

# Reinforcement Learning 1



한양대학교 ERICA  
소프트웨어융합대학  
COLLEGE OF COMPUTING

인공지능학과  
Department of  
Artificial Intelligence

정 우 환 (whjung@hanyang.ac.kr)

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# Supervised Learning

**Data:**  $(x, y)$

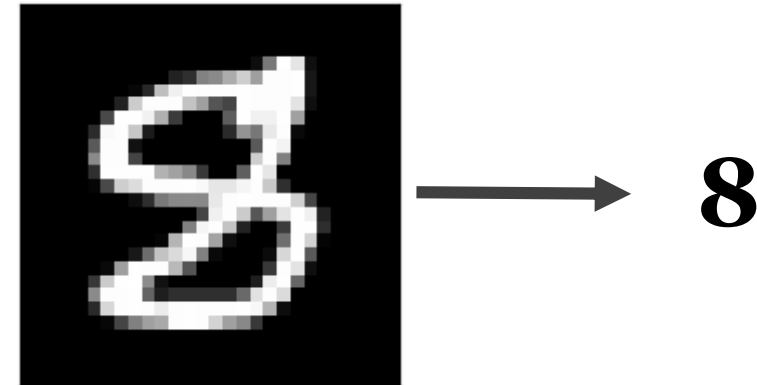
$x$  is data,  $y$  is label

**Goal:** Learn a function to map

$x \rightarrow y$

**Examples:**

Classification, regression, ...



# Unsupervised Learning

**Data:**  $x$

Just data, no labels!

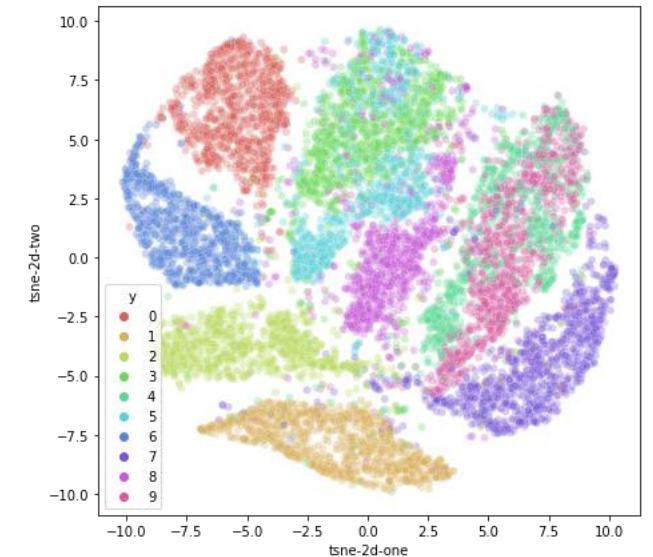
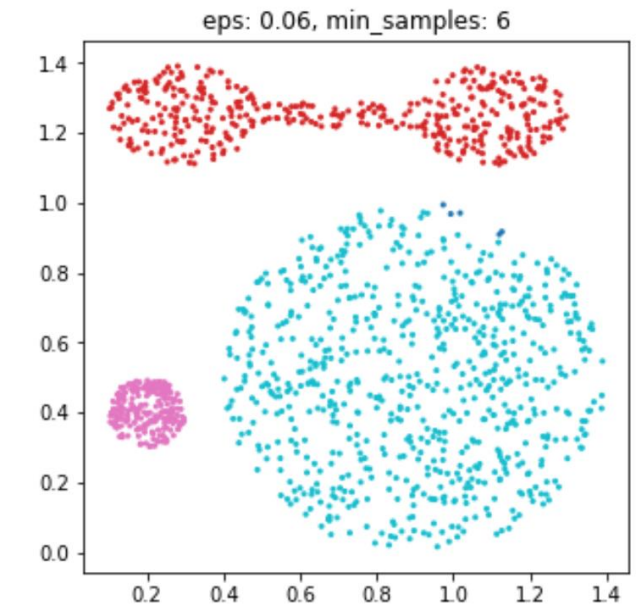
**Goal:** Learn some underlying hidden structure of the data

**Examples**

Clustering,

Dimensionality reduction,

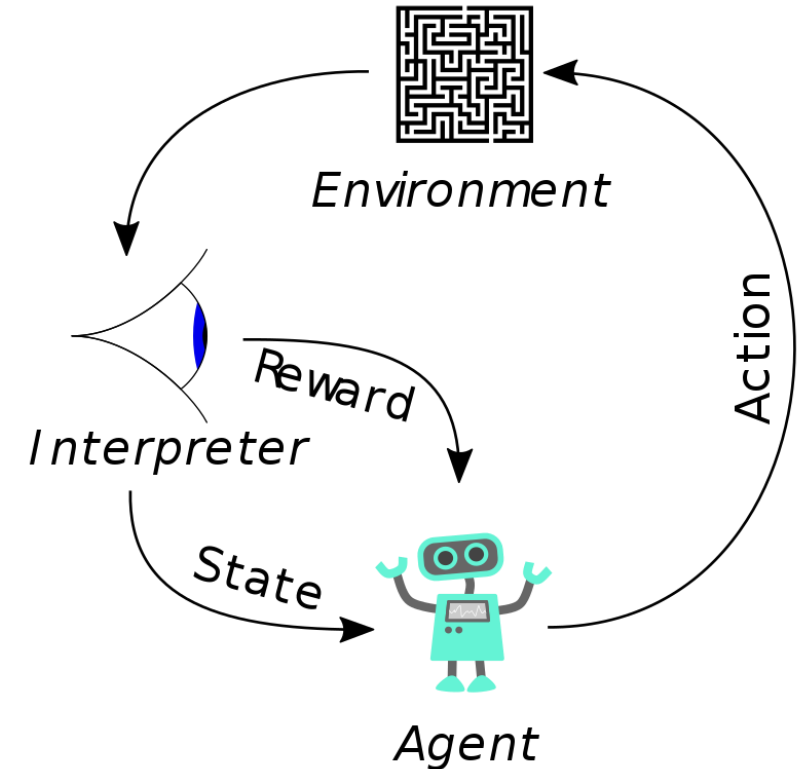
...



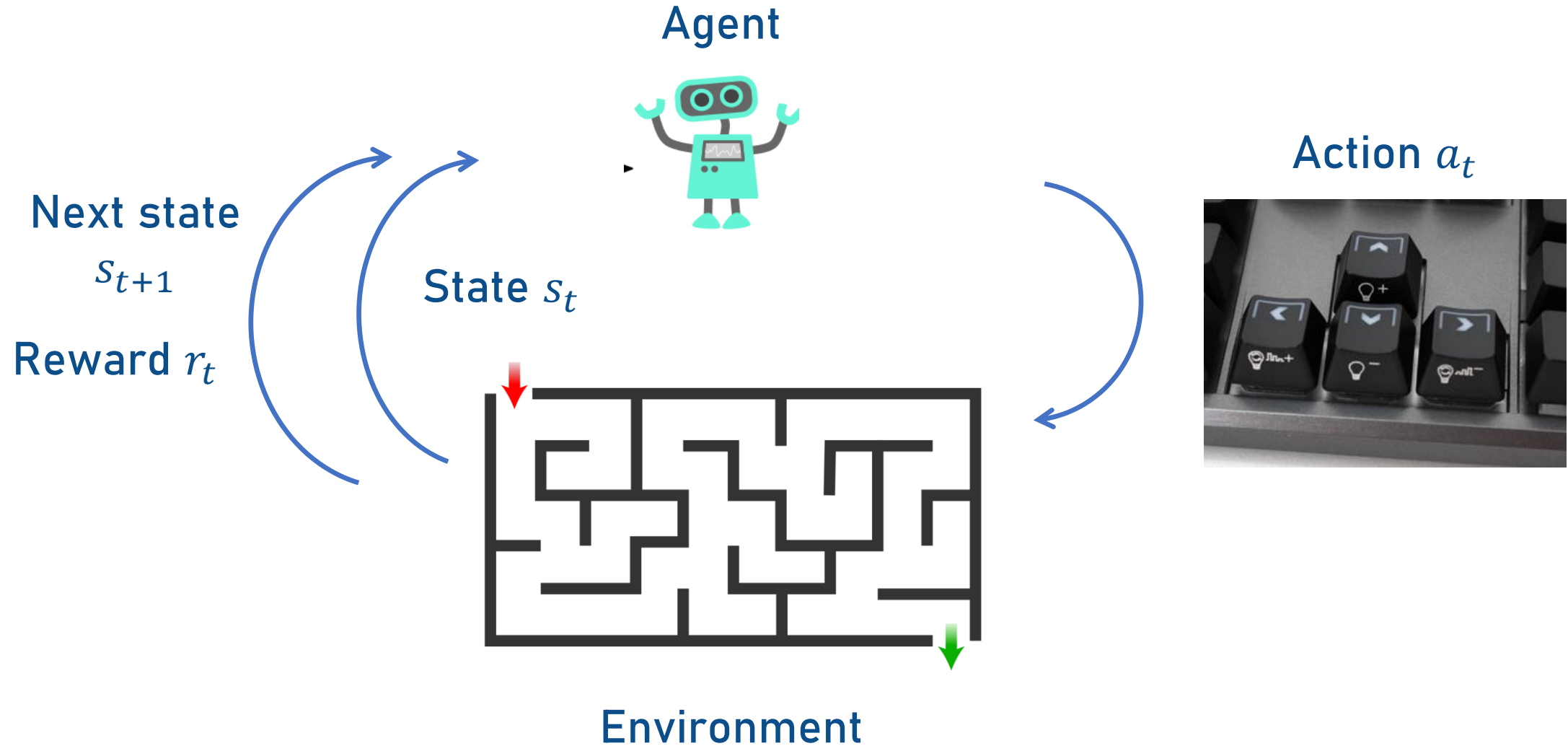
# Reinforcement Learning

Problems involving an **agent** interacting with an **environment**, which provides numeric **reward** signals

**Goal:** Learn how to take actions in order to **maximize reward**



# Reinforcement Learning



# Properties of RL

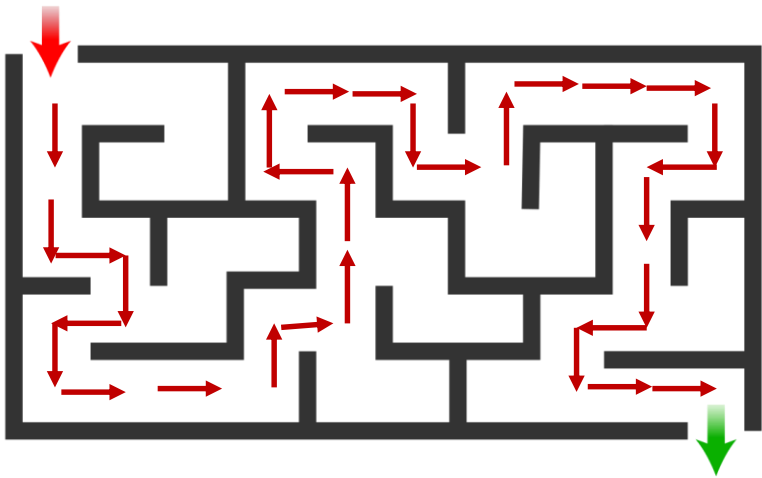
- Trial and Error (Agent chooses training data)
  - 행동 -> 보상
  - 보상 -> 행동의 수정
  - 행동 -> 보상
  - 보상 -> 행동의 수정
  - ...
- Sparse/delayed reward signal

<https://youtu.be/V1eYniJ0Rnk>

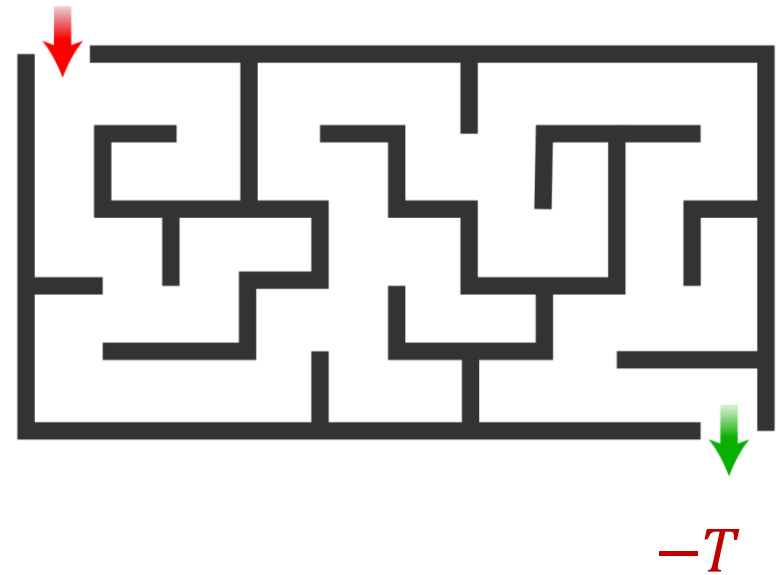


# Sparse signal

**Labels** in supervised learning



**Rewards** in reinforcement learning



# Reward design tip

- Example) (Reward) = -(걸린 시간)

$$a_1 \quad r_1 = 0$$

$$a_2 \quad r_2 = 0$$

$$a_3 \quad r_3 = 0$$

$$\vdots$$

$$a_T \quad r_T = -T$$

$$a_1 \quad r_1 = -1$$

$$a_2 \quad r_2 = -1$$

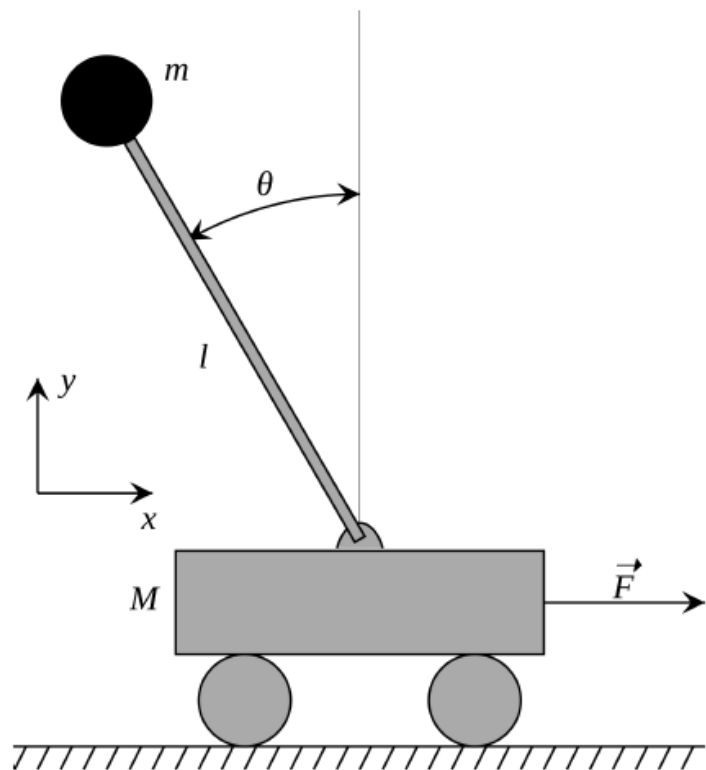
$$a_3 \quad r_3 = -1$$

$$\vdots$$

$$a_T \quad r_T = -1$$



# Cart-Pole Problem



**Objective:** Balance a pole on top of a movable cart

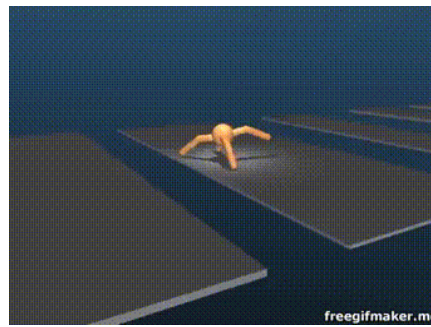
**State:** angle, angular speed, position, horizontal velocity

**Action:** horizontal force applied on the cart

**Reward:** 1 at each time step if the pole is upright

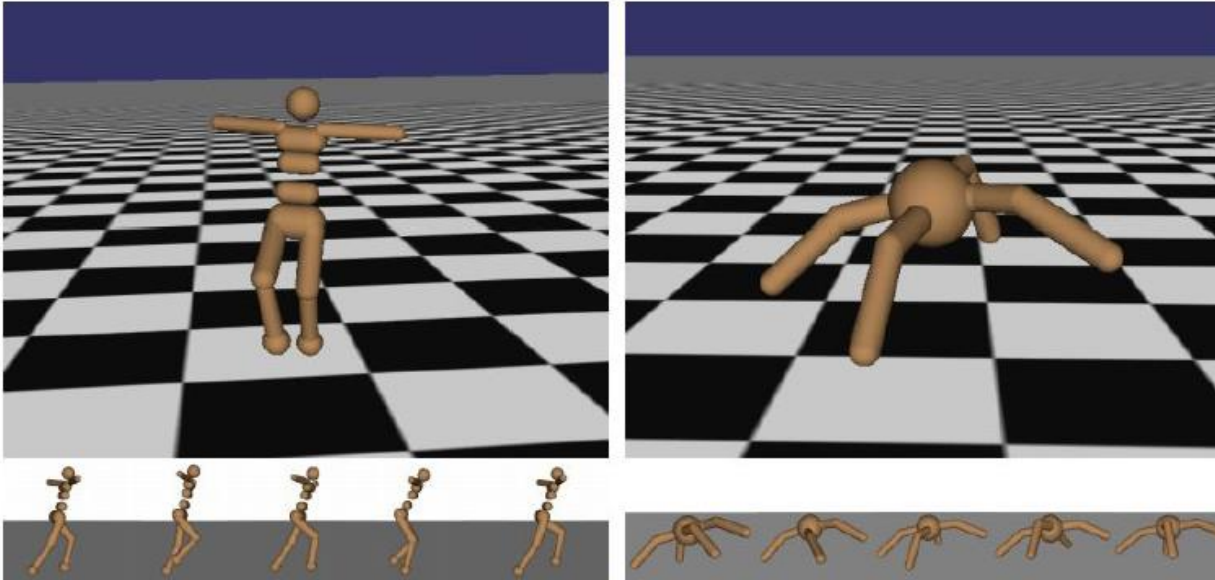
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# Robot Locomotion



[https://youtu.be/hx\\_bgoTF7bs](https://youtu.be/hx_bgoTF7bs)

2017 paper <https://arxiv.org/pdf/1707.02286.pdf>



**Objective:** Make the robot move forward

**State:** Angle and position of the joints

**Action:** Torques applied on joints

**Reward:** 1 at each time step upright + forward movement

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# Atari Games



**Objective:** Complete the game with the highest score

**State:** Raw pixel inputs of the game state

**Action:** Game controls e.g. Left, Right, Up, Down

**Reward:** Score increase/decrease at each time step

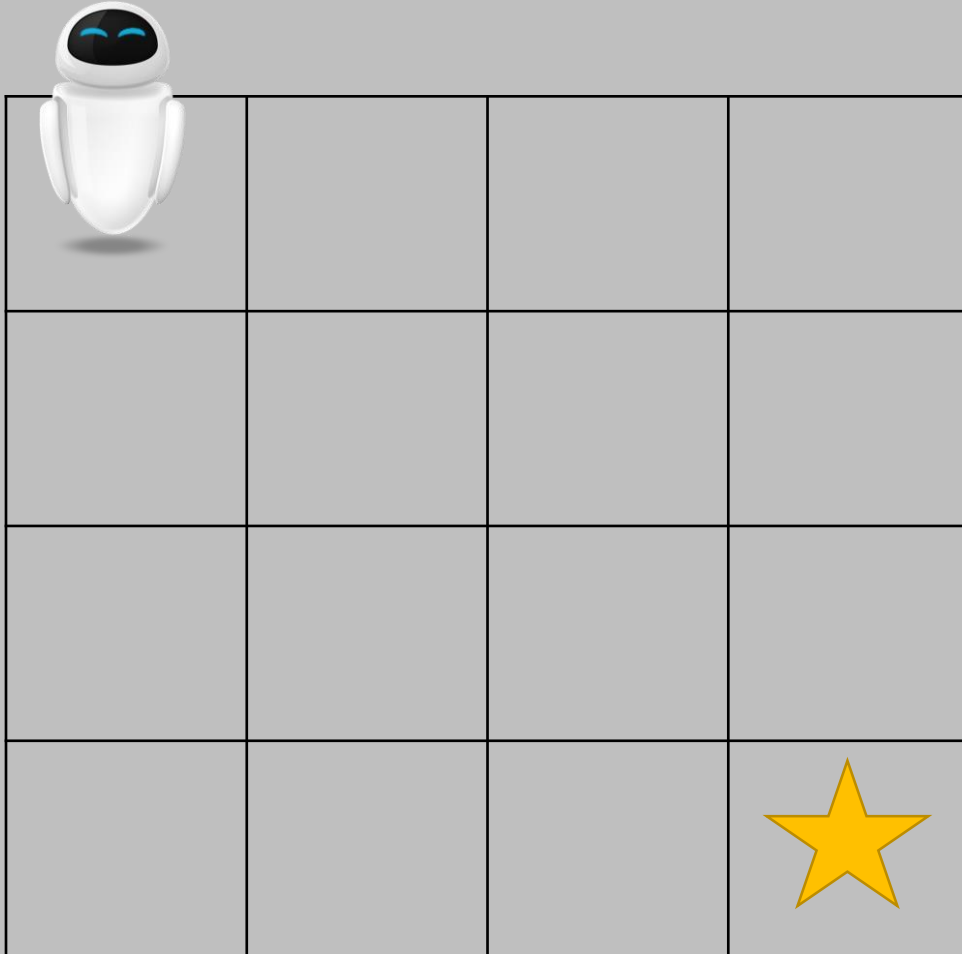
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# Q-learning

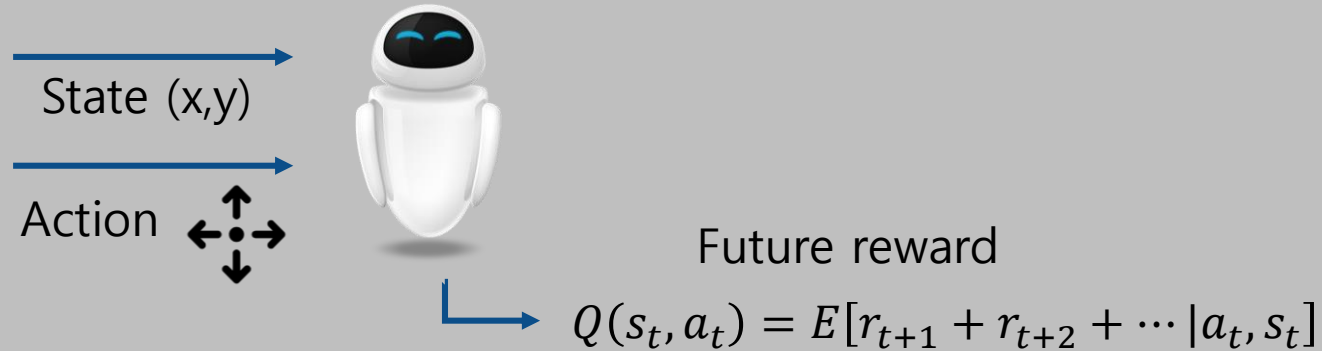
# Value functions

- Total reward  $R_t = r_{t+1} + r_{t+2} + \dots$
- Value functions measure the expected total reward after  $t$ 
  - Value of a **state**  $V(s)$ 
    - $V(s) = E[r_{t+1} + r_{t+2} + \dots | s_t = s]$
  - Value of **of taking an action in a state**  $Q(s, a)$ 
    - $Q(s, a) = E[r_{t+1} + r_{t+2} + \dots | a_t = a, s_t = s]$
- Policy  $\pi$ : mapping from **state** to **action**
  - Optimal policy  $\pi^*(s) = \arg \max_a Q(s, a)$

# Q-Learning



Q-Function (State-action value)       $Q(\text{state}, \text{action})$



$$Q((1,1), LEFT) = 0.0$$

$$Q((1,1), RIGHT) = 0.5$$

$$Q((1,1), UP) = 0.0$$

$$Q((1,1), DOWN) = 0.3$$

$$Q((3,4), LEFT) = 0.0$$

$$Q((3,4), RIGHT) = 0.0$$

$$Q((3,4), UP) = 0.0$$

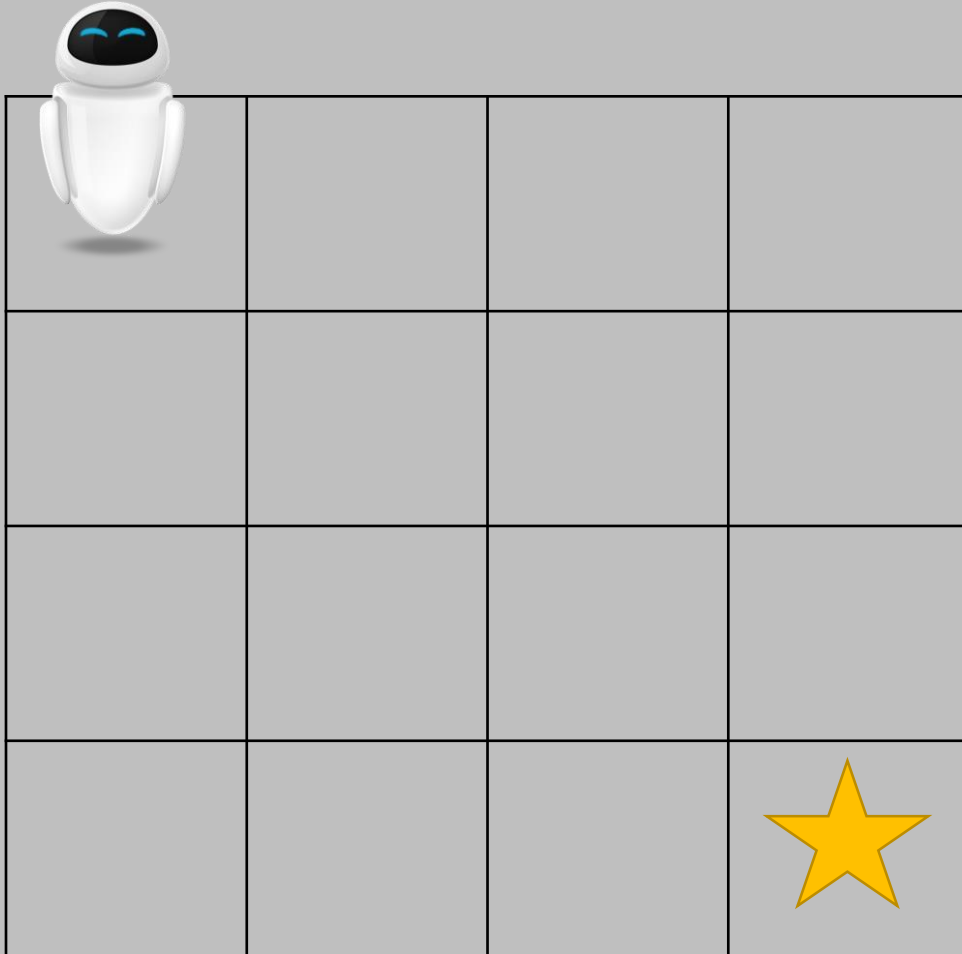
$$Q((3,4), DOWN) = 1.0$$

Optimal policy       $\pi^*(s) = \arg \max_a Q(s, a)$

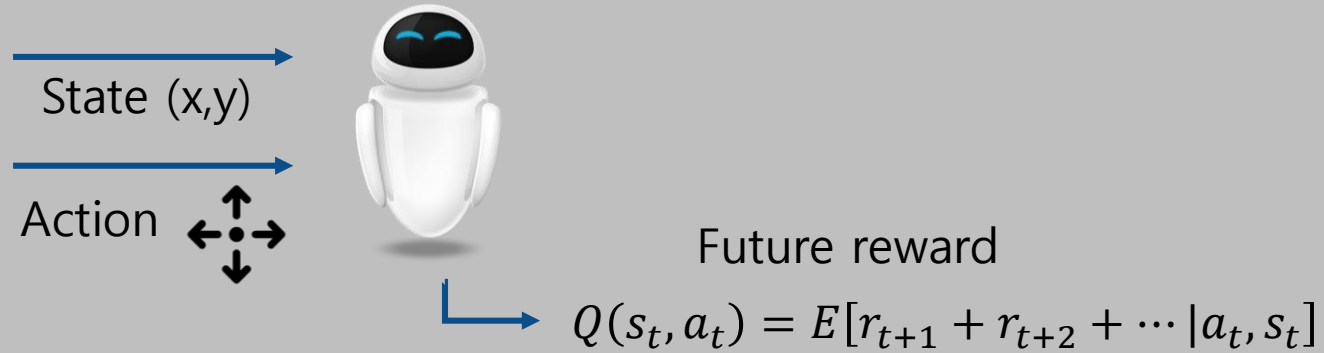
$$\pi * ((1,1)) \rightarrow RIGHT$$

$$\pi * ((3,4)) \rightarrow DOWN$$

# Q-Learning

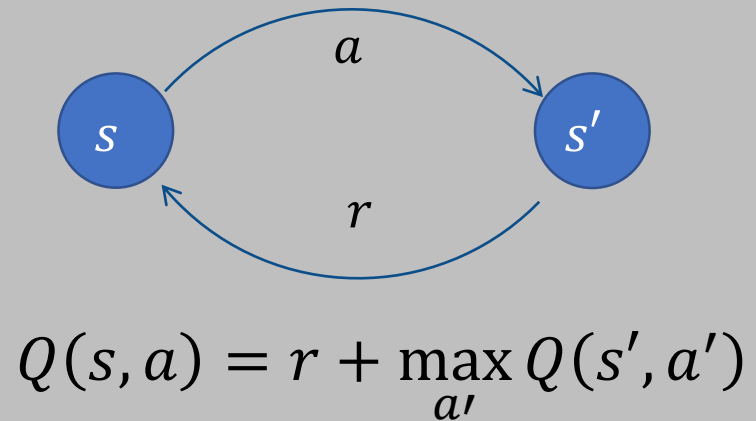


Q-Function (State-action value)       $Q(\text{state}, \text{action})$



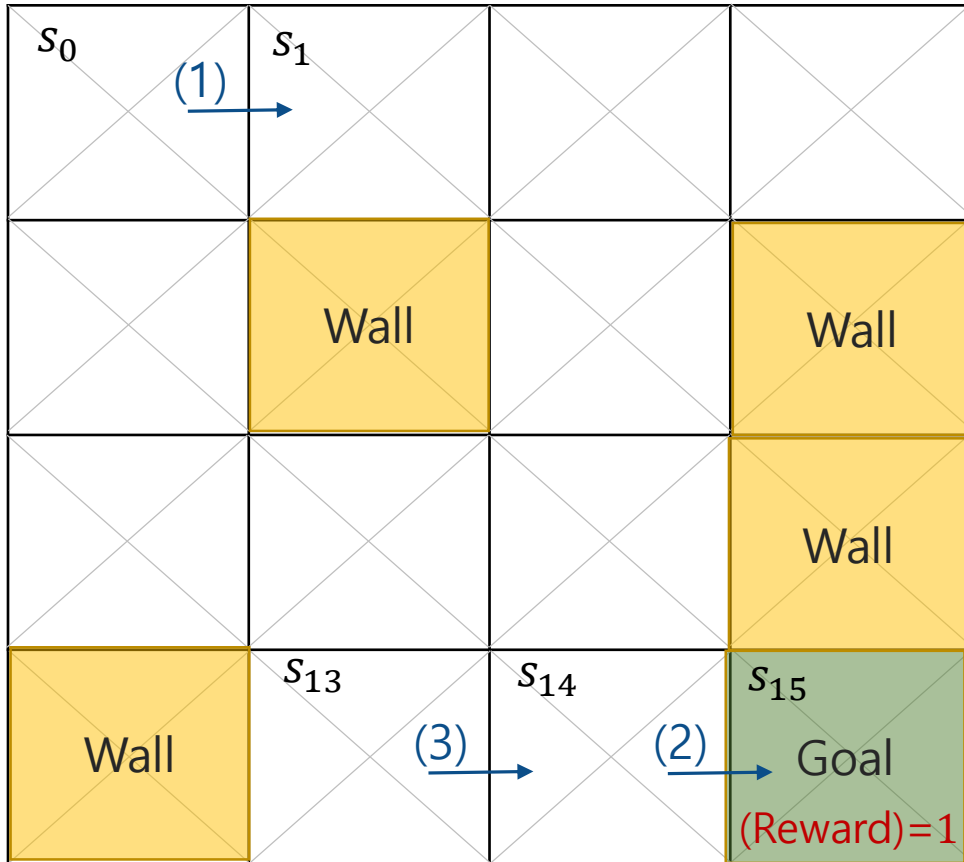
Optimal policy       $\pi^*(s) = \arg \max_a Q(s, a)$

Recurrence equation



# Q-Learning

16 states and 4 actions (U, D, L, R)



$$Q(s, a) = r + \max_{a'} Q(s', a')$$

- Initial status
  - $Q(s, a) = 0$  for all  $s, a$
  - Reward are all zero except in  $s_{15}$

Case (1) →

$$Q(s_0, R) = r + \max_{a'} Q(s_1, a') = 0 + \max_{a'} \{0, 0, 0, 0\} = 0$$

Case (2) →

$$Q(s_{14}, R) = 1 + \max_{a'} Q(s_{15}, a') = 0 + \max_{a'} \{0, 0, 0, 0\} = 1$$

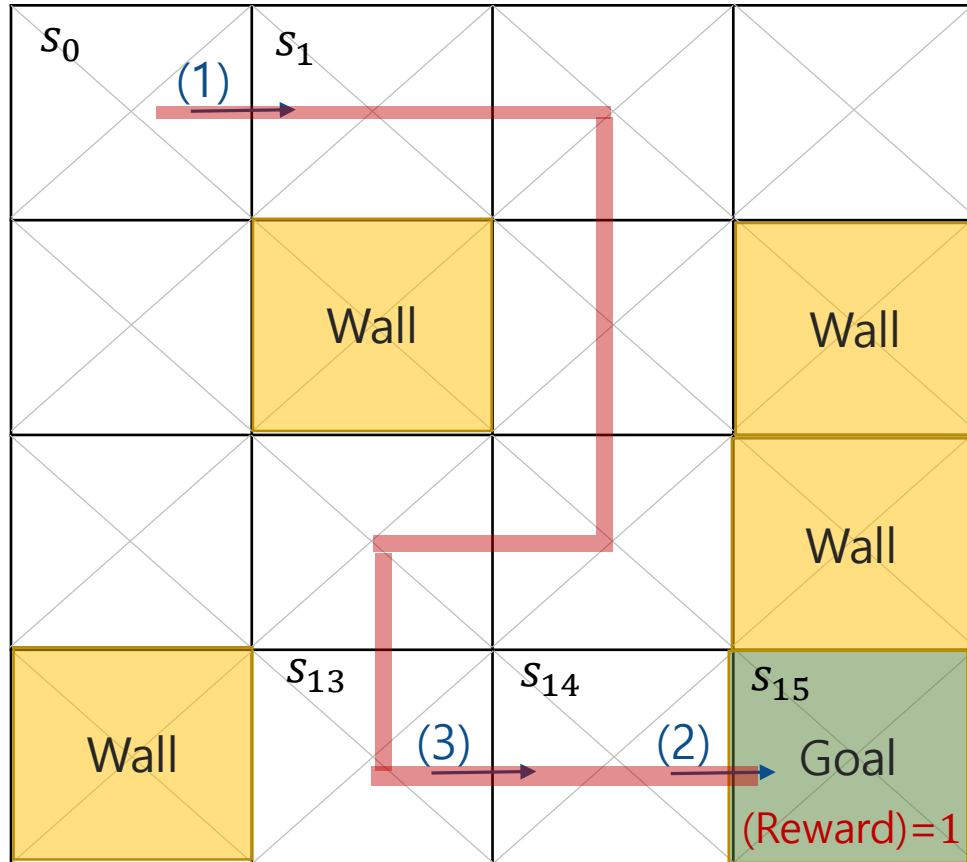
Case (3) →

$$Q(s_{13}, R) = r + \max_{a'} Q(s_{14}, a') = 0 + \max_{a'} \{0, 0, 1, 0\} = 1$$



# Q-Learning

16 states and 4 actions (U, D, L, R)



$$Q(s, a) = r + \max_{a'} Q(s', a')$$

- Initial status
  - $Q(s, a) = 0$  for all  $s, a$
  - Reward are all zero except in  $s_{15}$

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$$Q(s_0, R) = r + \max_{a'} Q(s_1, a') = 0 + \max_{a'} \{0, 0, 0, 0\} = 0$$

Case (2) →

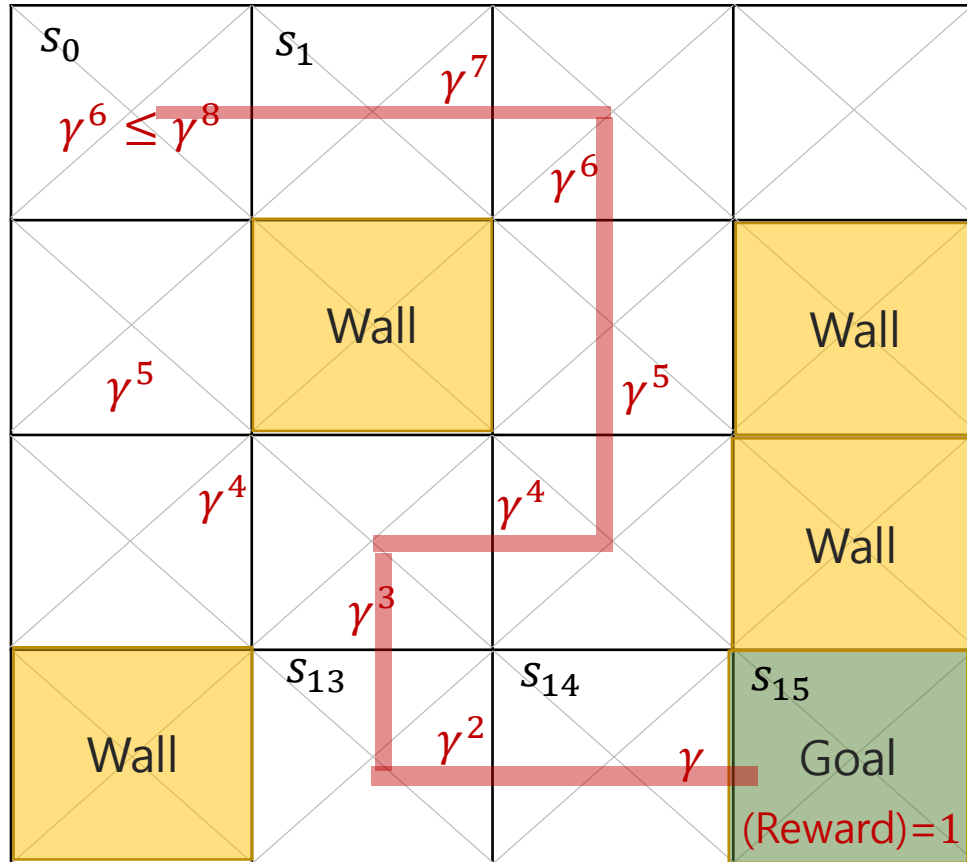
$$Q(s_{14}, R) = 1 + \max_{a'} Q(s_{15}, a') = 0 + \max_{a'} \{0, 0, 0, 0\} = 1$$

Case (3) →

$$Q(s_{13}, R) = r + \max_{a'} Q(s_{14}, a') = 0 + \max_{a'} \{0, 0, 1, 0\} = 1$$

# Q-Learning: Discounted reward

16 states and 4 actions (U, D, L, R)



$$Q(s, a) = r + \gamma \max_{a'} Q(s', a')$$

$$Q(s, a) \leftarrow r + \gamma \max_{a'} Q(s', a')$$

# Q-Learning: Temporal Difference

- Iterative update

$$Q^{new}(s_t, a_t) \leftarrow \underbrace{Q(s_t, a_t)}_{\text{old value}} + \underbrace{\alpha}_{\text{learning rate}} \cdot \left( \underbrace{\underbrace{r_t}_{\text{reward}} + \underbrace{\gamma}_{\text{discount factor}} \cdot \underbrace{\max_a Q(s_{t+1}, a)}_{\text{estimate of optimal future value}}}_{\text{new value (temporal difference target)}} - \underbrace{Q(s_t, a_t)}_{\text{old value}} \right)$$

temporal difference

# Q-learning

- For each  $s, a$ , initialize table entry  $Q(s, a) \leftarrow 0$
- Do until  $Q$  converges
  - Initialize  $s$
  - Do until  $s$  *is terminal*
    - Select an action  $a$  using policy  $\pi$  derived from  $Q$
    - Take action  $a$
    - Receive immediate reward  $r$
    - Observe the new state  $s'$
    - $Q(s, a) \leftarrow Q(s, a) + \alpha \left[ r + \gamma \max_{a'} Q(s', a') - Q(s, a) \right]$
    - $s \leftarrow s'$

# Policy

- Testing phase
  - Optimal policy  $\pi^*(s) = \arg \max_a Q(s, a)$
- Training phase (Exploration-Exploitation)
  - Exploration allows an agent to improve its current knowledge
  - Exploitation chooses the greedy action to get the most reward by exploiting the agent's current action-value estimates

# Exploration-Exploitation

- Restaurant Selection
  - **Exploitation** Go to your favorite restaurant
  - **Exploration** Try a new restaurant
- Game Playing
  - **Exploitation** Play the move you believe is best (META)
  - **Exploration** Play an experimental move



# Policy

- Testing phase
  - Optimal policy  $\pi^*(s) = \arg \max_a Q(s, a)$
- Training phase ( $\epsilon$ -greedy)
  - Exploration-Exploitation
  - Exploration allows an agent to improve its current knowledge
  - Exploitation chooses the greedy action to get the most reward by exploiting the agent's current action-value estimates

Action at time(t)  $\left\{ \begin{array}{ll} \max Q_t(a) & \text{with probability } 1-\epsilon \\ \text{any action (a)} & \text{with probability } \epsilon \end{array} \right.$

# Q-learning

- For each  $s, a$ , initialize table entry  $Q(s, a) \leftarrow 0$
- Do until  $Q$  converges
  - Initialize  $s$
  - Do until  $s$  is terminal
    - Draw a random value  $v \sim \text{Uniform}(0,1)$
    - If  $v < \epsilon$ 
      - Randomly select  $a$
    - Else:
      - $a = \underset{a'}{\operatorname{argmax}} Q(s, a')$
    - Take action  $a$
    - Receive immediate reward  $r$
    - Observe the new state  $s'$
    - $Q(s, a) \leftarrow Q(s, a) + \alpha \left[ r + \gamma \max_{a'} Q(s', a') - Q(s, a) \right]$
    - $s \leftarrow s'$

Testing phase  
 $\pi^*(s) = \arg \max_a Q(s, a)$



# References

- Fei-Fei Li & Justin Johnson & Serena Yeung, CS231n Lecture 14: Reinforcement Learning, Stanford University
- <https://sumniya.tistory.com/> **숨니의 무작정 따라하기**
- <https://youtu.be/m1FC3dMmY78> Joongheon Kim, Korea University