Q1

a. False

b. True

c. True

d. True

e. True

f. False

Q2

a.

$$\int_0^1 \int_0^{2-x^2} f(x,y)\ dy\ dx$$

b. 3

c. 4

d. 7

Q3

a. In circular coordinates

$$f(r,\theta,z) = \frac{1}{4\pi}(\sqrt{1-r^2} - z)$$

b.

$$\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{4-r^2}} 2zr\ dz\ dr\ d\theta = \frac{7}{2}\pi$$

c. Take volume of half a sphere minus a spherical cap, and divide that by 2

$$\frac{1}{2}\left(\frac{1}{2}\frac{4}{3}\pi(2)^3 - \frac{\pi(2-\sqrt{2})^2}{3}(6-(2-\sqrt{2}))\right)$$

Q4

a. Yes. The potential function

$$\frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2$$

works

b. Let E be the solid bounded by S and D. Note that the flux across D is 0, as $z = 0$ in D. Hence, the flux across S is the same as the flux across the surface of E, and we can apply Divergence Theorem.

$$\nabla \cdot F = 3$$

$$\iint_S F \cdot dA = \iiint_E 3\ dV = \pi$$

c. 20

Q5

a. Change coordinates to u, v, z, where $u = x + y, v = x - 2y$, then the volume has $-1 \leq v \leq 1, 0 \leq u \leq 2, \frac{2}{3}(2u + v) - 3 \leq z \leq \frac{2}{3}(2u + v) - 1$

$$\frac{\partial(u, v)}{\partial(x, y)} = \left|\begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix}\right| = -3$$

$$\int_{-1}^{1} \int_{0}^{2} \int_{\frac{2}{3}(2u+v)-3}^{\frac{2}{3}(2u+v)-1} \frac{1}{3} \, dz \, du \, dv = \frac{8}{3}$$

b.

$$-1 \leq y \leq 1$$
$$0 \leq z \leq 1 - y^2$$
$$0 \leq x \leq 1 - z$$

Hence

$$0 \leq x \leq 1 - z \leq 1$$
$$0 \leq z \leq 1 - x$$
$$y^2 \leq 1 - z \implies -\sqrt{1 - z} \leq y \leq \sqrt{1 - z}$$

Hence

$$\int_{-1}^{1} \int_{0}^{1-y^2} \int_{0}^{1-z} f(x, y, z) \, dx \, dz \, dy = \int_{0}^{1} \int_{0}^{1-x} \int_{-\sqrt{1-z}}^{\sqrt{1-z}} f(x, y, z) \, dy \, dz \, dx$$