Description of a Deterministic Unit Commitment Model with Probabilistic Reserve constraints and Network Representation

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Contents

1. Model description

1.1. Sets

- G - Generators
- G_n - Generators located at node n
- GD - Dispatchable / thermal / conventional generators
- GR - Renewable / variable generators
- N - Nodes or buses in the network
- B - Lines or branches
- T - Time steps / intervals / slices
- L^+ - Upward reserve levels
- L^- - Downward reserve levels

1.2. Parameters

- D_{nt} - Demand
- $PTDF_{nl}$ - Power Transfer Distribution Factor
- AF_{gt} - Availability Factor
- K_g - Capacity
- P_g^{min} - Minimum power output
- P_g^{max} - Maximum power output
- $D_{lt}^{L^+}$ - Upward reserve requirement
- $D_{lt}^{L^-}$ - Downward reserve requirement

1.3. Variables

All variables are positive apart from injetion variables which may also be negative.

- q_{gt} - Generation
- $\hat{q}_{gt}$ - Generation above the minimum stable operating point (0 in the case of renewables).
- ls_{nt} - Load shedding
- inj_{nt} - Node injection

- f_{lt} - Branch flow
- r^{+}_{gt} - Total upward reserve provision
- r^{-}_{gt} - Total downward reserve provision
- $r^{L^+}_{glt}$ - Upward reserve provision for reserve level l
- $r^{L^-}_{glt}$ - Downward reserve provision for reserve level l
- rs_{nlt} - Upward reserve shedding for reserve level l
- rc_{nlt} - Downward reserve provided by day ahead load shedding for reserve level l
- $rinj^{L^+}_{nlt}$ - Possible node injection due to activation of upward reserve level l
- $rinj^{L^-}_{nlt}$ - Possible node injection due to activation of downward reserve level l
- $rf^{L^+}_{nbt}$ - Possible branch flow due to activation of upward reserve level l
- $rf^{L^-}_{nbt}$ - Possible branch flow due to activation of downward reserve level l
- $d^{L^+}_{nlt}$ - Possible imbalance on node n for upward reserve level l
- $d^{L^-}_{nlt}$ - Possible imbalance on node n for downward reserve level l

1.4. Objective

$$\begin{aligned}\min \quad & \sum_{t\in T}\sum_{g\in G} C^{var}_g \cdot \hat{q}_{gt} \\ & + \sum_{t\in T}\sum_{l\in L^+}\sum_{n\in N} P^{L^+} \cdot \sum_{g\in G} C^{var}_g \cdot r^{L^+}_{glt} + C^{shed} \cdot rs_{nlt} \\ & - \sum_{t\in T}\sum_{l\in L^-}\sum_{n\in N} P^{L^-} \cdot C^{var}_g \cdot r^{L^-}_{gnlt} + C^{shed} \cdot rc_{nlt}\end{aligned} \tag{1}$$

From the top line to the bottom, the costs are those of dispatching generators and activating upwards or downwards reserves.

Costs related to unit commitment, z_{gt}, have been omitted for brevity.

1.5. Constraints

The power balance:

$$\sum_{g \in G_n} q_{gt} + ls_{nt} = D_{nt} + inj_{nt} \quad n \in N,\ t \in T \tag{2}$$

Note the use of the set G_n to only allow generators at node n to contribute to the power balance. Another way of describing this would have been through an incidence matrix.

Network constraints:

$$f_{bt} = \sum_{n \in N} PTDF_{nb} \cdot inj_{nt} \quad b \in B,\ t \in T \tag{3}$$

$$-F_b \leq f_{bt} \leq F_b \quad b \in B,\ t \in T \tag{4}$$

$$\sum_{n \in N} inj_{nt} = 0 \quad n \in N,\ t \in T \tag{5}$$

$$\tag{6}$$

Constraints on generator output:

$$q_{gt} - r_{gt}^{-} \geq 0 \quad g \in GR,\ t \in T \tag{7}$$

$$q_{gt} + r_{gt}^{+} \leq AF_{gt} \cdot K_g \quad g \in GR,\ t \in T \tag{8}$$

$$q_{gt} - r_{gt}^{-} \geq P^{min} \cdot z_{gt} \quad g \in GD,\ t \in T \tag{9}$$

$$q_{gt} + r_{gt}^{+} \leq P^{max} \cdot z_{gt} \quad g \in GD,\ t \in T \tag{10}$$

For brevity and clarity, constraints on ramping and minimum up and down times are omitted.

The constraints on reserve provision are as follows:

$$D_{lt}^{L^+} = \sum_{g \in G} r_{glt}^{L^+} + \sum_{n \in N} rs_{nlt} \quad l \in L^+,\ t \in T \tag{11}$$

$$D_{lt}^{L^-} = \sum_{g \in G} r_{glt}^{L^-} + \sum_{n \in N} rc_{nlt} \quad l \in L^-,\ t \in T \tag{12}$$

$$\sum_{l \in L^-} rc_{nlt} \leq ls_{nt} \quad n \in N,\ t \in T \tag{13}$$

$$r_{gt}^{+} = \sum_{l \in L^+} r_{gnlt}^{L^+} \quad g \in G,\ t \in T \tag{14}$$

$$r_{gt}^{-} = \sum_{l \in L^-} r_{gnlt}^{L^-} \quad g \in G,\ t \in T \tag{15}$$

There are several matters to note here:

- The operating reserve balance is performed over the entire network, not per node.
- The operating reserve balance is split into reserve levels. Higher reserve levels (values of l) are less likely to occur.
- It is possible to shed upward reserves, and this is more likely to occur for higher reserve levels. This model is therefore able to make a tradeoff between day ahead adequacy and real time operational security, albeit crudely.
- Shedding load in day ahead allows additional downward reserves to be provided through the variable rc_{nlt}. Implicitly this assumes that load can be 'activated' in real time to provide downward reserves.

The following constraints attempt to take network constraints into account (albeit very weakly):

$$\sum_{g\in G_n, l'=1:l} (r^{L^+}_{glt} + rs_{nlt}) = d^{L^+}_{nlt} + rinj^{L^+}_{nlt} \quad n \in N,\ l \in L^+,\ t \in T \tag{16}$$

$$\sum_{g\in G_n, l'=1:l} (r^{L^-}_{glt} + rc_{nlt}) = d^{L^-}_{nlt} + rinj^{L^-}_{nlt} \quad n \in N,\ l \in L^-,\ t \in T \tag{17}$$

$$\sum_{n\in N} d^{L^+}_{nlt} = \sum_{l'=1:l} D^{L^+}_{l't} \quad l \in L^+,\ t \in T \tag{18}$$

$$\sum_{n\in N} d^{L^-}_{nlt} = -\sum_{l'=1:l} D^{L^-}_{l't} \quad l \in L^-,\ t \in T \tag{19}$$

$$rf^{L^+}_{blt} = \sum_{n\in N} PTDF_{nb} \cdot rinj^{L^+}_{nlt} \quad b \in B,\ l \in L^+,\ t \in T \tag{20}$$

$$rf^{L^-}_{blt} = \sum_{n\in N} PTDF_{nb} \cdot rinj^{L^-}_{nlt} \quad b \in B,\ l \in L^-,\ t \in T \tag{21}$$

$$-F_b \leq f_{bt} + rf^{L^+}_{blt} \leq F_b \quad b \in B,\ l \in L^+,\ t \in T \tag{22}$$

$$-F_b \leq f_{bt} + rf^{L^-}_{blt} \leq F_b \quad b \in B,\ l \in L^-,\ t \in T \tag{23}$$

$$\sum_{n\in N} rinj^{L^+}_{nt} = 0 \quad l \in L^+,\ n \in N,\ t \in T \tag{24}$$

$$\sum_{n\in N} rinj^{L^-}_{nt} = 0 \quad l \in L^-,\ n \in N,\ t \in T \tag{25}$$

Since imbalances are aggregated across the network, a particular reserve level activation is not associated with an imbalance at the nodal level. The above constraints therefore enforce that for each reserve level l and node n, there exists some combination of nodal imbalance, node injections, generator dispatches and line flows which would satisfy the network constraints AND the imbalance across the entire network. Given the formulation here, which uses reserve levels, i.e. quantiles, over the entire network to represent forecast errors, it is difficult to come up with more stringent conditions.