t) \log p(j\,|\,t)$$

(NOTE: $p(j \mid t)$ is the relative frequency of class j at node t).

- Measures purity of a node
  - Maximum ($\log n_c$) when records are equally distributed among all classes implying least information
  - Minimum (0.0) when all records belong to one class, implying most information

Examples for computing Entropy

$$Entropy(t) = -\sum_j p(j \mid t) \log_2 p(j \mid t)$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0   P(C2) = 6/6 = 1

Entropy = – 0 log 0 – 1 log 1 = – 0 – 0 = 0

C1	**1**
C2	**5**

P(C1) = 1/6   P(C2) = 5/6

Entropy = – (1/6) $\log_2$ (1/6) – (5/6) $\log_2$ (5/6) = 0.65

C1	**2**
C2	**4**

P(C1) = 2/6   P(C2) = 4/6

Entropy = – (2/6) $\log_2$ (2/6) – (4/6) $\log_2$ (4/6) = 0.92

Splitting Criteria based on Classification Error

- Classification error at a node t :

$$\boxed{Error(t) = 1 - \max_i P(i \mid t)}$$

- Measures misclassification error made by a node.
  - Maximum ($1 - 1/n_c$) when records are equally distributed among all classes, implying least interesting information
  - Minimum (0.0) when all records belong to one class, implying most interesting information

Examples for Computing Error

$$Error(t) = 1 - \max_i P(i \mid t)$$

C1	**0**
C2	**6**

P(C1) = 0/6 = 0   P(C2) = 6/6 = 1

Error = 1 – max (0, 1) = 1 – 1 = 0

C1	**1**
C2	**5**

P(C1) = 1/6   P(C2) = 5/6

Error = 1 – max (1/6, 5/6) = 1 – 5/6 = 1/6

C1	**2**
C2	**4**

P(C1) = 2/6   P(C2) = 4/6

Error = 1 – max (2/6, 4/6) = 1 – 4/6 = 1/3

Computing Gain

- **Gain:**

$$GAIN_{split} = Measure(p) - \left(\sum_{i=1}^{k} \frac{n_i}{n} Measure(i) \right)$$

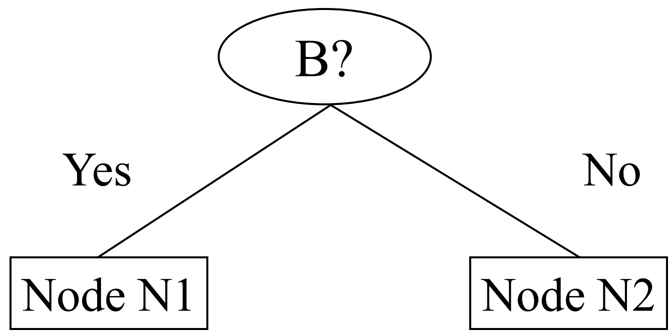
Parent Node p is split into k partitions;

n_i is number of records in partition i

– Measures reduction in impurity measure achieved because of the split. Choose the split that achieves most reduction (maximizes GAIN)

Binary Attributes: Computing GINI Index

- Splits into two partitions
- Effect of Weighing partitions:
  - Larger and Purer Partitions are sought for



	Parent
C1	**6**
C2	**6**
Gini = 0.500	

Gini(N1)
$= 1 - (5/7)^2 - (2/7)^2$
$= 0.408$

Gini(N2)
$= 1 - (1/5)^2 - (4/5)^2$
$= 0.32$

	N1	**N2**
C1	**5**	**1**
C2	**2**	**4**
Gini=0.333		

Gini(Children)
$= 7/12 * 0.408 +$
$5/12 * 0.32$
$= 0.371$

Gain=0.5-0.371=0.129

Tree Induction

- **Greedy strategy**
  - Split the records based on an attribute test that optimizes certain criterion.

- **Issues**
  - Determine how to split the records
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  - Determine when to stop splitting

Stopping Criteria for Tree Induction

- Stop expanding a node when all the records belong to the same class

- Stop expanding a node when all the attributes have been used

- Early termination (to be discussed later)