

Q1

- a. False
- b. True
- c. True
- d. True
- e. True
- f. False

Q2

a.

$$\int_0^1 \int_0^{2-x^2} f(x, y) \, dy \, dx$$

- b. 3
- c. 4
- d. 7

Q3

a. In circular coordinates

$$f(r, \theta, z) = \frac{1}{4\pi} (\sqrt{1-r^2} - z)$$

b.

$$\frac{10}{3}\pi$$

c. Take volume of half a sphere minus a spherical cap, and divide that by 2

$$\frac{1}{2} \left(\frac{1}{2} \frac{4}{3} \pi (2)^3 - \frac{\pi (2 - \sqrt{2})^2}{3} (6 - (2 - \sqrt{2})) \right)$$

Q4

a. Yes. The potential function

$$\frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}z^2$$

works

b. Let E be the solid bounded by S and D . Note that the flux across D is 0, as $z = 0$ in D . Hence, the flux across S is the same as the flux across the surface of E , and we can apply Divergence Theorem.

$$\nabla \cdot F = 3$$

$$\iint_S F \cdot dA = \iiint_E 3 \, dV = \pi$$

c. 20

Q5

- a. Change coordinates to u, v, z , where $u = x + y, v = x - 2y$, then the volume has $-1 \leq v \leq 1, 0 \leq u \leq 2, \frac{3}{2}(2u + v) - 3 \leq z \leq \frac{3}{2}(2u + v) - 1$

$$\frac{\partial(u, v)}{\partial(x, y)} = \left| \begin{pmatrix} 1 & 1 \\ 1 & -2 \end{pmatrix} \right| = -3$$

$$\int_{-1}^1 \int_0^2 \int_{\frac{3}{2}(2u+v)-3}^{\frac{3}{2}(2u+v)-1} \frac{1}{3} dz du dv = \frac{8}{3}$$