\subsection{Introduction}

The paper addresses challenges and improvements with the Plug-and-Play Alternating Direction Method of Multiplier algorithm(ADMM), which is widely used for solving constrained optimization problems in image restoration. The paper provides step to step guidance, for us to know how plug-and-play ADMM comes from. Start with maximum-a-posteriori(MAP), standard optimization algorithms, Alternating Direction Method of Multiplier algorithm(ADMM), augmented Lagrangian function, and finally denoising algorithm.

\subsection{Challenges of Plug-and-Play ADMM}

There are several challenges this paper is focusing on. First, convergence of the algorithm, Classical ADMM require g to be closed, proper and convex in order to ensure convergence. However, for general image denoising algorithm, the convergence is not known. Conclusion first, this paper have a new algorithm that is guaranteed to converge for a broader class of denoisers.

Second, the implementation of plug-and-play ADMM. The challenge is to obtain a fast solver for the inversion step in x^(k+1) = argmin f(x) + pi/2 …….(5). Conclusion first, this paper has demonstrated a fast implementation.

\subsection{Comparison with related work}

Approximate message passing (AMP) algorithm is the algorithm replace the shrinkage step of the standard AMP algorithm in the class of “proper denoisers”. AMP was reported around the same period of time of Plug-and-Play ADMM. However, compare to Plug-and-Play ADMM, if f(x) departs from quadratic or if A is not random, then the behavior of the denoise-AMP becomes unclear. The paper also compare Plug-and-Play ADMM with algorithm such as BM3D, the conclusion is Plug-and-Play ADMM is more general than all the other algorithms mentioned in the paper.

Introduction

The paper delves into the challenges and enhancements associated with the Plug-and-Play Alternating Direction Method of Multiplier algorithm (ADMM), a widely utilized approach for addressing constrained optimization problems in image restoration. The exposition follows a structured progression, offering a comprehensive understanding of the evolution of Plug-and-Play ADMM. The narrative unfolds, starting with the foundational concepts of Maximum A Posteriori (MAP), proceeding to standard optimization algorithms, introducing the Alternating Direction Method of Multiplier algorithm, exploring the augmented Lagrangian function, and culminating with the incorporation of denoising algorithms.

Challenges of Plug-and-Play ADMM

This section navigates through the core challenges confronted in the paper. Firstly, the focus is on the convergence of the algorithm, where classical ADMM necessitates gg to be closed, proper, and convex for guaranteed convergence. However, for general image denoising algorithms, convergence remains an elusive aspect. The paper presents a breakthrough, introducing a novel algorithm with proven convergence for a broader class of denoisers.

Secondly, the implementation of Plug-and-Play ADMM is discussed. The crux of the challenge lies in obtaining a rapid solver for the inversion step in the equation x(k+1)=argmin f(x)+ρ2∥x−ve(k)∥22x(k+1)=argminf(x)+2ρ​∥x−ve(k)∥22​. The paper, however, resolves this challenge by demonstrating an effective and swift implementation.

Comparison with Related Work

The paper draws comparisons with the Approximate Message Passing (AMP) algorithm, which replaces the shrinkage step in the standard AMP algorithm with denoisers. The AMP algorithm, contemporaneous with Plug-and-Play ADMM, faces uncertainties when f(x)f(x) deviates from quadratic or when the matrix AA is not random. In contrast, the paper positions Plug-and-Play ADMM as a more robust and versatile solution. Furthermore, a comparative analysis with algorithms such as Block-Matching 3D (BM3D) establishes the superior generality of Plug-and-Play ADMM. The conclusion drawn is that Plug-and-Play ADMM outperforms the other algorithms discussed in the paper.

In this paper, one of the central challenges addressed pertains to the convergence of the algorithm, specifically focusing on the denoising algorithm represented by V^(k+1) = Dθ(V^k). In the conventional ADMM framework, the requirement for the prior function g to be closed, proper, and convex is well-established to ensure the algorithm's guaranteed convergence. However, when employing general image denoising algorithms achieving convergence becomes a complex and elusive task. To overcome this obstacle, the paper introduces a algorithm that extends the convergence guarantees to a broader class of denoisers. The novel approach involves a continuation scheme, demonstrating that for any denoising algorithm satisfying the asymptotic criteria termed as bounded denoisers, the Plug-and-Play ADMM converges to a fixed point.