Hyperspectral image project

AS.110.445. Mathematical and Computational Foundations of Data Science

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1. Introduction

The different performances of different substances in different bands of spectral signals can be drawn into a curve between spectral bands and spectral values. According to the difference of the curve, different substances in hyperspectral images can be classified. From the perspective of spectral dimension, each pixel in three-dimensional data can be regarded as the performance of a substance under different spectral signals in different bands, such as soil, water and vegetation.

Knowing the composition, structure, and geographic distribution of objects and materials is made possible by the capacity to record precise spectral features. Each substance has a distinct spectral signature, or a recognizable pattern of reflectance or absorption over the range of wavelengths. It is possible to recognize a substance and its characteristics by comparing its spectral signature to a database of known signatures. Nevertheless, due to the large dimensionality of the data, noise, atmospheric interference, and the requirement for specific algorithms for analysis, hyperspectral photography also poses significant difficulties.

Hyperspectral imaging has some advantages such as providing more accurate pictures and spending less time in imaging compared to traditional RGB methods. As a result, people are developing sensor technology and data analysis techniques to make hyperspectral imaging useful in a variety of areas.

2. Basics about hyperspectral images

Spectral resolution: Hyperspectral imaging sensors can detect hundreds of narrow spectral bands, providing much higher spectral resolution than traditional RGB imaging. This allows for more accurate identification and differentiation of materials.

Spectral signatures: Each material has a unique spectral signature, or pattern of reflectance across the different spectral bands. Hyperspectral imaging can capture these signatures, allowing for material identification and analysis.

Processing of data: The hyperspectral imaging process produces a lot of data, from which it is necessary to use specific methods in order to be able to extract useful information. This may be accomplished using a variety of methods, such as feature extraction, classification, and spectral unmixing.

3. Data processing

Two methods could be used in order to reach the topic. One is the SVD method, another is NMF. The basic formulas and principles are listed below.

SVD

The SVD is a powerful tool for various applications in hyperspectral imaging, including noise reduction, data compression, dimension reduction, classification, and unmixing. Truncating the SVD can also serve as a way to compress hyperspectral datasets, which are often large in size. Compression can be achieved from two directions: for the first decompressing the data of the wavelength bands or compressing the data of the pixels.

We can define the SVD of a data matrix as:

$$X = U\Sigma V^T$$

.In the formula, U and V are orthogonal matrices, and Σ is a diagonal matrix containing the singular values of X. Along the diagonal of Σ, the singular values are placed in decreasing order. The columns of U are represented by the left singular vectors, while those of V are represented by the right singular vectors. The covariance matrix of X can be expressed as

$$C = XX^T$$

The squares of the singular values of X are the eigenvalues of C, and the eigenvectors of C are the left singular vectors of X. The amplitude of the variance in the data along each eigenvector is therefore represented by the singular values, according to this.

Dimensionality reduction is one of SVD s main uses. A reduced-dimensional representation of the original data may be made by choosing only the k most significant singular values and associated left and right singular vectors. So we can expressed it as follows:

$X_k = U_k \Sigma_k V_k^T$, $where\ U_k, V_k^T\ are\ orthogonal\ matrix\ and$, Σ_k are the singular matrix with K most significant singular value.

Using the SVD of a matrix in computations, rather than the original matrix, has the advantage of being more robust to numerical error. Additionally, the SVD exposes the geometric structure of a matrix, an important aspect of many matrix calculations. Also the SVD method is very efficient in dealing with really big matrices. Also SVD has an optimality property which means the truncated SVD produces the best rank-k approximation (in terms of squared distances).

NMF

The Nonnegative Matrix Factorization (NMF) method can be represented mathematically using the following formula:

$$X \approx WH$$

where X is the nonnegative data matrix that is to be factorized, W is the nonnegative basis matrix, and H is the nonnegative coefficient matrix. The aim of NMF is to factorize the matrix X into two smaller matrices W and H, such that the product of W and H approximates X.The NMF of a data matrix A is created by solving the following nonlinear optimization problem.

$$min||A_{m\times n} - W_{m\times k}H_{k\times n}||^2_F$$

, where $W \geq 0$ and $H \geq 0$

The Frobenius norm is often used to measure the error between the original matrix A and its low rank approximation WH, and the k is chosen by the user.

One important property of NMF is that it can provide a sparse representation of the original matrix. This means that many of the coefficients in the matrix H are zero, which can be useful for applications such as clustering or feature extraction.

In the context of hyperspectral imaging, NMF can be used for unsupervised classification, where the aim is to group similar pixels together based on their spectral signatures. By applying NMF to the hyperspectral data, the resulting factors can be used as features for clustering algorithms, such as k-means or hierarchical clustering.

Compared with SVD, NMF has sparsity and nonnegativity, which means the factorization maintains these properties of the original matrix. Also NMF usually needs less memory since the factors are sparse, which also results in easier application to new data

4. Difference between SVD and NMF

Using NMF, we attempt to divide the matrix R into the product of the two matrices U and V. The two matrices U and V must be obtained in order for their multiplication to result in the matrix R. This R is frequently quite sparse, which is common in many recommender systems. As the missing-values assumption is integrated into the algorithm in these circumstances, NMF performs better.

The SVD method makes no assumptions on missing data. In order to use SVD, you must provide some missing value imputation. This might generate extraneous noise. On the other hand, SVD could result in better outcomes if your ratings matrix is not very sparse. Results from SVD are more predictable than those from NMF. The approach you select may also depend on this. Strictly speaking, NMF looks for a product of two nonnegative matrices that closely resembles the original matrix. So, the primary distinction between NMF and other dimension reduction techniques (such as SVD) is that NMF only permits non-subtractive combinations of nonnegative components. The parts-based representation of NMF is finally produced by this nonnegativity requirement.

First, SVD is not a suitable matrix decomposition for unmixing problem, because its main property, orthogonality, does not match the physical mechanism of spectral mixing, i.e., the spectra of endmembers or abundance matrices in an HSI are not orthogonal to each other.

5. Modeling

Using online dataset Indian_pines.csv, we make several models predicting the pixels given certain features. The dataset, after reshaping to 2-dimension, has 21025 rows and 220 columns .Because the main point of the project is to compare the performance of SVD and NMF, we choose to stick to one classification method, which is the most suitable one in this case, K-means clustering. The main logic is to process the data using SVD and NMF separately, and then conduct K-means clustering. The results could be shown in graphs so that they are more interpretable.

The first figure is from the SVD-K Means model, while the second is from NMF-K Means model.

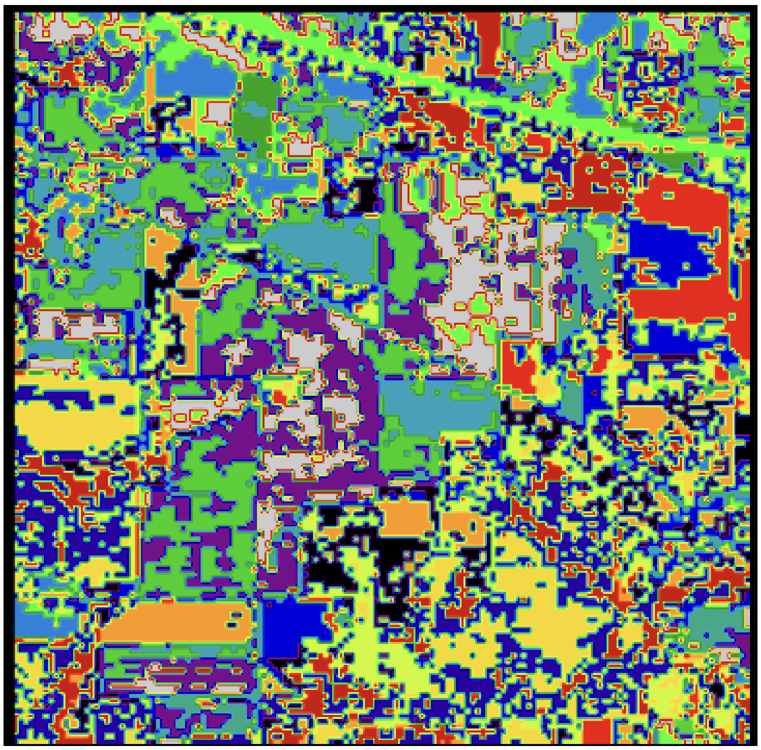


Fig1. SVD result

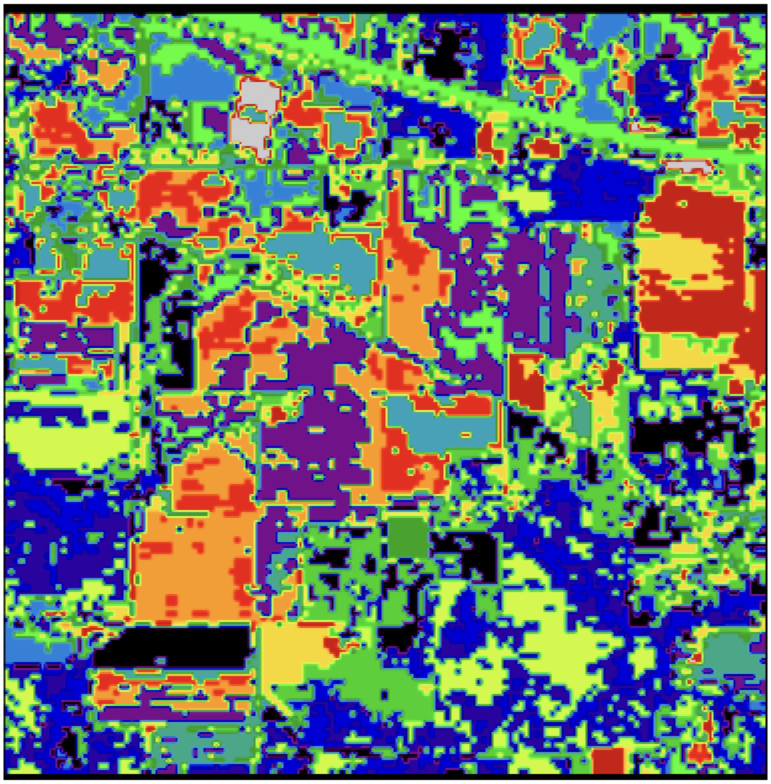


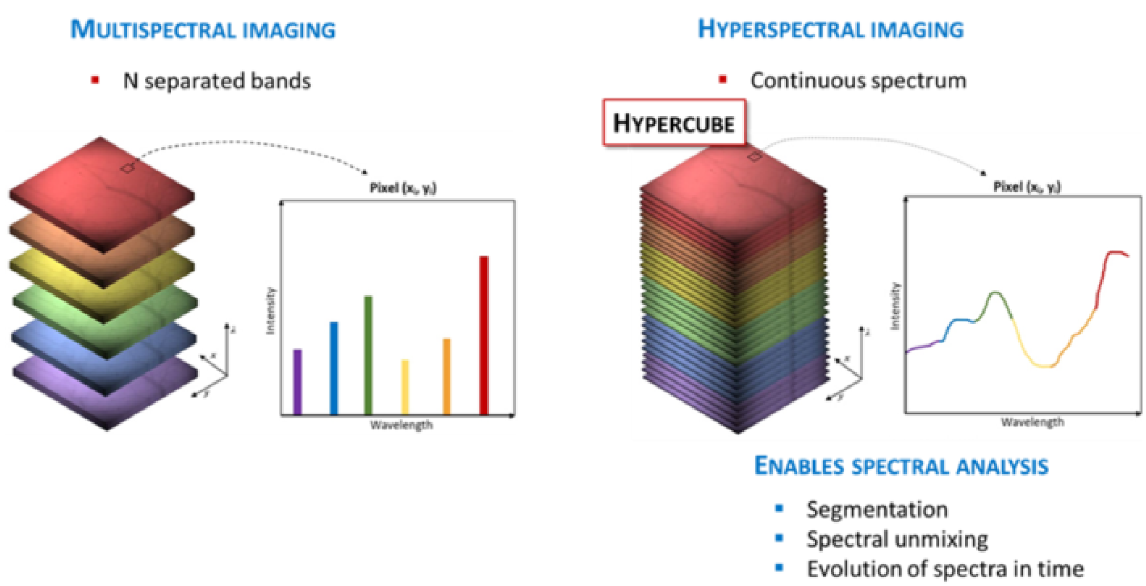
Fig2. NMF result

Note that colors here represent the 16 clusters we need to get. As we can see, the second model seems to perform a little bit better, as the colors are more distinct and have more of a sense of “boundary”.

Thus, we conclude that NMF has better performance on this dataset, probably because NMF is able to capture more diversity in the data. Also, another advantage of NMF is also testified in this project, that NMF has much shorter running time than SVD. This makes it more suitable for dealing with huge datasets like the one we use.

6.Applications Using Hyperspectral Images:

The hyperspectral imaging technique collects hundreds of photos for the same geographical region at various wavelengths. Hyperspectral imaging captures the continuous spectrum of the light for each pixel of the image with precise wavelength resolution, not just in the visible but also in the near-infrared, in contrast to the human eye's three color receptors (blue, green, and red). The gathered information is organized into a "hyperspectral cube," which has three dimensions: two of which indicate the scene's spatial extent and the third its spectral content.



Hyperspectral imaging has been used in various aspects in our daily life. At the top of the list, hyperspectral pictures may be used in agriculture to monitor crop health, identify crop species, detect crop stress and illnesses, and assess crop growth and production. There are several sensors that can be used to acquire hyperspectral pictures in agriculture, including airborne and satellite sensors. From the visible through the near-infrared and shortwave infrared areas of the electromagnetic spectrum, these sensors monitor the reflectance of light from the crop canopy. Researchers may distinguish between the distinct spectral fingerprints of healthy and stressed crops, as well as find the presence of plant diseases and pests, by studying the reflectance spectra of crops in hyperspectral photographs. Then, with the use of this knowledge, crop management choices may be made, such as altering irrigation or fertilization routines, treating sick regions specifically, or picking the right crop kinds.

Hyperspectral images could also be used in astronomy. Researchers collect the hyperspectral images through telescopes such as the Hubble Space Telescope or Keck Observatory in Hawaii. It allows astronomers to study the composition, temperature, and other properties of stars, galaxies, and other celestial objects in great detail.

7.Summary

Images are taken in a variety of small spectral bands during hyperspectral imaging. While hyperspectral imaging provides benefits over conventional imaging in many applications, it also creates a lot of data that can be difficult to handle and classify computationally. Both of the two methods were efficient in reducing the size of data. NMF is based on the decomposition of a nonnegative matrix into two lower-dimensional nonnegative matrices, which can be used to approximate the original matrix while the SVD method makes no assumptions on missing data. However, NMF provides more accuracy in capturing more diversity in data in the dataset above.