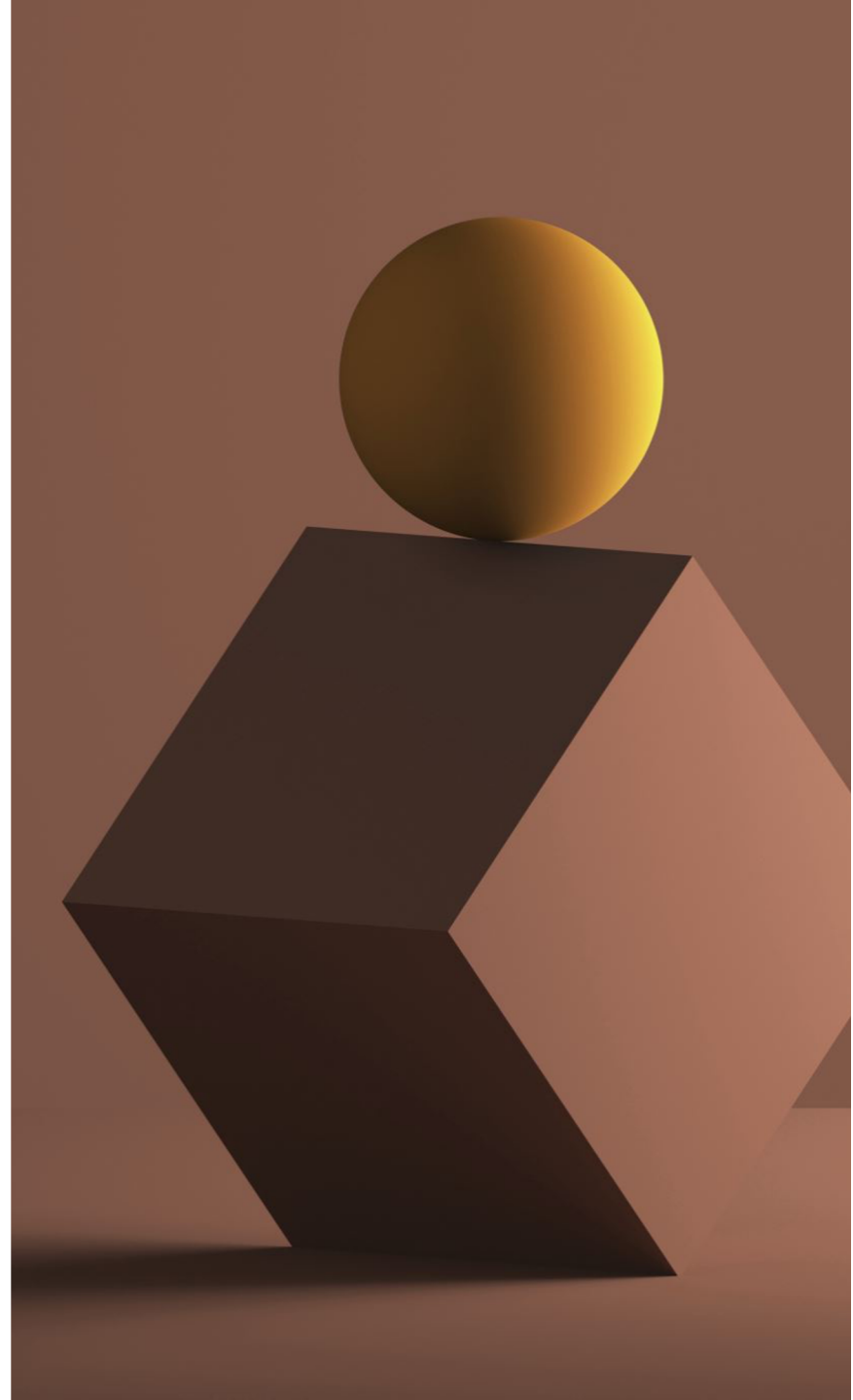
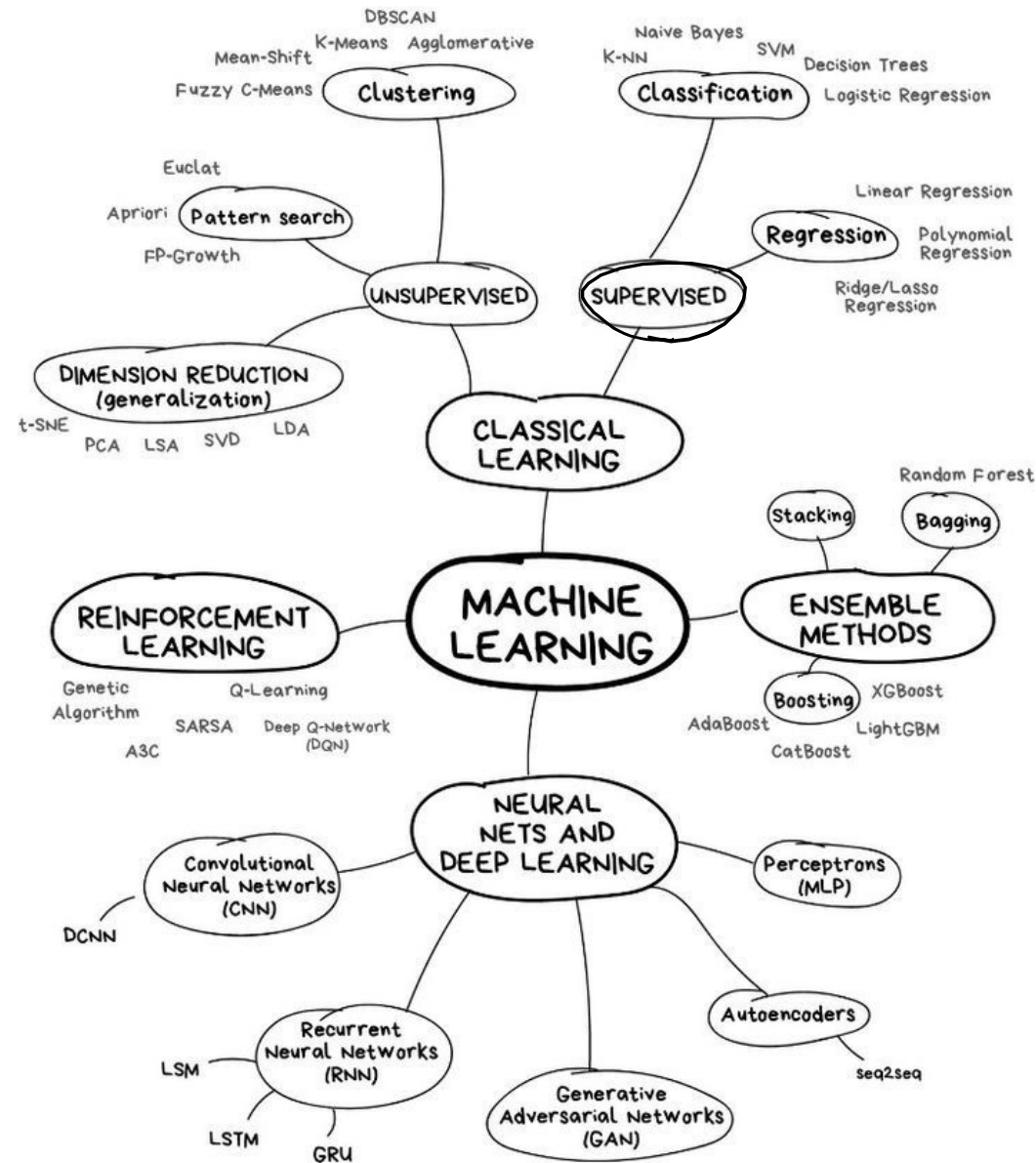


Basic models (2)

Artificial Intelligence

Woohwan Jung





	Linear Regression	Logistic Regression
Problem	Regression	Classification
Model	$\hat{y} = \mathbf{w}^\top \mathbf{x} + b$ Parameters: $\mathbf{w} \in \mathbb{R}^n, b \in \mathbb{R}$	$\hat{y} = \sigma(\mathbf{w}^\top \mathbf{x} + b)$ Parameters: $\mathbf{w} \in \mathbb{R}^n, b \in \mathbb{R}$
Loss	Squared Error $L(y, \hat{y}) = (y - \hat{y})^2$	Binary Cross Entropy (BCE) $L(y, \hat{y}) = -y \log \hat{y} - (1 - y) \log(1 - \hat{y})$

Cost function: $J(\mathbf{w}, b) = \frac{1}{m} \sum_{i=1}^m L(\hat{y}^{(i)}, y^{(i)})$

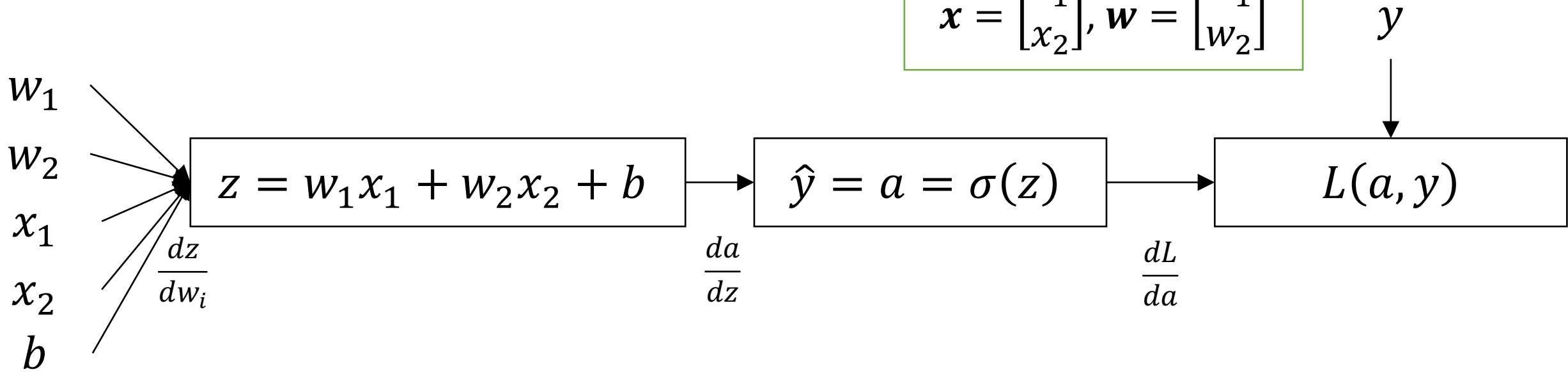
Gradient Descent :Linear Regression

$$L(a, y) = -y \log a - (1 - y) \log(1 - a)$$

Logistic Regression Recap

For the simplicity

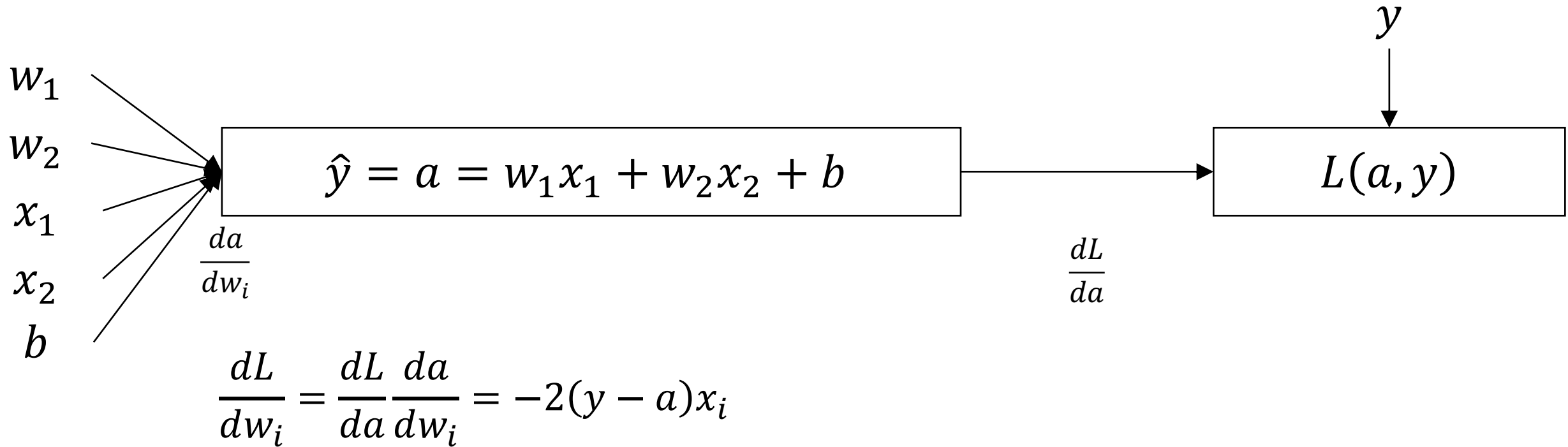
$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$$



$$\frac{dL}{dw_i} = \frac{dL}{da} \frac{da}{dz} \frac{dz}{dw_i}$$

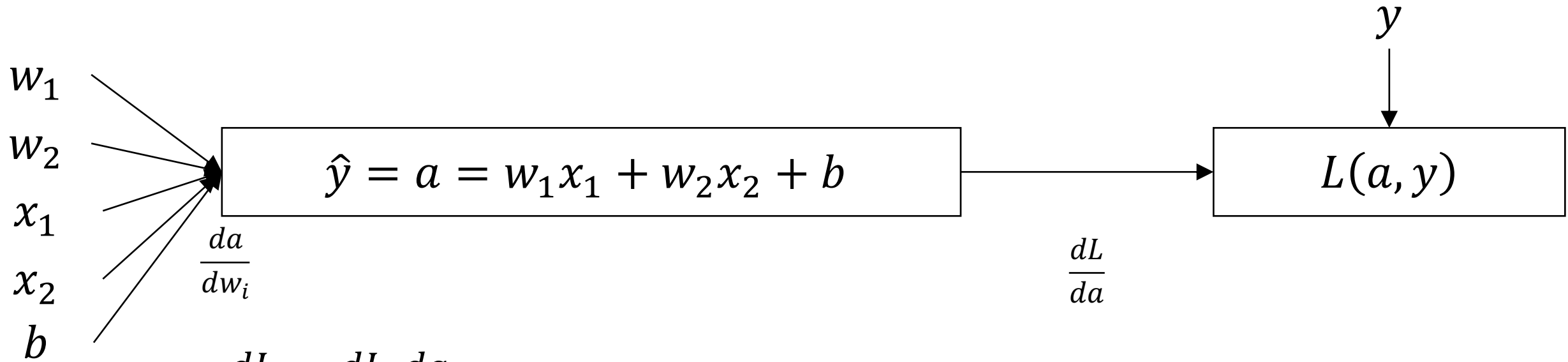
$$L(a, y) = (y - a)^2$$

Linear Regression Recap



$$L(a, y) = (y - a)^2$$

Linear Regression Recap



$$\frac{dL}{dw_i} = \frac{dL}{da} \frac{da}{dw_i} = -2(y - a)x_i$$

$$\frac{dL}{db} = \frac{dL}{da} \frac{da}{db} = -2(y - a)$$

Gradient descent on m examples

$$J(\mathbf{w}, b) = \frac{1}{m} \sum_{i=1}^m L(\hat{y}^{(i)}, y^{(i)})$$

$$\frac{d}{dw_k} J(\mathbf{w}, b) = \frac{1}{m} \sum_{i=1}^m \frac{d}{dw_k} L(\hat{y}^{(i)}, y^{(i)}) = \frac{2}{m} \sum_{i=1}^m (a - y) x_k$$

$$\frac{d}{db} J(\mathbf{w}, b) = \frac{1}{m} \sum_{i=1}^m \frac{d}{db} L(\hat{y}^{(i)}, y^{(i)}) = \frac{2}{m} \sum_{i=1}^m (a - y)$$

Gradient descent for training a Linear Regression model

- Randomly Initialize w, b
- $lr = 0.1$
- For $e = 1$ to n_{epoch} :
 - $J = 0; d_w1 = 0; d_w2 = 0; d_b = 0$
 - For $i = 1$ to m :
 - $z = w_1 x_1^{(i)} + w_2 x_2^{(i)} + b$
 - $a = \sigma(z)$
 - $d_w1 += 2(a - y)x_1^{(i)}$
 - $d_w2 += 2(a - y)x_2^{(i)}$
 - $d_b += 2(a - y)$
 - $w_1 -= lr * d_w1/m$
 - $w_2 -= lr * d_w2/m$
 - $b -= lr * d_b/m$

$$\frac{dL}{dw_k} = 2(a - y)x_k$$
$$\frac{dL}{db} = 2(a - y)$$