

# Lecture 14: Wage Differentials & Income Distribution

## Economía Laboral

Junghanss, Juan Cruz

Universidad del CEMA

2nd Semester 2023

Today's lecture content:

- Compensating Wage Differentials (Diferencias Igualadoras)
- Ejercicio TP N°4
- Income Distribution (Distribución del Ingreso)
- Ejercicio en R para TP N°4

# Compensating Wage Differentials

Intuition: free entry and exit of workers and firms in a competitive labor market leads to a single wage equilibrium *as long as all jobs are alike and all workers are alike* (ceteris paribus).

# Compensating Wage Differentials

Intuition: free entry and exit of workers and firms in a competitive labor market leads to a single wage equilibrium *as long as all jobs are alike and all workers are alike* (ceteris paribus).

As we all know, workers and jobs are different, so the real-world labor market is not characterized by a single wage.

- Workers differ in their skills.
- Jobs differ in the amenities they offer.

# Compensating Wage Differentials

Because workers care about the job's location, tasks, etc. we should think of a job offer not just in terms of how much money the job pays, but in terms of the entire job package that includes both *wages* and *working conditions*.

# Compensating Wage Differentials

Because workers care about the job's location, tasks, etc. we should think of a job offer not just in terms of how much money the job pays, but in terms of the entire job package that includes both *wages* and *working conditions*.

**“Compensating wage differentials”** arise to compensate workers for the nonwage characteristics of jobs. For example:

- Firms that have unpleasant working conditions must offer some offsetting advantages (such as a higher wage) to attract workers;
- Firms that offer pleasant working conditions can get away with paying lower wages (in effect, making workers pay for the pleasant environment).

# Compensating Wage Differentials: risky jobs

Consider a very simple example: **a market for risky jobs**.

Assumptions:

- There are only two types of jobs, and they are differentiated by the probability of getting injured on the job, denoted by  $\rho$ .
- Suppose the worker has complete information about the risk associated with every job, i.e. the worker knows if the job is “safe” ( $\rho = 0$ ) or “risky” ( $\rho = 1$ ).
- Workers care about the wage ( $w$ ) they earn on the job and also care about whether they will get hurt. Hence, the utility function is  $U(w, \rho)$ .

# Compensating Wage Differentials: risky jobs

In this case;

- the marginal utility of income  $U_w$  gives the change in utility resulting from a \$1 increase in the worker's income, holding constant the probability of injury. It's *positive*.
- the marginal utility of risk  $U_\rho$  gives the change in utility resulting from a one-unit change in the probability of injury, holding constant the worker's income. It's *negative*.



# Compensating Wage Differentials: risky jobs

Furthermore, we can define a worker's “**reservation price**” as the amount of money it would take to bribe him into accepting the risky job, i.e. the difference of both wages  $\Delta w = w_1 - w_0$ .

Intuition: the reservation price, therefore, is the answer to the age-old question: How much would it take for you to do something that you would rather not do?

# Compensating Wage Differentials: risky jobs

Furthermore, we can define a worker's “**reservation price**” as the amount of money it would take to bribe him into accepting the risky job, i.e. the difference of both wages  $\Delta w = w_1 - w_0$ .

Intuition: the reservation price, therefore, is the answer to the age-old question: How much would it take for you to do something that you would rather not do?

Different workers have different attitudes toward risk. Depending on how we draw a worker's indifference curves, the reservation price  $\Delta w$  could be a small number or a large number.

# Compensating Wage Differentials: Labor Supply

The labor supply curve will tell us how many workers are willing to offer their services to the risky job as a function of the wage differential between the risky job and the safe job.

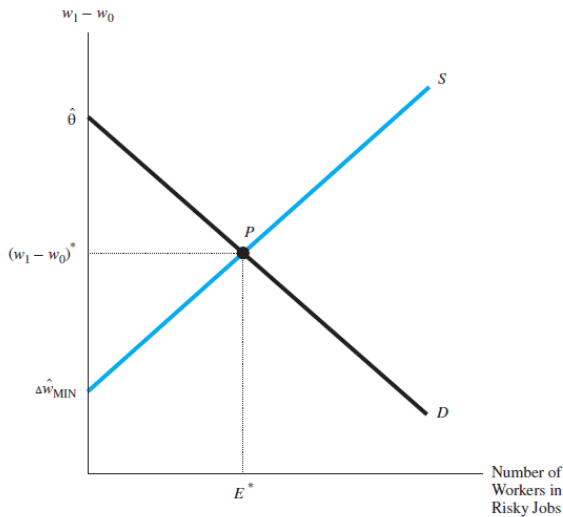
# Compensating Wage Differentials: Labor Supply

The labor supply curve will tell us how many workers are willing to offer their services to the risky job as a function of the wage differential between the risky job and the safe job.

Because we assumed that all workers dislike risk, no worker would be willing to work at the risky job when the wage differential is zero (or negative).

As the wage differential rises, there will come a point where the worker who dislikes risk the least is “bought off ” and decides to work in the risky job.

# Compensating Wage Differentials: Labor Supply



# Compensating Wage Differentials: Labor Demand

A firm must decide whether to provide a safe or risky work environment to its workers. The firm's choice depends on which is more profitable. We can illustrate it as follows:

- If the firm offers a safe environment, the production function is:  
 $q_0 = \alpha_0 \cdot L^*$
- If the firm offers a risky environment, the production function is:  
 $q_1 = \alpha_1 \cdot L^*$

where  $\alpha_i$  is the marginal product of labor in a safe environment ( $\alpha_0$ ) and a risky one ( $\alpha_1$ ).

# Compensating Wage Differentials: Labor Demand

We now must address a crucial question: How does the marginal product of labor differ between safe and risky environments? Safety is not cheap.

The firm must allocate labor and capital to produce a safe environment, diverting resources from the production of output.

# Compensating Wage Differentials: Labor Demand

We now must address a crucial question: How does the marginal product of labor differ between safe and risky environments? Safety is not cheap.

The firm must allocate labor and capital to produce a safe environment, diverting resources from the production of output.

This diversion of resources suggests that the marginal product of labor is higher in a risky environment, so that  $\alpha_1 > \alpha_0$ . Note that if the marginal product of labor were higher in safe firms, we would never observe any risky firms. After all, not only would workers be more productive in safe firms, but the firm could get away with paying them lower wages because workers value safety.



# Compensating Wage Differentials: Labor Demand

The firm's profits equal the difference between the firm's revenues ( $p \cdot q_i$ ) and the firm's costs ( $w \cdot L$ ). Profits depend on whether the firm offers a safe or a risky environment. The profits under each of the possibilities are given by:

$$\pi_0 = p \cdot \alpha_0 L^* - w_0 L^*$$

$$\pi_1 = p \cdot \alpha_1 L^* - w_1 L^*$$

where  $\pi_0$  gives the firm's profits if it chooses to be a safe firm, and  $\pi_1$  gives the profits if it chooses to be a risky firm. Both revenues and costs are affected by the firm's decision.

# Compensating Wage Differentials: Labor Demand

A profit-maximizing firm offers a risky environment if  $\pi_1 > \pi_0$ . Define  $\theta = p\alpha_1 - p\alpha_0$  as the per-worker dollar gain (that is, the difference in the value of marginal product) when the firm switches from a safe to a risky environment. Algebraic manipulations of equations show that the firm's decision rule is:

- Offer a safe work environment if  $w_1 - w_0 > \theta$
- Offer a risky work environment if  $w_1 - w_0 < \theta$

Different firms have different technologies for producing safety—implying that the parameter  $\theta$  differs across firms.

# Compensating Wage Differentials: Labor Demand

If the wage gap between risky and safe firms is very large, no firm would choose to become a risky firm and the demand for risky workers is zero. As the wage differential falls, there will come a point where the firm that has the most to gain by becoming a risky firm decides that it is worth incurring the additional labor cost.

# Compensating Wage Differentials

Question: what would happen if we drop the assumption of risk aversion?  
Some workers may prefer to work in jobs where they face a high probability of injury.

## Modelo de Diferencias Igualadoras: elección en el mercado de seguridad

Para trabajar de policía un trabajador tiene que estar dispuesto a utilizar la violencia como método de prevención y disuasión del delito. Asuma que una persona puede elegir entre ser policía y otra profesión, y que su función de utilidad está dada por

$$U_i(w_V, V) = w_V - \rho_i \cdot V$$

donde  $V$  es un indicador del nivel de violencia en el empleo y es igual a 1 en el trabajo de policía y 0 en otras profesiones, y  $w_V$  es el salario en el empleo con nivel de violencia  $V$  (i.e. el salario del policía es  $w_1$  y el salario en otras profesiones es  $w_0$ ). Asuma que  $\rho_i$  toma valores enteros entre -1 y 8 y se encuentra distribuido en la población de manera uniforme (e.g. para 10% de la población  $\rho_i$  es igual a -1, para 10% de la población  $\rho_i$  es igual a 0, etc.), y que hay 1000 trabajadores.

## a) Calcule la oferta de trabajo individual para el empleo de policía

Presentamos el problema de optimización considerando una restricción de a tramos (piecewise-defined function):

$$\begin{aligned} \max_{V \in \{0;1\}} \quad & U(w_V, V) = w_V - \rho_i \cdot V \\ \text{s.a.} \quad & w_V = w_0 \quad \text{si } V = 0 \\ & w_V = w_1 \quad \text{si } V = 1 \end{aligned}$$

# Compensating Wage Differentials: TP Exercise

El resultado de este problema es dicotómico: el trabajador elegirá trabajar como policía ( $V = 1$ ) si y solo si hacerlo le reporta más utilidad que la otra profesión ( $U(w_1, 1) > U(w_0, 0)$ ), y sino ( $U(w_0, 0) > U(w_1, 1)$ ) ofrecerá trabajo en otras profesiones ( $V = 0$ ).

Consideremos el desarrollo en el pizarron y obtenemos:

$$V_i^* = \begin{cases} 0 & \text{si } \rho_i \geq w_1 - w_0 \\ 1 & \text{si } \rho_i < w_1 - w_0 \end{cases}$$

Esa es la oferta de trabajo del individuo representativo, que trabajará como policía ( $V = 1$ ) solamente si el mercado lo compensa ( $w_1 - w_0$ ) más que lo mínimo que pide como diferencial para aceptar ese trabajo violento ( $\rho_i$ ).

## b) Calcule la oferta de trabajo de mercado para el empleo de policía.

La diferencia igualadora,  $\rho$ , pertenece al intervalo  $[a, b] = [-1, 8]$  y se distribuye uniformemente entre la población, de modo que sigue una función de distribución uniforme, cuya función de distribución por definición es:

$$F(x) = \begin{cases} 0 & \text{si } x < a \\ \frac{x-a}{b-a} & \text{si } x \in [a, b] \\ 1 & \text{si } x > b \end{cases}$$



# Compensating Wage Differentials: TP Exercise

Reemplazando en nuestro caso,

$$F(\rho) = \begin{cases} 0 & \text{si } \rho < -1 \\ \frac{\rho+1}{9} & \text{si } \rho \in [-1, 8] \\ 1 & \text{si } \rho > 8 \end{cases}$$

# Compensating Wage Differentials: TP Exercise

Como vimos, los trabajadores que se ofrecerán para trabajar como policía serán aquellos con  $\rho < w_1 - w_0$ . Teniendo en cuenta esto último y que sabemos que hay 1000 trabajadores simétricos, la oferta de mercado para el empleo de policía será:

$$N_1^s(w_1 - w_0) = \begin{cases} 0 & \text{si } w_1 - w_0 < -1 \\ 1000 \cdot \frac{w_1 - w_0 + 1}{9} & \text{si } w_1 - w_0 \in [-1, 8] \\ 1000 & \text{si } w_1 - w_0 > 8 \end{cases}$$

# Compensating Wage Differentials: TP Exercise

**Asuma que la demanda de trabajo para el empleo de policía es completamente inelástica e igual a 300 trabajadores.**

**c) Calcule el diferencial salarial de equilibrio (i.e.  $w_1 - w_0$ ).**

Si tenemos que  $N_1^d = 300$  y la condición de equilibrio  $N_1^s = N_1^d$  entonces

$$1000 \cdot \frac{w_1 - w_0 + 1}{9} = 300$$

$$w_1 - w_0 = \frac{300 \cdot 9}{1000} - 1$$

$$(w_1 - w_0)^* = 1,7$$

**d) Calcule la media en la población del nivel de aversión a la violencia,  $\rho_i$ .**

Como  $\rho_i$  se distribuye uniformemente en la población, podemos calcular la media poblacional haciendo uso de la esperanza matemática para el caso de la distribución uniforme, la cual por definición es

$$E(x) = \frac{a + b}{2}$$

por lo que en este ejercicio:

$$E(\rho) = \frac{-1 + 8}{2} = 3,5$$

# Compensating Wage Differentials: TP Exercise

- e) **Calcule el nivel de aversión a la violencia promedio entre los trabajadores empleados de policía.**
- f) **Calcule el nivel de aversión a la violencia promedio entre los trabajadores empleados en otras profesiones.**
- g) **De acuerdo a este modelo, ¿Qué tipo de selección se dará en las fuerzas policiales? Discuta.**

*Presentar resolución en la entrega del TP.*

**h) Qué grupos de trabajadores en el empleo de policías se estarán llevando una renta (i.e. individuos que están dispuestos a aceptar el trabajo a un diferencial salarial menor).**

Como el diferencial salarial de equilibrio es  $w_1 - w_0 = 1.7$ , y como las diferencias compensadoras  $\rho_i$  son números enteros entre -1 y 8, significa que los individuos que trabajarán como policías serán aquellos con  $\rho_i \in \{-1; 0; 1\}$ .

Dado que 1.7 es mayor a -1, 0 y 1, quiere decir que todos los trabajadores empleados como policías recibirán una renta, ya que  $w_1 - w_0 > \rho$ : lo que los compensa el mercado por trabajar como policías es más que lo mínimo que ellos necesitan para aceptar el trabajo.

# Compensating Wage Differentials: TP Exercise

La renta en cada caso se puede cuantificar como:

- $\rho_i = -1 \implies (w_1 - w_0) - \rho = 1,7 - (-1) = 2,7 > 0$
- $\rho_i = 0 \implies (w_1 - w_0) - \rho_i = 1,7 - 0 = 1,7 > 0$
- $\rho_i = 1 \implies (w_1 - w_0) - \rho = 1,7 - 1 = 0,7 > 0$

Como podemos observar, todas las rentas son positivas, lo que significa que todos los individuos que en equilibrio trabajan como policías se están llevando una renta, la cual es mayor cuanto menor es la aversión a la violencia  $\rho$ .

**i) ¿Qué grupo de trabajadores en los otros empleos se estarán llevando una renta?**

*Presentar resolución en la entrega del TP.*



# Income Distribution

The cumulative decisions we make about how much human capital to acquire, combined with the laws of supply and demand, determine the distribution of earnings in the labor market.

Inevitably, there will be some inequality in the allocation of rewards among workers; some workers will command higher earnings than others.

The observed wage distribution reflects two “fundamentals.”

- 1 **Productivity differences:** workers differ in their productivity, and these differences arise partly because we acquire different amounts of human capital. The greater the productivity differences, the more unequal the wage distribution.

The observed wage distribution reflects two “fundamentals.”

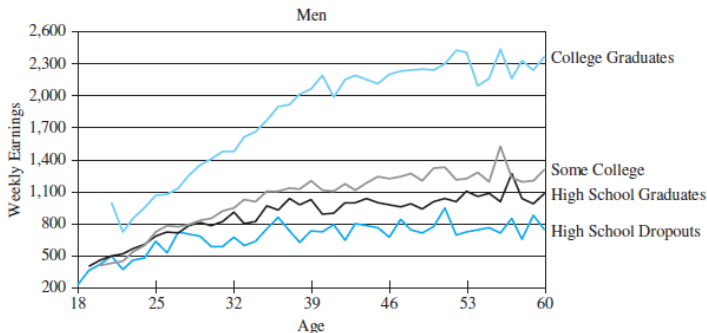
- 1 **Productivity differences:** workers differ in their productivity, and these differences arise partly because we acquire different amounts of human capital. The greater the productivity differences, the more unequal the wage distribution.
- 2 **Dynamic rate of return to skills:** as there will be changes in the supply and demand for specific skills, i.e. jobs, the rate of return to skills will vary across labor markets and over time. The greater the rate of return to skills, the greater the wage gap between skilled and unskilled workers, and the greater the inequality.

# Income Distribution: Postschool Human Capital Investments

Consider the following evolution of wages over the life cycle described by the age-earnings profile of Figure 7-1 in Borjas (2019) with US workers 2016 data:

**FIGURE 7-1** Age-Earnings Profiles, 2016

Source: U.S. Bureau of Labor Statistics, *Annual Social and Demographic Supplement of the Current Population Surveys*, pooled 2015–2017.



# Income Distribution: Postschool Human Capital Investments

The figure reveals three important properties of age–earnings profiles:

- ① *Highly educated workers earn more than less-educated workers.*

# Income Distribution: Postschool Human Capital Investments

The figure reveals three important properties of age–earnings profiles:

- ① *Highly educated workers earn more than less-educated workers.*
- ② *Earnings rise over time, but at a decreasing rate.* The wage increase suggests that a worker becomes more productive as she accumulates labor market experience, perhaps because of on-the-job or off-the-job training programs. But the rate of wage growth slows down as workers get older. Younger workers seem to add more to their human capital than older workers.

# Income Distribution: Postschool Human Capital Investments

The figure reveals three important properties of age–earnings profiles:

- ① *Highly educated workers earn more than less-educated workers.*
- ② *Earnings rise over time, but at a decreasing rate.* The wage increase suggests that a worker becomes more productive as she accumulates labor market experience, perhaps because of on-the-job or off-the-job training programs. But the rate of wage growth slows down as workers get older. Younger workers seem to add more to their human capital than older workers.
- ③ *The age–earnings profiles of different education groups diverge over time.* Earnings increase fastest for the most educated workers. The steeper slope of age–earnings profiles for the most educated suggests a complementarity education and postschool investments.

# Income Distribution: Postschool Human Capital Investments

We can see postschool human capital acquisitions are also important. However, which kind of post-school human capital investment can we make?

Most workers add to their human capital after they leave school, particularly through **on-the-job training** (OJT) programs:

- General training: it enhances productivity equally in all firms. These general skills, which include typing, learning how to drive, MS Excel skills, etc.
- Specific training: enhances productivity only in the firm where it is acquired, so that skill is useless once the worker leaves the firm. Example: learning how to use a specific machine in an industry.



# Income Distribution: Inequality

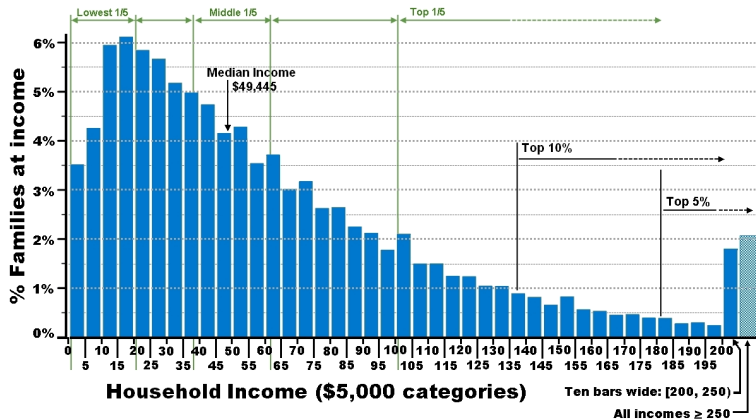
We have examined how workers acquire the human capital stock that maximizes the present value of earnings, both in school and through on-the-job training. These decisions inevitably create a lot of wage dispersion across workers.

Generally, the wage distribution exhibits two important properties.

- 1 There is a lot of inequality.
- 2 The distribution is not symmetric, with similar tails on both sides. Instead, it is positively skewed, with a long right tail.

# Income Distribution: Inequality

Regarding the 2nd property about its non symmetric distribution, a positively skewed income distribution implies that the bulk of workers earn relatively low wages and that a small number of workers in the upper tail receive a disproportionately large share of the rewards:



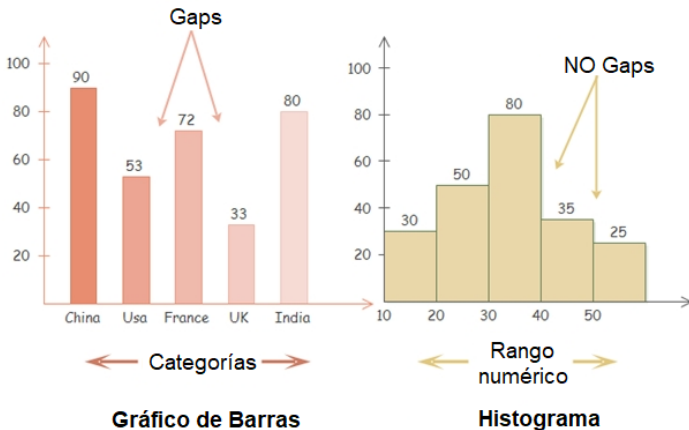
Data source: [http://www.census.gov/hhes/www/cpstable/032011/hhinc/new06\\_000.htm](http://www.census.gov/hhes/www/cpstable/032011/hhinc/new06_000.htm)

We'll examine inequality measures next class, so let us see first how can we plot income distribution:

- Histogram
- Boxplot
- Kernel Density Estimation

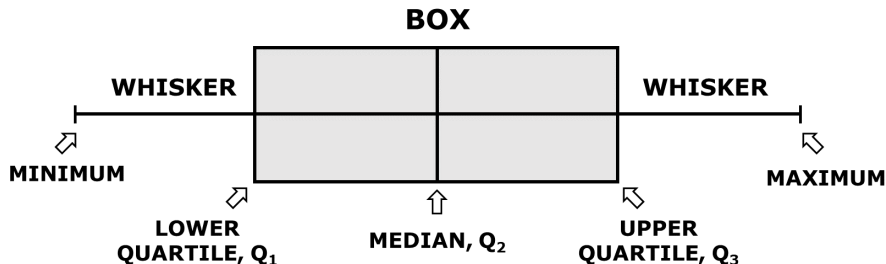
# Income Distribution: Histogram

Un **histograma** es la forma más usual de representar frecuencias de distribución. Si bien se parece a un gráfico de barras, es muy distinto: el histograma sirve para observar la forma de la distribución de la muestra o población de datos.



# Income Distribution: Boxplot

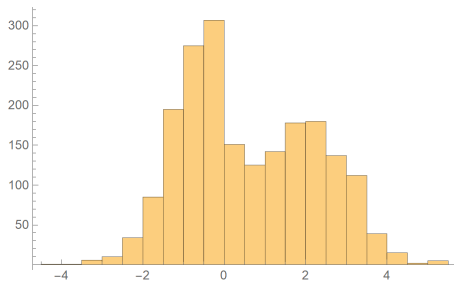
Un **boxplot** es un método estandarizado para graficar una serie de datos numéricos a través de sus **cuartiles**, por lo que permite visualizarlos rápidamente junto también con la mediana.



# Income Distribution: KDE

La **estimación de densidad kernel (KDE)** es una forma no paramétrica para estimar la función de densidad de probabilidad de una variable aleatoria. Si bien es menos usada que el histograma, puede ser una útil herramienta para mostrar la distribución de una variable.

Histograma



Kernel DE

